Minimum spanning tree

Outline

In this topic, we will
- Define a spanning tree
- Define the weight of a spanning tree in a weighted graph
- Define a minimum spanning tree
- Consider applications
- List possible algorithms

Spanning trees

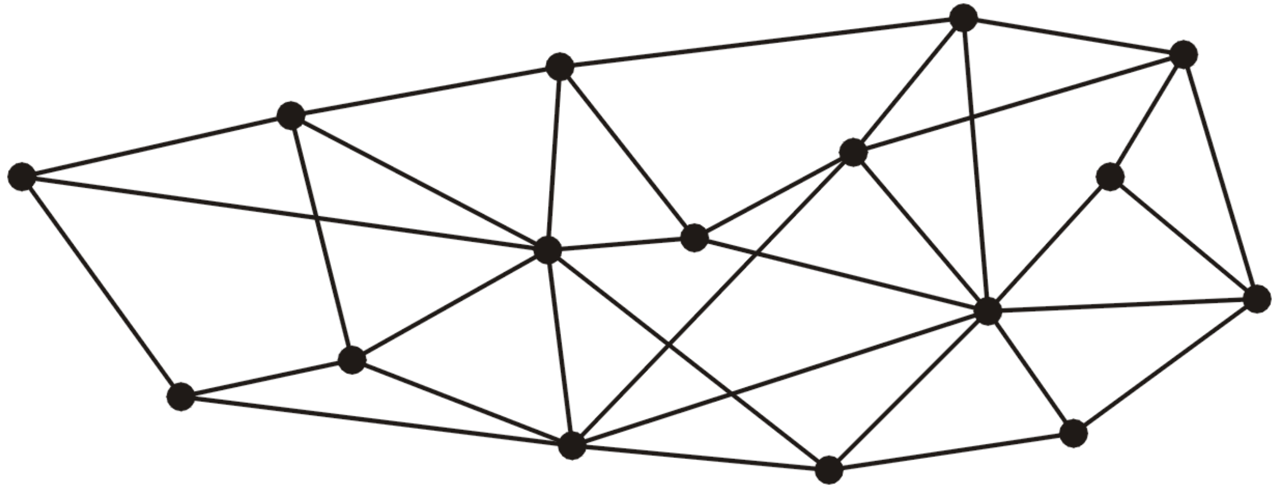
Given a connected graph with $|\mathrm{V}| = n$ vertices, a spanning tree is defined a collection of $n - 1$ edges which connect all n vertices

- The n vertices and $n - 1$ edges define a connected sub-graph

A spanning tree is not necessarily unique

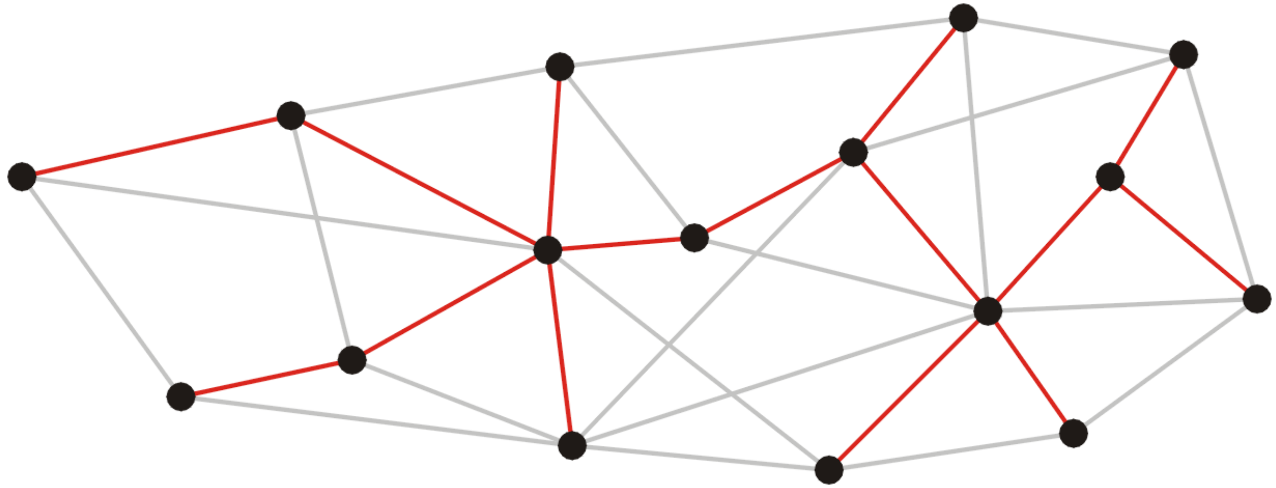
Spanning trees

This graph has 16 vertices and 35 edges



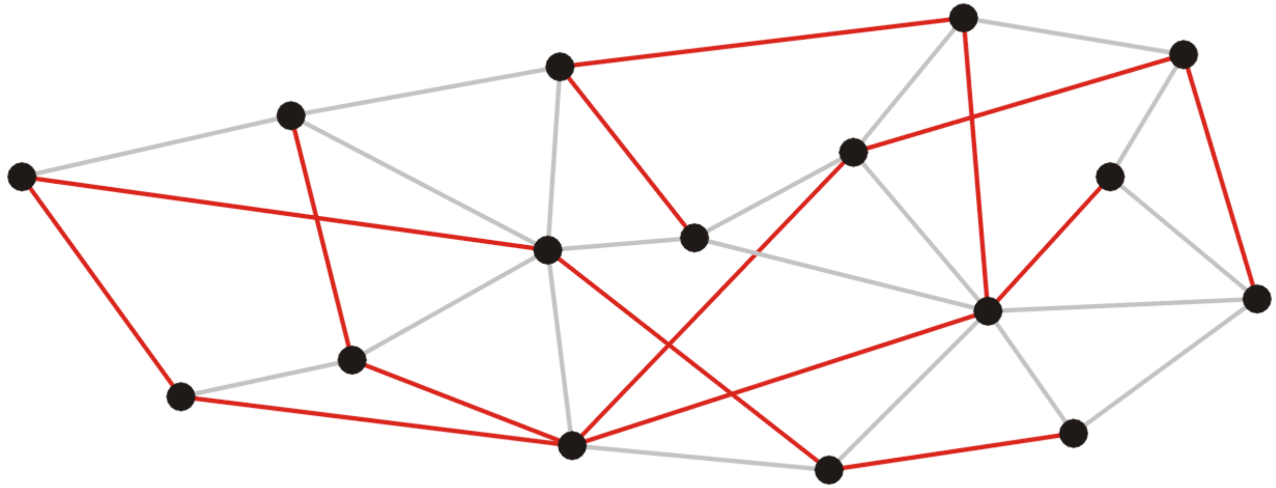
Spanning trees

These 15 edges form a minimum spanning tree



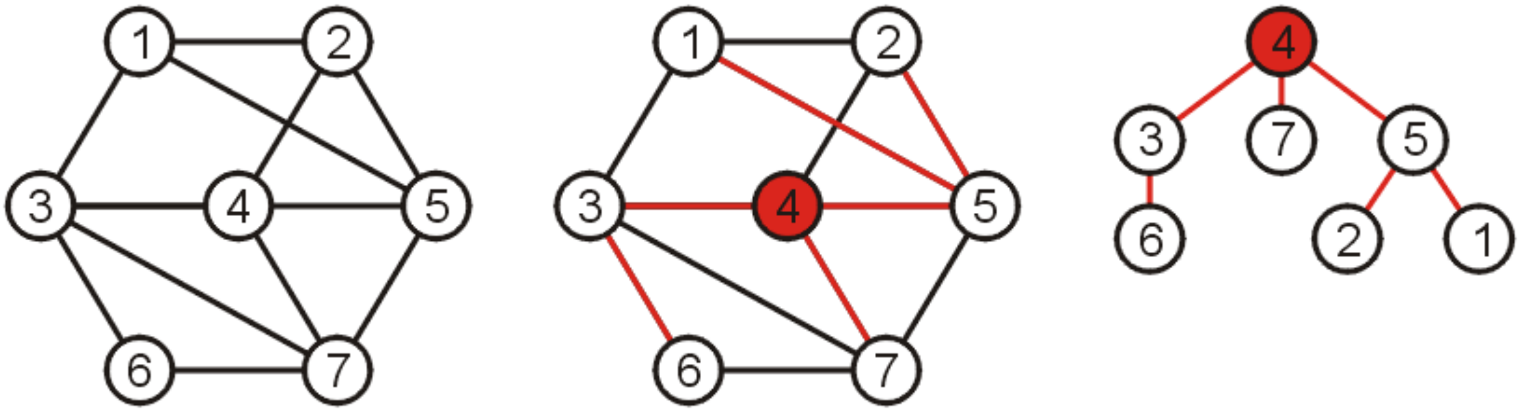
Spanning trees

As do these 15 edges:



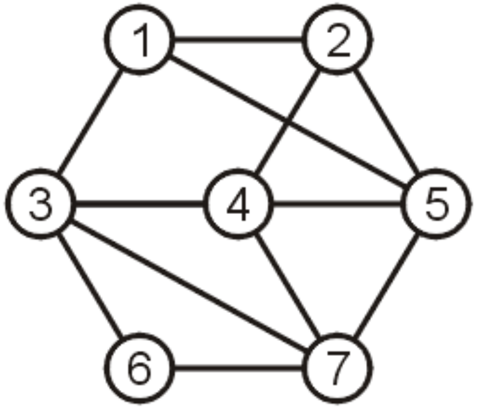
Spanning trees

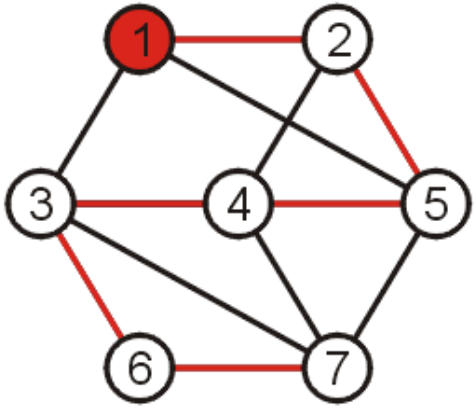
Such a collection of edges is called a *tree* because if any vertex is taken to be the root, we form a tree by treating the adjacent vertices as children, and so on...



Spanning trees

Nor do they necessarily have *nice* properties

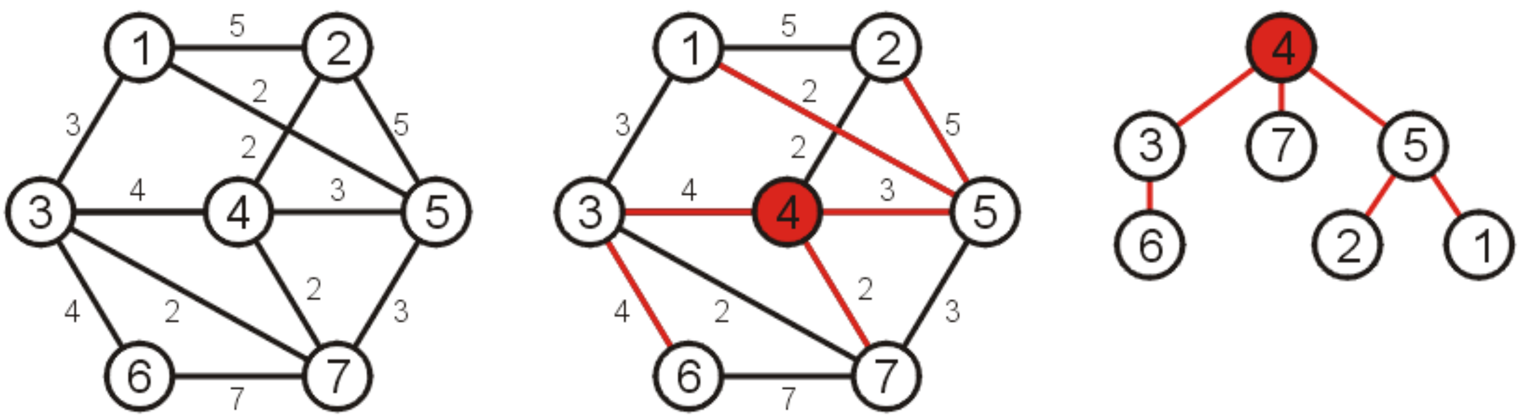




Spanning trees on weighted graphs

The weight of a spanning tree is the sum of the weights on all the edges which comprise the spanning tree

- The weight of this spanning tree is 20

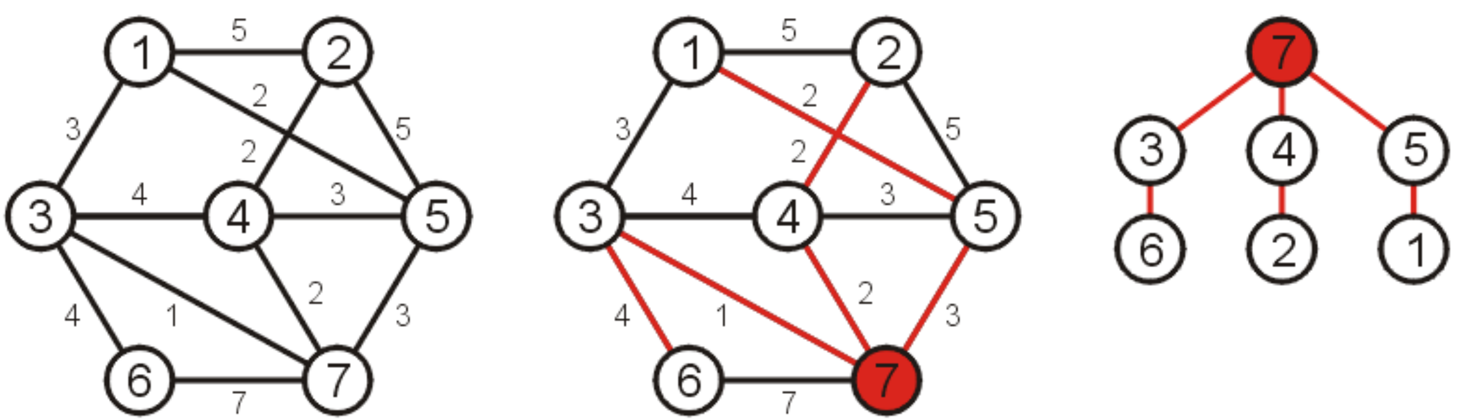


Minimum Spanning Trees

Which spanning tree which minimizes the weight?

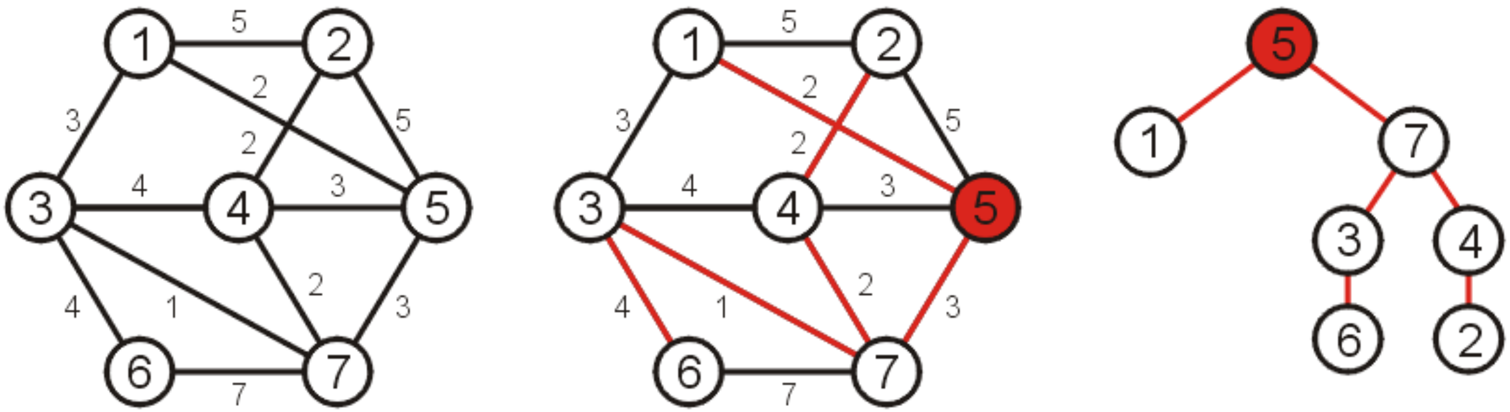
– Such a tree is termed a *minimum spanning tree*

The weight of this spanning tree is 14



Minimum Spanning Trees

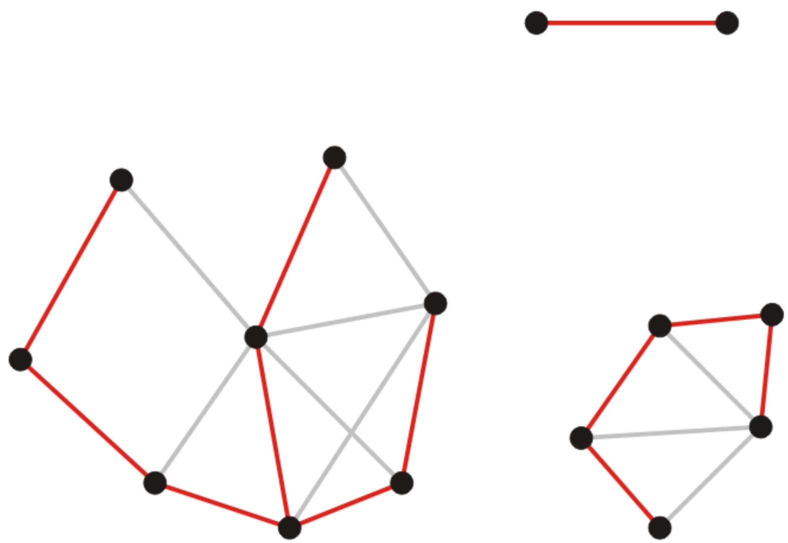
If we use a different vertex as the root, we get a different tree, however, this is simply the result of one or more rotations



Spanning forests

Suppose that a graph is composed of N connected vertex-induced sub-graphs

- In this case, we may define a *spanning forest* as a collection of N spanning trees, one for each connected vertex-induced sub-graph



- A *minimum spanning forest* is therefore a collection of N minimum spanning trees, one for each connected vertex-induced sub-graph

Unweighted graphs

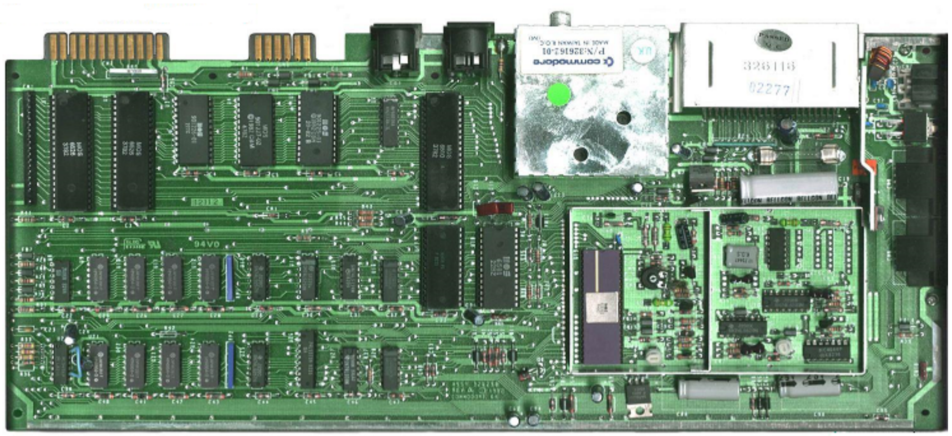
Observation

- In an unweighted graph, we nominally give each edge a weight of 1
- Consequently, all minimum spanning trees have weight $|V| - 1$

Application

Consider supplying power to
- All circuit elements on a board
- A number of loads within a building

A minimum spanning tree will give the lowest-cost solution



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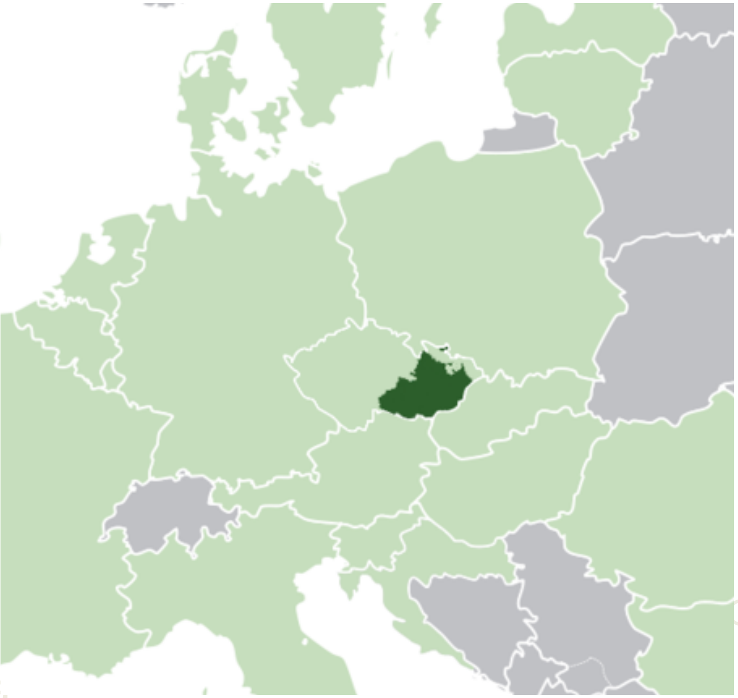
Application

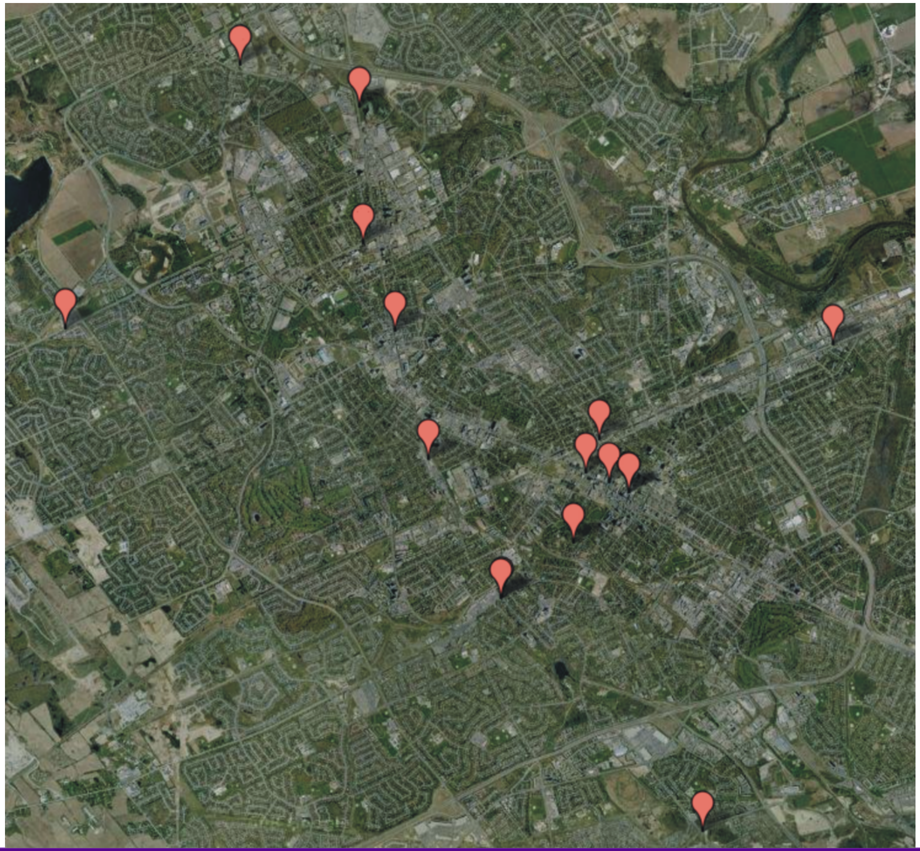
The first application of a minimum spanning tree algorithm was by the Czech mathematician Otakar Borůvka who designed electricity grid in Morovia in 1926



Application

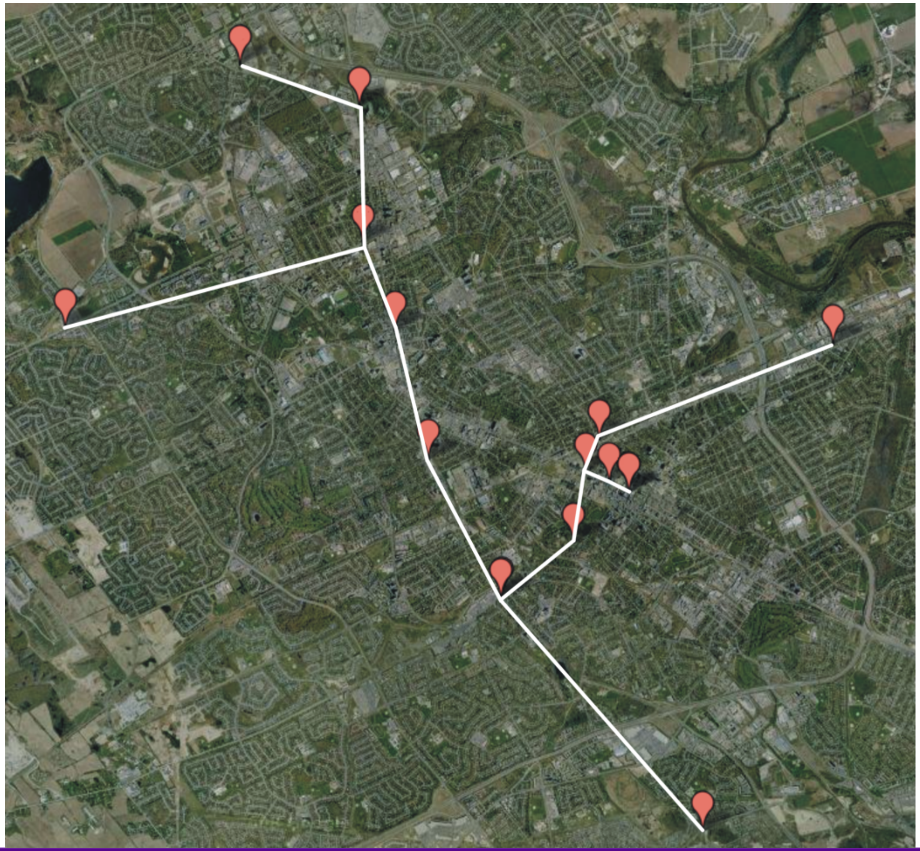
Consider attempting to find the best means of connecting a number of LANs

- Minimize the number of bridges
- Costs not strictly dependant on distances



Application

A minimum spanning tree will provide the optimal solution



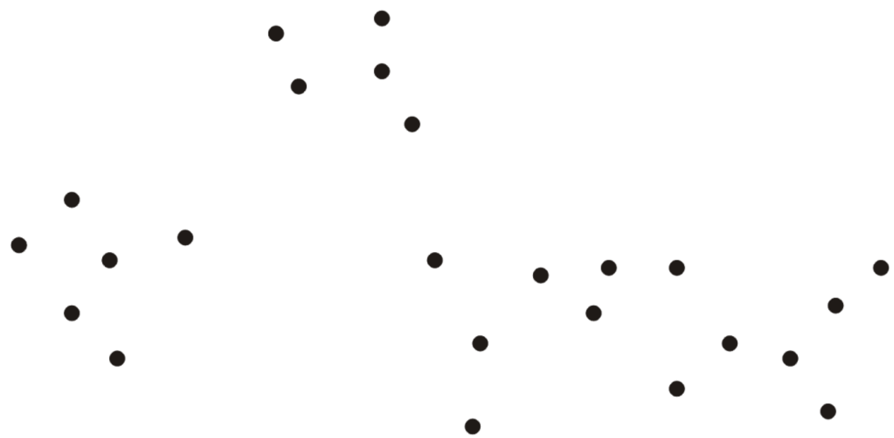
Application

Consider an *ad hoc* wireless network

- Any two terminals can connect with any others

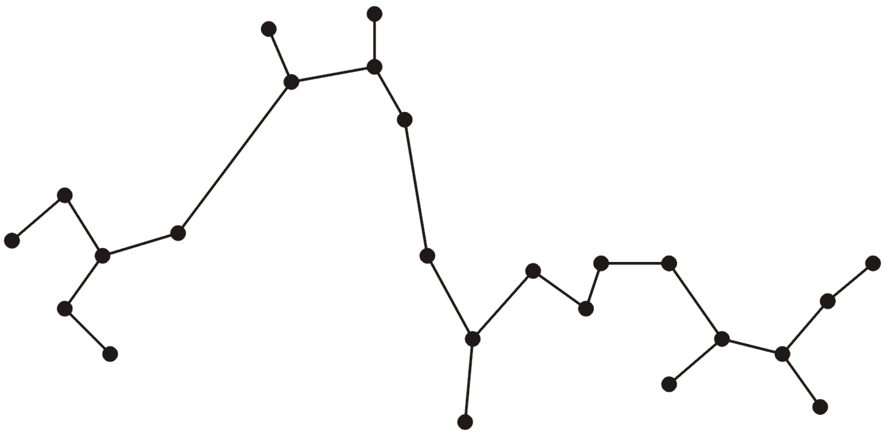
Problem:

- Errors in transmission increase with transmission length
- Can we find clusters of terminals which can communicate safely?



Application

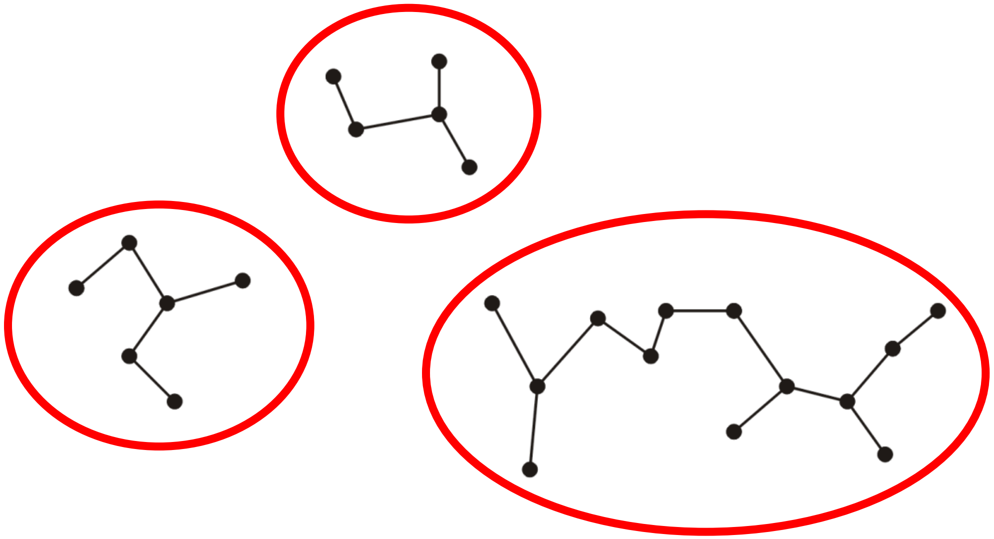
Find a minimum spanning tree



Application

Remove connections which are too long

This *clusters* terminals into smaller and more manageable sub-networks



Algorithms

Two common algorithms for finding minimum spanning trees are:

- Prim's algorithm
- Kruskal's algorithm

Summary

This topic covered

- The definition of spanning trees, weighted graphs, and minimum spanning trees
- Applications generally involve networks (electrical or communications)
- Two algorithms are Prim's and Kruskal's

References

Wikipedia, http://en.wikipedia.org/wiki/Minimum_spanning_tree