Journal of Physics: Conference Series

PAPER • **OPEN ACCESS**

Vehicle Routing Problem with Time Windows Arising in Urban Delivery

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Abstract. Delivery is an important part of logistics operation. The way to plan the vehicle route with consideration of the demand, time window and total travel cost will directly affect the operation cost and the efficiency of delivery. In this paper, a mathematical model of the vehicle routing with time windows arising in urban delivery was formulated, and Clarke and Wright saving algorithm was used to solve it. A real case of a company was used to test its performance. The result shows that the Clarke and Wright saving algorithm is more effective than the actual operation of the company.
Keywords: Vehicle routing problem with time windows; Clarke and Wright saving algorithm; Urban delivery.

1. Introduction

In recent years, E-commerce has grown rapidly around the world. Nowadays, online shopping has become very popular. Urban delivery, one of the fastest growing fields in the logistics industry, plays an important role in the last kilometer problem of online shopping. According to the operation of the postal industry in 2019 released by the State Postal Administration, the business volume of domestic express service in the city has reached 11.04 billion yuan, accounting for 17.4% of the total express business volume, and the business income accounts for 10% of the total express business revenue. It is also estimated that by 2020, the market size of China's urban delivery business will exceed 200 billion yuan. With the rapid rise in practice, as an important field of logistics, urban delivery has attracted extensive attention of scholars.

At present, the research on urban delivery mainly includes urban delivery network [1-3], urban delivery system [4-6], urban delivery fleet allocation [7], and urban delivery vehicle routing optimization [8][9]. Among them, the vehicle routing optimization refers to how to arrange the vehicles to complete the delivery to meet all customer demands while minimizing the total delivery cost under the constraints of the capacity of vehicle, the time windows of customers and so on. Clarke and Wright saving algorithm [10] was proposed by Clarke and Wright in 1964. The algorithm is a heuristic construction algorithm, which provides a simple and easy way to solve the route optimization problem. There are a lot of achievements in using Clarke and Wright saving algorithm to solve the route optimization. Recent literature such as a heuristic method based on Clarke-Wright algorithm was proposed to solve the open vehicle routing problem [11], and a modified Clarke-Wright saving algorithm was proposed to solve the capacitated vehicle routing problem [12].

This paper considers the optimization of vehicle routing problem with time window arising in urban delivery. We are intending to solve the problem by employing Clarke and Wright saving algorithm since the algorithm's application has succeeded in traditional vehicle routing problem.

Journal of Physics: Conference Series **1626** (2020) 012097 doi:10.1088/1742-6596/1626/1/012097

The remainder of this paper is organized as follows. Section 2 presents the definition and mathematical formulation for the vehicle routing with time windows arising in urban delivery. Section 3 introduces the solution approach based on Clarke and Wright saving algorithm. Section 4 provides the computational results and also analyzes the algorithm performance. Section 5 provides concluding remarks.

2. Problem Description and Mathematical Modeling

2.1. Problem Description

The problem of optimizing the delivery route with time window arising in urban delivery can be described as follows: in the same city, there are multiple vehicles starting from a depot. ach vehicle serves multiple customers in turn, and returns to the delivery center after completing the delivery task. The quantity that one vehicle can deliver in one turn is limited by its capacity, and the sum of all customer demands served by one vehicle shall not exceed the limit. Each customer has a certain demand and time window that allow the vehicle to start service, which means the vehicle can only start service for customers within the specified range of time. If the vehicle arrives early, it needs to wait until the beginning of the time window; if it is late, it will not be able to serve the customer. Each customer's needs must be met and each customer can only be served once.

The optimization problem of vehicle route problem with time window arising in urban delivery is about determining the routes of the vehicles to serve the customers' demands under conditions shown above, so as to minimize the total travel cost of serving all customer demands.

2.2. Mathematical Modelling

In order to formulate the mathematical model of vehicle routing problem with time window arising in urban delivery, the following symbols are defined:

- $N = \{0,1,2,\cdots,n\}$ represents the nodes set in the urban delivery, in which 0 represents the depot.
- $N' = N \setminus \{0\}$ represents the customer nodes set for urban delivery.
- $K = \{1,2,3,\cdots,k\}$ represents the assemble of the vehicles.
- Q represents the capacity limit of each vehicle.
- d_i represents the demand of customer i.
- s_i represents the service time lasted for customer i.
- $[e_i, l_i]$ represents the time window when customer i allows the vehicle to start providing services.
- c_{ij} represents the travel cost of the vehicle between node i and node j.
- t_{ij} represents the travel time of the vehicle between node i and node j.
- x_{ij} is the decision variable. If edge (i, j) is used, it is 1; otherwise, it is 0.
- y_{ik} is another decision variable. If the customer i is served by the vehicle k, it is 1; otherwise, it is 0.

Based on the problem description and symbol definition, the mathematical model of vehicle route problem with time window arising in urban delivery is established as follows:

$$\min f = \sum_{i \in N} \sum_{j \in N} c_{ij} x_{ij} \tag{1}$$

$$s.t. \sum_{k \in K} y_{ik} = 1, \forall i \in N' \tag{2}$$

$$\sum_{j \in N} x_{ij} = \sum_{m \in N} x_{mi}, \forall i \in N \tag{3}$$

$$x_{ij}y_{ik} = x_{ij}y_{jk}, \forall i, j \in N', \forall k \in K \tag{4}$$

$$\sum_{i \in N'} d_i y_{ik} \le Q, \forall k \in K \tag{5}$$

$$b_j > \max(e_j, b_i + s_i + t_{ij}) - M(1 - x_{ij}), \forall i, j \in N' \tag{6}$$

$$e_i \le b_i \le l_i, \forall i \in N' \tag{7}$$

$$\sum_{i \in S}\sum_{j \in S} x_{ij} \le |S| - 1, S \subset \{1,2,\cdots,n\}, |S| \ge 2 \tag{8}$$

$$x_{ij} \in \{0,1\}, \forall i, j \in N \tag{9}$$

$$y_{ik} \in \{0,1\}, \forall i \in N', \forall k \in K \tag{10}$$

In the model above, the objective function minimizes the travel cost of the vehicle. The objective function (1) minimizes the total travel cost for all vehicles to complete the delivery service [1]. Constraints (2) ensure that each customer has and only has one vehicle to provide services. Constraints (3) show that the departure edge of node i is equal to the arrival edge of node i. Constraints (4) indicate that if the customer i and customer j are on the route of the vehicle, the vehicle will provide services for both consumers [6]. Constraints (5) indicate that the sum of customer demands for any vehicle service shall not exceed the loading capacity. Constraints (6) define the relationship between the time when the vehicle starts to provide service to customer j, the lower time window and the time when the forward customer i to be served [10]. Constraints (7) ensure that the vehicle can only start the service within the time windows allowed by the customer. Constraints (8) are used to eliminate the sub loop [1]. Constraints (9) and (10) are used to define the value range of decision variables.

3. Clarke and Wright Saving Algorithm

Clarke and Wright saving algorithm is the most widely applied heuristic algorithm to solve vehicle routing problem due to its simple implementation and effectiveness of speed calculating [13]. The idea of this algorithm is to combine the two routes into one in turn, and compare those results in order to maximize the reduction of the combined total travel distance each time, until constraints are reached. Next, we start to optimize the route for the next vehicle.

For example, an optimization problem consisting of a depot and six customer points is shown in figure 1. According to the idea of Clarke and Wright saving algorithm, each customer is first connected with the depot to form a vehicle driving route starting from the depot and returning to the depot after the customer completes the delivery service. This gives you six routes, in which each customer is individually assigned a car to serve them. It is assumed that the routes of different customers can be merged with each other within the constraints of vehicle capacity and customer's time window. In the case that the sum of both sides of the triangle is always greater than the third side, it can be known that the combination will lead to the saving of the total driving distance of vehicles. After the merger of customer 2 and customer 3, the vehicle driving distance originally needed to serve these two customer points is $2d_{02} + 2d_{03}$, the merged vehicle driving distance is $d_{02} + d_{23} + d_{03}$, and the saving value of the merged vehicle driving distance is $d_{02} + d_{03} - d_{23}$. Since the sum of the two sides of the triangle is greater than the third side, $d_{02} + d_{03} - d_{23} > 0$. The vehicle travel distance is saved after the combination. Similarly, under the constraint conditions of customer 4, 5 and 6, the driving distance of the merged vehicle is further reduced. If the plan to merge customer 1 into another route would violate the constraint, the original route will not be changed. Thus, the optimized solution is obtained, as shown in figure 2.

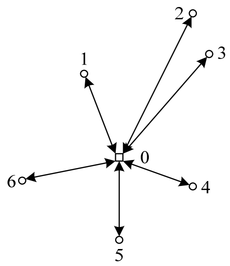

Figure 1. Initial solution.

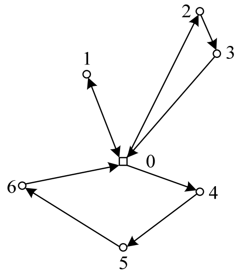

Figure 2. Merged solution.

The pseudo code of Clarke and Wright saving algorithm to solve vehicle routing problem with time windows arising in urban delivery is shown in algorithm 1.

Algorithm 1: Clarke and Wright saving algorithm.

Input distance between nodes in urban delivery, the demand, time window and service time of customers, the capacity of vehicle

Obtain the initial solution $route_i (i = 1,2,\cdots,n)$ with n routes by connecting each customer with the depot.

Calculate the savings $d(i, j) = d_{0i} + d_{0j} - d_{ij}$ $(\forall i, j \in N')$.

Sort the distance saving value from large to small, $SM = \{d'(i, j) \mid i, j \in N'\}$.

While $SM \neq \phi$ **do**

Select the first element $d'(i, j)$ of SM

delete $d'(i, j)$ from SM

If Constraints(5), (6) and (7) are true when connect customer i and customer j **then**

connect customer i and customer j to form a new route

If customer i is not connected with 0 **then**

delete $d'(i,k)$ from SM for all $k \in N'$

End if

If customer j is not connected with 0 **then**

delete $d'(k, j)$ from SM for all $k \in N'$

End if

End if

End while

4. Case Study

Shandong Jiajiayue Group Co., Ltd. is the largest supermarket chain enterprise in Shandong Province, which takes supermarket chain as its main business and is engaged in logistics delivery, food processing and wholesale of agricultural products. At present, it has more than 20000 employees and more than 500 direct chain stores, covering 40 cities and counties in Weihai, Yantai, Qingdao, Jinan, Weifang, Laiwu and other places in Shandong Province, China. Therefore, Shandong Jiajiayue chain supermarket fresh agricultural products delivery as the case. The earliest Songcun fresh food logistics depot located in Wendeng City of Weihai, which provides an important platform for the implementation of agricultural supermarket docking, is selected as the depot in the calculation. At the same time, 20 supermarket stores within 40 kilometers of Wendeng City served by the depot are selected as the customers in the case, and vegetables are delivered to them. The depot uses the same type of vehicles to provide services and adopts the road transportation mode. The capacity of vehicle is 9 tons, the travel speed is 50 km/h, and the vehicle starts from the depot at 5:00am. The relevant data of stores are shown in table 1 and the distances between nodes are shown in table 2 [14].

Table 1. Time windows, service time and demands of the stores.

Store i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
e_i	6:00	7:30	6:00	6:30	6:40	7:00	7:20	7:30	7:00	7:00	7:30	7:30	7:30	7:30	6:50	7:00	7:00	7:50	6:30	7:50
l_i	8:00	9:00	8:00	8:20	8:30	9:00	9:00	9:00	8:30	9:00	9:30	9:00	9:30	9:00	8:30	8:40	8:40	9:00	8:30	9:00
s_i (m)	20	10	30	25	30	30	30	20	25	20	20	15	15	20	40	15	20	10	40	20
d_i (t)	1.5	0.5	1.5	1.5	2	2	1.8	1	1	1	1	0.5	0.5	1.5	2	1.5	1.5	0.5	2.5	1

Table 2. Distance between nodes.

nodes	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	0	17.9	19.6	22.1	22.9	22.2	12.1	39.6	19.5	19	11.7	15.5	33.6	19.6	15.2	16.2	17.3	15.8	19	18.6	18.9
1		0	30.7	37.2	33.7	33.2	25.4	54.6	30.1	19	20	31.2	39.7	30.7	31.7	31.8	32.2	31.8	29.7	30.5	30.5
2			0	26.6	5.2	5.6	25.5	25.7	1.9	26.2	13.2	3.6	21.3	0.2	3.4	2.2	2.4	2.7	1	0.7	0.9
3				0	31.4	31.6	11.7	25.7	27.8	40	31.6	23.3	44.6	2.6	23.4	24.4	25.2	24	27.4	27	27.2
4					0	5.7	30.3	26.8	4.4	24.7	14	8.3	18	4.8	7.9	6.8	6	7.4	4.6	4.6	4.5
5						0	28.2	23.7	4.9	4.4	16.2	6.2	22.7	3	5.8	4.7	3.7	5.2	3.5	2.8	2.7
6							0	29.3	25.8	4.9	21.7	21.4	42.7	24.7	21.5	22.5	23.6	22	25.5	25.1	25.3
7								0	27.5	25.8	38.9	26.3	44.6	25.6	25.8	25.1	25	25.1	26.3	25.5	25.5
8									0	27.5	12.1	4.8	19.7	2.3	4.5	3.7	3.9	4.2	1.4	2.1	2.2
9										0	11	27.7	27	26.7	28.1	27.8	28.2	28.8	25.9	26.7	26.9
10											0	14.6	20	13.6	15.1	14.8	15.4	15.2	12.7	13.5	13.5
11												0	22.5	3.5	1	1.6	2.6	1.6	4.3	3.9	4.1
12													0	21.6	23.1	22.8	23.4	23.2	20.8	21.5	21.5
13														0	3.3	2.3	2.4	2.7	1.1	0.8	1
14															0	1.2	2.3	1.1	4	3.6	3.5
15																0	1.4	0.6	3.1	2.7	2.8
16																	0	1.7	3.3	2.2	2.3
17																		0	3.5	3.2	3.3
18																			0	1	1
19																				0	0.4
20																					0

The solution process of Clarke and Wright saving algorithm is as follows:

- **Step 1,** connect the depot to each customer to form n initial routes.
- **Step 2,** apply the expression $d(i, j) = d_{0i} + d_{0j} - d_{ij}$ to calculate the savings value of the new delivery route after connecting customer i and customer j compared with the initial route. The data on the right side of the expression are from table 2. The mileage saving values are shown in table 3.
- **Step 3,** the savings generated by two-point connections that do not meet the time window and capacity constraints are deleted from table 3. Sort the remaining connections according to the saving value from large to small. Table 4 lists only the connections and their savings values that still satisfy all constraints in the model when combined further.
- **Step 4,** according to the Constraints (5), (6) and (7) in the model, judge one by one, and connect the connections meeting the capacity and time windows constraints one by one to form a route. Until all connections in table 4 are processed, all routes constitute the final solution.

Table 3. Mileage saving value.

stores	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1		6.8	2.8	7.1	6.9	4.6	2.9	7.3	17.9	9.6	2.2	11.8	6.8	1.4	2.3	3	1.9	7.2	6	6.3
2			15.1	37.3	36.2	6.2	33.5	37.2	12.4	18.1	31.5	31.9	39	31.4	33.6	34.5	32.7	37.6	37.5	37.6
3				13.6	12.7	22.5	36	13.8	1.1	2.2	14.3	11.1	39.1	13.9	13.9	14.2	13.9	13.7	13.7	13.8
4					39.4	4.7	35.7	38	17.2	20.6	30.1	38.5	37.7	30.2	32.3	34.2	31.3	37.3	36.9	37.3
5						6.1	38.1	36.8	36.8	17.7	31.5	33.1	38.8	31.6	33.7	35.8	32.8	37.7	38	38.4
6							22.4	5.8	26.2	2.1	6.2	3	7	5.8	5.8	5.8	5.9	5.6	5.6	5.7
7								31.6	32.8	12.4	28.8	28.6	33.6	29	30.7	31.9	30.3	32.3	32.7	33
8									11	19.1	30.2	33.4	36.8	30.2	32	32.9	31.1	37.1	36	36.2
9										19.7	6.8	25.6	11.9	6.1	7.4	8.1	6	12.1	10.9	11
10											12.6	25.3	17.7	11.8	13.1	13.6	12.3	18	16.8	17.1
11												26.6	31.6	29.7	30.1	30.2	29.7	30.2	30.2	30.3
12													31.6	25.7	27	27.5	26.2	31.8	30.7	31
13														31.5	33.5	34.5	32.7	37.5	37.4	37.5
14															30.2	30.2	29.9	30.2	30.2	30.6
15																32.1	31.4	32.1	32.1	32.3
16																	31.4	33	33.7	33.9
17																		31.3	31.2	31.4
18																			36.6	36.9
19																				37.1
20																				

Table 4. Sequence of saved mileage.

sort	connections	save miles	sort	connections	save miles
1	4-5	39.4	9	9-7	32.8
2	3-13	39.1	10	15-17	31.4
3	13-2	39	11	20-11	30.3
4	5-20	38.4	12	19-14	30.2
5	8-4	38	13	17-12	26.2
6	2-18	37.6	14	6-9	26.2
7	18-19	36.6	15	1-10	9.6
8	16-8	32.9			

According to Table 4, connect store 4 and store 5 corresponding to sort 1 to form route 1 (0→4→5→0). Connect store 3 and store 13 corresponding to sort 2 to form route 2 (0→3→13→0). If the store 13 corresponding to sort 3 is already in route 2, the constraints (5), (6) and (7) in the model will be met. Add the store 2 corresponding to sort 3 into route 2, and then route 1 will be updated to 0→3→13→2→0. Similarly, add store 20 corresponding to sort 4 to route 1, and update route 1 as 0→4→5→20→0. This continues until store 1 and store 10 corresponding to sort 15 in Table 4 are processed. The solution of Clarke and Wright saving algorithm is: route1 (0→16→8→4→5→20→11→0), of which travel cost is 53.6; route2 (0→3→13→2→18→19→14→0), of which travel cost is 45.7; route3 (0→6→9→7→0), of which travel cost is 61.8; route4 (0→15→17→12→0), whose travel cost is 73.6; route5 (0→1→10→0), of which travel cost is 49.6. So the total travel cost is 284.3. The current delivery route of Jiajiayue supermarket is [14]: route1 (0→6→15→19→5→0), of which travel cost is 62.3; route 2(0→14→18→4→12→1→3→7→0), of which travel cost is 184; route3 (0→10→9→8→13→2→20→16→17→11→0), of which travel cost is 74.7. The total travel cost is 321. It can be seen that the objective function value obtained by Clarke and Wright saving algorithm is 11.43% lower than the real cost in the actual operation of the company.

5. Conclusions
With the rapid growth of e-commerce, online shopping is becoming more and more popular, which leads to more and more complex vehicle routing in urban distribution. Based on the establishment of mathematical model, this paper uses Clarke and Wright saving algorithm to solve a real case. Compared with the actual operation of the company, the vehicle running cost is greatly reduced.
Vehicle routing problem with time windows arising in urban delivery remains challenging, and the solution approaches are still evolving and call for further improvement. The Clarke and Wright saving algorithm in this paper is effective, but the potential enhancement can be obtained by introducing other meta-heuristic algorithms can be designed.

Acknowledgments
This research was supported in part by the ninth issue of China's Chongqing young eagle project "intelligent scheduling models and algorithms research of instant delivery". The authors would like to thank professor Jiumei Chen, Ph.D. supervisor of Chongqing Technology and Business University, for her guidance in the process of this research.

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