Sample Solution for Homework 12

Problem 1 Computing Meets and Joins (12 Points)

This is a warm-up exercise to make yourself familiar with the join and meet operations on our type language with subtyping.

For each of the following pairs of types τ_1 and τ_2, compute their join $\tau_1 \sqcup \tau_2$ and meet $\tau_1 \sqcap \tau_2$. If the meet does not exist, indicate this by writing $\tau_1 \sqcap \tau_2 = \bot$.

(a) $\tau_1 =$ **Num**, $\tau_2 =$ {**const** f**:** **Num**}

$\tau_1 \sqcup \tau_2 =$ **Any**

$\tau_1 \sqcap \tau_2 = \bot$

(b) $\tau_1 =$ { }, $\tau_2 =$ {**let** f**:** **Num**, **const** g**:** **Bool**}

$\tau_1 \sqcup \tau_2 =$ { }

$\tau_1 \sqcap \tau_2 =$ {**let** f**:** **Num**, **const** g**:** **Bool**}

(c) $\tau_1 =$ {**let** f**:** **Num**}, $\tau_2 =$ {**const** g**:** **Bool**}

$\tau_1 \sqcup \tau_2 =$ { }

$\tau_1 \sqcap \tau_2 =$ {**let** f**:** **Num**, **const** g**:** **Bool**}

(d) $\tau_1 =$ {**let** f**:** **Num**, **const** g**:** {**let** h**:** **Any**}},
$\tau_2 =$ {**let** f**:** **Num**, **const** g**:** {**const** h**:** **Bool**}}

$\tau_1 \sqcup \tau_2 =$ {**let** f**:** **Num**, **const** g**:** {**const** h**:** **Any**}}

$\tau_1 \sqcap \tau_2 = \bot$

(e) $\tau_1 =$ (**Any** => **Bool**), $\tau_2 =$ (**Bool** => **Any**)

$\tau_1 \sqcup \tau_2 =$ **Bool** => **Any**

$\tau_1 \sqcap \tau_2 =$ **Any** => **Bool**

(f) $\tau_1 =$ (**Bool** => **Num**), $\tau_2 =$ (**Num** => **Bool**)

$\tau_1 \sqcup \tau_2 =$ **Any**

$\tau_1 \sqcap \tau_2 = \bot$