Homework 3

Due: Sept 28, 2018

Write python functions to implement the FTCS (forward in time, centered in space) method for solving a 1D scalar advection equation

$$\frac{\partial u}{\partial t} = -v \frac{\partial u}{\partial x}$$

where u is a scalar and v is constant. We showed in lecture that the basic approach

$$u_j^{n+1} = u_j^n - \frac{v\Delta t}{2\Delta x}\left(u_{j+1}^n - u_{j-1}^n\right)$$

is unconditionally *unstable*. Test this by running the FTCS scheme on a Gaussian Wave of varying σ (e.g. $\sigma \gg \mathrm{d}x$ and $\sigma \sim \mathrm{d}x$) and show the instabilities growing over time. Then, repeat your analysis using the corrected and *stable* Lax-Friedrich Method and compare your results with those from FTCS. What is the restriction on Δt such that the Lax-Friedrich method remains stable? Show what happens if you violate this condition.

Write a LaTeX report discussing your results. It should include a short explanation of the algorithms and implementation with all relevant formula and a discusion of the following:

- Plots showing growing instabilities in the Gaussian Wave for the FTCS scheme for a few different σ.
- The same plots from the Lax-Friedrich method including what happens when the Courant Condition is violated.

Include the `.pdf` version of the report and all Python files in your homework repo.

Bonus: If you're feeling enthusiastic, try both of your advection schemes on a Gaussian Wavepacket!