

1. Suppose that  $x_0 < x_1 < \dots < x_n$  and  $y_0, y_1, \dots, y_n$  are given real numbers. Then there exists a unique polynomial  $p \in \mathcal{P}_n$  such that  $p(x_i) = y_i$  for all  $i \in \{0, \dots, n\}$ .
2. Suppose that  $f: [-1, 1] \rightarrow \mathbb{R}$  is given by  $f(x) = e^{-2x}$ , and for any  $n \in \mathbb{N}$ , let  $f_n \in \mathcal{P}_n$  denote the polynomial interpolating  $f$  at  $n+1$  equidistant points  $-1 = x_0 < x_1 < \dots < x_n = 1$ . Then

$$\lim_{n \rightarrow \infty} \left( \max_{-1 \leq x \leq 1} |f(x) - f_n(x)| \right) = 0.$$

3. Define  $\mathcal{F}: \mathcal{P}_n \rightarrow \mathbf{R}$  as the function which, to a polynomial  $p \in \mathcal{P}_n$ , associates its maximum absolute value over the interval  $[-1, 1]$ , that is

$$\mathcal{F}(p) = \max_{x \in [-1, 1]} |p(x)|.$$

Then it holds that  $\mathcal{F}(p) \geq \mathcal{F}(T_n)$  for all  $p \in \mathcal{P}_n$ , where  $T_n$  is the Chebyshev polynomial of degree  $n$ .

4. Suppose that  $f: [-1, 1] \rightarrow \mathbb{R}$  is the Runge function  $f(x) = \frac{1}{1+25x^2}$ . For any  $n \in \mathbb{N}$ , let  $f_n \in \mathcal{P}_n$  denote the polynomial interpolating  $f$  at  $n+1$  equidistant points, and let  $g_n \in \mathcal{P}_n$  denote the polynomial interpolating  $f$  at the  $n+1$  Chebyshev points (the roots of  $T_{n+1}$ ). Then it holds that

$$\lim_{n \rightarrow \infty} \left( \max_{-1 \leq x \leq 1} |f(x) - f_n(x)| \right) = +\infty, \quad \lim_{n \rightarrow \infty} \left( \max_{-1 \leq x \leq 1} |f(x) - g_n(x)| \right) = 0.$$

5. Suppose that  $x_0 < x_1 < \dots < x_n$  and  $y_0, y_1, \dots, y_n$  are given real numbers. Then for any  $m \in \{0, 1, \dots\}$ , there exists exactly one polynomial  $p \in \mathcal{P}_m$  such that

$$\frac{1}{n} \sum_{i=1}^n |p(x_i) - y_i|^2 = 0.$$

6. Given  $x_0 < x_1 < \dots < x_n \in \mathbb{R}$  and  $y_0, y_1, \dots, y_n \in \mathbb{R}$ , and let  $m \geq n$ . There exists a unique  $p \in \mathcal{P}_m$  such that the following quantity is minimized:

$$\frac{1}{n} \sum_{i=1}^n |p(x_i) - y_i|^2 = 0.$$

7. Given  $x_0 < x_1 < \dots < x_n \in \mathbb{R}$  and  $y_0, y_1, \dots, y_n \in \mathbb{R}$  with  $n = 10$ , there exists a unique polynomial  $p(x) = ax + b$  that minimizes

$$J(a, b) = \frac{1}{2} \sum_{i=1}^n |y_i - p(x_i)|^2.$$

8. In Julia, if `v` is a vector of size 5, then `v[[1, 2, 5]]` returns a vector with the first, second and fifth element of `v`.
9. In Julia, if `v` is a vector of size 5, then `v[[true, true, false, true, false]]` returns a vector with the first, second and fourth element of `v`.
10. In Julia, if `A` is a  $10 \times 10$  matrix, then `A[mod.(1:end, 2) .== 0, mod.(1:end, 2) .== 0]` gives the matrix obtained by removing the second column and the second row.