Sample Solution for Homework 4

Problem 1 Variable Binding and Scoping (16 Points)

Consider the following grammar describing the abstract syntax of the simple expression language with constant declarations that we considered in class:

$$
\begin{array}{rr}
n \in \mathit{Num} & \text{numbers} \\
x \in \mathit{Var} & \text{variables} \\
e \in \mathit{Expr} ::= n \mid x \mid e_1\, \mathit{bop}\, e_2 \mid \textbf{const}\ x = e_d; e_b & \text{expressions} \\
\mathit{bop} \in \mathit{Bop} ::= + \mid * & \text{binary operators}
\end{array}
$$

For each of the following expressions do the following:

(a) overline the defining variable occurrences and draw an arrow between each bound using occurrence of a variable x and the corresponding defining occurrence of x

(b) compute the set of free variables of the expression

(c) give the AST of the expression in tuple notation. Assume that the different types of expressions are assigned variant numbers in the order in which they appear in the grammar.

(d) evaluate the expression in the environment $env = \{x \mapsto 3, y \mapsto 2, z \mapsto 1\}$ using the evaluation function *eval* defined in class.

Expressions:

(i) $e_1 = x + 2$

(a) $x + 2$
(b) $fv(e_1) = \{x\}$
(c) $\langle \underline{3}, \langle \underline{2}, x\rangle, \langle \underline{1}\rangle, \langle \underline{1}, 2\rangle\rangle$
(d) $eval(env, e_1) = 5$

(ii) $e_2 = \textbf{const}\ x = 2; x * y$

(a) $\textbf{const}\ \overline{x}_1 = 2; x_1 * y$
(b) $fv(e_2) = \{y\}$
(c) $\langle \underline{4}, x, \langle \underline{1}, 2\rangle, \langle \underline{3}, \langle \underline{2}, x\rangle, \langle \underline{2}\rangle, \langle \underline{2}, y\rangle\rangle\rangle$
(d) $eval(env, e_2) = 4$

(iii) $e_3 = \textbf{const}\ y = y; \textbf{const}\ y = y; y$

(a) $\textbf{const}\ \overline{y}_1 = y; \textbf{const}\ \overline{y}_2 = y_1; y_2$

(b) $fv(e_3) = \{y\}$

(c) $\langle \underline{4}, y, \langle \underline{2}, y\rangle, \langle \underline{4}, y, \langle \underline{2}, y\rangle, \langle \underline{2}, y\rangle\rangle\rangle$

(d) $eval(env, e_3) = 2$

(iv) $e_4 = \textbf{const}\ x = (\textbf{const}\ z = 3; z + x); z + x$

(a) $\textbf{const}\ \overline{x}_1 = (\textbf{const}\ \overline{z}_1 = 3; z_1 + x); z + x_1$

(b) $fv(e_4) = \{x, z\}$

(c) $\langle \underline{4}, x, \langle \underline{4}, z, \langle \underline{1}, 3\rangle, \langle \underline{3}, \langle \underline{2}, z\rangle, \langle \underline{1}\rangle, \langle \underline{2}, x\rangle\rangle\rangle,$
$\langle \underline{3}, \langle \underline{2}, z\rangle, \langle \underline{1}\rangle, \langle \underline{2}, x\rangle\rangle\rangle$

(d) $eval(env, e_4) = 7$