

1. The computational cost of the forward substitution algorithm, for a matrix of size  $n \times n$ , scales as  $\mathcal{O}(n)$ .
2. The computational cost to calculate the LU decomposition of an invertible matrix scales as  $\mathcal{O}(n^2)$ .
3. Suppose that  $\mathbf{A}$  is a symmetric, positive definite matrix. Calculating the Cholesky decomposition of  $\mathbf{A}$  requires fewer floating point operations than calculating the LU decomposition of  $\mathbf{A}$ .
4. The condition number  $\kappa_2(\mathbf{A})$  of a square invertible matrix  $\mathbf{A}$  is always strictly larger than 1.
5. The spectral radius of a matrix square  $\mathbf{A}$  is zero if and only if  $\mathbf{A}$  is zero.
6. If the condition number of a matrix  $\mathbf{A}$  is very large ( $\gg 1$ ), then it is possible that rounding errors arising from floating point arithmetic will have a large impact on the accuracy of the numerical solution to the linear system  $\mathbf{A}\mathbf{x} = \mathbf{b}$  (calculated by LU decomposition followed by forward and backward substitution, for example).
7. Suppose that  $\mathbf{A} \in \mathbf{R}^{n \times n}$  is symmetric and positive definite, and let  $\mathbf{b} \in \mathbf{R}^n$ . Consider the following iterative method for solving the linear system  $\mathbf{A}\mathbf{x} = \mathbf{b}$ :

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \omega (\mathbf{b} - \mathbf{A}\mathbf{x}^{(k)}). \quad (1)$$

This iteration converges to the exact solution of the linear system for all  $\omega \in \mathbf{R}$ .

8. The convergence speed of the iteration (1), for the optimal value of  $\omega$ , is independent of the condition number  $\kappa_2(\mathbf{A})$ .
9. If  $\mathbf{A}$  is symmetric positive definite, there always exists  $\omega \in \mathbf{R}$  such that the iteration (1) converges.
10. If  $\mathbf{A}$  is a nonzero matrix, then its norm  $\|\mathbf{A}\|_2$  is strictly positive.
11. Suppose that we want to solve the linear systems  $\mathbf{A}\mathbf{x} = \mathbf{b}$  and  $\mathbf{A}\mathbf{x} = \mathbf{c}$  by a direct method. To this end, the LU decomposition of  $\mathbf{A}$  can be calculated only once.
12. Assuming that the matrix  $\mathbf{A}$  and the vector  $\mathbf{b}$  are already defined, write (on paper) Julia code implementing 100 iterations of (1).