FINAL EXAM

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May 29, 2019

customer. We evaluate this as follows:

$$\sum_{k=0}^{\infty} \rho_k (k+1)\frac{1}{\mu} = \sum_{k=0}^{\infty} \rho^k (1-\rho)(k+1)\frac{1}{\mu} = \frac{1-\rho}{\mu}\sum_{k=0}^{\infty}(k\rho^k + \rho^k) = \frac{1-\rho}{\mu}\sum_{k=0}^{\infty}(k\rho^k + \rho^k)$$

$$= \frac{1-\rho}{\mu}(\rho\frac{\partial}{\partial\rho}\sum_{k=0}^{\infty}\rho^k + \sum_{k=0}^{\infty}\rho^k) = \frac{1-\rho}{\mu}(\frac{\rho}{(1-\rho)^2} + \frac{1}{1-p}) = \frac{1}{\mu(1-\rho)}$$

However, this same result applies to all scheduling disciplines that have the property that the server is never idle when there are customers present. This is true only for the mean value and not for higher moments. We shall consider the distribution of response time momentarily.

Average Waiting Time W_q
We have, from Little's law,

$$L_q = \lambda W_q$$

and so

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{\rho}{\mu-\lambda}$$

Similar comments regarding the scheduling discipline, made in our discussion of the mean response time, also apply here.

Effective Queue Size for Nonempty Queue, $L_q^{'}$
In this case we ignore instants in which the queue is empty.

$$L_q^{'} = E[N_q | N_q \neq 0] = \sum_{n=1}^{\infty}(n-1)\rho_n^{'} = \sum_{n=2}^{\infty}(n-1)\rho_n^{'}$$

where $\rho_n^{'}$ = Prob $\{n$ in system $\mid n \geq 2\}$. It therefore follows that

$$\rho_n^{'} = \frac{\text{Prob}\{n \text{ in system and } n \geq 2\}}{\text{Prob}\{n \geq 2\}} = \frac{\rho}{\sum_{n=2}^{\infty}\rho_n}$$

$$= \frac{\rho_n}{1-\rho_0-\rho_1} = \frac{\rho_n}{1-(1-\rho)-(1-\rho)\rho} = \frac{\rho_n}{\rho^2}, \quad n \geq 2.$$

Notice that the probability distribution $\rho_n^{'}$ is the probability distribution ρ_n normalized when the cases $n = 0$ and $n = 1$ are omitted. It now follows that

$$L_q^{'} = \sum_{n=2}^{\infty}(n-1)\frac{\rho_n}{\rho^2} = \frac{1}{\rho^2}\left[\sum_{n=2}^{\infty} n\rho_n - \sum_{n=2}^{\infty}\rho_n\right] = \frac{1}{\rho^2}[(L-\rho_1) - (1-\rho_0-\rho_1)]$$

$$= \frac{1}{\rho^2}[\rho/(1-\rho) - (1-\rho)\rho - 1 + (1-\rho) + (1-\rho)\rho]$$

$$= \frac{1}{\rho^2}[\rho/(1-\rho) - \rho] = \frac{1}{\rho}[1/(1-\rho) - 1] = \frac{1}{1-\rho}.$$

Thus

$$L_q^{'} = \frac{1}{1-\rho} = \frac{\mu}{\mu-\lambda}$$

Collecting these results together in one convenient location, we have

$$L = E[N] = \frac{\rho}{1-\rho}, \quad L_q = E[N_q] = \frac{\rho^2}{1-\rho}, \quad \text{Var}[N] = \frac{\rho}{(1-\rho)^2}, \tag{11.6}$$

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$$= \frac{1-\rho}{\mu}\left(\rho\frac{\partial}{\partial\rho}\sum_{k=0}^{\infty}\rho^k + \sum_{k=0}^{\infty}\rho^k\right) = \frac{1-\rho}{\mu}\left(\frac{\rho}{(1-\rho)^2} + \frac{1}{1-\rho}\right) = \frac{1}{\mu(1-\rho)}.$$

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where $p'_n = \text{Prob}\{n \text{ in system} \mid n \geq 2\}$. It therefore follows that

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Notice that the probability distribution p'_n is the probability distribution p_n normalized when the cases $n=0$ and $n=1$ are omitted. It now follows that

$$L'_q = \sum_{n=2}^{\infty}(n-1)\frac{p_n}{\rho^2} = \frac{1}{\rho^2}\left[\sum_{n=2}^{\infty} np_n - \sum_{n=2}^{\infty} p_n\right] = \frac{1}{\rho^2}\left[(L-p_1)-(1-p_0-p_1)\right]$$

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\textbf{408 Elementary Queueing Theory}

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