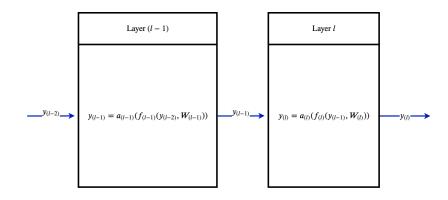
## Residual connections: a gradient highway

<u>Deep Residual Learning (https://arxiv.org/abs/1512.03385)</u>

Consider two layers of a Neural Network

$$egin{array}{lcl} \mathbf{y}_{(l-1)} &=& a_{(l-1)} & \left(f_{(l-1)}(\mathbf{y}_{(l-2)},\mathbf{W}_{(l-1)})
ight) \ \mathbf{y}_{(l)} &=& a_{(l)} & \left(f_{(l)}(\mathbf{y}_{(l-1)},\mathbf{W}_{(l)})
ight) \end{array}$$

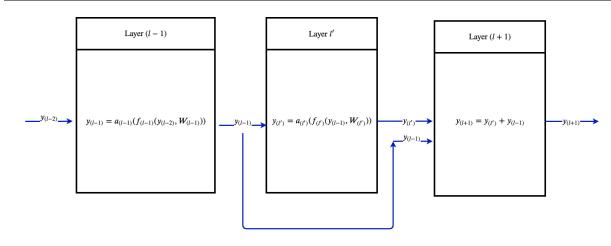
#### "Plain" Neural Network



Suppose we make a slight modification

- Allowing  $\mathbf{y}_{(l-1)}$ , the output of layer (l-1)
- ullet To both flow into layer l
- ullet And to "skip" over layer l
- ullet Where it is added, in layer (l+1) to the output of layer l

#### **Residual Network with Skip Connection**



Here are the equations defining the original network:

$$\mathbf{y}_{(l)} \;\; = \;\; a_{(l)} \;\; ig(f_{(l)}(\mathbf{y}_{(l-1)}, \mathbf{W}_{(l)})ig)$$

and the modified network:

$$egin{array}{lll} \mathbf{y}_{(l')} &=& a_{(l)} \left( f_{(l')}(\mathbf{y}_{(l-1)}, \mathbf{W}_{(l')}) 
ight) &+ \mathbf{y}_{(l-1)} \ \mathbf{y}_{(l+1)} &=& \mathbf{y}_{(l')} + \mathbf{y}_{(l-1)} \end{array}$$

Now suppose we wanted the two networks to compute the same mapping from input  $\mathbf{y}_{(l-1)}$  to output

$$\mathbf{y}_{(l+1)} = \mathbf{y}_{(l)}$$

Then

$$egin{array}{lll} \mathbf{y}_{(l+1)} &=& \mathbf{y}_{(l')} + \mathbf{y}_{(l-1)} & ext{definition of } \mathbf{y}_{(l+1)} \ \mathbf{y}_{(l)} &=& \mathbf{y}_{(l')} + \mathbf{y}_{(l-1)} & ext{requiring } \mathbf{y}_{(l+1)} = \mathbf{y}_{(l)} \ \mathbf{y}_{(l')} &=& \mathbf{y}_{(l)} - \mathbf{y}_{(l-1)} & ext{re-arranging terms} \end{array}$$

That is: layer l' computes the *residual* of  $\mathbf{y}_{(l)}$  with respect to  $\mathbf{y}_{(l-1)}$ .

This type of architecture is called a Residual Network or Residual Layer.

The connection of  $\mathbf{y}_{(l-1)}$  to the input of layer (l+1) is called a *Skip connection* 

This seems strange (and perhaps pointless) until you consider the Back Propagation process.

Recall how the Loss Gradient

$$\mathcal{L}_{(l)}' = rac{\partial \mathcal{L}}{\partial \mathbf{y}_{(l)}}$$

propagates backwards inductively

$$\mathcal{L}_{(l-1)}' = \mathcal{L}_{(l)}' rac{\partial \mathbf{y}_{(l)}}{\partial \mathbf{y}_{(l-1)}}$$

It is modulated by Local Gradient  $rac{\partial \mathbf{y}_{(l)}}{\partial \mathbf{y}_{(l-1)}}$ 

In the original Neural Network, the Local Gradient relating the output and input  $\mathbf{y}_{(l-1)}$  is

$$egin{array}{lll} rac{\partial \mathbf{y}_{(l)}}{\partial \mathbf{y}_{(l-1)}} &=& rac{\partial a_{(l)}(f_{(l)}(\mathbf{y}_{(l-1)},\mathbf{W}_{(l)})}{\partial \mathbf{y}_{(l-1)}} & ext{since } \mathbf{y}_{(l)} = a_{(l)}(f_{(l)}(\mathbf{y}_{(l-1)},\mathbf{W}_{(l)}) \ &=& a'_{(l)} * f'_{(l)} & ext{where } a'_{(l)} = rac{\partial a_{(l)}(\ldots)}{\partial f_{(l)}(\ldots)}; f'_{(l)} = rac{\partial f_{(l)}(\ldots)}{\partial \mathbf{y}_{(l-1)}} \end{array}$$

Whereas, in the modified neural network, the Local Gradient relating the output (now  $\mathbf{y}_{(l+1)}$ ) to input  $\mathbf{y}_{(l-1)}$  \$\$ \begin{array}[|||]\ \frac{\partial \y{(\|l+1)}}{\partial \y{(\|l-1)}} & = & \frac{\gamma((|l-1)|)}{\partial \y{(\|l-1)}} + \frac{\gamma((|l-1)|)}{\partial \y{(\|l-1)}} & \text{since } \y{(\|l+1)} = \y'(|l-1)} & \text{since } \y{(\|l+1)} = \y'(|l-1)} & \text{since } \\ \quad \qua

• \y{(\|I-1)}\ & = & \frac{\partial a{(\|I')} ( f{(\|I')}(\y{(\|I-1)}, \W{(\|I')}) }{\partial \y{(\|I-1)}} \ + 1 & \text{since } y'\|Ip = a{(\|I')}( f{(\|I')}( \y{(\|I-1)}, \W{(\|I')}) \ & = & a'{(\|I')} \* f'{(\|I')} + 1 & \text{where } a'{(\|I')} = \frac{\partial a{(\|I')}(\|Idots)}{\partial f{(\|I')} \ end{array} \$\$ (\|Idots)}; f'{(\|I')} = \frac{\partial f{(\|I')}(\|Idots)}{\partial \y\_{(\|I-1)}} \ end{array} \$\$

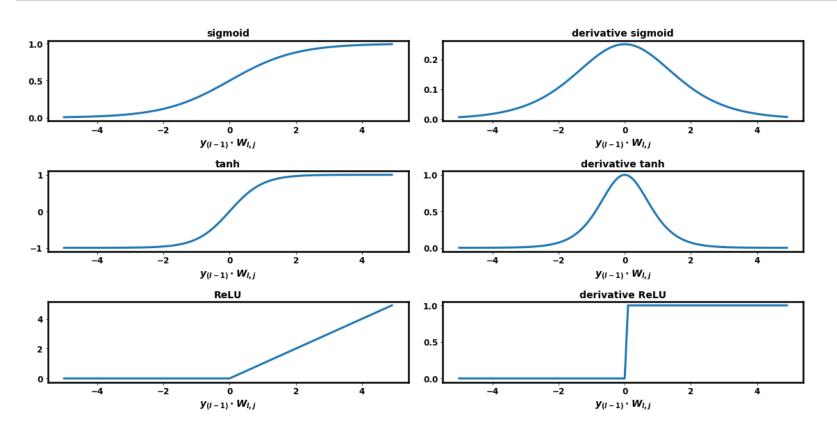
The main difference between the Local Gradients of the two networks

• is the "+1" term

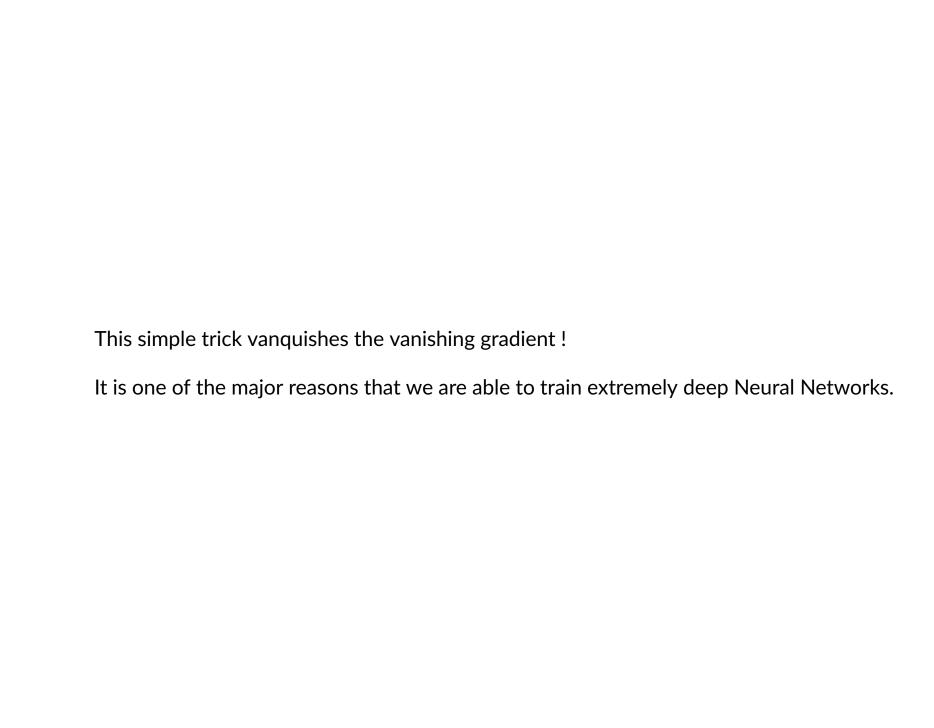
Since the derivative a' of most activation functions is less than one

- The Local Gradient of the original network diminishes the Loss Gradient
- The Local Gradient of the modified network *does not* diminish the Loss Gradient

```
In [5]: fig, _ = nnh.plot_activations( np.arange(-5,5, 0.1) )
```



The "+ 1" term in the Local Gradient of the Residual Network allows the Loss Gradient to flow backwards un-diminished • It is like an "express lane" on the backward pass highway!



#### Another important fact

Adding an additional residual layer to a Neural Network cannot increase the Loss!

Because there exists a set of weights 
$$\mathbf{W}'_{(l')}$$
 for which  $a_{(l')}\left(f_{(l')}(\mathbf{y}_{(l-1)},\mathbf{W}'_{(l')})
ight)=0$ 

The residual layer would choose these weights rather than suffering an increased loss!

By choosing these weight  $\mathbf{W}'_{(l')}$  the residual layer computes the identity function

$$\mathbf{y}_{(l+1)} = \mathbf{y}_{(l-1)}$$

Thus, adding more residual layers cannot hurt performance.

Without the skip connection, it has been observed empicially

- That it is difficult for a Neural Network to learn the identity function
- So the skip connection is quite useful

### Technical aside

There is an unresolved debate where to place the "head" of the skip connection

- insider the activation function
- outside the activation function

We choose the latter to simplify the derivative expression for the loss gradient.

# Conclusion and Preview: Skip connections in LSTM's, GRU's

The gradient highway also turns out to be useful in RNN's.

There are more powerful variants of the RNN called LSTM and GRU which avoid vanishing gradients, partially through the use of skip connections.

These variants enhance the power of skip connections by allowing selective skipping via the use of "gates".

We will see this shortly.

```
In [6]: print("Done")
```

Done