Part A:

The Fundamentals of MHD

Nothing can be more fatal to progress than a too confident reliance on mathematical symbols; for the student is only too apt to take the easier course, and consider the formula and not the fact as the physical reality.

Kelvin (1879)

Introduction: The Aims of Part A

Magnetohydrodynamics (MHD for short) is the study of the interaction between magnetic fields and moving, conducting fluids. In the following seven chapters we set out the fundamental laws of MHD. The discussion is restricted to incompressible flows, and we have given particular emphasis to the elucidation of physical principles rather than detailed mathematical solutions to particular problems.

We presuppose little or no background in fluid mechanics or electromagnetism, but rather develop these topics from first principles. Nor do we assume any knowledge of tensors, the use of which we restrict (more or less) to Chapter 7, in which an introduction to tensor notation is provided. We do, however, make extensive use of vector analysis and the reader is assumed to be fluent in vector calculus.

The subjects covered in Part A are:

1. A qualitative overview of MHD
2. The governing equations of electrodynamics
3. The governing equations of fluid mechanics
4. The kinematics of MHD: advection and diffusion of a magnetic field
5. Dynamics at low magnetic Reynolds' number
6. Dynamics at high magnetic Reynolds' number
7. MHD turbulence at low and high magnetic Reynolds' numbers

One point is worth emphasising from the outset. The governing equations of MHD consist simply of Newton's laws of motion and the pre-Maxwell form of the laws of electrodynamics. The reader is likely to be familiar with elements of both sets of laws and many of the phenomena associated with them. Thus, while the mathematical formulation of MHD may often seem daunting, the underlying physical phenomena are usually fairly straightforward. It pays, therefore, when confronted with a welter of mathematical detail, to follow the advice of Kelvin and keep asking the question: 'What is really going on?'

In line with this principle, we start, in §1.3, not with fully fledged MHD, but rather with a simple laboratory experiment. This consists of a static magnetic field at right angles to a conducting rod which in turn slides along two conducting rails. Such an apparatus is commonly used in high schools to illustrate Faraday's law of induction. However, when the *dynamics* of the sliding rod are investigated we discover a lot more than just Faraday's law. In fact, this simple experiment illustrates many of the key physical phenomena to be found in MHD. That is to say, a magnetic field, **B**, and a moving, conducting medium interact in such a way as to restrain the relative motion of the field and medium.

We start our formal analysis in Chapters 2 and 3, where we set out the governing equations of MHD. These consist of the Navier–Stokes equation and a simplified version of Maxwell's equations from which Gauss's law is omitted and displacement currents are neglected.

In Chapter 4 we consider one half of the coupling between **B** and the medium. Specifically, we look at the influence of a prescribed fluid velocity, **u**, on the magnetic field without worrying about the origin of the velocity field or the back-reaction of the Lorentz force on the fluid. In effect, we take **u** to be prescribed, dispense with the Navier–Stokes equation, and focus on the rôle of **u** when using Maxwell's equations.

We finally introduce dynamics in Chapters 5 and 6. We start, in Chapter 5, by considering weakly conducting or slowly moving fluids in which the magnetic field greatly influences the motion of the conductor but there is little back-reaction on the imposed magnetic field. This typifies much of liquid-metal MHD. Next, in Chapter 6, we consider highly conducting, or rapidly moving, fluids in which the two-way coupling of **B** and **u** is strong. Here interest focuses on stability theory, which is important in plasma containment, and on dynamo theory, a phenomenon which is of considerable importance in geophysics. We end, in Chapter 7, with a discussion of MHD turbulence.

Throughout Part A emphasis is placed on physical phenomena, rather than mathematical rigor, or engineering applications. This is not so much because we particularly share Rutherford's view of the commanding rôle of physics,[1] although he had a point, but rather that it provides a convenient way of introducing the diverse range of phenomena we call MHD.

[1] Ernest Rutherford is reputed to have said: 'Science is divided into two categories, physics and stamp collecting.'