Implementing the Boltzmann Collision Operator for DM-Baryon Scattering

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Computational Physics Project Presentation

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BOLTZMANN COLLISION OPERATOR (FOR DM)

$$\frac{df_\chi}{dt} = C_{\chi s}[f_\chi](v_\chi) = \int d^3v'_\chi \, \{ \underbrace{f_\chi(v'_\chi)\Gamma_{\chi s}(v'_\chi \to v_\chi)}_{\text{“in”}} - \underbrace{f_\chi(v_\chi)\Gamma_{\chi s}(v_\chi \to v'_\chi)}_{\text{“out”}} \} \quad (*)$$

minus

- Scattering between DM and baryons

$$\chi(v_\chi) + s(v_s) \to \chi(v'_\chi) + s(v'_s)$$

- $f(v_\chi) \Rightarrow$ distribution function in velocity (phase) space

- $\Gamma_{\chi s}(v'_\chi \to v_\chi) \Rightarrow$ Determines the probability of scattering $v' \to v$ for two specific species

COMPUTATIONAL ASPECTS

Form for this is determined from eq (*)

$$\boxed{\frac{df(u)}{dt}} = \frac{\partial f}{\partial t} + \frac{du}{dt}\boxed{\frac{\partial f}{\partial u}} \qquad u \equiv \sqrt{\frac{m}{T(t)}}v$$

Runge Kutta + Centered difference

$$N \equiv \left(\frac{T(t)}{m}\right)^{3/2} u^2 f(u)$$

$$\frac{\partial N_i}{\partial t} = \underbrace{\left(\frac{T(t)}{m}\right)^{3/2} 4\pi\, du \sum_j \left(\Gamma_{ji} N_j u_i^2 - N_i u_j^2 \Gamma_{ij}\right)}_{\text{RK-4}} + \frac{1}{2}\frac{d\, lnT}{dt}\left(u_i \underbrace{\frac{\partial N_i}{\partial u}}_{\text{Centered diff.}} + N_i\right)$$

COMPUTATIONAL ASPECTS

$$\frac{\partial N_i}{\partial t} = \left(\frac{T(t)}{m}\right)^{3/2} 4\pi\ du \sum_j \left(\Gamma_{ji} N_j u_i^2 - N_i u_j^2 \Gamma_{ij}\right) + \frac{1}{2}\frac{d\ lnT}{dt}\left(u_i \frac{\partial N_i}{\partial u} + N_i\right)$$

- T(t) and hence its derivative is a known function
- Γ is a matrix that must satisfy *detailed balance*, i.e., if

the distribution is equilibrated, Γ should not alter it.

- In equilibrium, *f(u)* or *N(u)* should be Maxwell-Boltzmann
- My code inputs any form of Γ and T(t), and evolves *N(u)*.

TIME EVOLVING TEMPERATURE

In an expanding universe,

$$v \propto 1/a(t) \qquad\qquad a(t) \equiv \text{scale factor}$$

Recall, $$u \propto \sqrt{\frac{1}{T}}\, v$$

$$\Rightarrow u \propto 1/a(t)$$

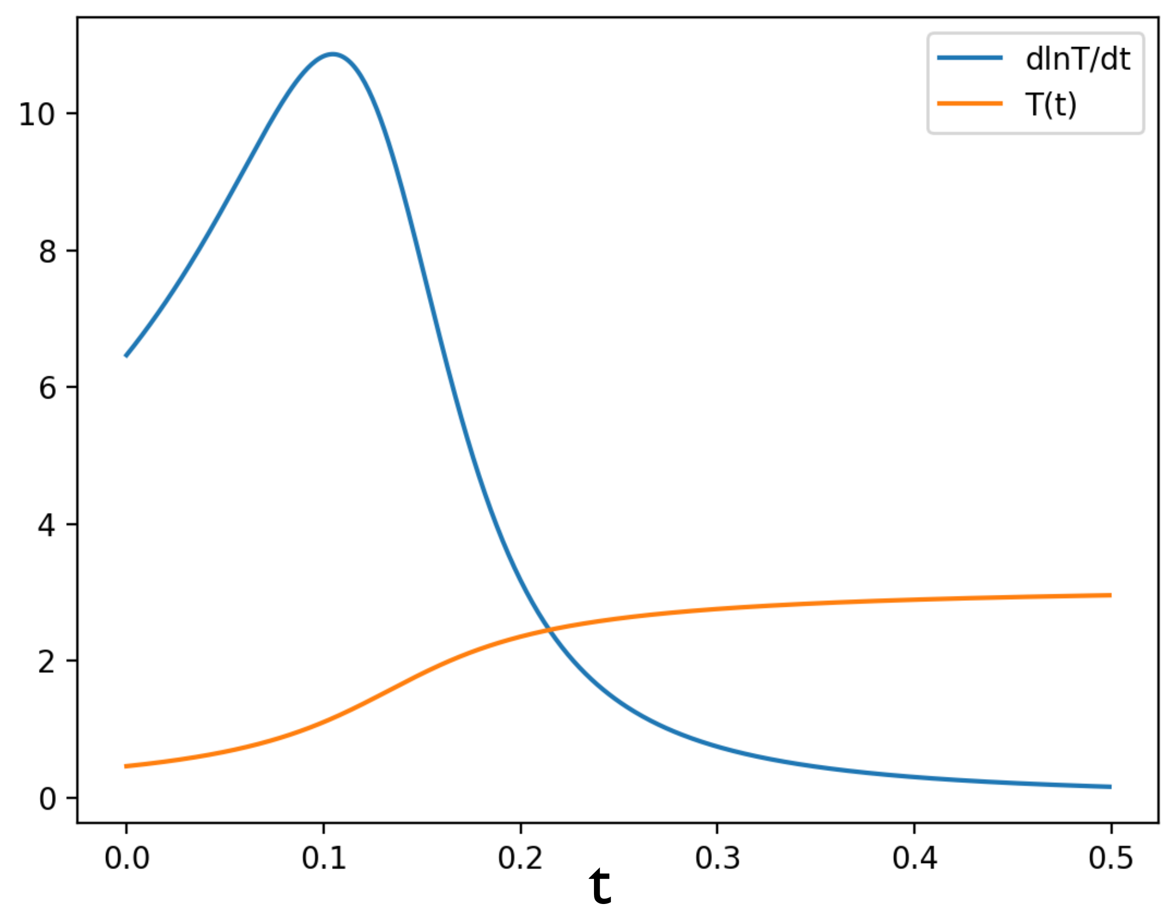
So for expansion, i.e., $u \propto 1/a$ we can equivalently have

$$T(t) \propto a(t)^2$$

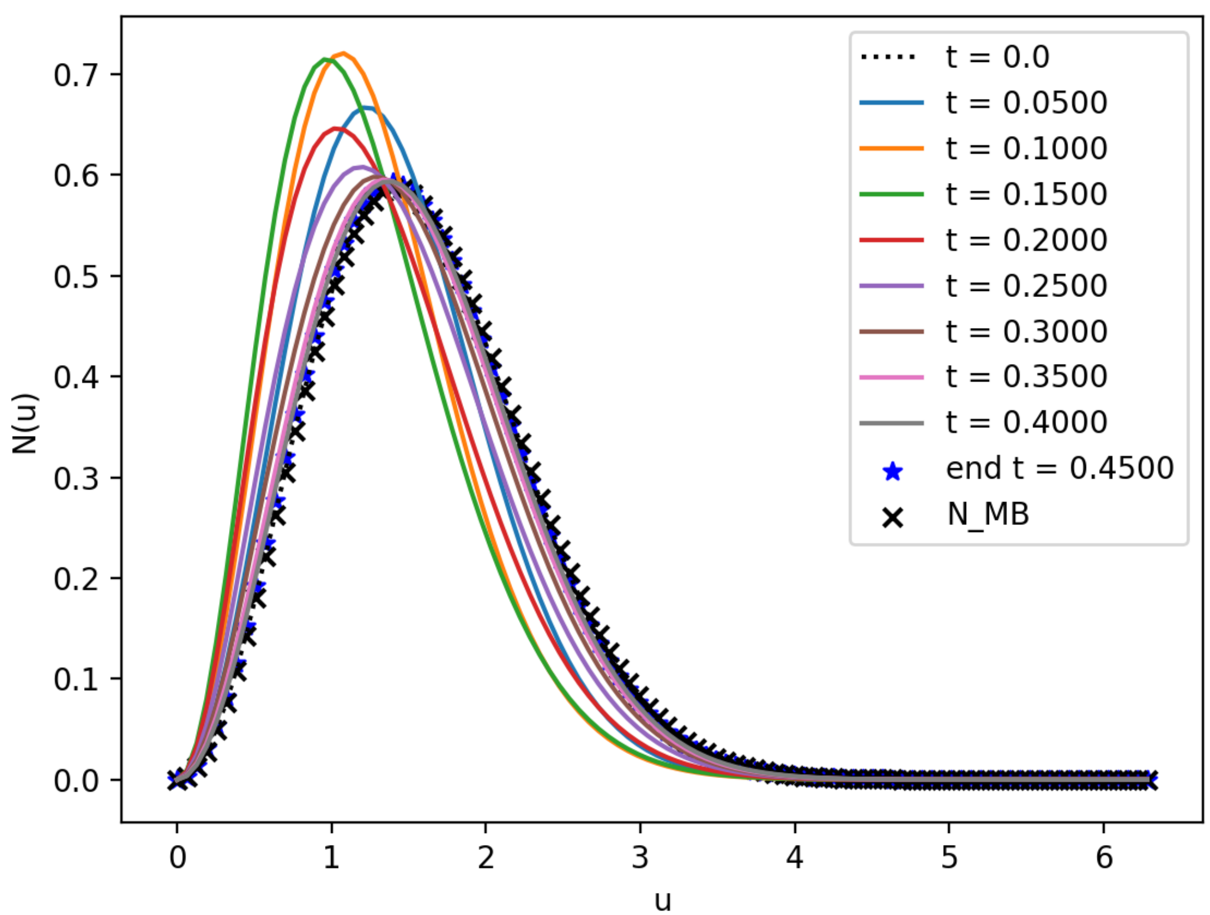
i.e., alternatively put the time dependence in T

RESULTS

Expect MB distribution to change only in absence of thermal equilibrium (ie., only when T(t) is not constant)



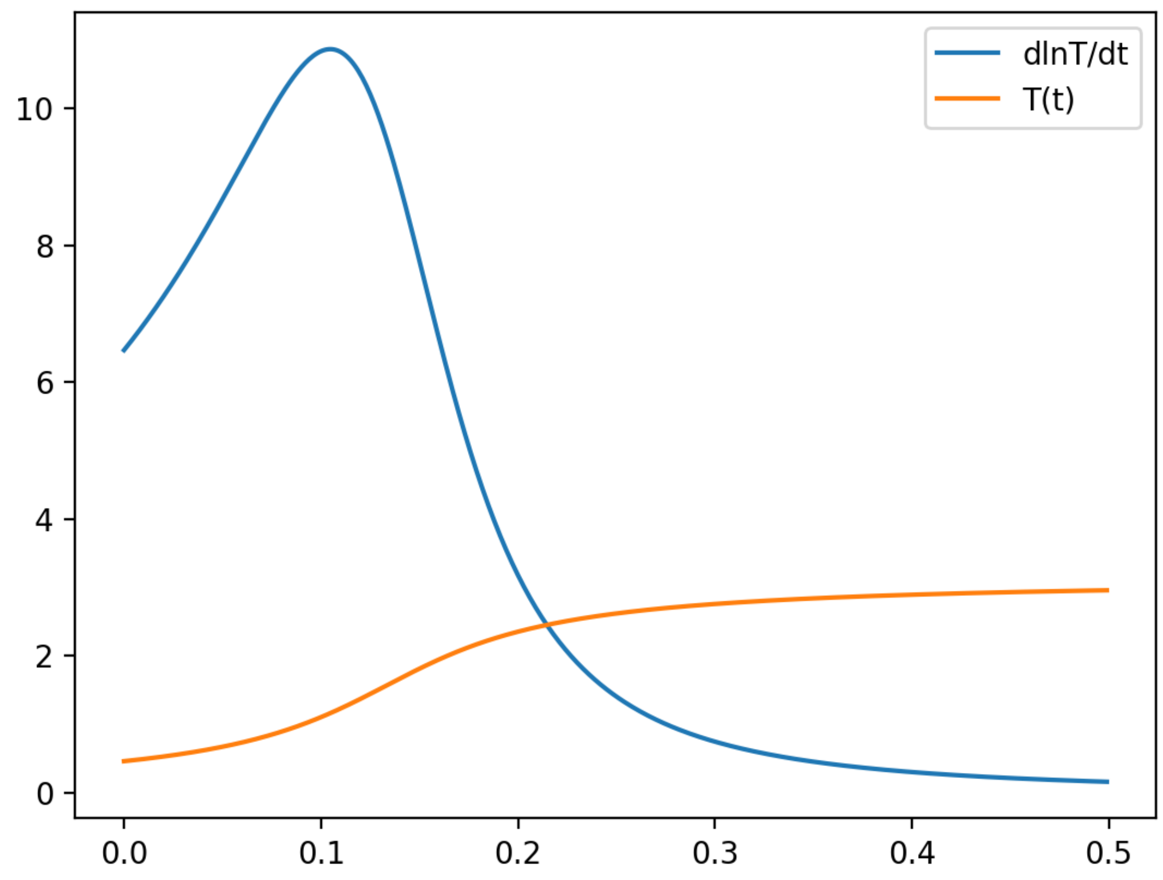
Temperature profile (artificially selected)



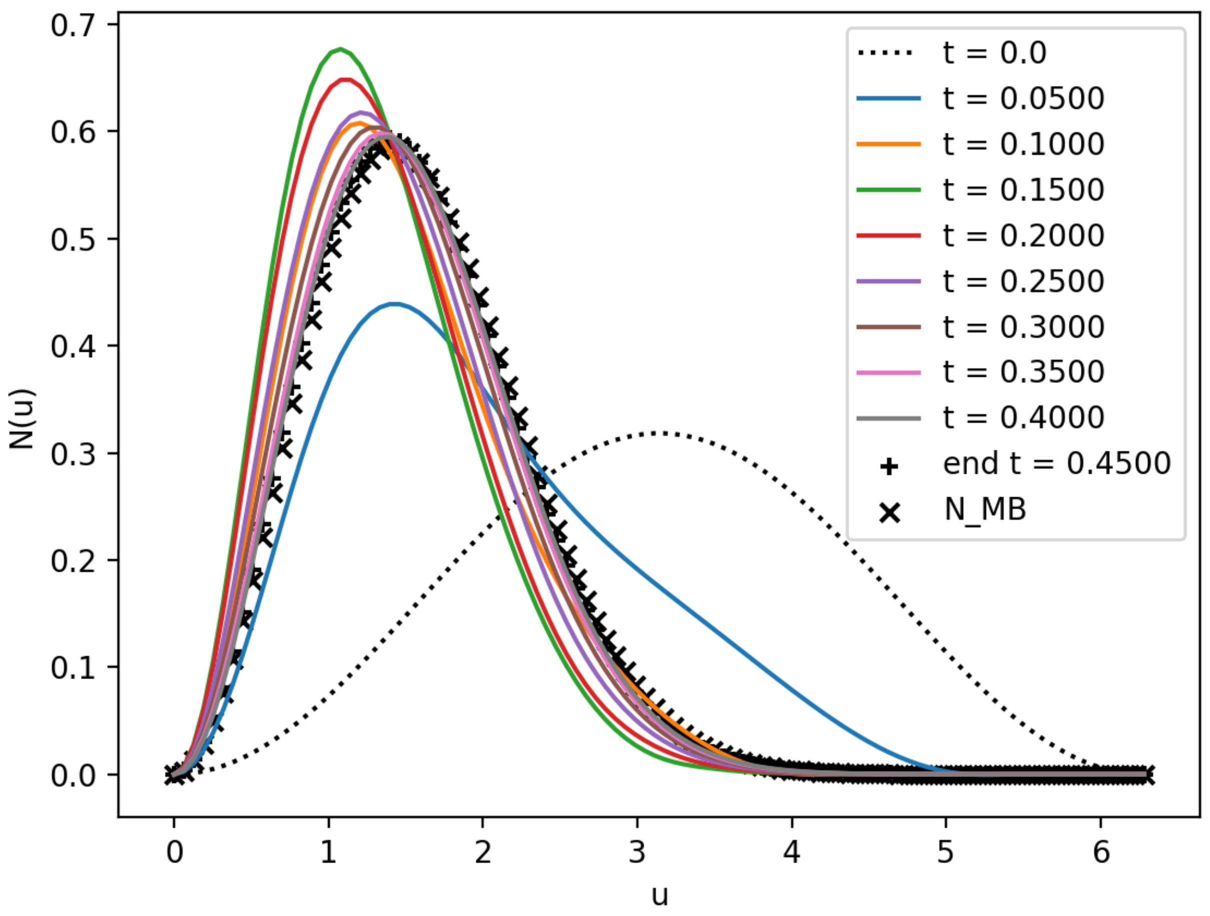
Evolution of initially Maxwellian distribution function

RESULTS

Expect <u>any</u> distribution to approach MB in <u>absence</u> of thermal equilibrium



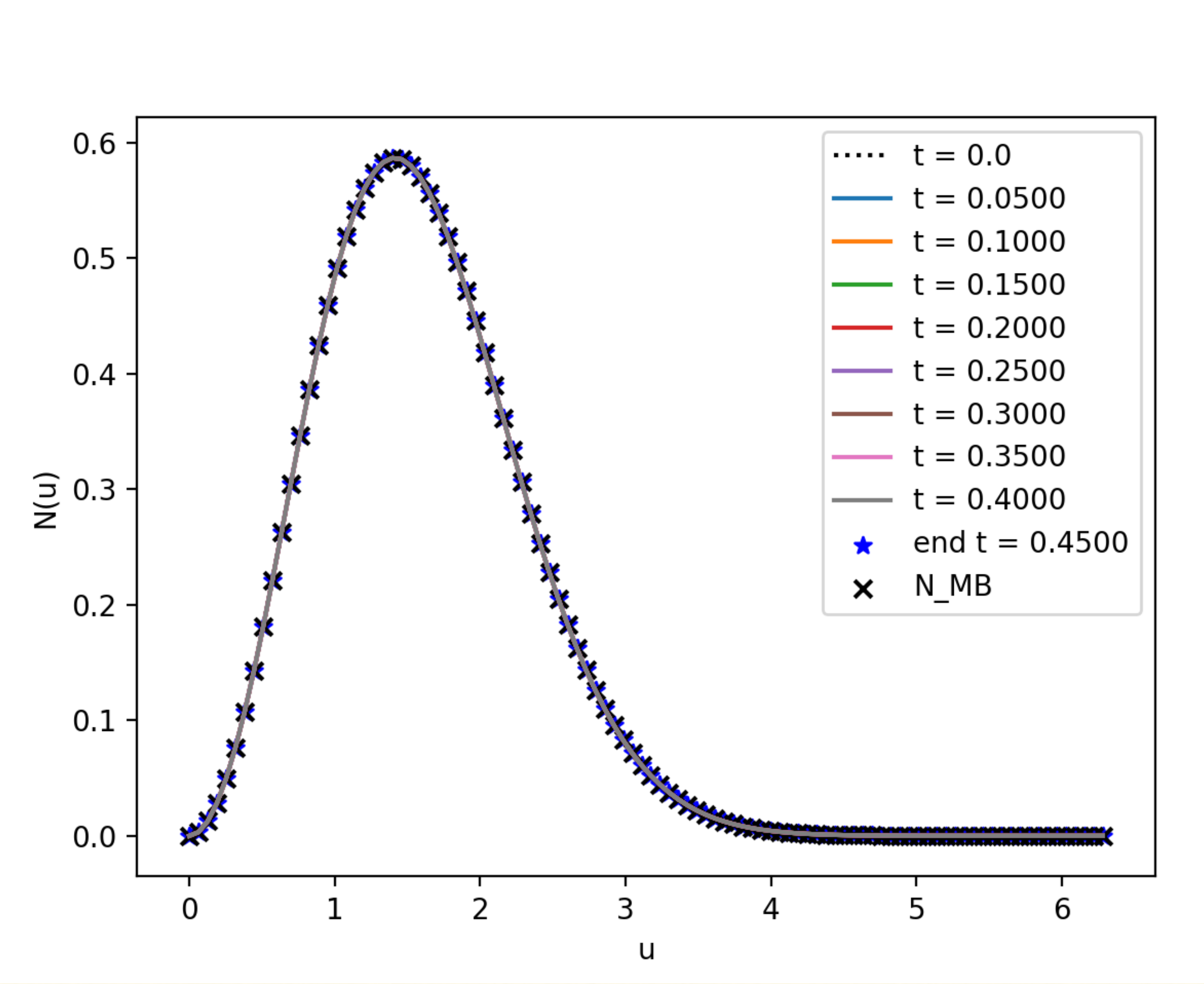
Temperature profile (artificially selected)



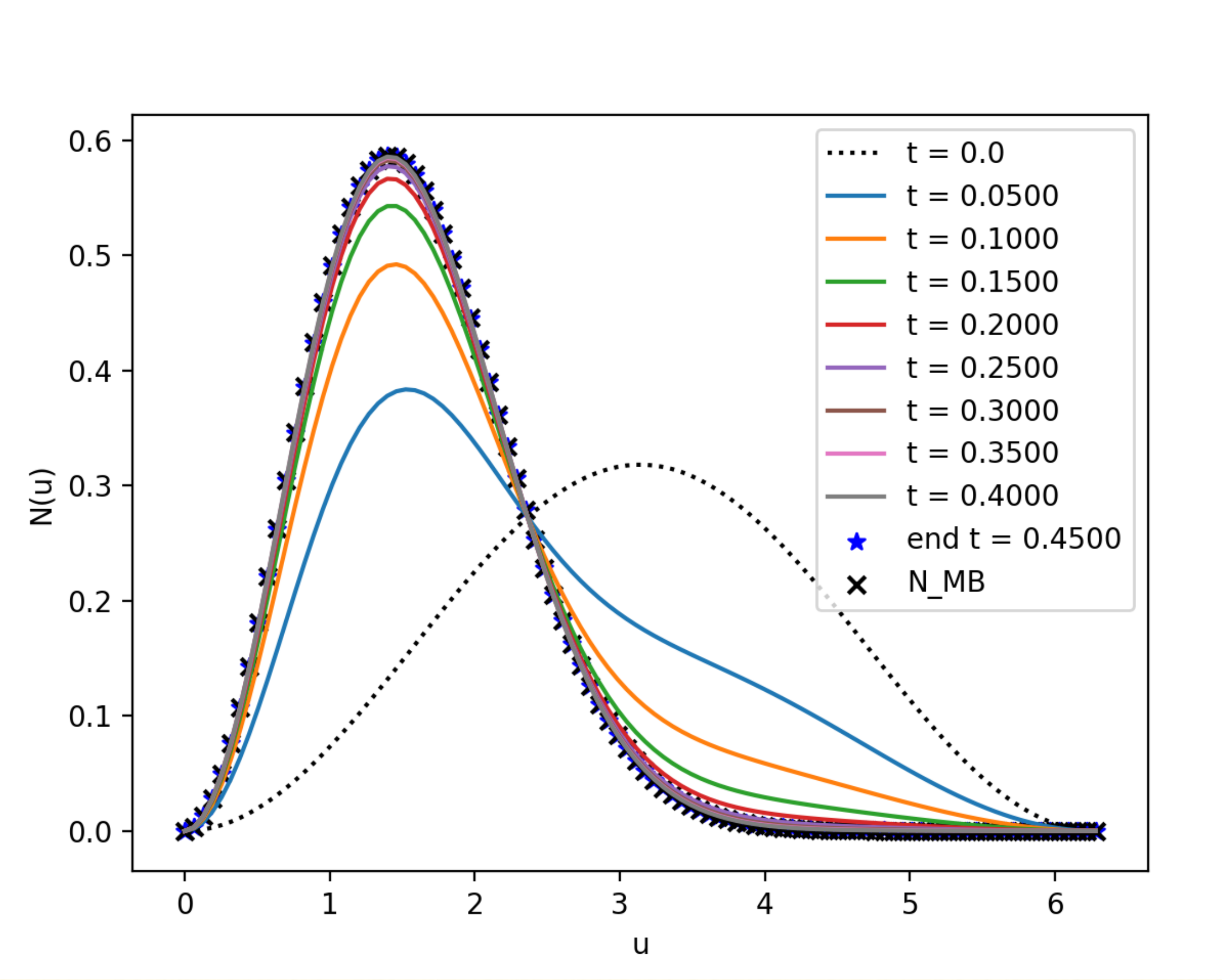
Evolution of non-MB initial distribution

TESTS

Expect any distribution to approach & stay MB in presence of thermal equilibrium



MB distribution stays MB in thermal equilibrium



non-MB distribution approaches MB in thermal equilibrium

FUTURE DIRECTION

- Usually DM is assumed to have MB distribution, which assumes efficient self-scattering
- Prof Yacine Ali-Haïmoud has done an approximation of the Boltzmann collision operator using the Fokker-Planck equation
- My code implements the full Boltzmann operator, so that we can compare how different the results are from the Fokker-Planck approximation

THANK YOU . . . any questions?

- Prof Yacine Ali-Haïmoud, NYU
- Prof Andrew Macfadyen, NYU
- Maaz ul Haq, NYU