1. The computational cost of the forward substitution algorithm, for a matrix of size $n \times n$, scales as $\mathcal{O}(n)$.

2. The computational cost to calculate the LU decomposition of an invertible matrix scales as $\mathcal{O}(n^2)$.

3. Suppose that A is a symmetric, positive definite matrix. Calculating the Cholesky decomposition of A requires fewer floating point operations than calculating the LU decomposition of A.

4. The condition number $\kappa_2(\mathsf{A})$ of a square invertible matrix A is always strictly larger than 1.

5. The spectral radius of a matrix square A is zero if and only if A is zero.

6. If the condition number of a matrix A is very large ($\gg 1$), then it is possible that rounding errors arising from floating point arithmetic will have a large impact on the accuracy of the numerical solution to the linear system $\mathsf{A}\boldsymbol{x} = \boldsymbol{b}$ (calculated by LU decomposition followed by forward and backward substitution, for example).

7. Suppose that $\mathsf{A} \in \mathbf{R}^{n \times n}$ is symmetric and positive definite, and let $\boldsymbol{b} \in \mathbf{R}^n$. Consider the following iterative method for solving the linear system $\mathsf{A}\boldsymbol{x} = \boldsymbol{b}$:

$$\boldsymbol{x}^{(k+1)} = \boldsymbol{x}^{(k)} + \omega \left(\boldsymbol{b} - \mathsf{A}\boldsymbol{x}^{(k)}\right). \tag{1}$$

   This iteration converges to the exact solution of the linear system for all $\omega \in \mathbf{R}$.

8. The convergence speed of the iteration (1), for the optimal value of ω, is independent of the condition number $\kappa_2(\mathsf{A})$.

9. If A is symmetric positive definite, there always exists $\omega \in \mathbf{R}$ such that the iteration (1) converges.

10. If A is a nonzero matrix, then its norm $\|\mathsf{A}\|_2$ is strictly positive.

11. Suppose that we want to solve the linear systems $\mathsf{A}\boldsymbol{x} = \boldsymbol{b}$ and $\mathsf{A}\boldsymbol{x} = \boldsymbol{c}$ by a direct method. To this end, the LU decomposition of A can be calculated only once.

12. Assuming that the matrix `A` and the vector `b` are already defined, write (on paper) Julia code implementing 100 iterations of (1).