MA-UY 2224 Final A

Bridget Kegelman

TOTAL POINTS

62 / 100

QUESTION 1

1 Q1 **11 / 20**

- **0 pts** Hints: part a) 12 points. approximation :0.8686.   part b) 8 points. p is about 0.9772
- ✓ **- 1 pts part a) small mistake(s)**
- **4 pts** part a) partial
- **8 pts** part a) mistakes OR incomplete.
- **12 pts** part a) wrong OR not showing work.
- **1 pts** part b) mistake.
- **3 pts** part b) partial.
- **5 pts** part b) partial
- ✓ **- 8 pts part b) wrong work /not showing work.**

QUESTION 2

2 Q2 **13 / 16**

- **0 pts** Hints: a) $$(1.4778, 11.8899)$$ b) Fail to reject $$Ho$$.   8 points for each part.
- **1 pts** Part a) Small mistake
- ✓ **- 3 pts Part a) mistake(s).**
- **7 pts** Part a) wrong work. Need to use the Chi-Square values to find the interval.
- **8 pts** No work for part a)
- **2 pts** Part b) mistake
- **4 pts** Part b) partial.
- **7 pts** Part b) wrong work. Need to show $$ f$$-value for testing.
- **8 pts** part b)

QUESTION 3

3 Q3 **18 / 24**

- **0 pts** Correct
- **8 pts** (a) i missing or wrong
- **8 pts** (a) ii missing or wrong
- **8 pts** (b) missing or wrong
- **6 pts** a (i) mostly wrong
- **6 pts** a (ii) mostly wrong
- ✓ **- 6 pts (b) mostly wrong; or you did a signed rank test or t test instead of sign test. and it was not done correctly.**
- **4 pts** a(i) completely the opposite
- **5 pts** big error in a(i) and (ii)
- **3 pts** (a) ii confused the power and type II error
- **4 pts** (b) p-value is calculated wrong.
- **2 pts** (b) is 2-sided, p-value *2
- **2 pts** (b) n=11 after dropping 75.
- **2 pts** (b) error in p-value
- **1 pts** small error
- **2 pts** error.
- **3 pts** You were asked to do a sign test, not signed rank test
- **3 pts** a (i) what you get is not alpha, but 1-alpha.
- **6 pts** many inconsistencies in (a)
- **5 pts** You were asked to do a sign test, not signed rank test or t-test.
- **4 pts** a(ii)
- **2 pts** a(i) is an one-sided test.
- **4 pts** (b)
- **3 pts** a(i)

QUESTION 4

4 Q4 **10 / 20**

(a) graded out of 10 points.

+ **10 pts** Essentially or completely correct.
+ ✓ **+ 0 pts Methods discussed in class are not used.**
+ **1 pts** Computed row sums.
+ **1 pts** Computed column sums.
+ **3 pts** Correctly computed expected values.
+ **1 pts** Identified correct degrees of freedom.
+ **1 pts** Used correct (corresponding to correct significance) value from (correct) table.
+ **1 pts** Made decision about hypothesis correctly.

+ **1 pts** State conclusion about decision (e.g. in order for "reject $$H_0$$" to be accepted, $$H_0$$ must be stated).

+ **1 pts** State conclusion in an understandable way.

(b) graded out of 10 points.

✓ + **10 pts** **Essentially or completely correct.**

+ **0 pts** Partial correct (essentially) not possible.

+ **1 pts** State correct null hypothesis.

+ **2 pts** State correct alternative hypothesis.

+ **1 pts** Correctly computed $$\hat{p}$$.

+ **1 pts** Use $$Z$$ distribution.

+ **1 pts** Use (only) null hypothesis value in standard deviation.

+ **1 pts** Compute $$z$$-value correctly.

+ **1 pts** Correctly find $$p$$-value or find correct critical region.

+ **1 pts** Made correct decision based on $$p$$-value or critical region.

+ **1 pts** Made a clear and correct statement about conclusion of test.

QUESTION 5

5 Q5 **10 / 20**

- **0 pts** All correct
- **20 pts** Question all wrong or missing
- **10 pts** a) All wrong or missing
- **4 pts** a) Didnt Pair
- **3 pts** a) T-value wrong or missing

✓ - **2 pts** **a) Mean and/or sd miscalculated**

- **1 pts** a) Small calculation error
- **10 pts** b) All wrong or missing
- **3 pts** b) Hypotheses wrong or not stated

✓ - **2 pts** **b) Critical value wrong**

✓ - **2 pts** **b) Ranks incorrect**

✓ - **3 pts** **b) Decision wrong or missing**

✓ - **1 pts** **b) Small calculation or symbol error**

- **4 pts** Big calculation error

NYU - Mathematics at Tandon

MA-UY 2224 FINAL (A) MAY 13, 2022

Print Name: Bridget Kegelman Signature: Bridget Kegelman

Section:

Instructor: Malcom

Net ID #: bk2427

Directions: You have **120 minutes** to answer the following questions. ***You must show all your work*** as neatly and clearly as possible and indicate the final answer clearly. You may use only a TI-30 calculator.

If you are feeling ill you should inform the proctor. The proctor will note your name, student ID and accept any written statement(s) that you may wish to make regarding your illness. Cell phones and other electronic devices may **NOT** be used during the exam.

Problem	Possible	Points
1	20	
2	16	
3	24	
4	20	
5	20	
Total	100	

(1) A random sample of size $n = 100$ is taken from an exponential population with $\theta = 15$, i.e., $X_1, X_2, \cdots, X_{100}$ are independent and identically distributed random variables with the probability density function $f(x) = \frac{1}{15}e^{-x/15}$ for $x \geq 0$.

(a) Approximate the probability that at most 50 of the 100 sample points will have a value greater than 12.

$$p = \int_{12}^{100} \frac{1}{15} e^{-\frac{x}{15}} = -e^{-\frac{x}{15}}\Big|_{12}^{100} = -e\left(1^{-\frac{100}{15}} - 1^{-\frac{50}{15}}\right)$$

$$= -e\left(1^{-\frac{20}{3}} - 1^{-\frac{12}{15}}\right) = \cancel{0.0034} \; p(x>12) = 0.45$$

$$P(X \leq 50 | p = 0.45) = \frac{50.5 - 45}{\sqrt{24.75}} = \frac{5.5}{\sqrt{24.75}} = 1.10$$

$$P(Z < 1.10) = \boxed{0.8643}$$

(b) Based on the central limit theorem, what's the probability that the sample mean will be greater than 12?

$$p(\mu < 12) = \cancel{pdf(12)} \left(\int_{12}^{100} \frac{1}{15} e^{-\frac{x}{15}}\right).$$

mean = $n \cdot p$

mean = 12 fn binomial (

$p = 0.45$ for ~~picking~~ val of 12

mean ~~= 0.45 · 12 =~~

$P(\mu < 12] =$

$p(np > 12) =$

(2) A study of the number of business lunches that executives in the insurance and banking industries claim as deductible expenses per month was based on random samples and yielded the following samples:

$$\text{Insurance}: \quad n_1 = 9, \qquad \overline{x}_1 = 9.1, \qquad s_1 = 1.8$$
$$\text{Banking}: \quad n_2 = 6, \qquad \overline{x}_2 = 8, \qquad s_2 = 2.1$$

Assume both population can be approximated by normal distributions.

(a) Find a 95% confidence interval for σ_1^2, the variance of number of business lunches that executives in the insurance industries claim as deductable expenses.

$$\frac{(n_1-1)s_1^2}{\sigma^2} \sim \chi^2(n-1)$$

$$\chi^2_{(0.0975)}(8) < \frac{(8)(1.8)^2}{\sigma^2} < \chi^2_{(0.025)}(\cancel{n-1}\ 8) \rightarrow 2.18 < \frac{25.92}{\sigma^2} < 17.54$$

$$\frac{2.18}{25.92} < \frac{1}{\sigma^2} < \frac{17.54}{25.92} \rightarrow \frac{25.92}{2.18}(11.88) < \sigma^2 < \frac{2.18}{25.92}(0.84)$$

$$\sigma^2 \in (0.84, 11.88)$$

(b) Test $H_0 : \sigma_1^2 = \sigma_2^2$ against $H_1 : \sigma_1^2 \neq \sigma_2^2$ at $\alpha = 0.1$.

$\cancel{F_{crit}(5,8) = 3.69}$ $\quad F_{crit}(n_1-1, n_2-1) = F(8,5) = 4.82$

$$F_{stat} = \frac{s_1^2/\sigma_1^2}{s_2^2/\sigma_2^2} = \frac{s_1^2}{s_2^2} = \frac{1.8^2}{2.1^2} = \cancel{4.82} = 0.734$$

Reject if $F_{stat} > F_{crit}$

$\therefore F_{stat} < crit$

$$\boxed{\text{Fail to Reject}}$$

R SRT

R (data)

(then CR = R

(3) (a) Assume that the compressive strength of a certain type of cement approximately follows a normal distribution $N(\mu, 100^2)$. To test the hypothesis $H_0 : \mu = 5000$ against the alternative $H_1 : \mu < 5000$, a random sample of 50 pieces of cement is tested. The critical region is defined to be $C = \{\overline{X} < 4970\}$. use for +1

(i) Find α, the probability of committing the type I error.

$P(\bar{x} < 4970 \mid \mu = 5000)$ $\sigma^2 = 100^2$ $\sigma = 100$

$P\left(Z < \frac{4970 - 5000}{100/\sqrt{50}}\right) = P(Z < -2.12) =$

$1 - \Phi(2.12) = 1 - 0.9830 = 0.017$

$\alpha = 0.017$

(ii) Evaluate β, the probability of committing the type II error, for the alternative $\mu = 4950$.

$\beta = 1 - \text{Power}$

$\text{Power} = P(\bar{x} < 4970 \mid \mu = 4950) = P\left(Z < \frac{4970 - 4950}{100/\sqrt{50}}\right)$

$P(Z < 1.414) = 0.9207$

$\beta = 1 - 0.9207$ $\beta = 0.079$

W ~~n(n+1)~~ / K

(b) The following data represents a random sample of size 12 from the grades of an exam given to a big freshmen class.

+ : 9

9		2	5	1	7		8		4	3	6
96	68	78	83	75	86	48	92	~~70~~	82	79	84
+	-	+	+	+	+	-	+		+	+	+

- : 2

Use the sign test to test the null hypothesis that the median grade in the class is 70 against the alternative that the median is not 70. Use $\alpha = 0.05$. State your conclusion clearly and report the p-vlaue.

med: 83

$H_0: \text{med} = 70$

$H_1: \text{med} \neq 70$

H_0:

$\chi^2_{stat} = \frac{(70-83)^2}{70} = 2.41$, $\chi^2_{crit} = 21.92$

$\text{crit} = \chi^2_{(0.025)}(11)$

?

$P_{val} = 2(P(2.41)) = 2(0.99)$

Reject H_0

(4) (a) A college infirmary conducted an experiment to determine the effectiveness provided by three cough remedies. Each cough remedy was tried on 100 students and following data recorded:

Cough Remedy

	NyQuil	Robitussin	Triaminic
No Relief	22	26	18
At least some relief	78	74	82

$\frac{p_j - p}{\sqrt{\frac{pc}{n}}}$

Test the hypothesis that the three cough remedies are equally effective at the 0.01 significance level.

$\bar{p} = \frac{22+26+18}{78+74+82} = 0.282$

$p_1 = p_2 = p_3$

$\frac{0-\bar{p}}{\frac{\bar{p}}{\sqrt{n}}}$

$\frac{p_i - p^2}{\sqrt{\frac{pq}{n}}}$

Crit: $t_{(0.005)}(2) = 9.925$

$\boxed{p_{stat} = \frac{0.282}{\sqrt{\frac{p}{n}}}} = \boxed{1.92}$

$P_{stat} < \text{Crit}$ and $\text{pval} \leq \alpha$

$\boxed{\text{Reject}}$

$\cdot\, p(z < -1.92) = 1 - 0.9207$ $\quad 2.828$

$\cdot$ pval $< \alpha$

(b) The manufacturer of a spot remover claims that his product removes at least 90% of all spots. If, in a random sample, only 174 of 200 spots were removed with the product, is there enough evidence against the manufacturer's claim at the 0.10 level of significance? Please clearly state both hypotheses and conclusion of the test.

$H_0: p = 0.9 \qquad H_1: p < 0.9$

Critical R $= t_{(0.10)}(\cancel{200-1}) = t < 1.282$

$P_{stat} = \frac{\frac{174}{200} - 0.9}{\sqrt{\frac{(0.9)(0.1)}{200}}} = -1.414$

$P_{val} = 1 - 0.9207 \approx 0.08 < \alpha$

$\boxed{\text{reject } H_0}$

(5) The weight of 8 people before they stopped smoking and 5 weeks after they stopped smoking, in kilograms, are as follows:

Individual

	1	2	3	4	5	6	7	8
Before	66	80	69	52	75	63	71	56
After	71	82	68	56	73	65	72	53
diff	5	2	-1	4	-2	2	1	3

$\bar{X} = 1.75$ $S = 2.375$

(a) If we can assume that the difference of their weights before and after quitting smoking roughly follows the normal distribution, find a 95% confidence interval for the mean difference of their weights.

$\bar{X} \pm t_{(0.025)}(7) \cdot \frac{S}{\sqrt{n}}$

$1.75 \pm 2.365 \left(\frac{2.375}{\sqrt{8}}\right)$

1.75 ± 1.986

$\boxed{\bar{X} \in [-0.236, 3.736]}$

$S = \sqrt{\frac{(5-1.75)^2 + (2-1.75)^2 + (-1-1.75)^2 + (4-1.75)^2 + (-2-1.75)^2 + (2-1.75)^2 + (1-1.75)^2 + (3-1.75)^2}{n-1}} = \sqrt{5.643}$

(b) If we do not have a good reason to make the normal assumption that we made in part (a), use the signed rank test to test that giving up smoking has no effect on a person's weight, against the alternative that one's weight increases if they quit smoking. Use $\alpha = 0.05$.

14.07

$H_0: \bar{X} = 0$ $\boxed{H_1: \bar{X} > 0}$ reject

1.895

	R
-2 -	
-1 -	
1 +	1
2 +	1.5
2 +	1.5
3 +	3
4 +	4
5 +	5

$W_+ = 16$

$W_+ = 16$

$X_{crit(0.05)}(7) = 14.07$

$\boxed{W_+ > 14.07 \therefore \text{Reject } H_0}$