# Context-Free Parsing

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# Logistics

- Homework 3 released
- Project proposal due next week (one page)
  - What problem are you tackling and why is it important?
  - What's your approach?
  - How do you plan to evaluate it?

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# Langauge is a set of strings

**Formal language**:

- A set of **strings** consisting of **words** from an **alphabet**
- *Well-formed* according to a set of rules
- Studies the *syntactical* aspects of a language

Examples:

- Formulas (logic): $(p_1 \wedge p_2) \vee (\neg p_3)$
- Programming languages: `int a, b = 0;`
- Sequences from the alphabet $\{a, b\}$ that ends with two $a$'s

Questions:

- Formal language theory: How to describe languages (expressive power, recognizability etc.)
- Linguistics: Can we design formal languages that capture syntactic properties of natural language?

# Natural language syntax

Construct a formal language to represent the syntax of natural language

- *Expressivity*: how many syntactic phenomena can it cover?
- *Computation*: how fast can we parse a sentence?

Context-free grammars for natural language

- Captures nested structures which are common in natural language

  [I told Mary that [John told Jane that [Ted told Tom a secret]]].

- Captures long-range dependencies

  *the* burnt and badly-ground Italian *coffee*
  *these* burnt and badly-ground Italian *coffees*

- Strikes a good balance between expressivity and computation

# Context-free language

**Context-free languages (CFL)** are generated by a **context-free grammar** $G = (\Sigma, N, R, S)$:

- a finite alphabet $\Sigma$ of **terminals** (words)
- a finite set of **non-terminals** $N$ disjoint from $\Sigma$ (word groups)
- a set of **production rules** $R$ of the form $A \to \beta$, where $A \in N, \beta \in (\Sigma \cup N)^*$ (how to group words)
- a start symbol $S \in N$ (root of derivation)

Example:

$$S \to SS$$

$$S \to (S)$$

$$S \to ()$$

# Phrase-structure grammar for English

Sentences are broken down into **constituents**.

A constituent works as a single unit in a sentence.

- Can be moved around or replaced without breaking grammaticality.
  (Abigail) and (her younger brother) (bought a fish).

Construct CFG for English

- Each word is a terminal, derived from its POS tag.
- Each sentence is derived from the start symbol $S$.
- Each phrase type is a non-terminal.
- Each constituent is derived from a non-terminal.

Grammar design: choose the right set of non-terminals that produces different constituents.

# A toy example CFG

$$
\begin{aligned}
N &= \{\mathrm{S}, \mathrm{NP}, \mathrm{VP}, \mathrm{PP}, \mathrm{DT}, \mathrm{Vi}, \mathrm{Vt}, \mathrm{NN}, \mathrm{IN}\} \\
S &= \mathrm{S} \\
\Sigma &= \{\text{sleeps}, \text{saw}, \text{man}, \text{woman}, \text{dog}, \text{telescope}, \text{the}, \text{with}, \text{in}\}
\end{aligned}
$$

$R =$

| | | | |
|---|---|---|---|
| S | $\rightarrow$ | NP | VP |
| VP | $\rightarrow$ | Vi | |
| VP | $\rightarrow$ | Vt | NP |
| VP | $\rightarrow$ | VP | PP |
| NP | $\rightarrow$ | DT | NN |
| NP | $\rightarrow$ | NP | PP |
| PP | $\rightarrow$ | IN | NP |

| | | |
|---|---|---|
| Vi | $\rightarrow$ | sleeps |
| Vt | $\rightarrow$ | saw |
| NN | $\rightarrow$ | man |
| NN | $\rightarrow$ | woman |
| NN | $\rightarrow$ | telescope |
| NN | $\rightarrow$ | dog |
| DT | $\rightarrow$ | the |
| IN | $\rightarrow$ | with |
| IN | $\rightarrow$ | in |

**Lexicon**: rules that produce the terminals

(Example from Mike Collins' notes)

# Parsing

$R =$

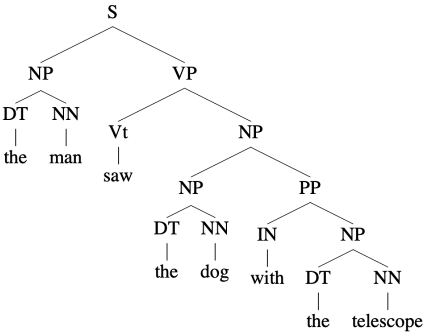
| | | | |
|---|---|---|---|
| S | $\rightarrow$ | NP | VP |
| VP | $\rightarrow$ | Vi | |
| VP | $\rightarrow$ | Vt | NP |
| VP | $\rightarrow$ | VP | PP |
| NP | $\rightarrow$ | DT | NN |
| NP | $\rightarrow$ | NP | PP |
| PP | $\rightarrow$ | IN | NP |

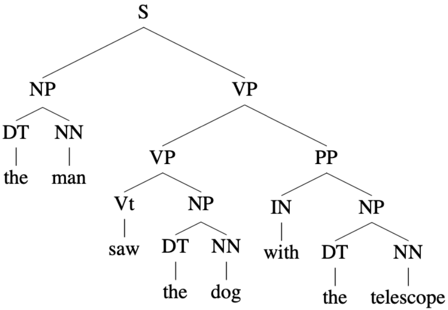
| | | |
|---|---|---|
| Vi | $\rightarrow$ | sleeps |
| Vt | $\rightarrow$ | saw |
| NN | $\rightarrow$ | man |
| NN | $\rightarrow$ | woman |
| NN | $\rightarrow$ | telescope |
| NN | $\rightarrow$ | dog |
| DT | $\rightarrow$ | the |
| IN | $\rightarrow$ | with |
| IN | $\rightarrow$ | in |

Can we derive the sentence “the man sleeps”?

# Ambiguity

Can a sentence have multiple parse trees?




Exercise: find parse trees for
"She announced a program to promote safety in trucks and vans".

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# PCFG

Notation: let $\mathcal{T}_G$ be the set of all possible left-most parse trees under the grammar $G$.

Goal: define a probability distribution $p(t)$ over parse trees $t \in \mathcal{T}_G$

Parsing: pick the most likely parse tree for a sentence $s$

$$\arg\max_{t \in \mathcal{T}_G(s)} p(t)$$

Three questions:

- Modeling: how to define $p(t)$ for trees?
- Learning: how to estimate parameters of the distribution $p(t)$?
- Inference: how to find the most likely tree efficiently?

# Modeling

Generate parse trees: iteratively sample a production rule to expand a non-terminal

$R =$

| | | | |
|---|---|---|---|
| S | $\rightarrow$ | NP | VP |
| VP | $\rightarrow$ | Vi | |
| VP | $\rightarrow$ | Vt | NP |
| VP | $\rightarrow$ | VP | PP |
| NP | $\rightarrow$ | DT | NN |
| NP | $\rightarrow$ | NP | PP |
| PP | $\rightarrow$ | IN | NP |

| | | |
|---|---|---|
| Vi | $\rightarrow$ | sleeps |
| Vt | $\rightarrow$ | saw |
| NN | $\rightarrow$ | man |
| NN | $\rightarrow$ | woman |
| NN | $\rightarrow$ | telescope |
| NN | $\rightarrow$ | dog |
| DT | $\rightarrow$ | the |
| IN | $\rightarrow$ | with |
| IN | $\rightarrow$ | in |

# PCFG

A **PCFG** consists of

- A CFG $G = (\Sigma, N, R, S)$
- Probabilities of production rules $q(\alpha \to \beta)$ for each $\alpha \to \beta \in R$ such that

$$\sum_{\beta :\, X \to \beta \in R} q(X \to \beta) = 1 \quad \forall X \in N$$

$R, q =$

| | | | | |
|---|---|---|---|---|
| S | $\to$ | NP | VP | 1.0 |
| VP | $\to$ | Vi | | 0.3 |
| VP | $\to$ | Vt | NP | 0.5 |
| VP | $\to$ | VP | PP | 0.2 |
| NP | $\to$ | DT | NN | 0.8 |
| NP | $\to$ | NP | PP | 0.2 |
| PP | $\to$ | IN | NP | 1.0 |

| | | | |
|---|---|---|---|
| Vi | $\to$ | sleeps | 1.0 |
| Vt | $\to$ | saw | 1.0 |
| NN | $\to$ | man | 0.1 |
| NN | $\to$ | woman | 0.1 |
| NN | $\to$ | telescope | 0.3 |
| NN | $\to$ | dog | 0.5 |
| DT | $\to$ | the | 1.0 |
| IN | $\to$ | with | 0.6 |
| IN | $\to$ | in | 0.4 |

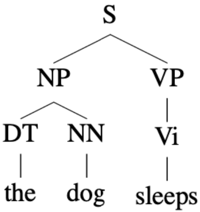
# From HMM to PCFG

# Probabilities of parse trees

Given a parse tree $t$ consisting of rules $\alpha_1 \rightarrow \beta_1, \ldots, \alpha_n \rightarrow \beta_n$, its probabilities under the PCFG is

$$p(t) = \prod_{i=1}^{n} q(\alpha_i \rightarrow \beta_i)$$

Example:



# Learning

Training data: treebanks

```
((S
   (NP-SBJ (DT That)
     (JJ cold) (, ,)
     (JJ empty) (NN sky) )
   (VP (VBD was)
     (ADJP-PRD (JJ full)
       (PP (IN of)
         (NP (NN fire)
           (CC and)
           (NN light) ))))
   (. .) ))
```

(a)

```
((S
   (NP-SBJ The/DT flight/NN )
   (VP should/MD
     (VP arrive/VB
       (PP-TMP at/IN
         (NP eleven/CD a.m/RB ))
       (NP-TMP tomorrow/NN )))))
```

(b)

**Figure 12.7** Parsed sentences from the LDC Treebank3 version of the Brown (a) and ATIS (b) corpora.

Given a set of trees (production rules), we can estimate rule probabilities by MLE.

$$q(\alpha \to \beta) = \frac{\text{count}(\alpha \to \beta)}{\sum_{\beta' : \alpha \to \beta' \in R} \text{count}(\alpha \to \beta')}$$

- Similar to estimate word probabilities ($\to \beta$) given the document class ($\alpha$).

# Parsing

Input: sentences, (P)CFG
Output: derivations / parse trees (with scores/probabilities)

Total number of parse trees for a sentence?

Consider a minimal CFG:

$$X \rightarrow XX$$
$$X \rightarrow \text{aardvark}|\text{abacus}|\ldots|\text{zyther}$$

Given a string, # of parse trees = # of strings with balanced brackets

$((w_1 w_2)(w_3 w_4))$, $(((w_1 w_2) w_3) w_4)$, ...

# of strings with $n$ pairs of brackets:

$$\text{Catalan number } C_n = \frac{1}{n+1}\binom{2n}{n}$$

# Chomsky normal form (CNF)

A CFG is in **Chomsky normal form** if every production rule takes one of the following forms:

- Binary non-terminal production: $A \rightarrow BC$ where $A, B, C \in N$.
- Unary terminal production: $A \rightarrow a$ where $A \in N, a \in \Sigma$.

Grammars in CNF produces *binary* parse trees.

Binarize a production rule: VP $\rightarrow$ VBD NP PP

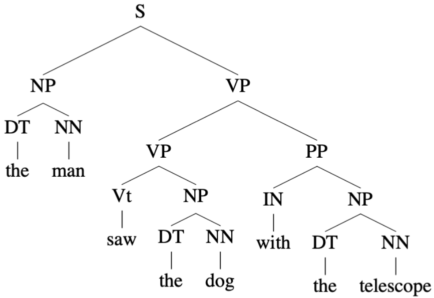
VP $\rightarrow$ VBD @VP-VBD

@VP-VBD $\rightarrow$ NP PP

We assume the grammar are in CNF.

# Dynamic programming on the tree

$$p(t) = \underbrace{q(A \rightarrow BC)}_{\text{top rule}} \times \underbrace{p(t_B)}_{\text{left child}} \times \underbrace{p(t_C)}_{\text{right child}}$$




What are the variables when constructing a tree rooted at $A$ spanning $x_i, \ldots, x_j$?

- The production rule $A \rightarrow BC$
- The splitting point $s$: $B$ spans $x_i, \ldots, x_s$ and $C$ spans $x_{s+1}, \ldots, x_j$

# The CYK algorithm

Notation: $\mathcal{T}(i, j, X)$ is the set of trees with root node $X$ spanning $x_i, \ldots, x_j$

Subproblem:

$$\pi(i, j, X) = \max_{t \in \mathcal{T}(i,j,X)} p(t)$$

Base case:

$$\pi(i, i, X) = \begin{cases} q(X \to x_i) & \text{if } X \to x_i \in R \\ 0 & \text{otherwise} \end{cases}$$

Recursion:

$$\pi(i, j, X) = \max_{\substack{Y,Z \in N \\ s \in \{i,\ldots,j-1\}}} q(X \to YZ) \times \pi(i, s, Y) \times \pi(s+1, j, Z)$$

Use backtracking to find the argmax tree.

# Bottom-up parsing

$R =$

| | | | |
|---|---|---|---|
| S | $\rightarrow$ | NP | VP |
| VP | $\rightarrow$ | Vi | |
| VP | $\rightarrow$ | Vt | NP |
| VP | $\rightarrow$ | VP | PP |
| NP | $\rightarrow$ | DT | NN |
| NP | $\rightarrow$ | NP | PP |
| PP | $\rightarrow$ | IN | NP |

| | | |
|---|---|---|
| Vi | $\rightarrow$ | sleeps |
| Vt | $\rightarrow$ | saw |
| NN | $\rightarrow$ | man |
| NN | $\rightarrow$ | woman |
| NN | $\rightarrow$ | telescope |
| NN | $\rightarrow$ | dog |
| DT | $\rightarrow$ | the |
| IN | $\rightarrow$ | with |
| IN | $\rightarrow$ | in |

| 0 | the<br>0 | man<br>1 | saw<br>2 | the<br>3 | dog<br>4 |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |

# Variants of CYK

**Argmax**: find the most likely tree (analogous to Viterbi).

$$\pi(i, j, X) = \max_{\substack{Y,Z \in N \\ s \in \{i,\ldots,j-1\}}} q\,(X \to YZ) \times \pi(i, s, Y) \times \pi(s+1, j, Z)$$

**Recognition**: does the string belong to the language?

$$\pi(i, j, X) = \bigvee_{\substack{Y,Z \in N \\ s \in \{i,\ldots,j-1\}}} \mathbb{I}\,[X \to YZ \in R] \wedge \pi(i, s, Y) \wedge \pi(s+1, j, Z)$$

**Marginalization**: what's the probability of the string being generated from the grammar? (the **inside algorithm**)

$$\pi(i, j, X) = \sum_{\substack{Y,Z \in N \\ s \in \{i,\ldots,j-1\}}} q\,(X \to YZ) \times \pi(i, s, Y) \times \pi(s+1, j, Z)$$

# Summary

| | NB | HMM | PCFG |
|---|---|---|---|
| output structure | category | sequence | tree |
| learning | | | |
| decoding | bruteforce | Viterbi | CKY |
| marginalization | | $p(y_i \mid x)$,<br>$p(y_i, y_{i-1} \mid x)$ | $p(i, j, N \mid x)$ |
| unsupervised learning | | | |

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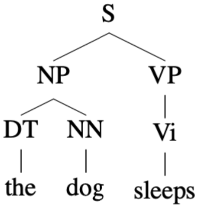
# CRF for trees

*Input*: sequence of words $x = (x_1, \dots, x_n)$
*Output*: parse tree $y \in \mathcal{T}(x)$
*Model*: decompose by production rules

$$p(y \mid x; \theta) \propto \prod_{(r,s)} \psi(r, s \mid x; \theta)$$

- $r$: production rule
- $s$: start, split, end indices of the rule $r$



# CRF parsing

Potential functions:

$$\psi(r, s \mid x; \theta) = \exp\left(\theta \cdot \phi(r, s, x)\right)$$

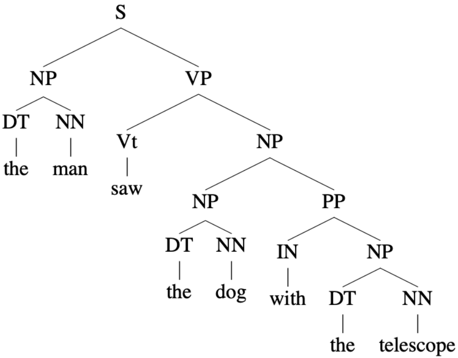
$$\prod_{(r,s) \in \mathcal{T}(x)} \psi(r, s \mid x; \theta) = \exp\left( \sum_{(r,s) \in \mathcal{T}(x)} \theta \cdot \phi(r, s, x) \right)$$

Learning: MLE

1. Compute the partition function by the inside algorithm
2. Call autograd to compute the gradient (backpropagation)

Inference: CYK

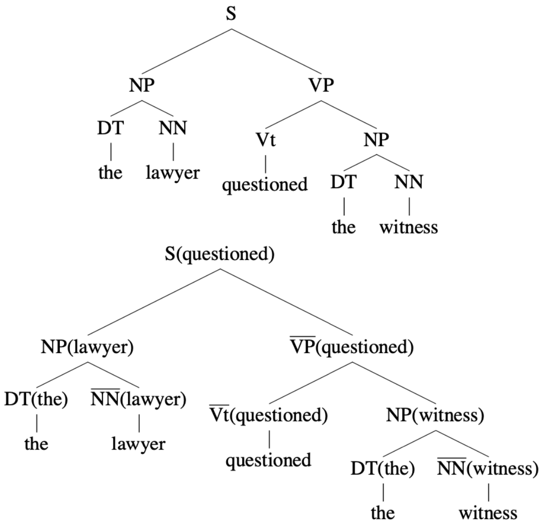
# Limitations of PCFG




Limited lexical information

# Lexicalized PCFG

Attach the "head" (most important child in a rule) of the span to each non-terminal



# Features

Easy to incorporate lexical information in features!

$$\text{local score} = \theta \cdot \phi(\text{VP} \to \text{VBD NP}, (5,6,8), \text{...averted financial disaster...})$$

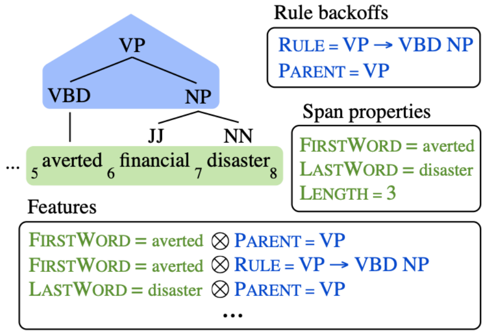



Figure: Less grammar, more features. [Hall+ 14]

# Neural CRF parser

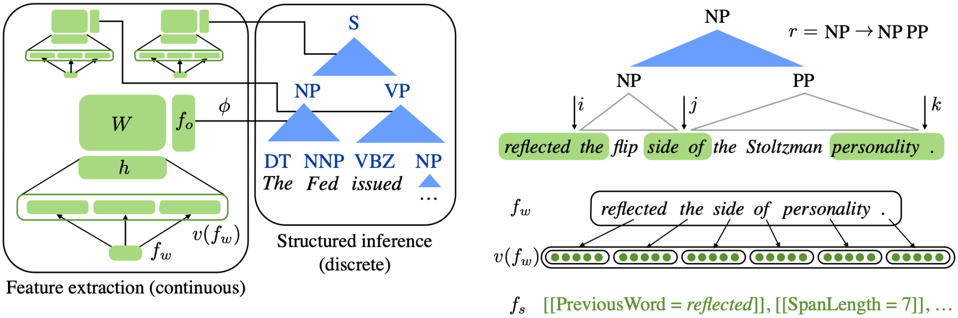



Figure: Neural CRF Parsing. [Durrett+ 15]

- $f_w$: lexical features
- $f_o$: rule features
- $h^T W f_o$: interaction between lexical and rule features

# Evaluation

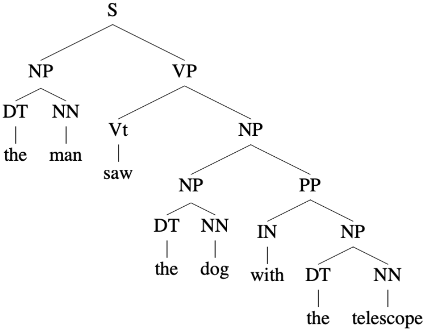
$$\text{recall} = \frac{\#\text{correct constituents}}{\#\text{total constituents in gold trees}}$$

$$\text{precision} = \frac{\#\text{correct constituents}}{\#\text{total constituents in predicted trees}}$$

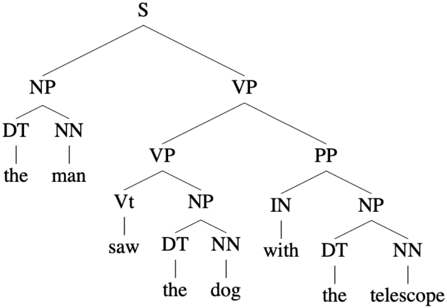
$$\text{F1} = \frac{2 \times \text{precision} \times \text{recall}}{\text{precision} + \text{recall}}$$

- Constituent: $(i, j, X)$
- Labeled F1: the non-terminal node label must be correct
- Unlabeled F1: just consider the tree structure

# Example




(a) Gold.




(b) Predicted.