

1. Suppose that $x_0 < x_1 < \dots < x_n$ and y_0, y_1, \dots, y_n are given real numbers. Then there exists a unique polynomial $p \in \mathcal{P}_n$ such that $p(x_i) = y_i$ for all $i \in \{0, \dots, n\}$.
2. Suppose that $f: [-1, 1] \rightarrow \mathbb{R}$ is given by $f(x) = e^{-2x}$, and for any $n \in \mathbb{N}$, let $f_n \in \mathcal{P}_n$ denote the polynomial interpolating f at $n+1$ equidistant points $-1 = x_0 < x_1 < \dots < x_n = 1$. Then

$$\lim_{n \rightarrow \infty} \left(\max_{-1 \leq x \leq 1} |f(x) - f_n(x)| \right) = 0.$$

3. Define $\mathcal{F}: \mathcal{P}_n \rightarrow \mathbf{R}$ as the function which, to a polynomial $p \in \mathcal{P}_n$, associates its maximum absolute value over the interval $[-1, 1]$, that is

$$\mathcal{F}(p) = \max_{x \in [-1, 1]} |p(x)|.$$

Then it holds that $\mathcal{F}(p) \geq \mathcal{F}(T_n)$ for all $p \in \mathcal{P}_n$, where T_n is the Chebyshev polynomial of degree n .

4. Suppose that $f: [-1, 1] \rightarrow \mathbb{R}$ is the Runge function $f(x) = \frac{1}{1+25x^2}$. For any $n \in \mathbb{N}$, let $f_n \in \mathcal{P}_n$ denote the polynomial interpolating f at $n+1$ equidistant points, and let $g_n \in \mathcal{P}_n$ denote the polynomial interpolating f at the $n+1$ Chebyshev points (the roots of T_{n+1}). Then it holds that

$$\lim_{n \rightarrow \infty} \left(\max_{-1 \leq x \leq 1} |f(x) - f_n(x)| \right) = +\infty, \quad \lim_{n \rightarrow \infty} \left(\max_{-1 \leq x \leq 1} |f(x) - g_n(x)| \right) = 0.$$

5. Suppose that $x_0 < x_1 < \dots < x_n$ and y_0, y_1, \dots, y_n are given real numbers. Then for any $m \in \{0, 1, \dots\}$, there exists exactly one polynomial $p \in \mathcal{P}_m$ such that

$$\frac{1}{n} \sum_{i=0}^n |p(x_i) - y_i|^2 = 0.$$

6. Given $x_0 < x_1 < \dots < x_n \in \mathbb{R}$ and $y_0, y_1, \dots, y_n \in \mathbb{R}$, and let $m \geq n$. There exists a unique $p \in \mathcal{P}_m$ such that the following quantity is minimized:

$$\frac{1}{n} \sum_{i=0}^n |p(x_i) - y_i|^2.$$

7. Given $x_0 < x_1 < \dots < x_n \in \mathbb{R}$ and $y_0, y_1, \dots, y_n \in \mathbb{R}$ with $n = 10$, there exists a unique polynomial $p(x) = ax + b$ that minimizes

$$J(a, b) = \frac{1}{2} \sum_{i=0}^n |y_i - p(x_i)|^2.$$

8. In Julia, if `v` is a vector of size 5, then `v[[1, 2, 5]]` returns a vector with the first, second and fifth element of `v`.
9. In Julia, if `v` is a vector of size 5, then `v[[true, true, false, true, false]]` returns a vector with the first, second and fourth element of `v`.
10. In Julia, if `A` is a 10×10 matrix, then `A[mod.(1:end, 2) .== 0, mod.(1:end, 2) .== 0]` gives the matrix obtained by removing the second column and the second row.