

Homework 6

Due: Oct 12, 2018

In `Python`, write a 1D hydrodynamics code! Follow the “How to write a Hydro Code” instructions included in this homework repo.

1. **First Order - Shocks** Write a first order code following Section 2 of “How to write a Hydro Code.” As a test problem, compute the Sod shock tube. Do a convergence test on the Sod by varying the number of grid cells N_x , show the code is indeed first order.
2. **“High” Order - Waves** Make your code high order (second order in space, third order in time) by following Section 3 of “How to write a Hydro Code.” Repeat the convergence test with the Sod problem, due to the discontinuities the code is still first order on this problem. A smooth solution is the *isentropic wave*. The initial conditions for an isentropic wave are:

$$\rho(x) = \rho_0 (1 + \alpha f(x)) \quad (1)$$

$$P(x) = P_0 \left(\frac{\rho}{\rho_0} \right)^\gamma \quad (2)$$

$$v(x) = \frac{2}{\gamma - 1} (c_s - c_{s,0}) \quad (3)$$

where:

$$c_s = \sqrt{\frac{\gamma P}{\rho}} \quad , \quad c_{s,0} = \sqrt{\frac{\gamma P_0}{\rho_0}} \quad (4)$$

For an initial wave profile, use:

$$f(x) = \begin{cases} \left(1 - \left(\frac{x-x_0}{\sigma}\right)^2\right)^2 & |x - x_0| < \sigma \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

Run this problem on the domain $x \in [0, 2]$ with the parameters: $\rho_0 = 1$, $P_0 = 0.6$, $\gamma = 5/3$, $\alpha = 0.2$, $x_0 = 0.5$, and $\sigma = 0.4$. These waves eventually steepen into shocks, but for short times remain smooth. To compute the error, check that the specific entropy s remains constant:

$$L_1(t) = \int_0^2 |s(x, t) - s_0| dx \quad (6)$$

$$s(x, t) - s_0 = \frac{1}{\gamma - 1} \log \left[\frac{P(x, t)}{P_0} \left(\frac{\rho(x, t)}{\rho_0} \right)^{-\gamma} \right] \quad (7)$$

Perform a convergence test with this error measure, and demonstrate the code is second order accurate.

Write a report summarizing your work, showing all plots, giving your results, and discussing the questions. Include the report `.tex` file and all Python files in the repo. Also include either the `.pdf` version of the report, or all figures necessary to compile it from the `.tex` file.