Deep Learning Part II

Instructor: Prof. Yi Fang

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Python tutorial: http://learnpython.org/

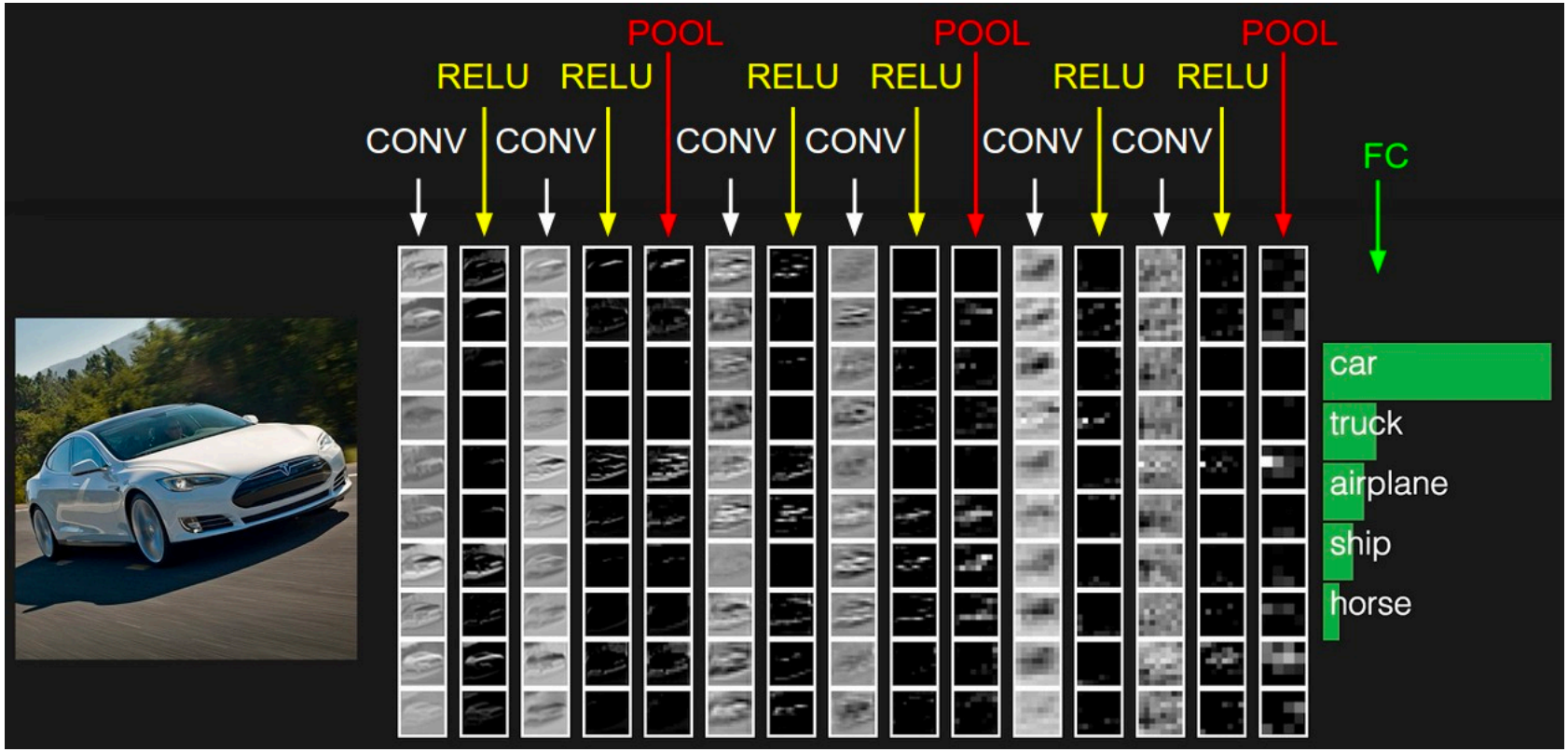
TensorFlow tutorial: https://www.tensorflow.org/tutorials/

PyTorch tutorial: https://pytorch.org/tutorials/

Acknowledge: The slides are partially referred to coursera online machine learning course by Prof. Andrew Ng, and NYU machine learning course. All copyrights owned by original authors. (i.e. http://u.cs.biu.ac.il/~89-570/), and Prof. Hongyi Lee ML lectures at Youtube.

Convolution Neural Network

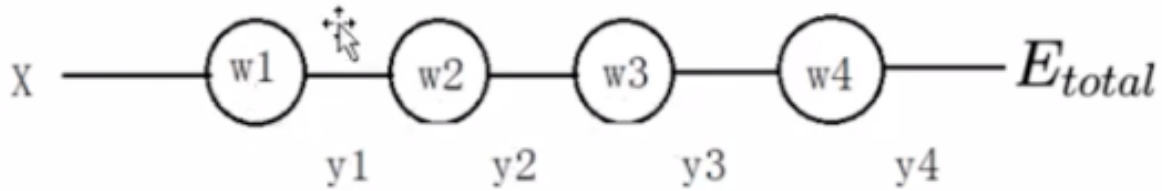
- An example of ConvNet architecture.



Gradient Vanishing and Exploding

$$w_1^+ = w_1 - \eta \frac{\partial E_{total}}{\partial w_1}$$

$$y_i = \sigma(z_i) = \sigma(w_i x_i + b_i)$$

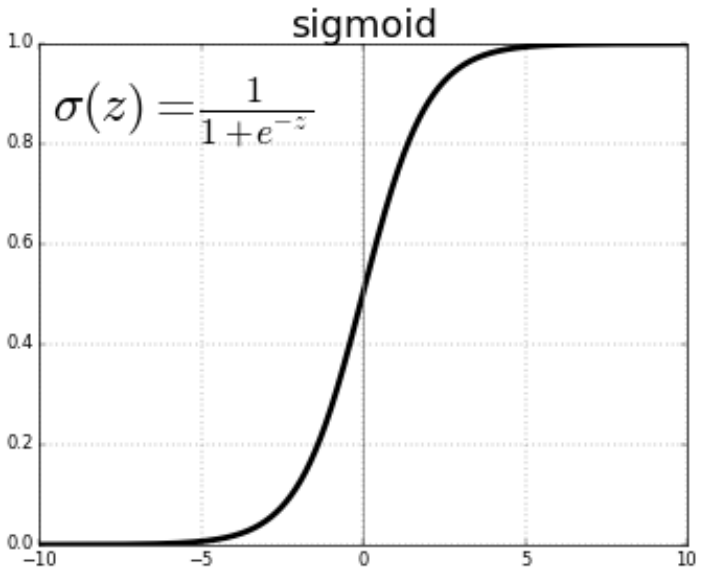


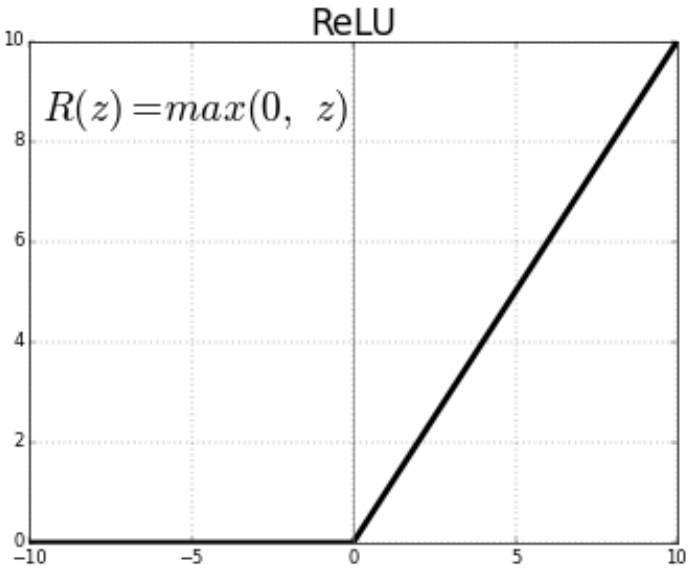
$$\frac{\partial E_{total}}{\partial w_1} = \frac{\partial E_{total}}{\partial y_4}\frac{\partial y_4}{\partial z_4}\frac{\partial z_4}{\partial x_4}\frac{\partial x_4}{\partial z_3}\frac{\partial z_3}{\partial x_3}\frac{\partial x_3}{\partial z_2}\frac{\partial z_2}{\partial x_2}\frac{\partial x_2}{\partial z_1}\frac{\partial z_1}{\partial w_1}$$

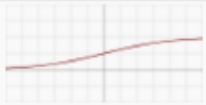
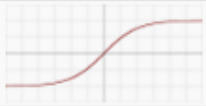
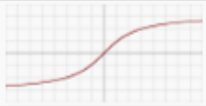
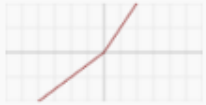
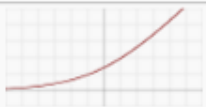
$$= \frac{\partial E_{total}}{\partial y_4}\sigma'(z_4)\, w_4 \sigma'(z_3)\, w_3 \sigma'(z_2)\, w_2 \sigma'(z_1) x_1$$

Activation Functions

- Sigmoid Function
- ReLU Function
- More

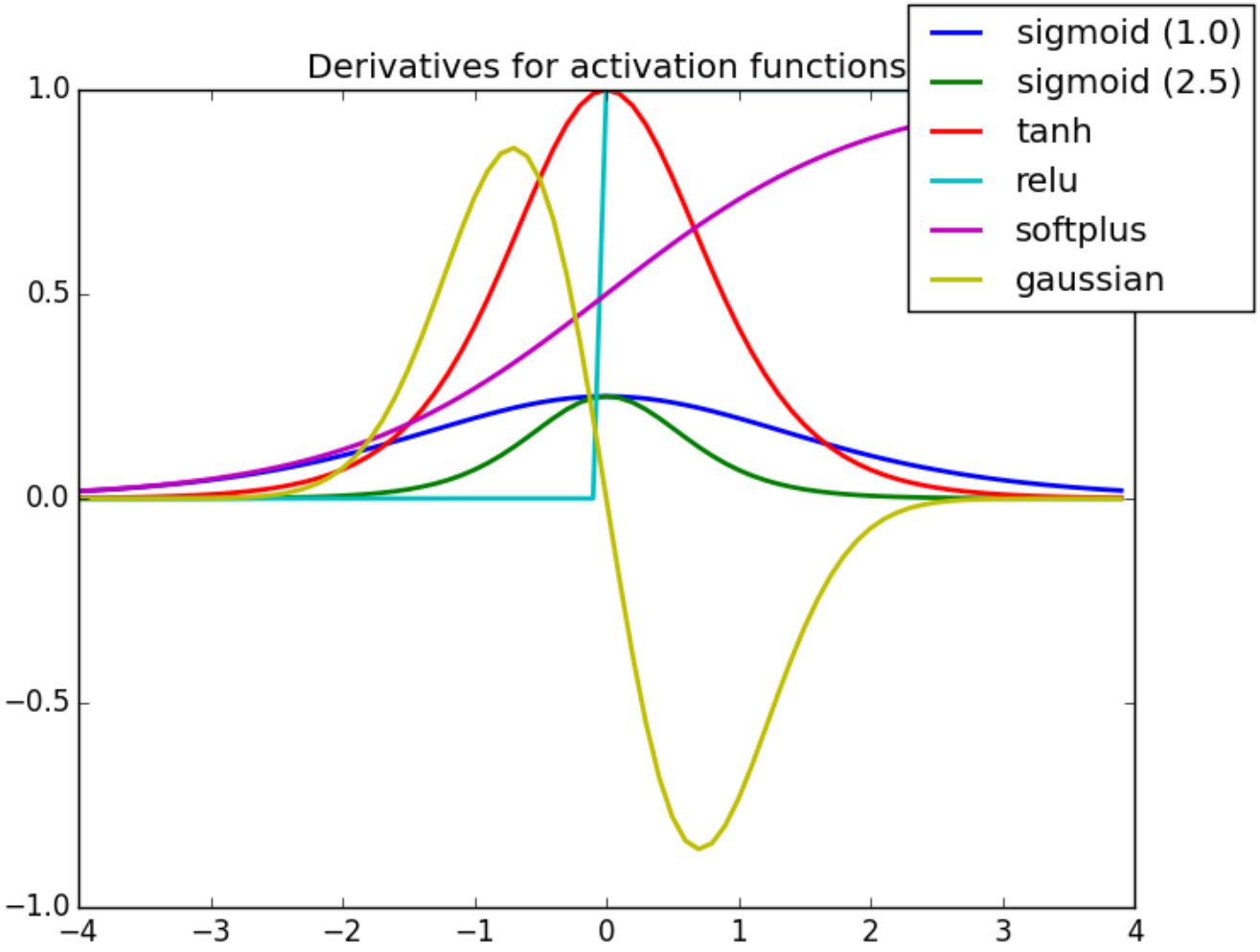




Name	Plot	Equation	Derivative
Identity		$f(x) = x$	$f'(x) = 1$
Binary step		$f(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} 0 & \text{for } x \neq 0 \\ ? & \text{for } x = 0 \end{cases}$
Logistic (a.k.a Soft step)		$f(x) = \frac{1}{1+e^{-x}}$	$f'(x) = f(x)(1-f(x))$
TanH		$f(x) = \tanh(x) = \frac{2}{1+e^{-2x}} - 1$	$f'(x) = 1 - f(x)^2$
ArcTan		$f(x) = \tan^{-1}(x)$	$f'(x) = \frac{1}{x^2+1}$
Rectified Linear Unit (ReLU)		$f(x) = \begin{cases} 0 & \text{for } x < 0 \\ x & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$
Parameteric Rectified Linear Unit (PReLU) [2]		$f(x) = \begin{cases} \alpha x & \text{for } x < 0 \\ x & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} \alpha & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$
Exponential Linear Unit (ELU) [3]		$f(x) = \begin{cases} \alpha(e^x - 1) & \text{for } x < 0 \\ x & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} f(x) + \alpha & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$
SoftPlus		$f(x) = \log_e(1 + e^x)$	$f'(x) = \frac{1}{1+e^{-x}}$

Source: https://towardsdatascience.com/activation-functions-neural-networks-1cbd9f8d91d6

Derivatives of Activation Functions



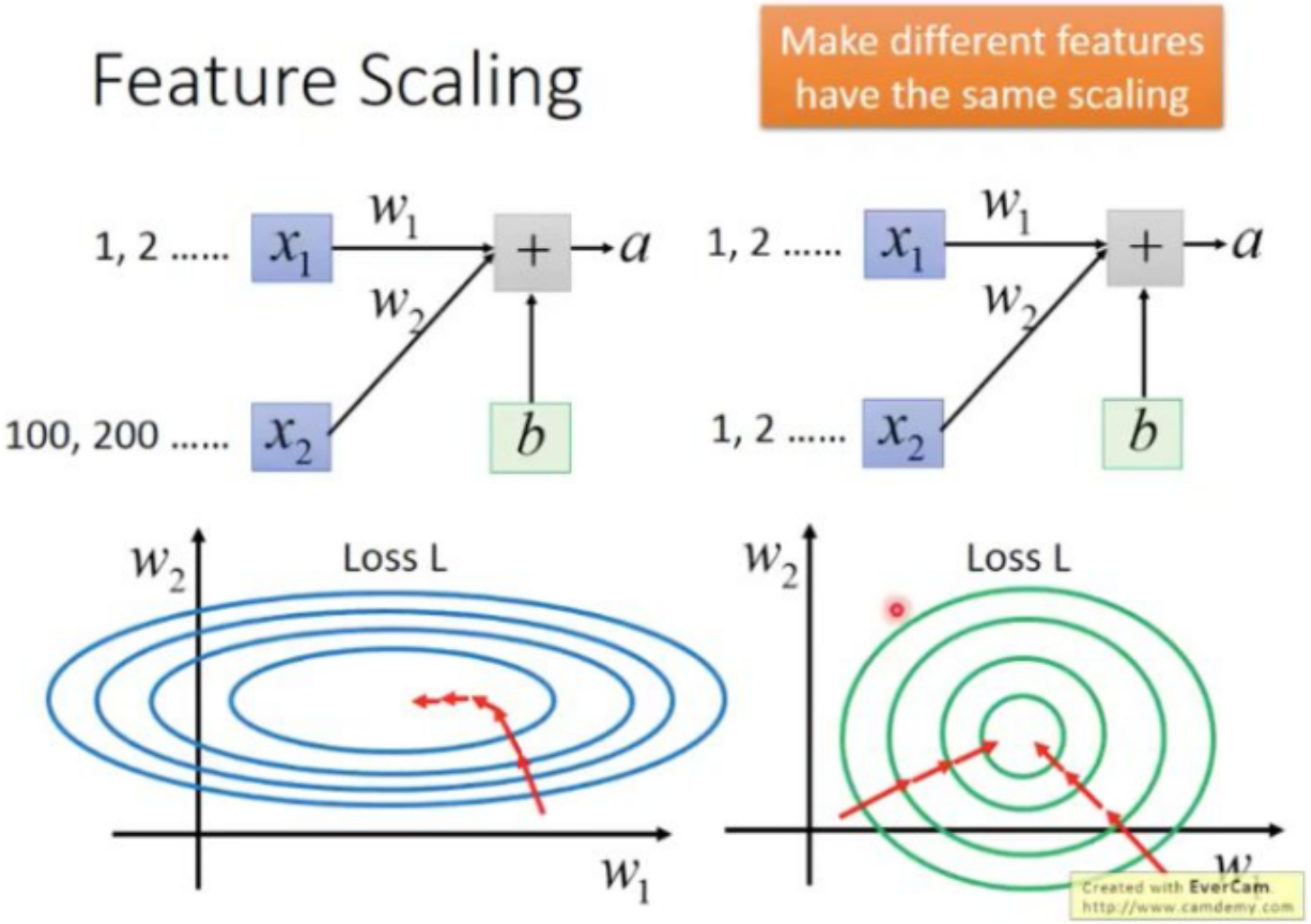
Source: https://www.picswe.com/pics/relu-activation-7d.html

Batch Normalization

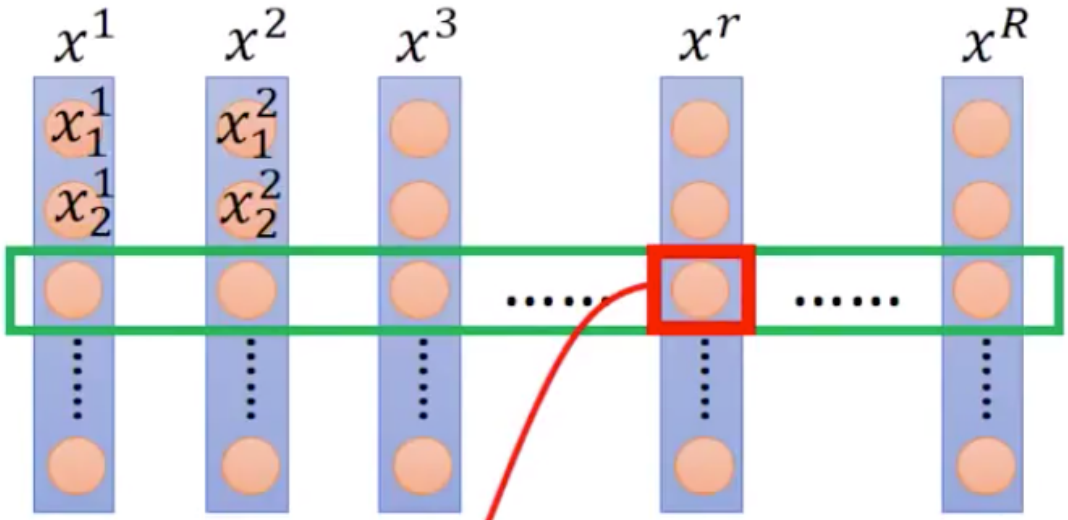
- Batch normalization speeds up the training by setting high learning rate
- Batch normalization prevents the gradients from getting too small or too large by normalizing data across each batch, as the name suggests. It also acts as a regularization method, similar to dropout.

Cited: https://zhuanlan.zhihu.com/p/34480619 and Prof. Lee from TW Univ.

Feature Scaling



Feature Scaling



For each dimension i:

mean: m_i

standard deviation: σ_i

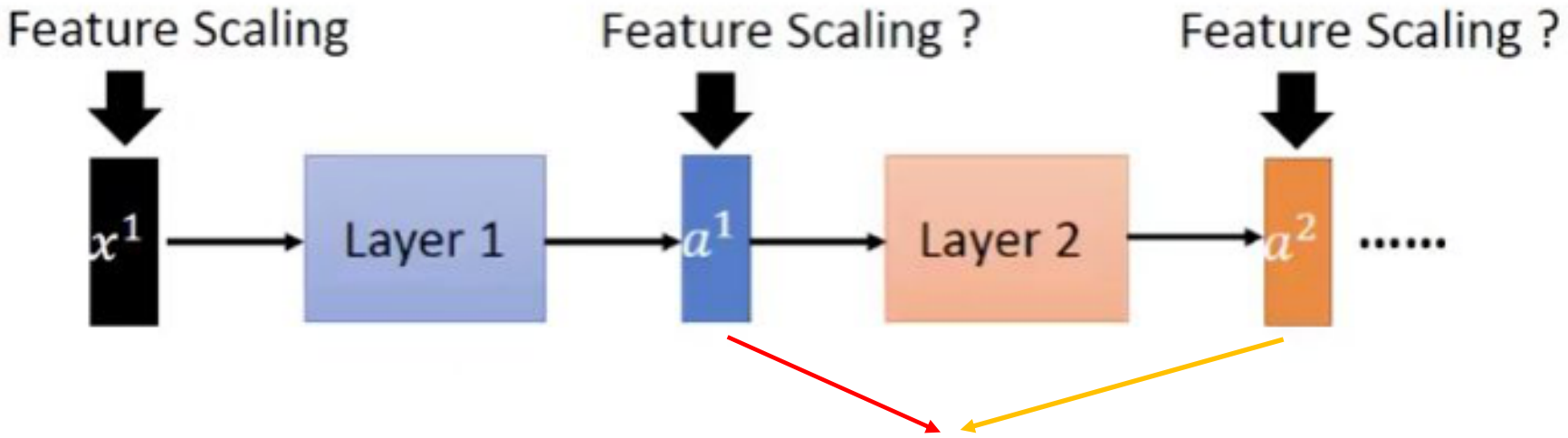
$$x_i^r \leftarrow \frac{x_i^r - m_i}{\sigma_i}$$

The means of all dimensions are 0, and the variances are all 1

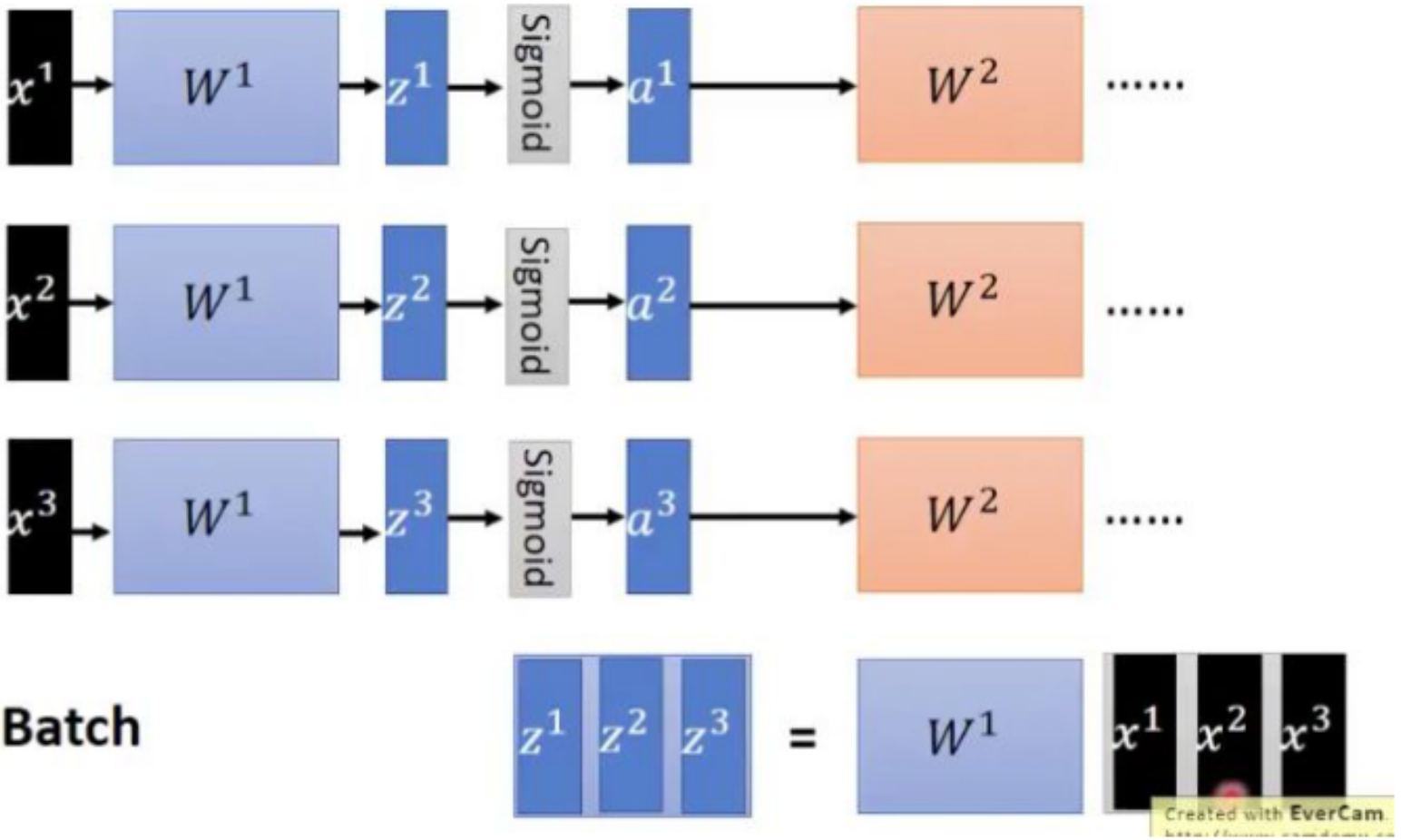
In general, gradient descent converges much faster with feature scaling than without it.

Scaling at Hidden Layer

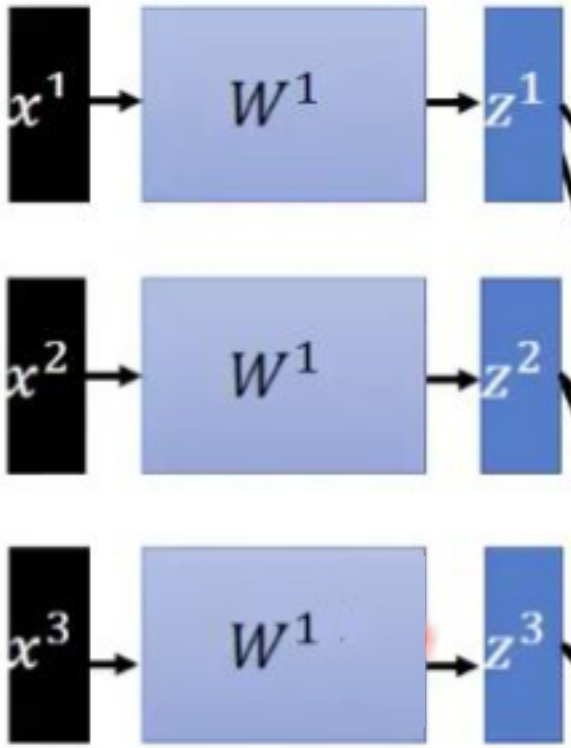
- Internal Covariate Shift



Batch of Data



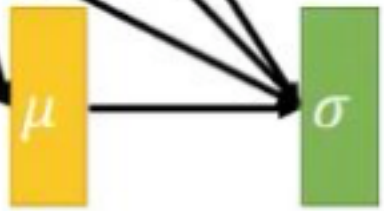
Batch normalization



$$\mu = \frac{1}{3}\sum_{i=1}^{3} z^i$$

$$\sigma = \sqrt{\frac{1}{3}\sum_{i=1}^{3}(z^i - \mu)^2}$$

Note: Batch normalization cannot be applied on small batch.

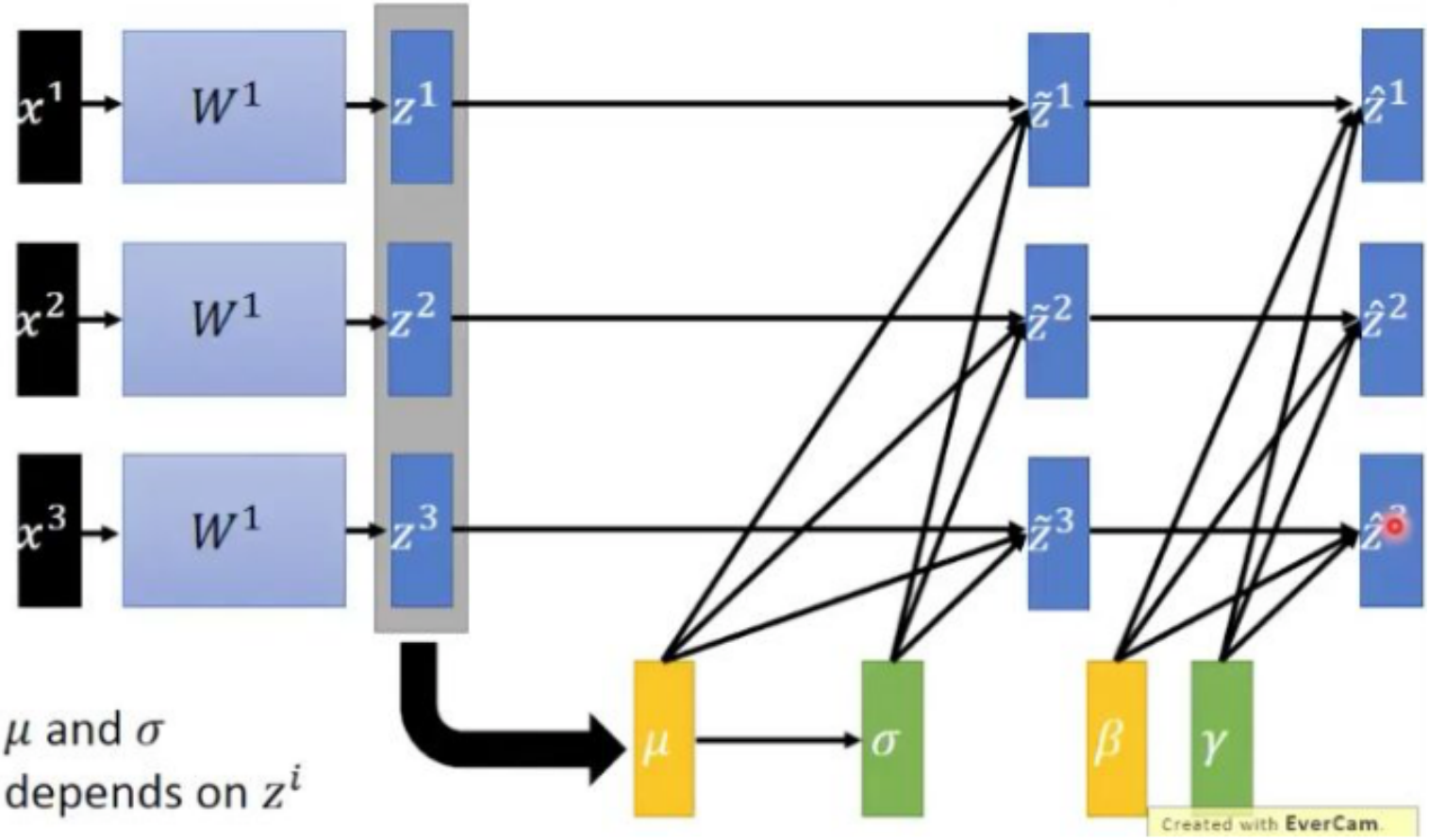


μ and σ depends on z^i

Batch normalization

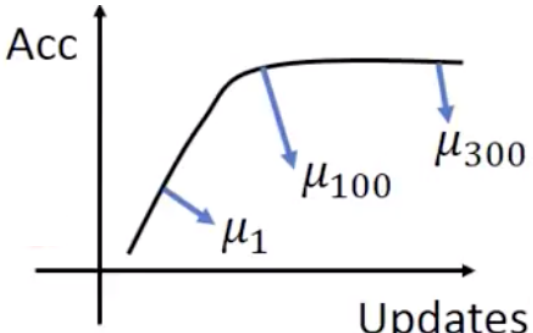
$$\tilde{z}^i = \frac{z^i - \mu}{\sigma}$$

$$\hat{z}^i = \gamma \odot \tilde{z}^i + \beta$$

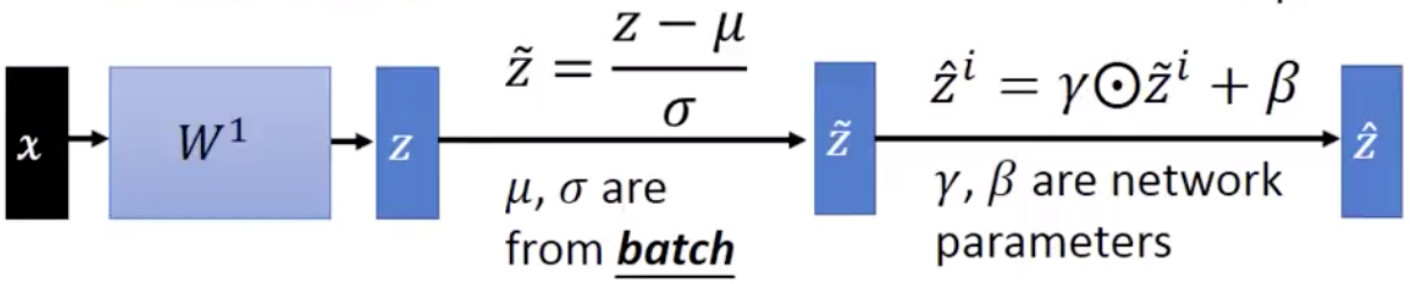


BN at Testing

Batch normalization



- At testing stage:



We do not have ***batch*** at testing stage.

Ideal solution:

Computing μ and σ using the whole training dataset.

Practical solution:

Computing the moving average of μ and σ of the batches during training.

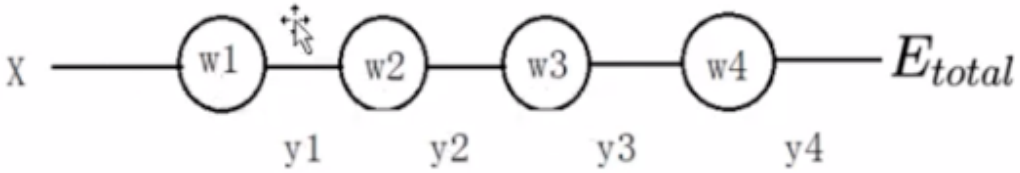
BN: Benefits

- Reduce the covariate shift, speed up the training
- Reduce the vanishing/exploding gradients
- Less affected by weights initialization

Gradient Vanishing and Exploding

$$w_1^+ = w_1 - \eta \frac{\partial E_{total}}{\partial w_1}$$

$$y_i = \sigma(z_i) = \sigma(w_i x_i + b_i)$$



$$\frac{\partial E_{total}}{\partial w_1} = \frac{\partial E_{total}}{\partial y_4} \frac{\partial y_4}{\partial z_4} \frac{\partial z_4}{\partial x_4} \frac{\partial x_4}{\partial z_3} \frac{\partial z_3}{\partial x_3} \frac{\partial x_3}{\partial z_2} \frac{\partial z_2}{\partial x_2} \frac{\partial x_2}{\partial z_1} \frac{\partial z_1}{\partial w_1}$$

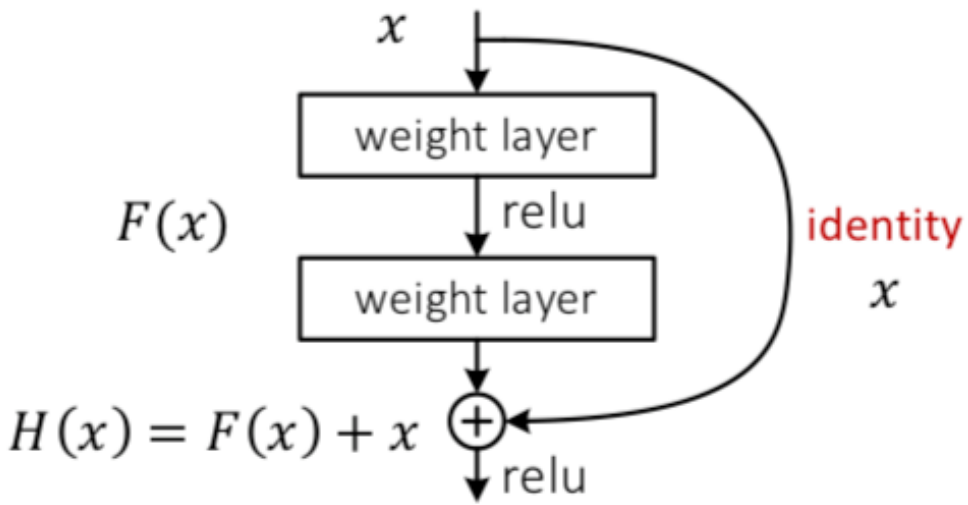
$$= \frac{\partial E_{total}}{\partial y_4} \sigma'(z_4) w_4 \sigma'(z_3) w_3 \sigma'(z_2) w_2 \sigma'(z_1) x_1$$

Cite: ResNet: https://www.youtube.com/watch?v=Bu9A_-M5OZk

Residual Training

Deep Residual Learning

- Residual net



$H(x)$ is any desired mapping,

~~hope the 2 weight layers fit $H(x)$~~

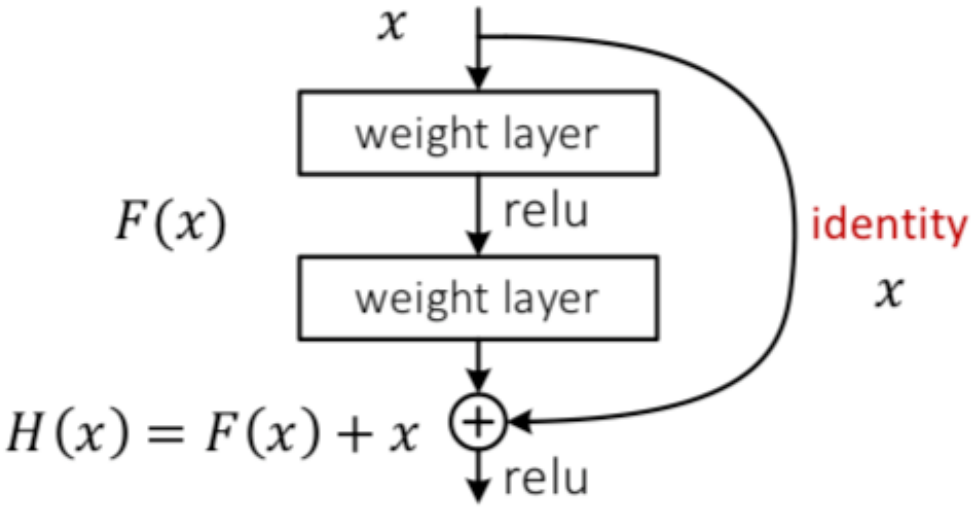
hope the 2 weight layers fit $F(x)$

let $H(x) = F(x) + x$

Cite: Kaiming He, etc, Deep Residual Learning for Image Recognition, CVPR 21016

Deep Residual Learning

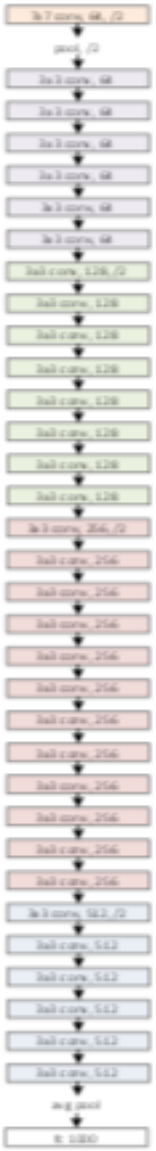
- $F(x)$ is a residual mapping w.r.t. identity



- If identity were optimal, easy to set weights as 0
- If optimal mapping is closer to identity, easier to find small fluctuations

Network Design

plain net

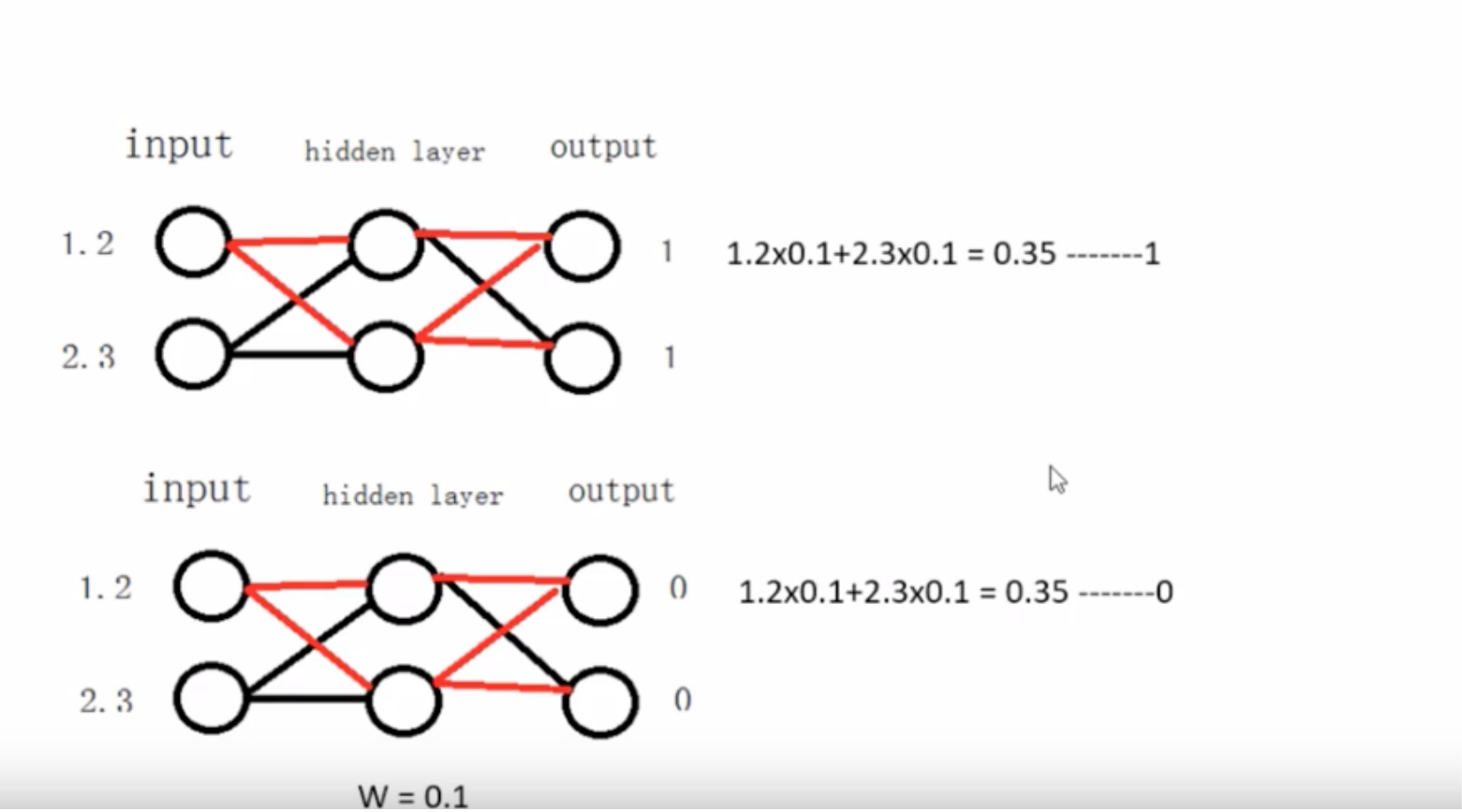


ResNet

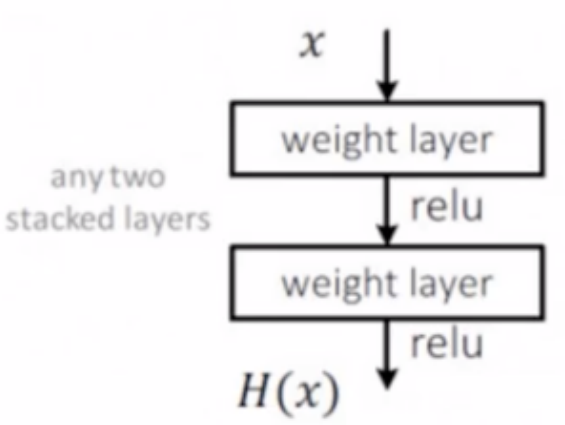
Benefits

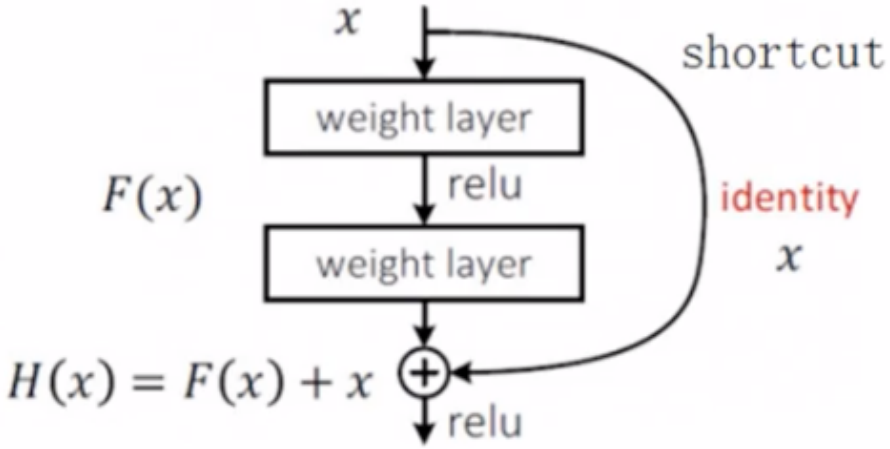
- Speed up the training
- Deeper network with reduced gradient vanishing

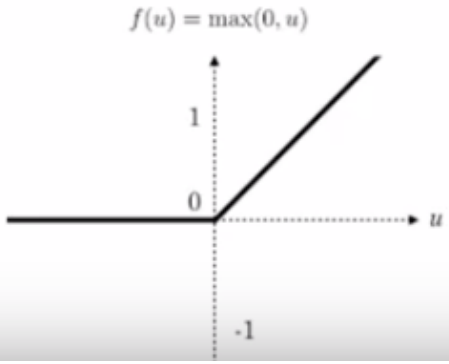
Easy to Approximate a Function



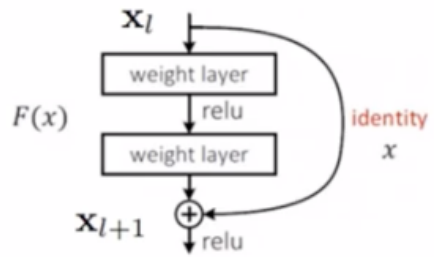
More zeros at ReLU







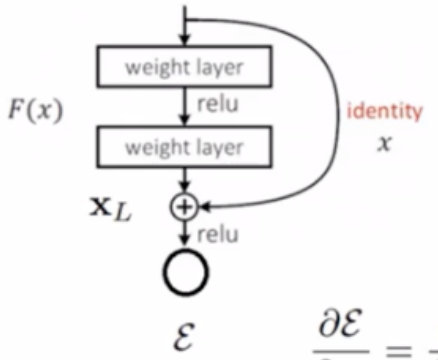
Less Gradient Vanishing



$$\mathbf{x}_{l+1} = \mathbf{x}_l + \mathcal{F}(\mathbf{x}_l, \mathcal{W}_l)$$

a term of $\frac{\partial \mathcal{E}}{\partial \mathbf{x}_L}$ that propagates information directly without concern ing any weight layers, and another term of $\frac{\partial \mathcal{E}}{\partial \mathbf{x}_L}\left(\frac{\partial}{\partial \mathbf{x}_l}\sum_{i=l}^{L-1}\mathcal{F}\right)$ that propagate through the weight layers. The additive term of $\frac{\partial \mathcal{E}}{\partial \mathbf{x}_L}$ ensures that information i directly propagated back to *any shallower unit* l.

$$\mathbf{x}_{l+2} = \mathbf{x}_{l+1} + \mathcal{F}(\mathbf{x}_{l+1}, \mathcal{W}_{l+1}) = \mathbf{x}_l + \mathcal{F}(\mathbf{x}_l, \mathcal{W}_l) + \mathcal{F}(\mathbf{x}_{l+1}, \mathcal{W}_{l+1})$$



$$\mathbf{x}_L = \mathbf{x}_l + \sum_{i=l}^{L-1} \mathcal{F}(\mathbf{x}_i, \mathcal{W}_i)$$

This "1" will back propagate the gradient at deeper layer

$$\frac{\partial \mathcal{E}}{\partial \mathbf{x}_l} = \frac{\partial \mathcal{E}}{\partial \mathbf{x}_L}\frac{\partial \mathbf{x}_L}{\partial \mathbf{x}_l} = \frac{\partial \mathcal{E}}{\partial \mathbf{x}_L}\left(1 + \frac{\partial}{\partial \mathbf{x}_l}\sum_{i=l}^{L-1}\mathcal{F}(\mathbf{x}_i, \mathcal{W}_i)\right)$$

References

- CNN: ML Lecture 10: Convolutional Neural Network by Hongyi Li (youtube)
- Chapter 20 from Berkeley Artificial Intelligence: A Modern Approach (link: http://aima.eecs.berkeley.edu/)
- Module 2: Convolutional Neural Networks from course notes of Stanford CS231n (link: http://cs231n.github.io/convolutional-networks/)
- Full connection blog (link: https://www.superdatascience.com/blogs/convolutional-neural-networks-cnn-step-4-full-connection)
- http://aima.eecs.berkeley.edu/
- http://cs231n.github.io/convolutional-networks/