Other decompositions of ${f X}$

Eigen decomposition of covariance matrix of $oldsymbol{X}$

There is another matrix factorization method known as Eigen Decomposition.

Eigen decomposition, unlike SVD, only works on symmetric matrices M:

$$M = W\Lambda W^T$$

where $WW^T = I$

We can obtain the PCA from the Eigen Decomposition of $\mathbf{X}\mathbf{X}^T$

- ullet the covariance matrix of ${f X}$ (i.e., original feature covariance)
- the covariance matrix is symmetric, as required

We can relate the SVD of ${\bf X}$ to the Eigen decomposition of ${\bf X}{\bf X}^T$ as follows:

$$egin{array}{lll} \mathbf{X}^T\mathbf{X} &=& V\Sigma U^TU\Sigma V^T & ext{from SVD } \mathbf{X} = U\Sigma V^T \ \mathbf{X}^T\mathbf{X} &=& V\Sigma \Sigma^T V^T & ext{since } U^TU = I \end{array}$$

Similarly, we can show

$$\mathbf{X}\mathbf{X}^T = U\Sigma\Sigma^TU^T$$
since $VV^T = I$

Setting

- $\Lambda = \Sigma \Sigma^T$
- ullet W=U=V we get ${f X}=W\Lambda W^T$, the Eigen Decomposition of ${f X}{f X}^T$.

The V that transforms ${f X}$ (original features) to $ilde{{f X}}=XV$ (synthetic features)

- Can be computed directly from SVD
- ullet Or by creating covariance matrix ${f X}{f X}^T$ and using Eigen decomposition.

SVD is more commonly used

- There are many fast implementations of SVD
- ullet There is no need to compute the big covariance matrix ${f X}{f X}^T$

Other factorization methods

- $ext{CUR} ext{ method}$ $ext{CUR}(\mathbf{X}) = C \cdot U \cdot R$
- ullet C chosen from Columns of ${f X}$
- ullet R chosen from Rows of ${f X}$

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In [4]: print("Done")
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Done