

# Dense representation for categorical variables

One drawback that we flagged with One Hot Encoding of a categorical variable

- Tokens that seem to be related in the input domain ("dog", "dogs")
- Become *unrelated* when One Hot Encoded

Because each value is a long vector

- With a single non-zero element
- That is *different* across the two values
- The OHE vectors are *orthogonal*

This means that there is no useful measure of the distance between two tokens

To illustrate the "lack of distance" issue, let  $\text{rep}$  be a mapping from tokens to their One Hot Encodings.

Using dot product (cosine similarity) as a measure of similarity to the token "dog"

word	$\text{rep}(\text{word})$	Similarity
dog	[1,0,0,0]	$\text{rep}(\text{word}) \cdot \text{rep}(\text{dog}) = 1$
dogs	[0,1,0,0]	$\text{rep}(\text{word}) \cdot \text{rep}(\text{dog}) = 0$
cat	[0,0,1,0]	$\text{rep}(\text{word}) \cdot \text{rep}(\text{dog}) = 0$
apple	[0,0,0,1]	$\text{rep}(\text{word}) \cdot \text{rep}(\text{dog}) = 0$

All words other than "dog" are equidistant from "dog".

Intuitively, we observe similarity between "dog" and other words

- ("dog", "dogs"): Same root, different Singular/Plural form
- ("dog", "cat"); Same concept: pet

and complete lack of similarity between "dog" and "apple".

Yet all have the same distance measure from "dog": 0.

We can consider an alternate encoding to OHE.

Suppose each dimension of the encoded vector

- Measured the intensity of the token against some concept
  - Singular/Plural
  - Domestic Animal
  - Edible

This type of representation is called *continuous*

- As the strength is a continuous value
- Compared to the *discrete* encoding of OHE as binary 0/1

It is also called a *dense* representation

- Multiple non-zero elements in the vector
- Compared to the single non-zero element in the OHE vector

In a continuous, dense representation two values expressing similar concepts will be "closer" than two values that do not share concepts

- "Cats", "Dogs", "Apples"
  - Share the concept "Plural"
- "Cat", "Dog"
  - Share the concept "Domestic animal"
- "good", "bad"
  - Share the concept "Opposite"

# Doing math with words

Let's explore the implication and power of dense vector representation of words.

If each element of the vector

- Expresses a concept
- And the number of concepts is small compared to  $||\mathbf{V}||$
- And the concepts are fairly independent
- Then we have found an alternate basis (compared to the  $||\mathbf{V}||$  basis vectors of OHE) of smaller dimension
- For representing words

This concept is sometimes called *word embeddings*



Let  $\mathbf{v}_w$  denote the dense representation of token  $w$ :

$w$	$\mathbf{v}_w$
cat	[.7, .5, .01 ]
cats	[.7, .5, .95 ]
dog	[.7, .2, .01 ]
dogs	[.7, .2, .95 ]
apple	[.1, .4, .01 ]
apples	[.1, .4, .95 ]

Notice that "dogs" and "apples"

- Are similar along one dimension (the last, perhaps encoding "Is Plural")
- Are dissimilar along one dimension (the first, perhaps encoding "Is Pet")

Also notice that "dog" and "cat"

- Are similar along the first dimension (reinforcing the notion that this dimension may be "Is Pet")

Taking this a step further: we can perform element-wise math on dense vector representations:

$$\mathbf{v}_{\text{cats}} - \mathbf{v}_{\text{cat}} \approx \mathbf{v}_{\text{dogs}} - \mathbf{v}_{\text{dog}} \approx \mathbf{v}_{\text{apples}} - \mathbf{v}_{\text{apple}}$$

because

- "cats" and "cat" are similar in all concepts *except* "Plural".
- As are "dogs" and "dog"
- As are "apples" and "apple"

If that's the case, we can approximate the vector that expresses the "pure" concept "Is Plural"

- Without expressing any other concept

as  $(\mathbf{v}_{\text{cats}} - \mathbf{v}_{\text{cat}})$

Then we can construct the Plural form of "apple"

- By adding the pure vector for Plural to the vector for "apple"

$$\mathbf{v}_{\text{apples}} \approx \mathbf{v}_{\text{apple}} + (\mathbf{v}_{\text{cats}} - \mathbf{v}_{\text{cat}})$$

we can create the Plural form of "apple"

## Word analogies

The implications of doing math on words is even more powerful.

Consider solving the analogy problem

*king:man :: ?:woman*

That is: what is the female analog of "king" ?

Suppose the concepts ("dimensions") of the dense representation were

- Gender (man or woman)
- Regal (Royal or commoner)

Then

$$\mathbf{v}_{\text{king}} - \mathbf{v}_{\text{man}} + \mathbf{v}_{\text{woman}} \approx \mathbf{v}_{\text{queen}}$$

because

$\mathbf{v}_{\text{king}}$	$=$	$(\text{Man}, \text{Royal})$	vector representation
$\mathbf{v}_{\text{king}} - (\text{Man}, 0)$	$=$	$(0, \text{Royal})$	subtract vector for "Man"
$\mathbf{v}_{\text{king}} - (\text{Man}, 0) + (\text{Woman}, 0)$	$=$	$(\text{Woman}, \text{Royal})$	add vector for "Woman"
	$=$	Queen	the word having con

We can use math on dense vectors to compute analogies!

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Let's formalize the "math" of word vectors

For tokens  $w, w'$  with dense vectors  $\mathbf{v}_w, \mathbf{v}_{w'}$

- Define a metric  $d(\mathbf{v}_w, \mathbf{v}_{w'})$  of the distance between the words
- For example:

$$d(\mathbf{v}_w, \mathbf{v}_{w'}) = 1 - \text{cosine similarity}(\mathbf{v}_w, \mathbf{v}_{w'})$$

Define the set of tokens  $N_{n',d}(w)$  in vocabulary  $\mathbf{V}$

- That are among the  $n'$  "closest" to a token  $w$
- According to distance metric  $d$

$$\begin{aligned}\mathbf{wv}_{n',d}(w) &= \{ \mathbf{v}_{w'} \mid \text{rank}_V(d(\mathbf{v}_w, \mathbf{v}_{w'})) \leq n' \} \quad \text{the dense vectors of the } n' \text{ to } w \\ N_{n',d}(w) &= \{ w' \mid \mathbf{v}_{w'} \in \mathbf{wv}_{n',d}(w) \}\end{aligned}$$

This is the "neighborhood" of token  $w$  as defined by the distance metric.

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Token  $w'$  is defined to be *approximately equal* to token  $w$

- Denoted as  $w \approx_{n',d} w'$
- If  $w'$  is in the neighborhood of  $w$

$$w \approx_{n',d} w' \text{ if } \mathbf{w}' \in N_{n',d}(w)$$

Thus, the analogy

$$a:b :: c:d$$

implies

$$\mathbf{v}_a - \mathbf{v}_b \approx_{n',d} \mathbf{v}_c - \mathbf{v}_d$$

So to solve the word analogy for  $c$ :

$$\mathbf{v}_c \approx_{n',d} \mathbf{v}_a - \mathbf{v}_b + \mathbf{v}_d$$

## **GloVe: Pretrained embeddings**

Fortunately, you don't have to create your own word-embeddings from scratch.

There are a number of precomputed embeddings freely available.

GloVe is a family of word embeddings that have been trained on large corpora

- GloVe6b
  - Trained on 6 Billion tokens
  - 400K words
  - Corpus: Wikipedia (2014) + GigaWord5 (version 5, news wires 1994-2010)
  - Many different dense vector lengths to choose from
    - 50, 100, 200, 300

We will illustrate the power of word embeddings using GloVe6b vectors of length 100.

king- man + woman	$\approx_{n',d}$	queen
man - boy + girl	$\approx_{n',d}$	woman
Paris - France + Germany	$\approx_{n',d}$	Berlin
Einstein - science + art	$\approx_{n',d}$	Picasso

You can see that the dense vectors seem to encode "concepts", that we can manipulate mathematically.

You may discover some unintended bias

doctor - man + woman	$\approx_{n',d}$	nurse
mechanic - man + woman	$\approx_{n',d}$	teacher

## Domain specific embeddings

Do we speak Wikipedia English in this room ?

Here are the neighborhoods of some financial terms, according to GloVe:

$N(\text{bull})$  = [cow, elephant, dog, wolf, pit, bear, rider, lion, horse]

$N(\text{short})$  = [rather, instead, making, time, though, well, longer, shorter, l

$N(\text{strike})$  = [workers, struck, action, blow, striking, protest, stoppage, wal

$N(\text{FX})$  = [showtime, cnbc, ff, nickelodeon, hbo, wb, cw, vh1]

It may be desirable to create word embeddings on a narrow (domain specific) corpus.

This is not difficult provided you have enough data.

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# Obtaining Dense Vectors: Transfer Learning

How do we obtain Dense Vector representations that seem to have these wonderful properties ?

- Through Machine Learning !
- As a by-product of solving a specific Source task
- Once we have the embeddings, we can re-use them in many other Target tasks.



This is exactly what we called Transfer Learning

- We train a Source Task
- The layers and associated weights learned in training the Source Task
- Are re-used for a different Target Task

The layer and associated weights that implement the dense vector encoding are re-used

- Called an *Embedding Layer*

We will show code that trains an Embedding Layer shortly.

# Word prediction problems: high-level

The Source Task we will use to create word embeddings is from a class of *Word Prediction* tasks

- Given a set of tokens (the "context")
- Predict a related token

For example

- Given prefix  $\mathbf{w}_{(1)} \dots \mathbf{w}_{(t-1)}$  of a sequence of tokens  $\mathbf{w}$
- Predict the next token  $\mathbf{w}_{(t)}$ .

*"Machine Learning is  $\langle ??? \rangle$ "*

Or a similar problem

- Predict token  $\mathbf{w}_{(t)}$
- From surrounding tokens  
 $\mathbf{w}_{(t-o)}, \dots, \mathbf{w}_{(t-1)}, \langle ??? \rangle, \mathbf{w}_{(t+1)} \dots, \mathbf{w}_{(t+o)}$

*"Machine  $\langle ??? \rangle$  is easy"*

The inspiration behind using a Word Prediction task to learn embeddings

- Is that meaning of a word can be inferred by context
- "You are known by the company that you keep"

For example

- "I ate an apple"
- "I ate a blueberry"
- "I ate a pie"

"apple", "blueberry", "pie" concept: things that you eat

The Word Prediction task is thus a form of Classification.

We need a large number of training examples as this is a Supervised Learning problem.

One reason that Word Prediction is used is that it is fairly easy to obtain training examples

- From any source of raw text
- Just reformat
- That is: the target/label for an example is just an adjacent token

Since targets can be derived from examples, this is sometimes called *Semi-Supervised Learning*



Let  $\mathbf{w}$  be the sequence of  $n_{\mathbf{w}}$  words

*A word prediction* is a mapping

- from input  $\mathbf{w}$
- to a probability distribution  $\hat{\mathbf{y}}$  over all words in vocabulary  $\mathbf{V}$ 
  - $\hat{\mathbf{y}}_j = p(V_j)$
  - That is: it assigns a probability to each word in the vocabulary

Here are some simple word prediction problems:

predict next word from context      $p(\mathbf{w}_{(t)} \mid \mathbf{w}_{(t-o)} \dots, \mathbf{w}_{(t-1)})$

predict a surrounding word      $p(\mathbf{w}_{(t')} \mid \mathbf{w}_{(t)})$

$$t' = \{t - o, \dots, t + o\} - \{t\}$$

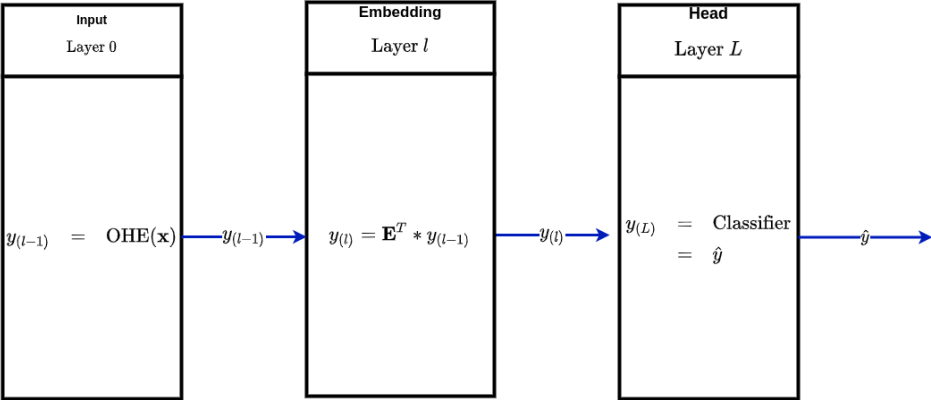
predict center word from context      $p(\mathbf{w}_{(t)} \mid [\mathbf{w}_{(t-o)} \dots \mathbf{w}_{(t-1)} \mathbf{w}_{(t+1)} \dots \mathbf{w}_{(t+o)}])$

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Here is the Neural Network we construct for the Source Task that will learn embeddings

- Ignoring for the moment the issue of converting variable length sequences to a fixed length

Embedding Layer



$\text{OHE}(\mathbf{x}) : (|\mathbf{V}| \times 1)$

$\mathbf{E} = \mathbf{W}_{(l)}$   
 $\mathbf{W}_{(l)} : (|\mathbf{V}| \times n_e)$

$\mathbf{W}_{(L)} : (n_e \times |\mathbf{V}|)$

Layers:

- One Hot Encoded token
- Embedding: converts sparse encoding to dense encoding
- Classifier: operating on dense encodings

The only "new" layer type is the Embedding layer

- This is nothing more than Matrix Multiplication
- The mapping can be implemented as an  $(|\mathbf{V}| \times n_e)$  matrix  $\mathbf{E}$
- Where  $n_e$  is the length chosen for the dense vector

That is because

- The OHE vector for the  $j^{th}$  word  $\mathbf{V}_j$  in vocabulary  $\mathbf{V}$
- Is the  $(|\mathbf{V}| \times 1)$  vector of all 0's except at index  $j$   
 $V^{(j)} = 1$
- $\mathbf{E}^T * \text{OHE}(\mathbf{V}_j)$
- Selects row  $j$  of  $\mathbf{E}$ , which is the  $(n_e \times 1)$  *dense vector* encoding of  $\mathbf{V}_j$

Matrix **E** are *weights to be learned* by training

- Along with the weights of the Classifier layer



In other words

- We train the Neural Network
- To create an embedding
- That makes it easy for a Classifier
- To solve the Source Task

# Conclusion

Categorical variables (such as tokens/words) are easily represented as One Hot Encoded values.

This is perfectly adequate when there is no relationship between tokens.

Word embeddings/Dense representations create a representation

- Not just of a token is isolation
- But a token with multiple dimensions of meaning
- Which enable inter-token relationships

We showed how to create dense representation of words as a by-product of solving a Source Task.

The Source Task we used was Word Prediction, but other tasks may work as well.

The embeddings learned for the Source Task may be useful in other tasks

- This is Transfer Learning in the real world

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In [ ]: print("Done")
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