Machine Learning, Spring 2020

Regression

Reading Assignment: Chapter 5 & 6

Python tutorial: http://learnpython.org/

TensorFlow tutorial: https://www.tensorflow.org/tutorials/

PyTorch tutorial: https://pytorch.org/tutorials/

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Polynomial Linear Regression

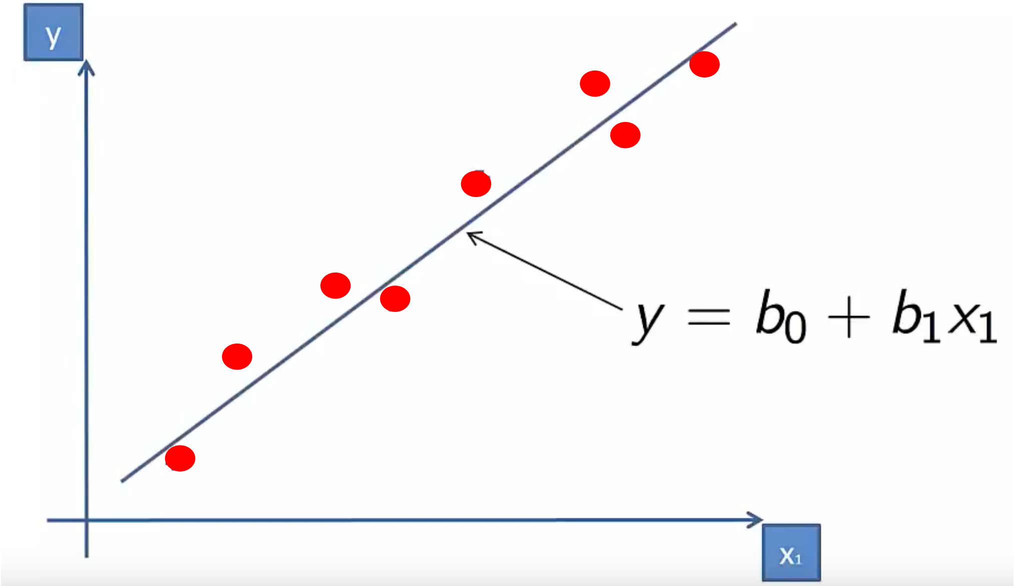
Linear Regression:

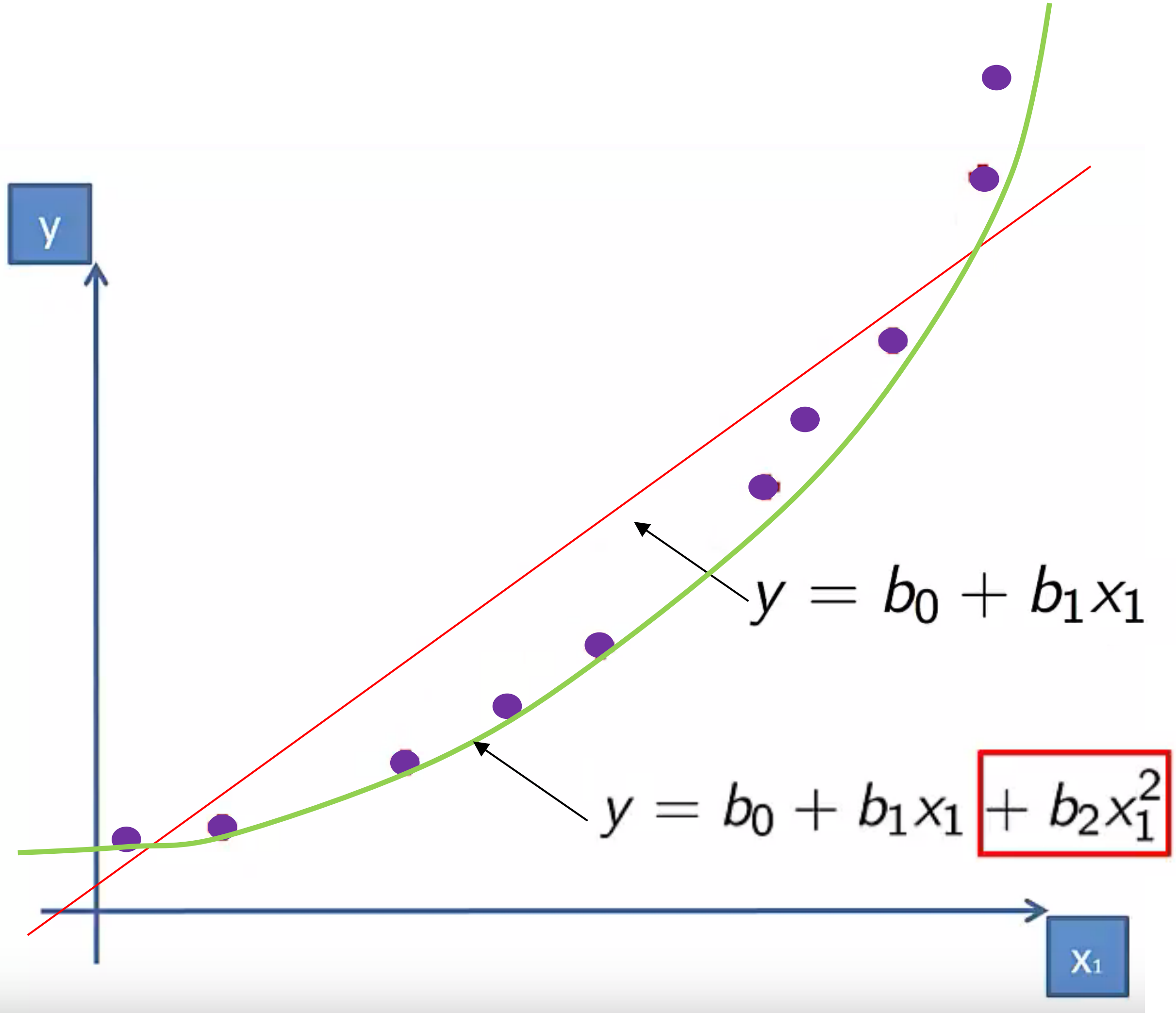
$$y = b_0 + b_1 x_1 + b_2 x_2 + \ldots + b_n x_n$$

Polynomial Linear Regression:

$$y = b_0 + b_1 x_1 + b_2 x_1^2 + \ldots + b_n x_1^n$$

Why polynomial regression




y
$y = b_0 + b_1 x_1$
$y = b_0 + b_1 x_1 \boxed{+ b_2 x_1^2}$
x1

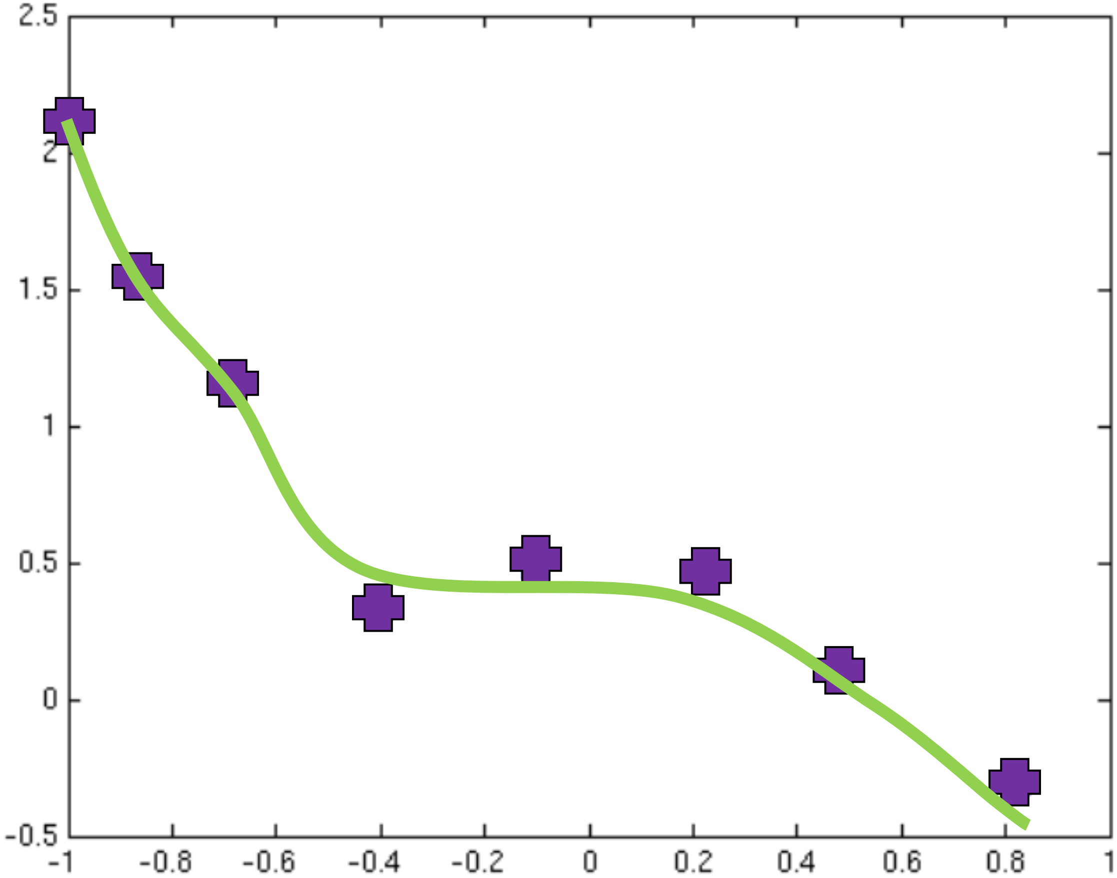
Why still called "Linear"?

Polynomial Linear Regression:

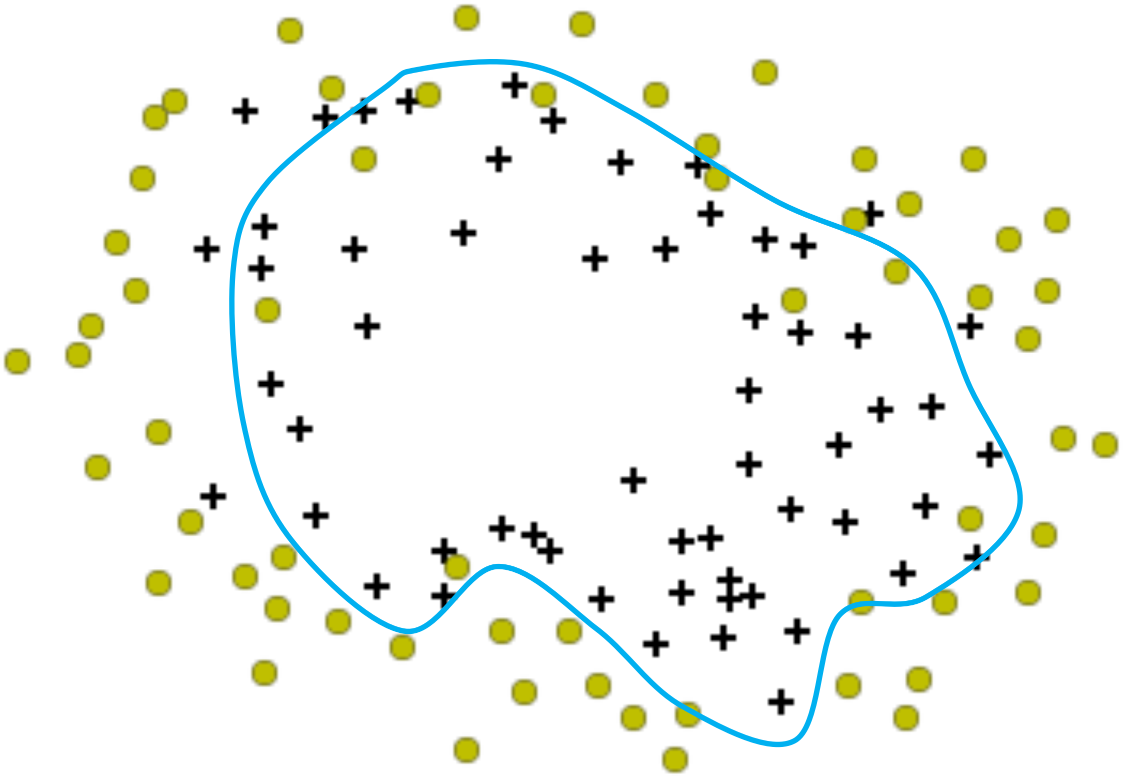
$$y = b_0 + b_1 x_1 + b_2 x_1^2 + \dots + b_n x_1^n$$

The function is expressed as the linear combination of unknowns (i.e. coefficients)?

Polynomial Regression

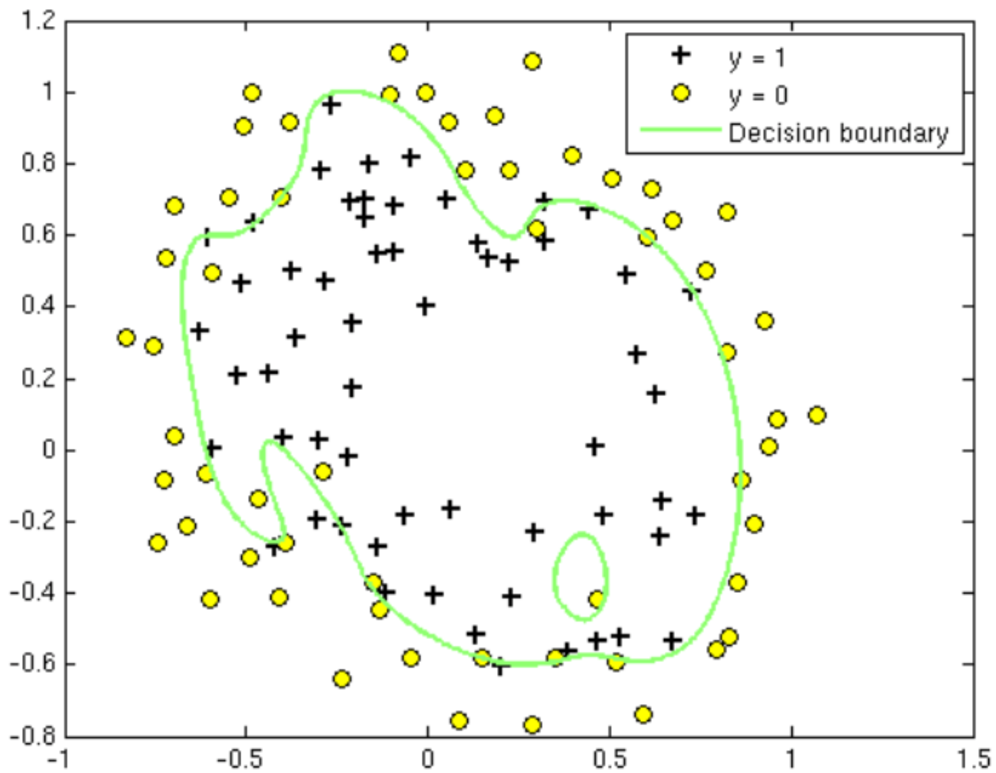


$$Y = \theta^T \mathrm{x}$$



$$Y = \sigma(\theta^T \mathbf{x})$$

Over Fitting



Regularized logistic regression

Regularized logistic regression Loss

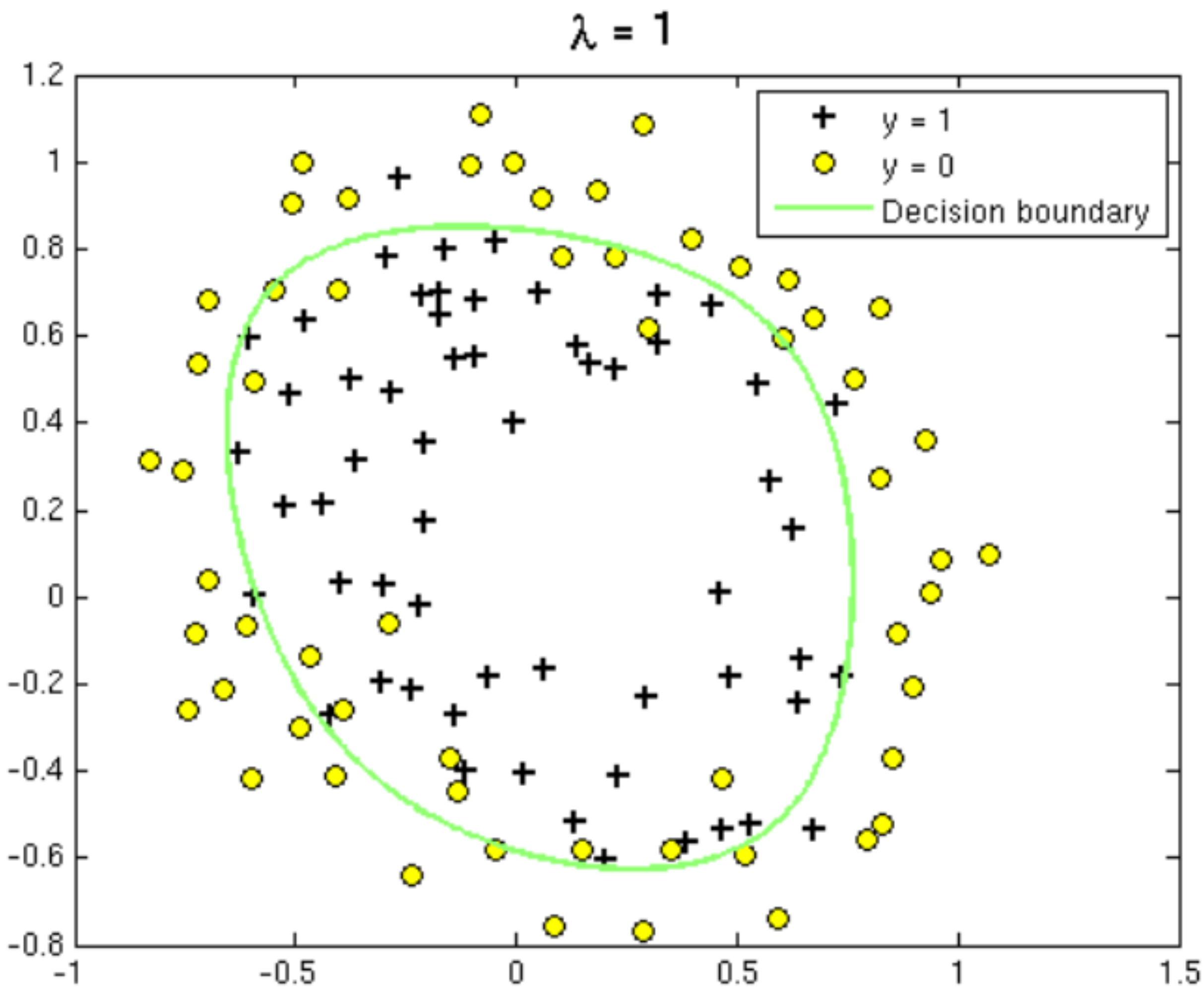
Loss function:

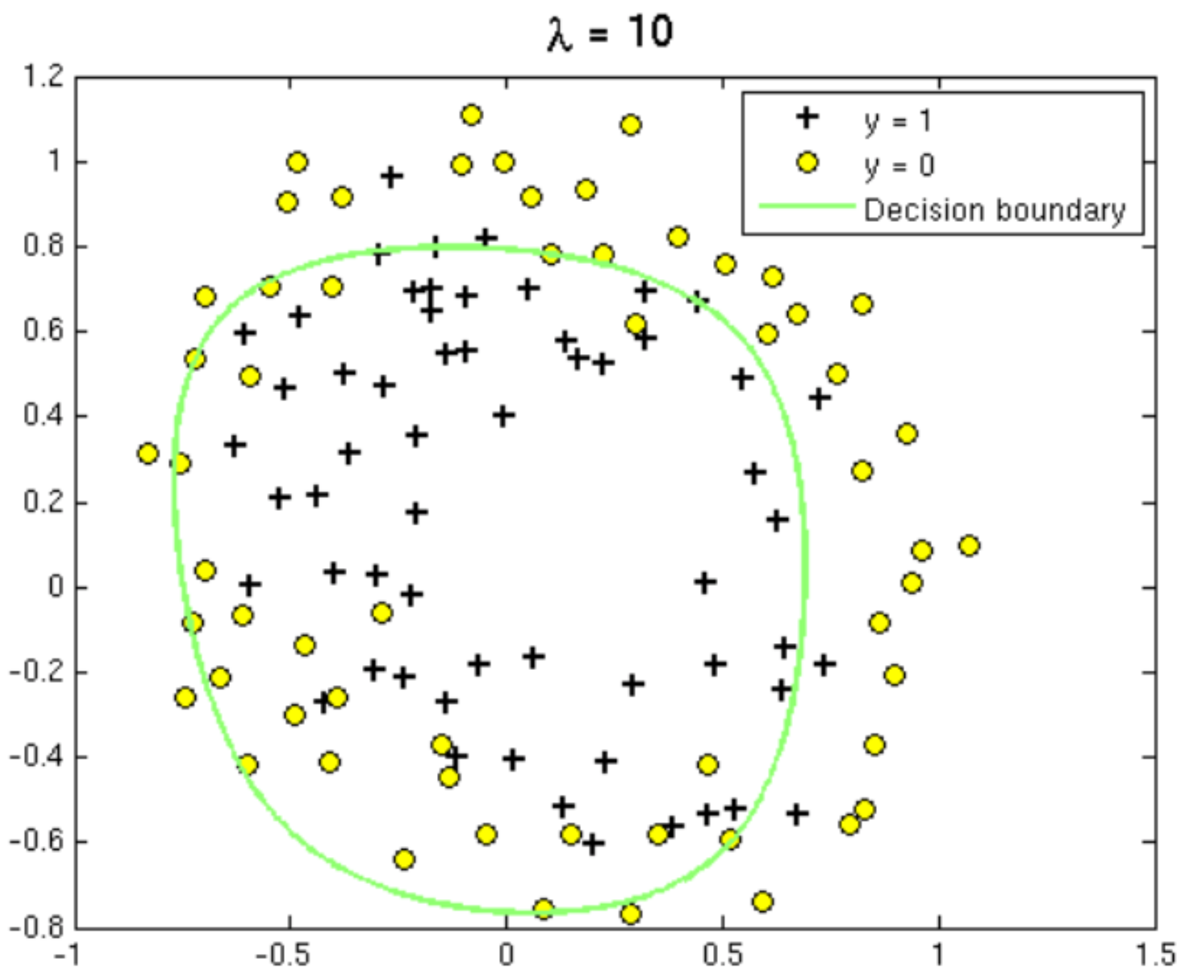
$$J(\theta) = -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}\log(h_\theta(x^{(i)})) + (1-y^{(i)})\log(1-h_\theta(x^{(i)}))\right] + \frac{\lambda}{2m}\sum_{j=1}^{n}\theta_j^2$$

Gradient function:

$$\frac{\partial}{\partial\theta_j}J(\theta) = \frac{\partial}{\partial\theta_j}\left[-\frac{1}{m}\sum_{i=1}^{m}\left(y^{(i)}\,log(h_\theta(x^{(i)}) + (1-y^{(i)})\,log(1-h_\theta(x^{(i)}))\right) + \frac{\lambda}{2m}\sum_{j=1}^{n}\theta_j^2\right]$$

$$= \frac{1}{m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)x_j^{(i)} + \frac{\lambda}{m}\theta_j$$


λ = 1
y = 1
y = 0
Decision boundary
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-0.5
0
0.5
1
1.5


λ = 10
y = 1
y = 0
Decision boundary
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-0.5
0
0.5
1
1.5

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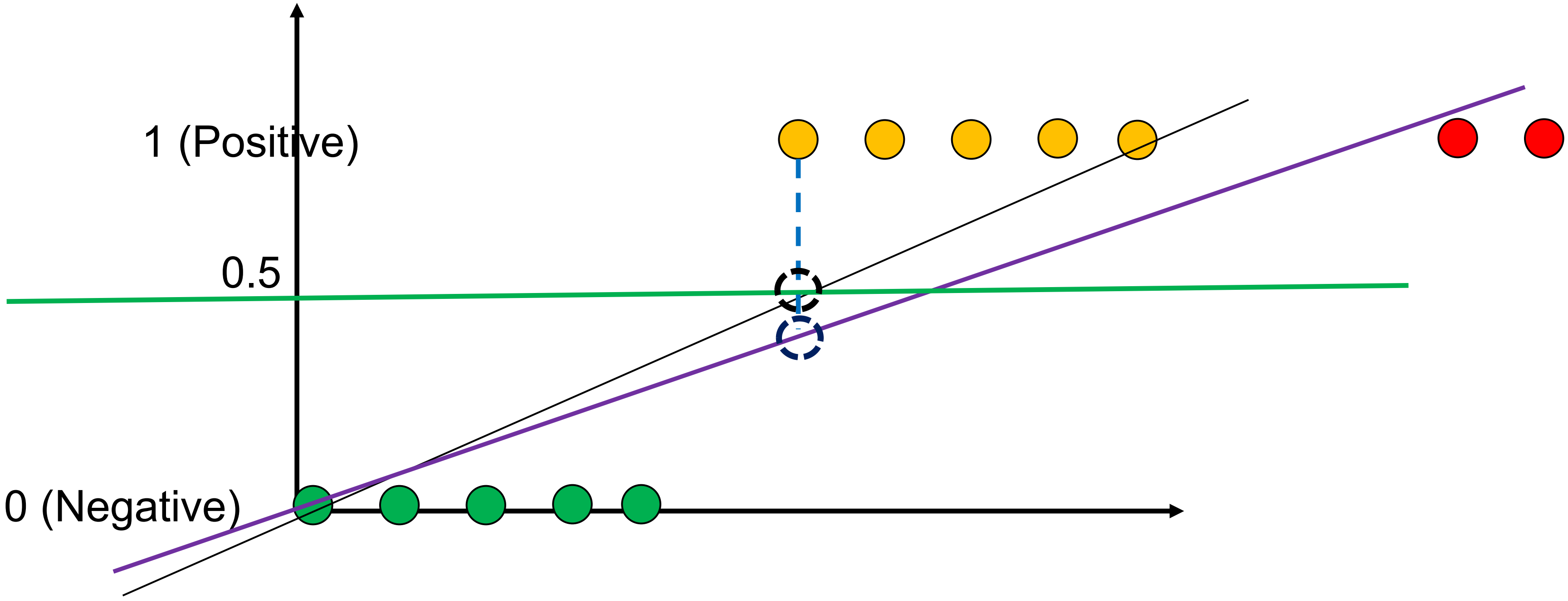
Classification

Overview

Problems:

1. Spam or Non-spam emails
2. Person re-identification (same or different person)

Linear Regression or Logistic Regression?



Hypothesis

- Hypothesis: A hypothesis is a certain function that we believe (or hope) is similar to the true function, the target function that we want to model. In context of email spam classification, it would be the rule we came up with that allows us to separate spam from non-spam emails.

Classifier

- Classifier: A classifier is a special case of a hypothesis (nowadays, often learned by a machine learning algorithm). A classifier is a hypothesis or discrete-valued function that is used to assign (categorical) class labels to particular data points. In the email classification example, this classifier could be a hypothesis for labeling emails as spam or non-spam. However, a hypothesis must not necessarily be synonymous to a classifier. In a different application, our hypothesis could be a function for mapping study time and educational backgrounds of students to their future SAT scores.

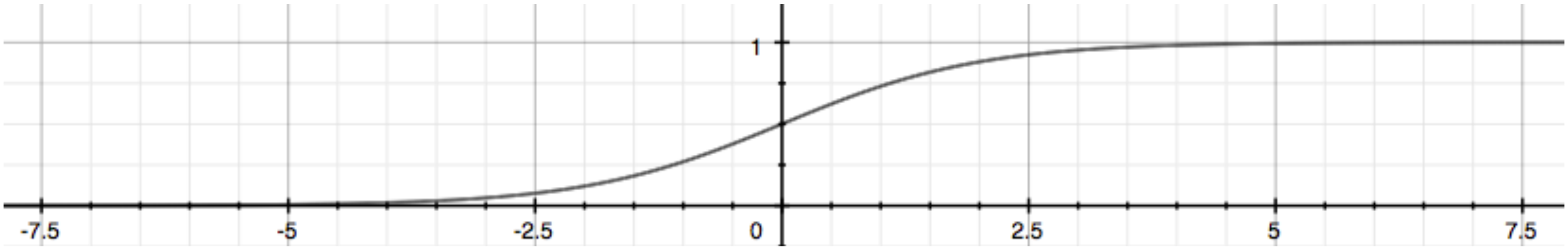
Logistic Regression Hypothesis

$$h_\theta(x) = g(\theta^T x) \tag{1}$$

$$g(z) = \frac{1}{1+e^{-z}} \tag{2}$$

$$z = \theta^T x \tag{3}$$

$$g(\theta^T x) = \frac{1}{1+e^{-\theta^T x}} \tag{4}$$



Hypothesis Function

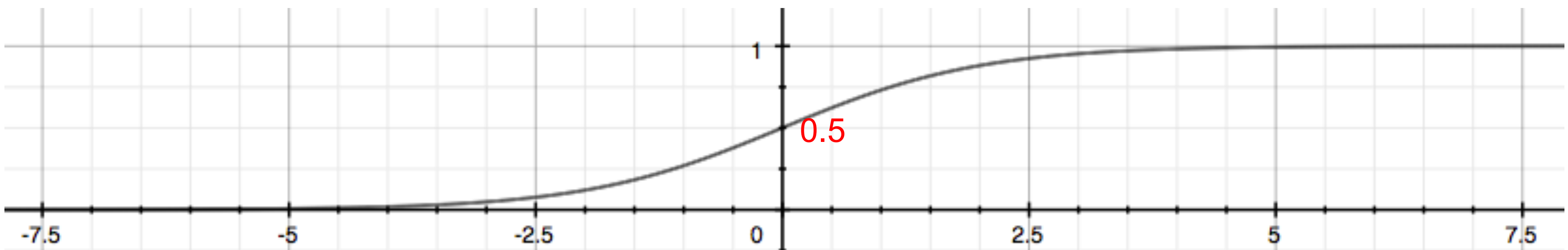
We have: $$h_\theta(x) = P(y = 1|x;\theta)$$

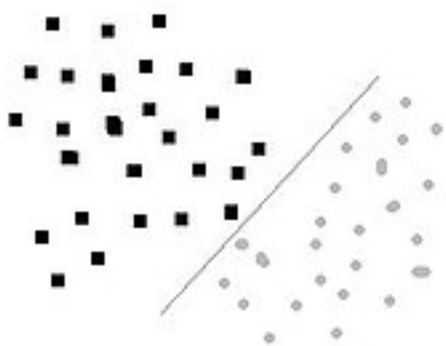
Then:

$$P(y = 0|x;\theta) + P(y = 1|x;\theta) = 1$$

$$h_\theta(x) = P(y = 1|x;\theta) = 1 - P(y = 0|x;\theta)$$

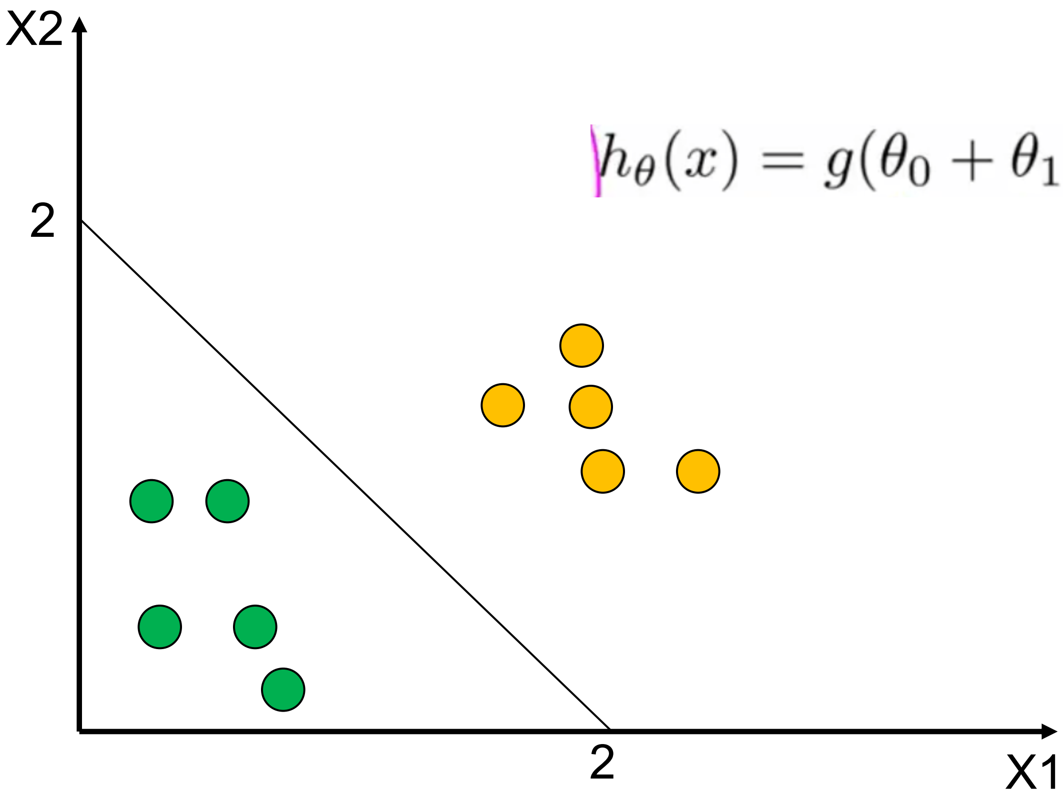
Decision Boundary





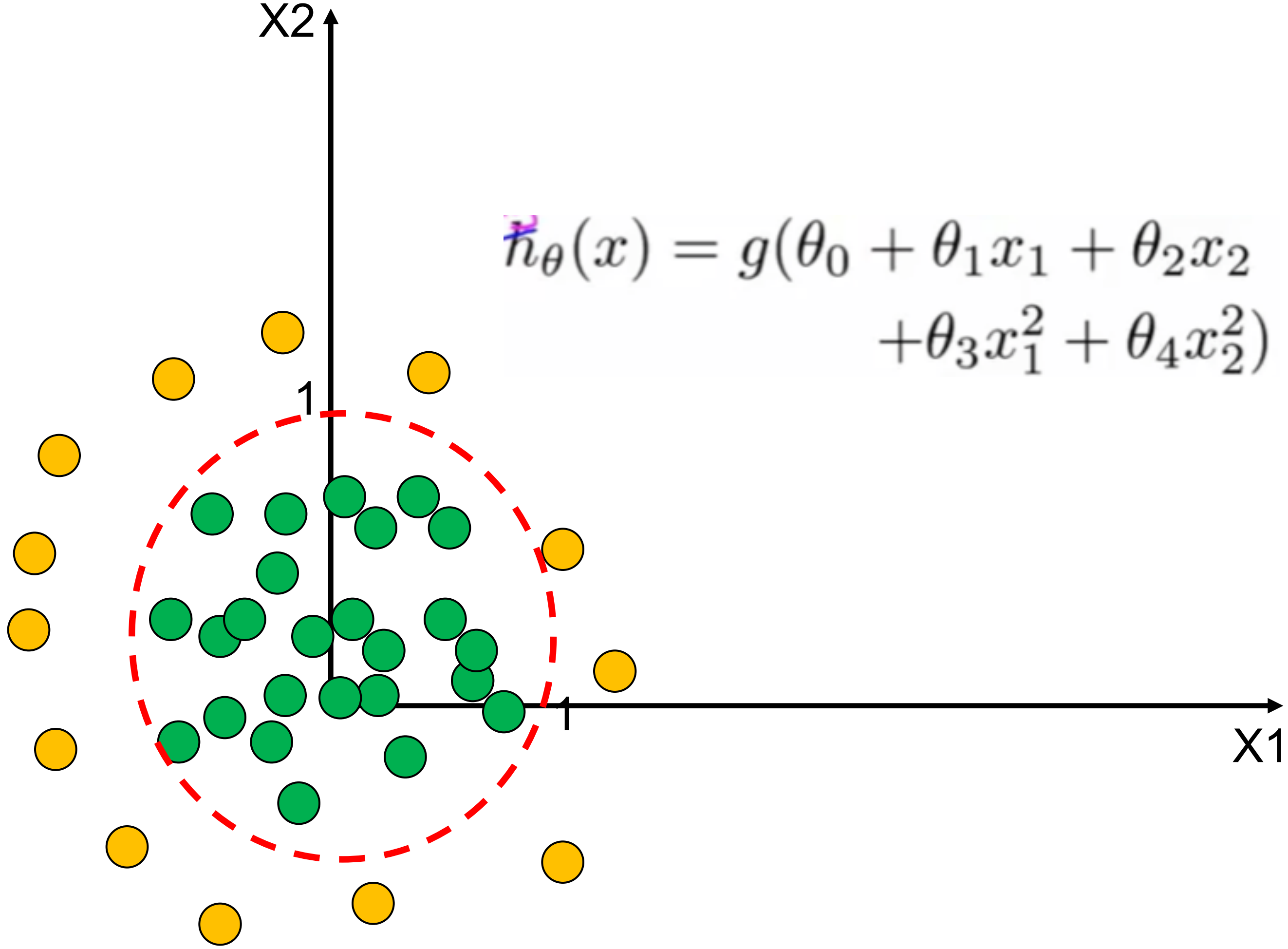
$$Y = \theta^T x$$

Classification: Linearly Separable



$$h_\theta(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2)$$

Classification: Non-Linearly Separable



$$h_\theta(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1^2 + \theta_4 x_2^2)$$

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Classification Cost Function

Design of Cost Function

Training Samples:

$$\left\{\left(x^{(1)}, y^{(1)}\right), \left(x^{(2)}, y^{(2)}\right), \ldots, \left(x^{(m)}, y^{(m)}\right)\right\}$$

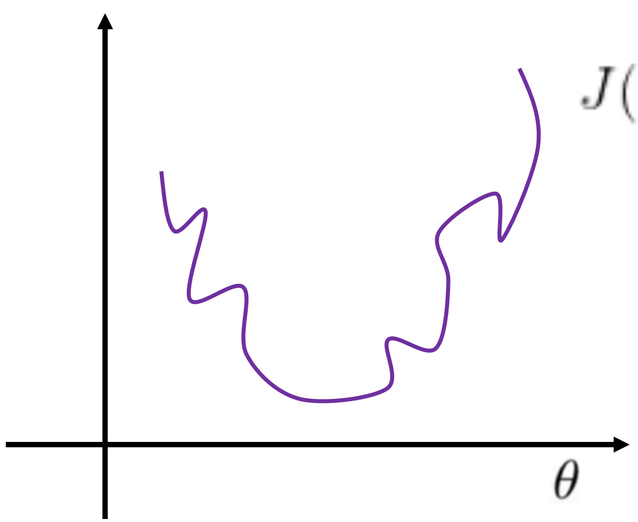
Hypothesis:

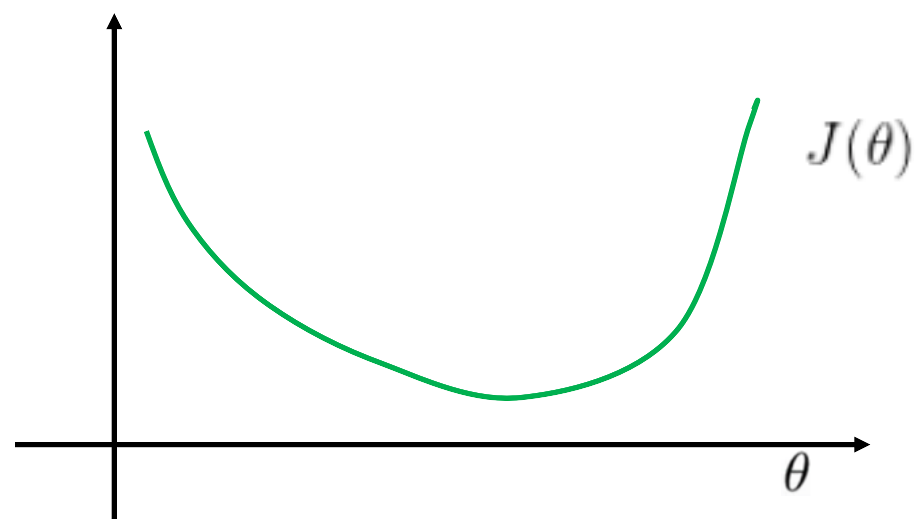
$$h_\theta(x) = \frac{1}{1 + e^{-\theta^T x}}$$

Direction Difference function:

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m}\frac{1}{2}(h_\theta(x^{(i)}) - y^{(i)})^2$$

Where: $$h_\theta(x) = \frac{1}{1+e^{-\theta^T x}}$$





Log Likelihood function:

$$J(\theta) = \frac{1}{m} \sum_{i=1}^{m} Cost(h_\theta(x^{(i)}), y^{(i)})$$

$$Cost(h_\theta(x), y) = \begin{cases} -log(h_\theta(x)) & if \quad y = 1 \\ -log(1 - h_\theta(x)) & if \quad y = 0 \end{cases}$$

Where: $$h_\theta(x) = \frac{1}{1 + e^{-\theta^T x}}$$

We can write it as one function as:

$$cost(h_\theta(x), y) = -ylog(h_\theta(x)) + [-(1-y)log(1 - h_\theta(x))]$$

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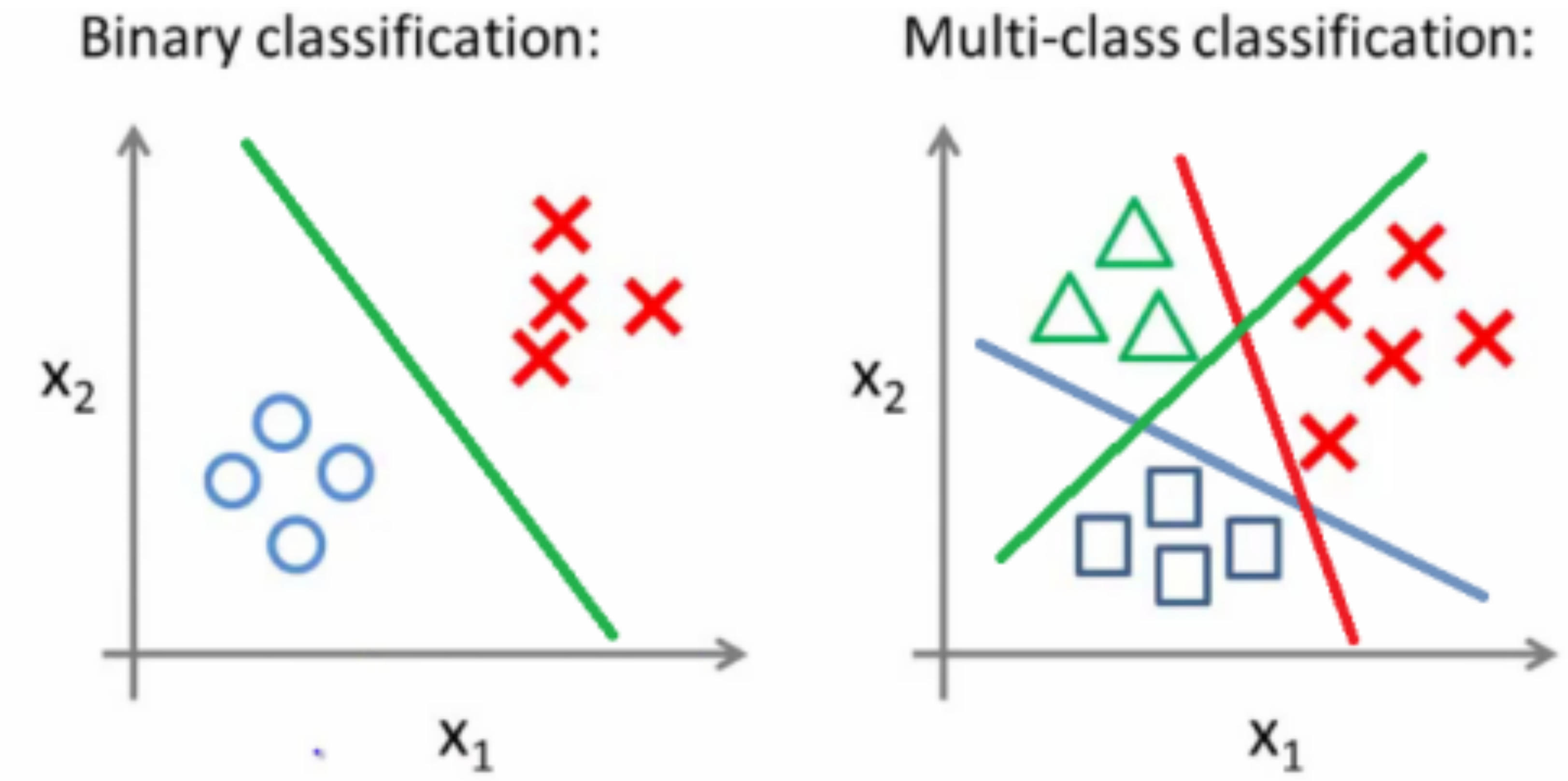
One-vs-All Classification

Multi-class classification



Hypothesis

Logistic Regression Model:

$$h_\theta(x) = \frac{1}{1 + e^{-\theta^T x}}$$

Cost Function

Log Likelihood function:

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m} Cost(h_\theta(x^{(i)}), y^{(i)})$$

Where: $$h_\theta(x) = \frac{1}{1 + e^{-\theta^T x}}$$

$$\begin{aligned} y &\in \{0, 1, \ldots, n\} \\ h_\theta^{(0)}(x) &= P(y = 0|x;\theta) \\ h_\theta^{(1)}(x) &= P(y = 1|x;\theta) \\ &\cdots \\ h_\theta^{(n)}(x) &= P(y = n|x;\theta) \end{aligned}$$

$$Cost(h_\theta(x), y) = -log(h_\theta(x))$$