NUMERICAL REALIZATIONS OF GALAXIES IN COSMOLOGICAL PERTURBATION THEORY

Student: Joan Manuel La Madrid

Advisor: Roman Scoccimarro

NEW YORK UNIVERSITY

WHAT IS COSMOLOGY?

Cosmology: κόσμος, *kosmos* "world" and -λογία, *-logia* "study of"

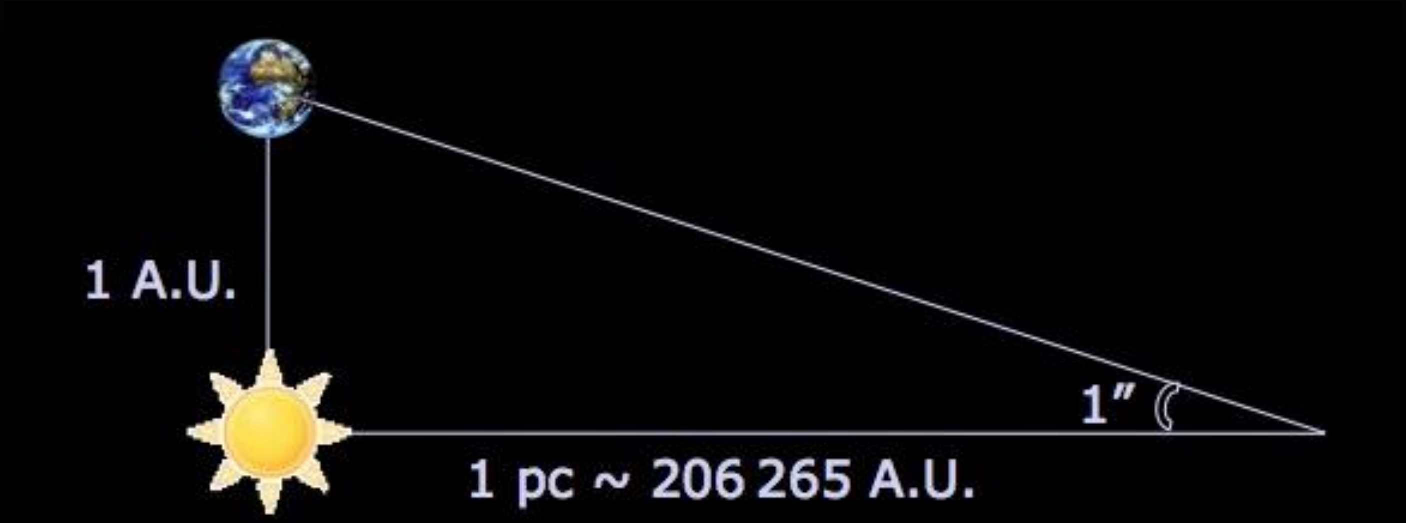
- Universe's origin and expansion
- Dark Matter
- Dark Energy
- Gravitational Waves
- Primordial Black Holes
- ✓ Formation of Structures

FORMATION OF STRUCTURES

- What do we want to describe?

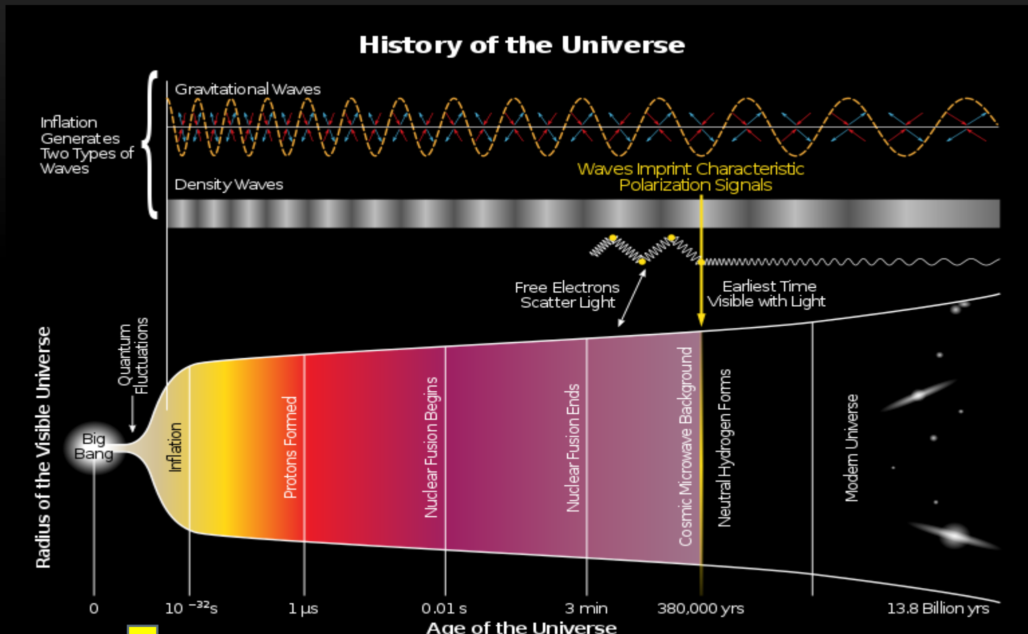
FORMATION OF STRUCTURES

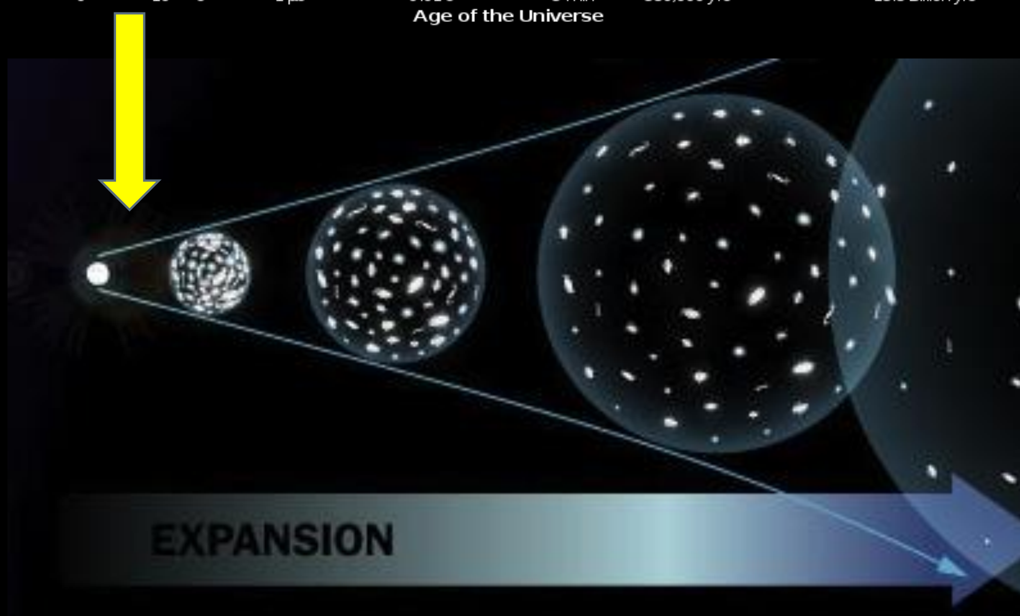
- Cosmological scale: Mpc, Gpc



- 1pc = 3.26 light-years
- Milky Way's stellar disk = 34 Kpc
- Virgo Cluster's radius: 2.2 Mpc

FORMATION OF STRUCTURES





INFLATION

- Inflaton: φ
- $G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$
- No isotropic and homegenous universe: primordial density fluctuations!
- Gravitational amplification of fluctuations to macroscopic scales!

FORMATION OF STRUCTURES

- Measurements of primordial fluctuations? NO!

- Density Fluctuations
  - Not deterministic
  - Are statistical in nature
  - Properties are result of quantum fluctuations of the Inflaton.

FORMATION OF STRUCTURES

STATISCAL APPROACH

- PDF: Statistically homogeneous and isotropic
  - $< \delta_{\overrightarrow{r1}}\ \delta_{\overrightarrow{r2}} >=< \xi_{(\overrightarrow{r1}-\overrightarrow{r2})}>=< \xi_{|\overrightarrow{r1}-\overrightarrow{r2}|}>$
  - $P_{(k)} \approx \int dr^3\ e^{-ikx}\xi$
  - Gaussian Fields: characterized by $P_{(k)}$

FORMATION OF STRUCTURES

SUMMARY

STRUCTURES IN UNIVERSE → DENSITY PERTURBATIONS → GAUSSIAN FIELDS: POWER SPECTRUM

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WHAT MODELS WE PROPOSE? EQUATIONS?

FORMATION OF STRUCTURES

2nd ORDER LAGRANGIAN NON-LINEAR PERTURBATION THEORY

- $\vec{q}$: **INITIAL POSITIONS ➡ INPUT!**
- $P_{(k)}$: **POWER SPECTRUM ➡ INPUT!**
- $\vec{X}$: **FINAL POSITIONS ➡ RESULT!**

FORMATION OF STRUCTURES

2nd ORDER LAGRANGIAN NON-LINEAR PERTURBATION THEORY

- $\vec{X} = \vec{q} + \overrightarrow{\psi_1} + \overrightarrow{\psi_2}$ ➡ **NEW POSITIONS!**
- $\overrightarrow{\psi_1} = -\nabla\Phi_1,\ \overrightarrow{\psi_2} = -\frac{3}{14}\nabla\Phi_2$

- $\nabla^2\Phi_1 = \delta$ ➡ $P_{(k)}$**: CODE'S INPUT!**
- $\nabla^2\Phi_2 = \sum_{i>j}\Phi_{,ii}^{(1)}\,\Phi_{,jj}^{(1)} - (\Phi_{,ij}^{(1)})^2$

Large-scale structure of the Universe and cosmological perturbation theory, Physics Reports367(2002) 1-248


BOOM!!
TIME TO CODE!!

NUMERICAL REALIZATION

SCHEME OF THE CODE

- ✓ $P_{(k)}$ → Gaussian Field: $\delta_{\vec{k}}$
- ✓ $\delta_{\vec{k}}$ → Solve Laplace with FFT → $\Phi_{\vec{k}}^{(1)}$
- ✓ $\Phi_{\vec{k}}^{(1)}$ → Solve Laplace with FFT → $\Phi_{\vec{k}}^{(2)}$
- ✓ $\Phi^{(1,2)}$ → Gradient in F. Space → $\overrightarrow{\psi_{1,2}}$
- ✓ $\vec{q}$ → New positions → $\vec{q} + \overrightarrow{\psi_1} + \overrightarrow{\psi_2}$

NUMERICAL REALIZATION

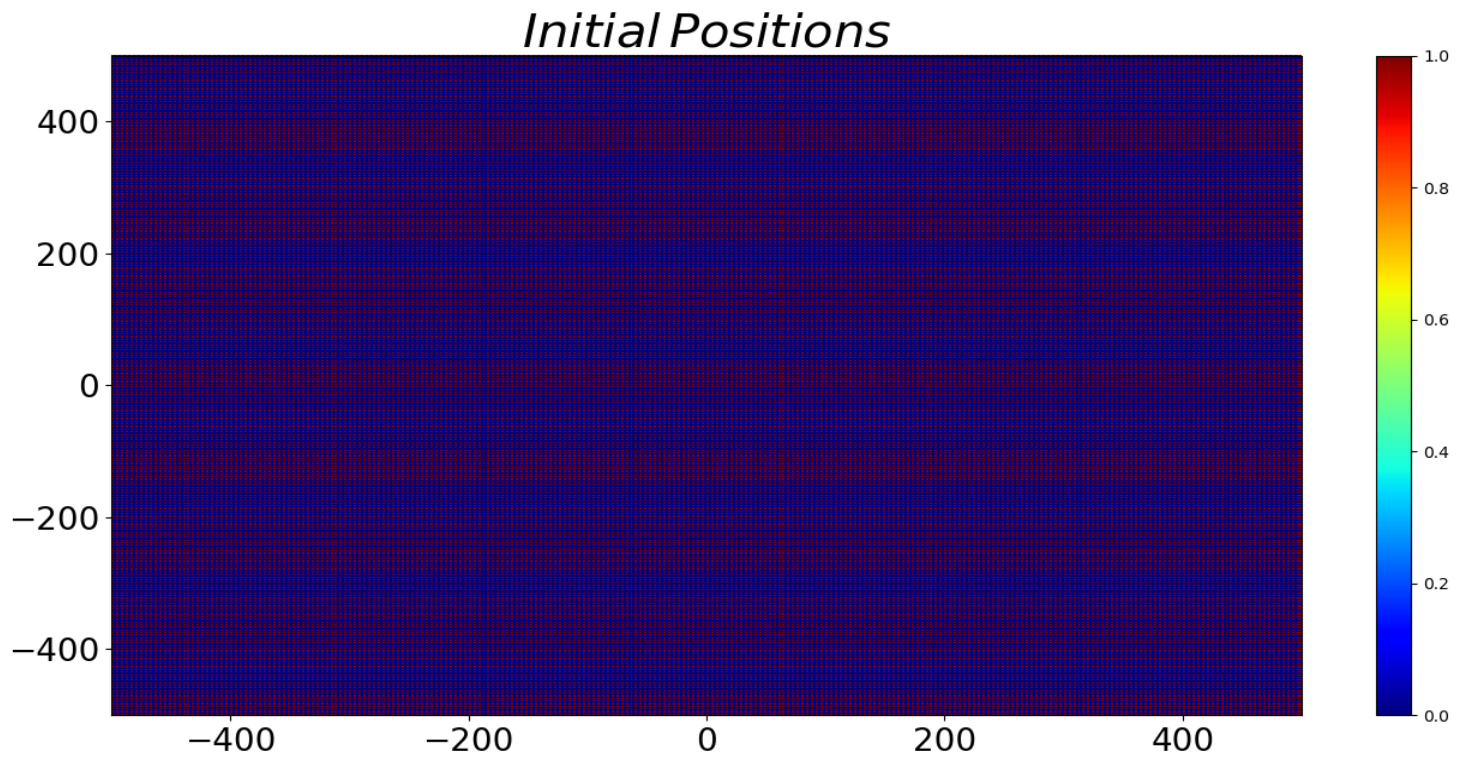
GAUSSIAN FIELDS

- ✓ $\delta_{\vec{k}} = \sqrt{\frac{P_{(k)}}{2}}(A_{\vec{k}} + iB_{\vec{k}})$ / $\delta_{\vec{k}} = \delta^*_{-\vec{k}}$

- ✓ $A_{\vec{k}}, B_{\vec{k}} = r_{\vec{k}}\cos(\theta_{\vec{k}}), r_{\vec{k}}\sin(\theta_{\vec{k}})$
- ✓ $r_{\vec{k}}, \theta_{\vec{k}} = \sqrt{-2\ln(1 - v_{\vec{k}})}, 2\pi u_{\vec{k}}$
- ✓ $u_{\vec{k}}, v_{\vec{k}}$: Uniform random variables $\in (0,1)$

- ✓ $< A >< B >< AB >= 0, < A^2 >=< B^2 >= \frac{P_{(k)}}{2}$

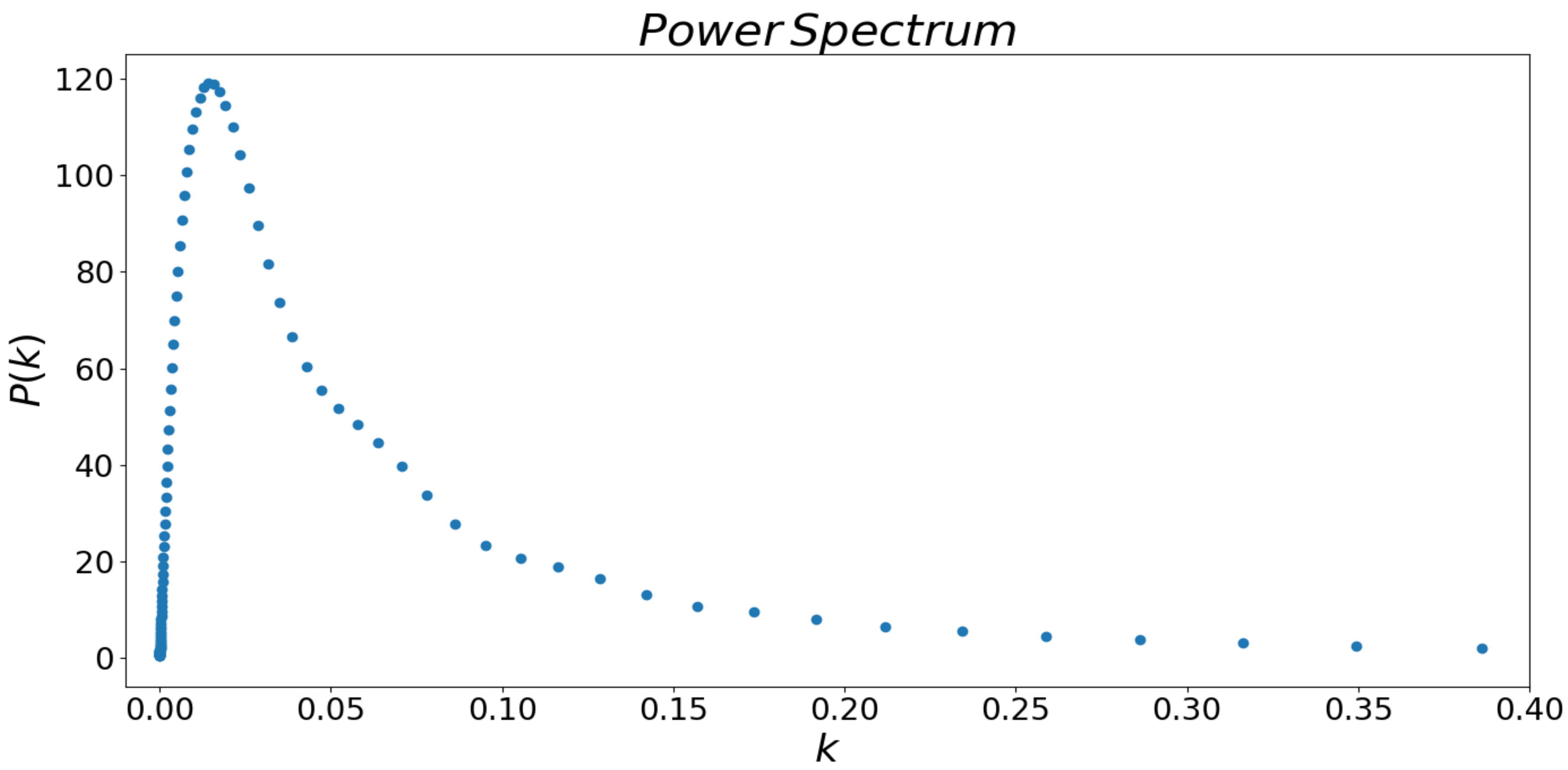
NUMERICAL REALIZATION
INPUTS OF THE CODE



CUBIC LATTICE: $L = 1000Mpc, N_{grid} = 250;\ \vec{k} = \frac{2\pi}{L}\vec{m}$

CELL $= 4Mpc$ $\quad m_i \in (-\frac{N_{grid}}{2}, \frac{N_{grid}}{2})$

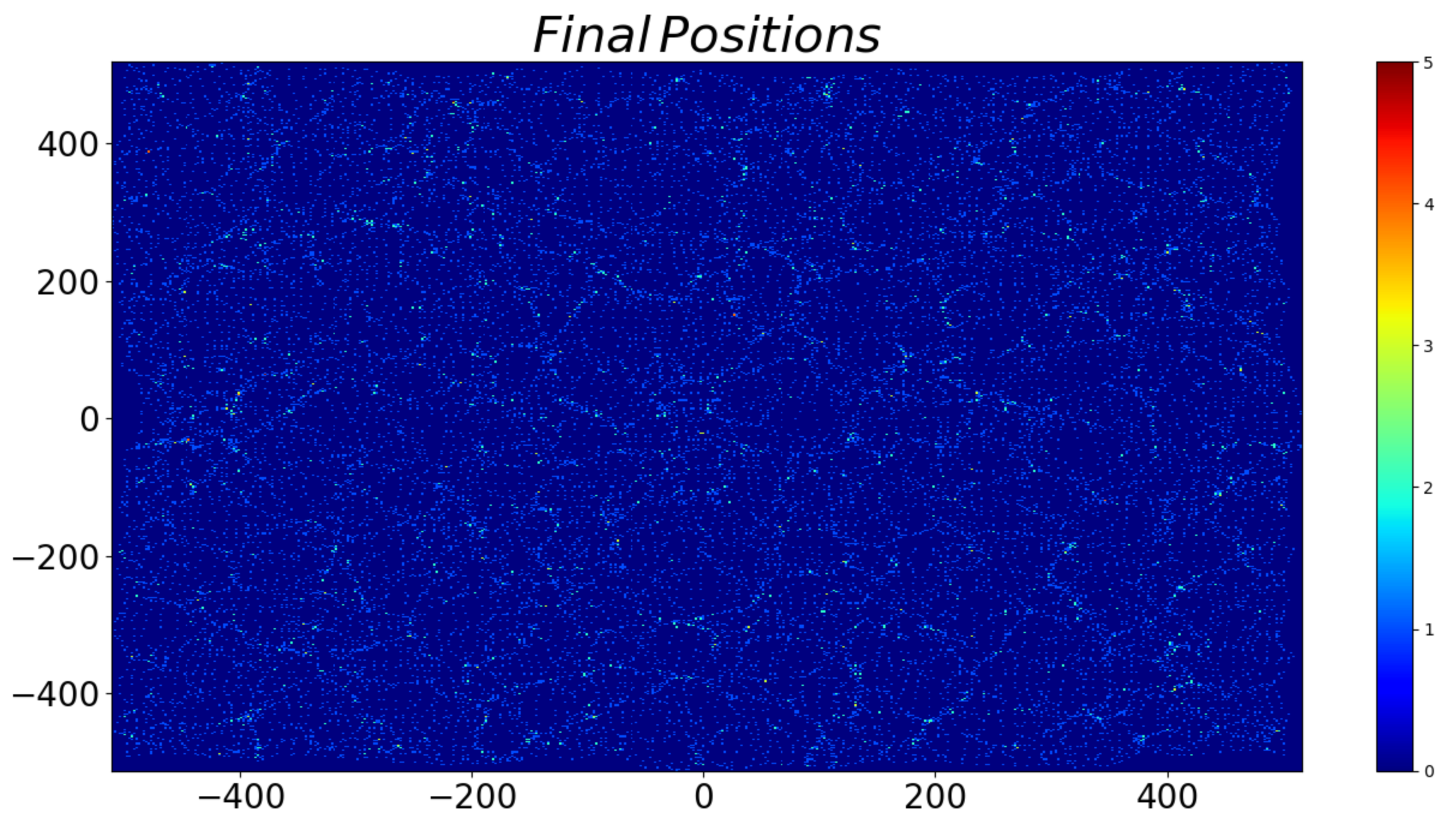
NUMERICAL REALIZATION
INPUTS OF THE CODE



CUBIC LATTICE: $L = 1000Mpc$, CELL $= 4Mpc$

$\overrightarrow{\psi_1} \approx$ 0.5, **1, 2**, 4 Mpc $\overrightarrow{\psi_2} \approx 10^{-2}\ Mpc$

NUMERICAL REALIZATION
RESULTS OF THE CODE



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NUMERICAL REALIZATION

CONCLUSIONS

- STUDIED GAUSSIAN FIELDS

- MODELS FOR FORMATION OF STRUCTURES
  - 2ORDDER LAGRANGIAN NON-LINEAR PT

- SOLVED EQUATIONS: SENSIBLE RESULTS

$$L = 1000Mpc, \text{CELL} = 4Mpc$$

$$\overrightarrow{\psi_1} \approx 0.5, 1, 2, 4\ Mpc \quad \overrightarrow{\psi_2} \approx 10^{-2}\ Mpc$$

NUMERICAL REALIZATION
POSSIBLE STEPS

- BETTER MODEL FOR GALAXY FORMATION
  - Consider extra parameters
- BETTER SCALE RESOLUTION: $N_{grid} \approx 400$
  - More efficient code
- 3 DIMENSIONAL PLOTS
  - Better visualization of galaxies


THANK YOU!!
QUESTIONS?