

# 1D Special Relativistic Hydro

Due: Oct. 15, 2017

## 1 1D SRHD

In this assignment, you will extend your 1D Hydro code from HW 3 to be special relativistic. You will use the same 1D HLL scheme from HW 3 found in the How to Write a Hydro Code document:

[https://github.com/nyu-compphys-2016/howToWriteAHydroCode/blob/master/hydro\\_code.pdf](https://github.com/nyu-compphys-2016/howToWriteAHydroCode/blob/master/hydro_code.pdf)

In the relativistic case, the conserved quantities are...

$$U = (D, S, \tau)^T$$

with corresponding fluxes...

$$(Dv, Sv + p, S - Dv)^T$$

where  $D$  is the rest mass density,  $S$  is the momentum density, and  $\tau$  is the energy density, **ALL MEASURED IN THE LABORATORY FRAME!** These conserved variables are related to the primitive variables (which are all measured in the fluid frame) by the following equations...

$$D = \rho W \tag{1}$$

$$S = \rho h W^2 v \tag{2}$$

$$\tau = \rho h W^2 - p - \rho W \tag{3}$$

$$W^2 = \frac{1}{1 - v^2} \tag{4}$$

where  $W$  is the Lorentz factor and  $h = 1 + \epsilon + p/\rho$  is the relativistic specific enthalpy and  $\epsilon$  is the specific internal energy. The system is closed by the equation of state for an ideal gas...

$$p = (\Gamma - 1) \rho \epsilon \tag{5}$$

In order to calculate the  $F^{\text{HLL}}$  flux, you will need to calculate the values of the primitive variables. In the non-relativistic case, this was very easy. In the relativistic case, the primitive variables are non-trivially coupled to the laboratory conserved quantities through factors of  $W$ . You will need to use Newton-Raphson to find the primitive variables each time you calculate an FHLL flux. Equations 1-5 form a system of equations that you can use to solve for the primitive variables. The system reduces to the following quartic equation for  $v$ ...

$$0 = [\Gamma v (\tau - Sv + D) - S (1 - v^2)]^2 - v^2 (1 - v^2) D^2 (\Gamma - 1)^2$$

Some simple algebra will give you expressions for the other primitive variables. The above quartic expression for velocity may occasionally yield unphysical results, in which case you should try a smaller time step, using a lower limit of  $p = 0$  for pressure and an upper limit of  $v = 1$  for velocity.

## 2 Test Problems

- Test your code on the 1D Riemann problem from section 4.1 of the MacFadyen paper linked in the resources below, and compare your results with their Figure 1.
- Test your code on the 1D Isentropic Smooth Flows from section 4.6 of the MacFadyen paper, and compare your results to Figure 6.
- Write up your results, showing plots of the Riemann problem and Isentropic Flows problem.

## 3 Useful Resources

Here are some papers that you may find useful, or at least very interesting:

- <http://iopscience.iop.org/article/10.1086/500792/pdf> – MacFadyen paper
- <https://link.springer.com/article/10.12942/lrr-2003-7> – Review paper
- [https://ac.els-cdn.com/S0021999183710569/1-s2.0-S0021999183710569-main.pdf?\\_tid=ce873b92-c33e-11e7-94a5-00000aacb361&acdnat=1510006198\\_95b9daf8ff8b6f3c163f0c1770e](https://ac.els-cdn.com/S0021999183710569/1-s2.0-S0021999183710569-main.pdf?_tid=ce873b92-c33e-11e7-94a5-00000aacb361&acdnat=1510006198_95b9daf8ff8b6f3c163f0c1770e)
- <http://iopscience.iop.org/article/10.1086/340382/pdf>