

## Sample Solution for Homework 4

### Problem 1 Variable Binding and Scoping (16 Points)

Consider the following grammar describing the abstract syntax of the simple expression language with constant declarations that we considered in class:

$n \in Num$	numbers
$x \in Var$	variables
$e \in Expr ::= n \mid x \mid e_1 \text{ bop } e_2 \mid \mathbf{const} \ x = e_d; e_b$	expressions
$\text{bop} \in Bop ::= + \mid *$	binary operators

For each of the expressions given below do the following:

- (a) Overline the defining variable occurrences and draw an arrow between each bound using occurrence of a variable  $x$  and the corresponding defining occurrence of  $x$ .
- (b) Give the set of free variables of the expression.
- (c) Give the AST of the expression in tuple notation. Assume that the different types of expressions are assigned variant numbers in the order in which they appear in the grammar.
- (d) Evaluate the expression in the environment  $env = \{x \mapsto 1, y \mapsto 2, z \mapsto 3\}$  using the evaluation function  $eval$  defined in class.

#### Expressions:

(i)  $e_1 = x + 4$

(a)  $x + 4$

(b)  $fv(e_1) = \{x\}$

(c)  $\langle \underline{3}, \langle \underline{2}, x \rangle, \langle \underline{1} \rangle, \langle \underline{1}, 4 \rangle \rangle$

(d)  $eval(env, e_1) = 5$

(ii)  $e_2 = \mathbf{const} \ x = 2; y * x$

(a)  $\mathbf{const} \ \bar{x}_1 = 2; y * x_1$

(b)  $fv(e_2) = \{y\}$

(c)  $\langle \underline{4}, x, \langle \underline{1}, 2 \rangle, \langle \underline{3}, \langle \underline{2}, y \rangle, \langle \underline{2} \rangle, \langle \underline{2}, x \rangle \rangle \rangle$

(d)  $eval(env, e_2) = 4$

(iii)  $e_3 = \mathbf{const} \ z = z; \mathbf{const} \ z = z; z$

(a)  $\mathbf{const} \ \bar{z}_1 = z; \mathbf{const} \ \bar{z}_2 = z_1; z_2$

- (b)  $fv(e_3) = \{z\}$
  - (c)  $\langle \underline{4}, z, \langle \underline{2}, z \rangle, \langle \underline{4}, z, \langle \underline{2}, z \rangle, \langle \underline{2}, z \rangle \rangle \rangle$
  - (d)  $eval(env, e_3) = 3$
- (iv)  $e_4 = \mathbf{const} \ x = (\mathbf{const} \ x = 3; z + x); z + x$
- (a)  $\mathbf{const} \ \bar{x}_1 = (\mathbf{const} \ \bar{x}_2 = 3; z + x_2); z + x_1$
  - (b)  $fv(e_4) = \{z\}$
  - (c)  $\langle \underline{4}, x, \langle \underline{4}, x, \langle \underline{1}, 3 \rangle, \langle \underline{3}, \langle \underline{2}, z \rangle, \langle \underline{1} \rangle, \langle \underline{2}, x \rangle \rangle \rangle, \langle \underline{3}, \langle \underline{2}, z \rangle, \langle \underline{1} \rangle, \langle \underline{2}, x \rangle \rangle \rangle$
  - (d)  $eval(env, e_4) = 9$