Single source unweighted path lengths

Outline

This topic looks at another problem solved by breadth-first traversals

- Finding all path lengths in an unweighted graph

Determining Distances

Problem: find the distance from one vertex v to all other vertices

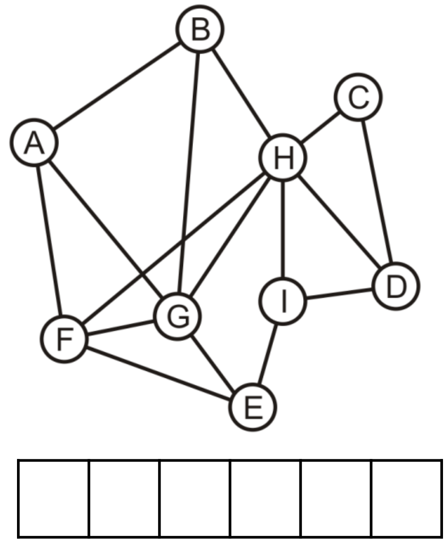
- Use a breadth-first traversal
- Vertices are added in *layers*
- The starting vertex is defined to be in the zeroeth layer, L_0
- While the k^{th} layer is not empty:
  - All unvisited vertices adjacent to verticies in L_k are added to the $(k + 1)^{\text{st}}$ layer

Any unvisited vertices are said to be an infinite distance from v

Reference: Kleinberg and Tardos

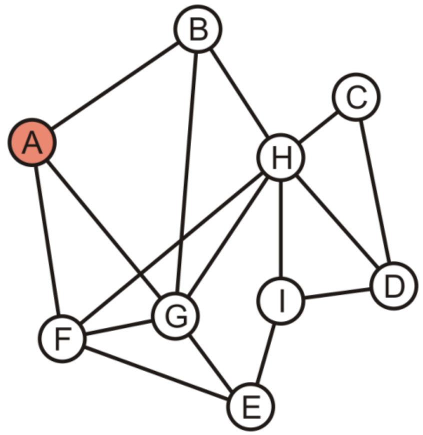
Determining Distances

Consider this graph: find the distance from A to each other vertex



Determining Distances

A forms the zeroeth layer, L_0

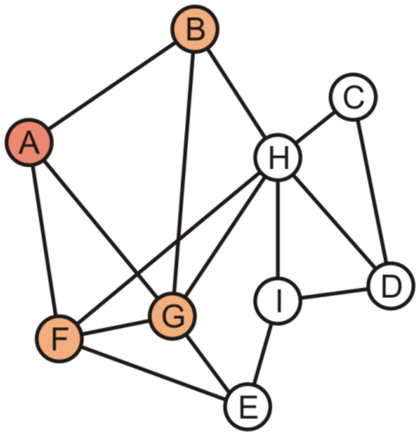


A					

Determining Distances

The unvisited vertices B, F and G are adjacent to A

- These form the first layer, L_1

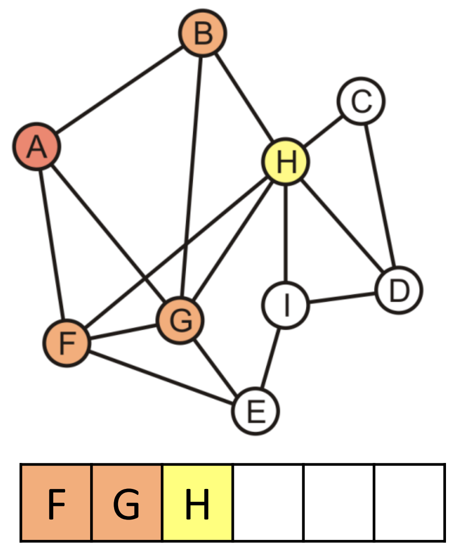


B	F	G			

Determining Distances

We now begin popping L_1 vertices: pop B

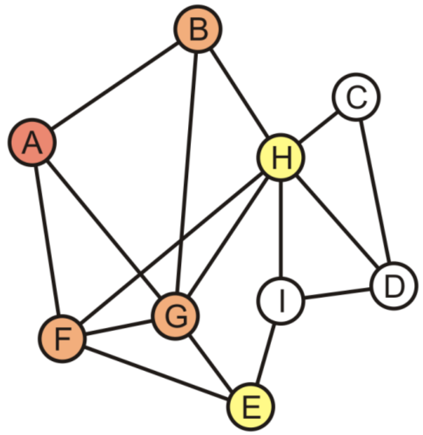
- H is adjacent to B
- It is tagged L_2

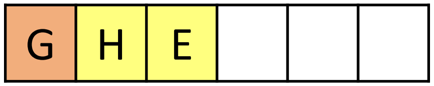


Determining Distances

Popping F pushes E onto the queue

– It is also tagged L_2

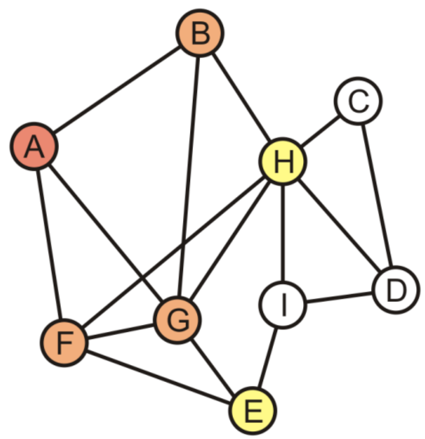




Determining Distances

We pop G which has no other unvisited neighbours

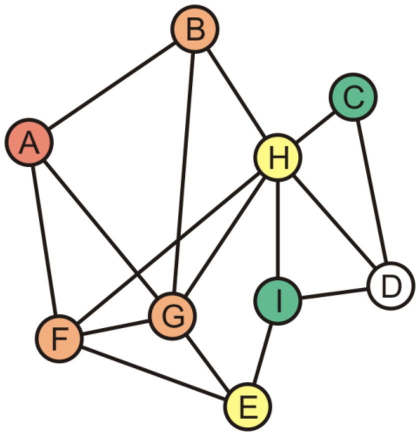
- G is the last L_1 vertex; thus H and E form the second layer, L_2



H	E				

Determining Distances

Popping H in L_2 adds C and I to the third layer L_3

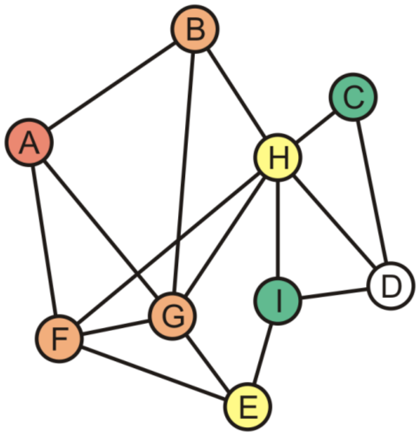


E	C	I			

Determining Distances

E has no more adjacent unvisited vertices

- Thus C and I form the third layer, L_3

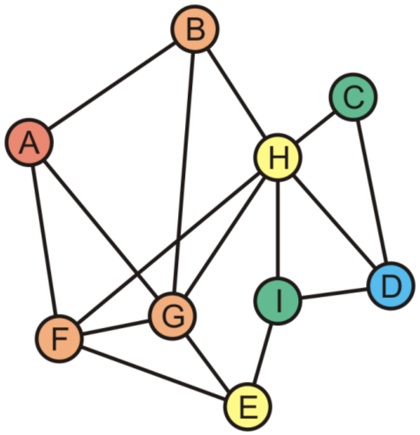


C	I				

Determining Distances

The unvisited vertex D is adjacent to vertices in L_3

– This vertex forms the fourth layer, L_4



Determining Distances

Theorem:

- If, in a breadth-first traversal of a graph, two vertices v and w appear in layers L_i and L_j, respectively and $\{v, w\}$ is an edge in the graph, then i and j differ by at most one

Proof:

If $i = j$, we are done

If $i \neq j$, without loss of generality, assume $i < j$

Because $v \in L_i$, w does not appear in any previous layer, and $\{v, w\}$ is an edge in the graph, it follows that $w \in L_{i+1}$

Thus, $j = i + 1$

Therefore, i and j differ by at most one

Reference: Kleinberg and Tardos

Sumary

This topic found the unweighted path length from a single vertex to all other vertices

- A breadth-first traversal was used
- The first vertex is marked as layer 0
- Vertices added to the queue by one in layer k are marked as layer $k + 1$
- Later, we will see different algorithms for finding the shortest path length in weighted graphs

References

Wikipedia, http://en.wikipedia.org/wiki/Shortest_path
http://en.wikipedia.org/wiki/Breadth-first_search

[1] Jon Kleinberg and Éva Tardos, *Algorithm Design,* Addison Wesley, 2006, §§3.2-5, pp.78-99.