Homework 5

Due: Oct. 26, 2018

Write a `Python` program to study orbits by solving ordinary differential equations with a variety of methods. Validate your methods and demonstrate their convergence using the Kepler problem. Apply your methods to study the motion of Halley's comet and the star S2.

1. **Convergence** Write `Python` functions to evolve a system of first-order ODEs $\dot{x}_i(t) = f_i(t, x)$ using Forward-Euler, RK2, RK4, and Verlet. For each method, using a variety of time steps, compute the orbit of Mars for $T = 5$ (Earth) years, beginning from aphelion. Ignore the effect of other planets, and assume the sun is fixed. Compute the exact position $\vec{r}(T)$ of Mars at this time by solving the Kepler problem. Make a convergence plot of the L_1 error $|\vec{r}(T)_{numerical} - \vec{r}(T)_{exact}|$ versus number of steps N for each method. Plot the energy versus time for each method as well.
   - Do the methods converge as expected? How many time steps per orbit are required for the theoretical scaling to begin?
   - How well do the methods conserve energy? When may this be a problem?
2. **Halley's Comet** Write an *adaptive* RK4 method, and use it to compute the orbit of Halley's comet for a few orbital periods. Compare its performance to non-adaptive RK4 and Verlet. How long does it take to run each and get similar errors?
3. **S2** The star S2 orbits the black hole at the centre of the Milky Way, Sagittarius A*, sufficiently close that general relativity is important. One can treat general relativity in an approximate way by replacing the Newtonian potential $\phi_N = -GM/r$ with the Paczyński-Wiita potential $\phi_{PW} = -GM/(r - r_S)$, where $r_S = 2GM/c^2$ is the Schwarzschild radius of the mass. Compute several orbits of S2 using both ϕ_N and ϕ_{PW}. What is the difference? Using your results, calculate the precession of the periapse of S2. How many time steps per orbit should you use to ensure an accurate answer? Earth-based telescopes in the coming years are going to measure this!

Write a report summarizing your work, showing all plots, giving your results, and discussing the questions. Include the `.pdf` version of the report and all Python files in the repo.

Reference: See Newman Chapter 8 for discussion of ODE solvers and Chapter 6 for methods of solving non-linear equations (like the kepler problem!).