Classification: choosing the threshhold

Recall our methodology for Classification:

- Compute a "score" that our example is in each of the target classes
- Construct a probability distribution (over the target classes) from the scores
    - convert the per class score into a probability via the sigmoid/softmax function
- Compare the probability to a threshhold

$$\hat{p} = \sigma(\Theta^T \mathbf{x})$$

where σ, the *logistic function*, is:

Convert $\hat{p}^{\mathbf{(i)}}$ into Classification prediction $\hat{y}^{\mathbf{(i)}}$

$$\hat{y}^{\mathbf{(i)}} = \begin{cases} 0 & \text{if } \hat{p}^{\mathbf{(i)}} < 0.5 & \text{Negative} \\ 1 & \text{if } \hat{p}^{\mathbf{(i)}} \geq 0.5 & \text{Positive} \end{cases}$$

But does the threshhold *need* to be 0.5 ?

We will motivate other choices for the threshhold.

Let's examine our predictions at a fine granularity via the following table

- the row labels correspond to the predicted class
- the column labels correspond to the target (actual) class

	P	**N**
P	TP	FP
N	FN	TN

The correct predictions

- True Positives (TP) are test examples predicted as Positive that were in fact Positive
- True Positives (TN) are test examples predicted as Negative that were in fact Negative

The incorrect predictions

- False Positives (FP) are test examples predicted as Positive that were in fact Negative
- False Positives (FN) are test examples predicted as Negative that were in fact Positive

Unconditional Accuracy can thus be written as

$$\text{Accuracy} = \frac{\text{TP} + \text{TN}}{\text{TP} + \text{FP} + \text{TN} + \text{FN}}$$

We can also define some conditional Accuracy measures

Recall

- Conditioned on Positive test examples

$$\text{Recall} = \frac{\text{TP}}{\text{TP} + \text{FN}}$$

- The fraction of Positive examples that were correctly classified
- Also goes by the names: True Positive Rate (TPR), Sensitivity

We can affect the prediction of Positive/Negative by varying the choice of Threshhold.

We can increase the number of Positive predictions by lowering the threshhold

- this will increase TP
    - degenerate case: *always* predict Positive !
    - increase Recall by increasing numerator
- but also increase FP
    - which *does not* appear in denominator

Why would we want to increase Recall (at the potential cost of decreased unconditional Accuracy) ?

It depends on your task.

Consider a diagnostic test for an extremely dangerous, infectious disease

- It might very important to have high Recall (catch truly infected patients)
- Even at the expense of incorrectly labelling some healthy patients as infected

Specificity

- conditioned on Negative examples

$$\text{Specificity} = \frac{\text{TN}}{\text{TN} + \text{FP}}$$

- The fraction of Negative examples that were correctly classified
- Also goes by the name: True Negative Rate (TNR)

By *raising* the threshhold, we can increase the number of Negative prediction.

Why would we want to increase Specificity (potentially decreasing unconditional Accuracy) ?

- by increasing the False Negatives (FN)

Consider a diagnostic test for a mild, non-infectious disease

- A Positive prediction might entail an expensive/painful remedy, which we want to avoid
- Even at the expense of incorrectly labelling some non-healthy patients as healthy

So the choice of threshhold affects all measures, not just Unconditional Accuracy.

Choosing the threshhold involves a tradeoff.

In [4]:
```
print("Done")
```

```
Done
```