## Linear Model with higher order features

Our error analysis of the toy problem suggested that a straight line was perhaps not the best fit

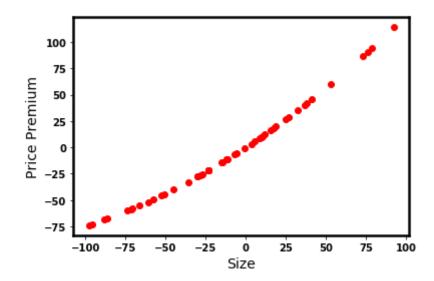
- positive errors in the extremes
- negative errors in the center

Perhaps a "curve" would be a better hypothesis? What if our data is not linear?



```
In [4]: (xlabel, ylabel) = ("Size", "Price Premium")

# I will give you the data via a function (so I can easily alter the data in sub sequent examples)
v1, a1 = 1, .005
lin = recipe_helper.Recipe_Helper(v = v1, a = a1)
X_lin, y_lin = lin.gen_data(num=50)
```

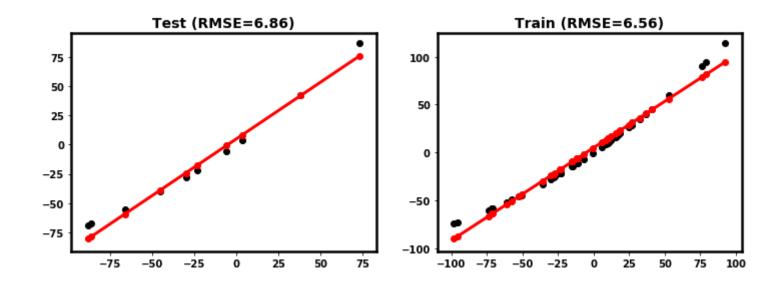


In [6]: \_= lin.run\_regress(X\_orig, y\_orig)

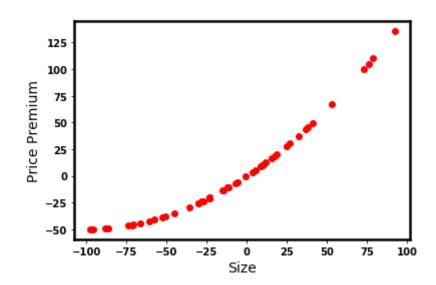
Coefficients: [4.93224426] [[0.96836946]]

R-squared (test): 0.98 Root Mean squared error (test): 6.86

R-squared (train): 0.98 Root Mean squared error (train): 6.56



```
In [7]: v2, a2 = v1, a1*2
    curv = recipe_helper.Recipe_Helper(v = v2, a = a2)
    X_curve, y_curve = curv.gen_data(num=50)
    _= curv.gen_plot(X_curve,y_curve, xlabel, ylabel)
```

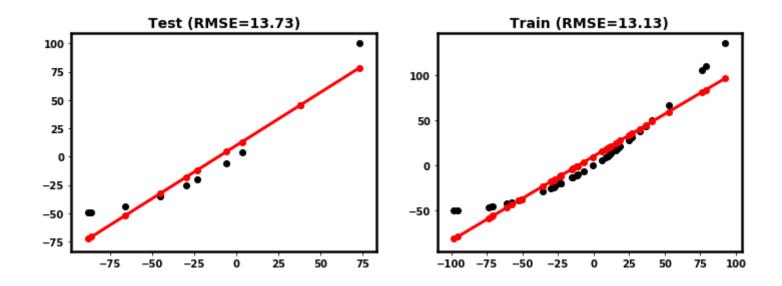


In [8]: \_= curv.run\_regress(X\_curve, y\_curve)

Coefficients: [9.86448852] [[0.93673892]]

R-squared (test): 0.91 Root Mean squared error (test): 13.73

R-squared (train): 0.91 Root Mean squared error (train): 13.13



Compared to the original, the "curvy" data set has a lot more curvature

- the  $R^2$  is still over 90%
- but the Performance Metric (RMSE) is twice as big

## Curvature in a linear model

Our (first-order) linear model was

$$y = \Theta_0 + \Theta_1 x$$

We can create a second order linear model by adding a feature  $x^2$ :

$$y=\Theta_0+\Theta_1x+\Theta_2x^2$$

y is a second order polynomial, whose plot is a curve

• but it is linear in features  $x, x^2$ 

In other words, we are performing feature iteration

 $\, \bullet \,$  in this case: adding the missing feature  $x^2$ 

Let's modify  $\mathbf{x^{(i)}}$  from a vector of length 1:

$$\mathbf{x^{(i)}} = (\mathbf{x_1^{(i)}})$$

to a vector of length 2:

$$\mathbf{x^{(i)}} = (\mathbf{x}_1^{(i)}, \mathbf{x}_1^{(i)^2})$$

by adding a squared term to the vector  $\mathbf{x^{(i)}}$ , for each i.

The modified X' becomes:

$$\mathbf{X} = egin{pmatrix} 1 & \mathbf{x}_1^{(1)} & (\mathbf{x}_1^{(1)})^2 \ 1 & \mathbf{x}_1^{(2)} & (\mathbf{x}_1^{(2)})^2 \ dots & dots \ 1 & \mathbf{x}_1^{(m)} & (\mathbf{x}_1^{(m)})^2 \end{pmatrix}$$

Note that this modified  $\mathbf{X}'$  fits perfectly within our Linear hypothesis

$$\hat{\mathbf{y}} = \mathbf{X}'\Theta$$

The requirement is that the model be linear in its *features*, **not** that the features be linear !

What we have done is added a second feature, that just so happens to be related to the first.

We can now run our linear model with the modified feature vectors

## A word about our module

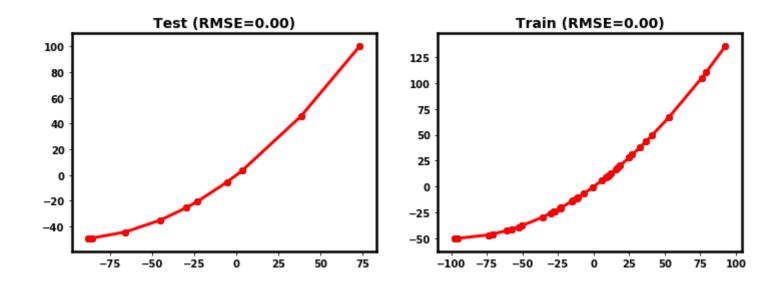
- we add the  $x^2$  column by setting optional parameter run\_transform to True

In [9]: \_= curv.run\_regress(X\_curve, y\_curve, run\_transforms=True)

Coefficients: [-3.55271368e-15] [[1. 0.005]]

R-squared (test): 1.00
Root Mean squared error (test): 0.00

R-squared (train): 1.00 Root Mean squared error (train): 0.00



Perfect fit!

## TIP

- Don't stop just because you scored 91%. And don't give up if the score was awful.
- Examining the errors (residuals) reveals a lot about how to improve your model.
  - Where was the fit good? Where was it bad?
  - Is there a pattern to the badly fit observations that points to a missing feature?

One of the real arts of ML is diagnosing model deficiencies and knowing how to improve them.

We will have a separate module on this topic.

```
In [10]: print("Done !")
```

Done!