Throughout the quiz, we consider the following scalar, nonlinear equation, where the function f is not necessarily continuous:

$$f(x) = 0, \qquad f\colon \mathbf{R} \to \mathbf{R}, \qquad x \in \mathbf{R}. \tag{1}$$

1. **(T/F)** If f is strictly increasing, then there exists a unique solution to (1).

   If you answer false, justify with an counterexample.

2. **(T/F)** There may exist infinitely many solutions to (1), depending on the specific form of f.

   If you answer true, justify with an example.

3. **(T/F)** Suppose that f is continuous and that $f(0)f(1) < 0$. Then the bisection method, initialized with $a = 0$ and $b = 1$, is guaranteed to converge towards a root of f.

4. **(T/F)** Suppose that $f(x) = 3x$, and consider the chord method to find a solution to (1):

$$x_{k+1} = x_k - \frac{f(x_k)}{\alpha}.$$

   Then, for the function f given, this method converges for $\alpha = 1$.

5. **(T/F)** Suppose f is differentiable with a fixed point at x_*, and consider the Newton–Raphson method to find a solution to (1):

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}. \tag{2}$$

   If $f'(x_*) \neq 0$ and $x_k \to x_*$, then it holds that

$$\lim_{k\to\infty} \left| \frac{x_{k+1} - x_*}{x_k - x_*} \right| = 0.$$

6. **(T/F)** Sir Isaac Newton was the first person to produce an ultra-efficient implementation of the Newton–Raphson method in *Python*, which earned him the title of Fellow of the Royal Society.

7. The Newton–Raphson method may be rewritten as a fixed point iteration of the form

$$x_{k+1} = F_{\rm NR}(x_k)$$

   for an appropriate function $F_{\rm NR}$. Write the expression of the function $F_{\rm NR}$:

$$F_{\rm NR}(x) =$$

8. Suppose that $f = x^2$, and that the Newton–Raphson method is employed to find a root of f starting from $x_0 = 1$. Calculate an explicit expression for x_k:

$$x_k =$$

9. **(2 marks)** Write a short Julia code to compute $\sqrt[3]{2}$ to machine precision, using the method of your choice and without resorting to the function `cbrt` or `^{1/3}`.