Homework 3

Due: Sept 28, 2018

Write python functions to implement the FTCS (forward in time, centered in space) method for solving a 1D scalar advection equation

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial x} = 0$$

where u is a scalar and v is constant, applied to a Gaussian Wave. We showed in lecture that the basic approach

$$u_j^{n+1} = u_j^n - \frac{v\Delta t}{2\Delta x}\left(u_{j+1}^n - u_{j-1}^n\right)$$

is unconditionally *unstable*. Show the instabilities growing over time in your Gaussian Wave. Compare this to the corrected, and unconditionally *stable*, Lax-Friedrich Method.

Write a LaTeX report discussing your results. It should include a short explanation of the algorithms and implementation with all relevant formula and a discusion of the following:

- Plots showing growing instabilities in the Gaussian Wave for the FTCS scheme.
- Similar plots for the stable wave from the Lax-Friedrich method.
- What, if any, are the restrictions on Δt in order for Lax-Friedrichs to be stable?

Include the `.pdf` version of the report and all Python files in your homework repo.