### *30.11.3 Marginal and conditional distributions*

Given a bivariate distribution $f(x, y)$, we may be interested only in probability function for $X$ irrespective of the value of $Y$ (or vice versa). This marginal distribution of $X$ is obtained by summing of integratingas appropriate, the joint probability distribution over all allowed values of $Y$. Thus, the marginal distribution of $X$ (for example) is given by

$$f_x(x) = \begin{cases} \sum_j f(x, y_j) & \text{for a discrete distribution,} \\ \int f(x, y)dy & \text{for a continuous distribution.} \end{cases}$$

It is clear that an analogous definition exists for the marginal distribution of $Y$.
Alternatively, one might be interested in the probability function of $X$ given that $Y$ takes some specific value of $Y = y_0$,i.e. $Pr(X = x|Y = y_0)$. This conditional distribution of $X$ is given by

$$g(x) = \frac{f(x, y_0)}{f_y(y_0)}$$

where $f_y(y)$ is the marginal distribution of $Y$. The division by $f_y(y_0)$ is necessary in order that $g(x)$ is properly normalised

### *30.12 Properties of joint distributions*

The probability density function $f(x, y)$ contains all the information on the joint probability of two random variables $X$ and $Y$. In a similar manner to that presented for univariate distributions, however, it is conventional to characterise $f(x, y)$ by certain of its properties, which we now discuss. Once again, most of these properties are based on the concept of expectation values, which are defined for joint distributions in an analogous way to those for single-variable distributions (30.46). Thus, the expectation value of any function $g(X, Y)$ of the random variables $X$ and $Y$ is given by

$$E[g(X, Y)] = \begin{cases} \sum_i \sum_j g(x_i, y_j) f(x_i, y_j) & \text{for the discrete case,} \\ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x, y) f(x, y) dxdy & \text{for the continuous case.} \end{cases}$$

### *30.12.1 Means*

The means of $X$ and $Y$ are defined respectively as the expectation values of the variables $X$ and $Y$.Thus, the mean of $X$ is given by

$$E[X] = \mu_x = \begin{cases} \sum_i \sum_j x_i f(x_i, y_j) & \text{for the discrete case,} \\ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x f(x, y) dxdy & \text{for the continuous case.} \end{cases}$$

E[Y] is obtained in a similar manner.