1D Special Relativistic Hydro

Due: 5:00 pm, Oct. 16, 2017

1 1D SRHD

In this assignment, you will extend your 1D Hydro code from HW 3 to be special relativistic. You will use the same 1D HLL scheme from HW 3 found in the How to Write a Hydro Code document:

`https://github.com/nyu-compphys-2016/howToWriteAHydroCode/blob/master/hydro_code.pdf`

In the relativistic case, the conserved quantities are...

$$U = (D, S, \tau)^T$$

with corresponding fluxes...

$$(Dv, Sv + p, S - Dv)^T$$

where D is the rest mass density, S is the momentum density, and τ is the energy density, **ALL MEASURED IN THE LABORATORY FRAME!** These conserved variables are related to the primitive variables (which are all measured in the fluid frame) by the following equations...

$$D = \rho W \tag{1}$$

$$S = \rho h W^2 v \tag{2}$$

$$\tau = \rho h W^2 - p - \rho W \tag{3}$$

$$W^2 = \frac{1}{1 - v^2} \tag{4}$$

where W is the Lorentz factor and $h = 1 + \epsilon + p/\rho$ is the relativistic specific enthalpy and ϵ is the speicific internal energy. The system is closed by the equation of state for an ideal gas...

$$p = (\Gamma - 1)\,\rho\epsilon \tag{5}$$

In order to calculate the $\mathrm{F}^{\mathrm{HLL}}$ flux, you will need to calculate the values of the primitive variables. In the non-relativistic case, this was very easy. In the relativistic case, the primitive variables are non-trivially coupled to the laboratory conserved quantities through factors of W. You will need to use Newton-Raphson to find the primitive variables each time you calculate an FHLL flux. Equations 1-5 form a system of equations that you can use to solve for the primitive variables. The system reduces to the following quartic equation for v...

$$0 = \left[\Gamma v\,(\tau - Sv + D) - S\left(1 - v^2\right)\right]^2 - v^2\left(1 - v^2\right) D^2\,(\Gamma - 1)^2$$

Some simple algebra will give you expressions for the other primitive variables. The above quartic expression for velocity may occasionally yield unphysical results, in which case you should try a smaller time step, using a lower limit of $p = 0$ for pressure and an upper limit of $v = 1$ for velocity.

2 Test Problems

- Test your code on the 1D Riemann problem from section 4.1 of the MacFadyen paper linked in the resources below, and compare your results with their Figure 1.
- Test your code on the 1D Isentropic Smooth Flows from section 4.6 of the MacFadyen paper, and compare your results to Figure 6.
- Write up your results, showing plots of the Riemann problem and Isentropic Flows problem.

3 Useful Resources

Here are some papers that you may find useful, or at least very interesting:

- `http://iopscience.iop.org/article/10.1086/500792/pdf` – MacFadyen paper
- `https://link.springer.com/article/10.12942/lrr-2003-7` – Review paper
- `https://ac.els-cdn.com/S0021999183710569/1-s2.0-S0021999183710569-main.pdf?_tid=ce873b92-c33e-11e7-94a5-00000aacb361&acdnat=1510006198_95b9daf8ff8b6f3c163f0c1770e`
- `http://iopscience.iop.org/article/10.1086/340382/pdf`