Adjusted Clustering Clarke-Wright Saving Algorithm for Two Depots-N Vehicles

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Abstract – **In this paper we modeled the distribution of a single type of products, which are distributed from two depots and use N-vehicles. This problem can be modeled using Capacitated Vehicle Routing Problems (CVRP), and the common algorithm to solve that model is Clarke and Wright Saving Algorithm (CWSA). The needed computational time for finding the nearly global optimum of CWSA grows exponentially with the numbers of the existed routes. Therefore, in this paper, we proposed to combine the clustering algorithm with CWSA. Additionally, we consider the largest item in the cluster, which has to be transported, as the starting point to find the optimum solution.**

Keywords - **CVRP, Clarke and Wright Saving Algorithm, Clustering**

I. INTRODUCTION

Capacitated Vehicle Routing Problem (CVRP) is a common method used to solve the routing problem for distributing goods using several limited capacity fleets. The classical version of CVRP was defined by Dantzig and Ramser[1]. CVRP aims to find *m* trips so that all customers are served and the total distance traveled by the fleet is minimized. It is well known, that this problem is classified as an NP-hard problem (Lenstra and Rinnooy, [2]). Therefore, it attracts many researchers to study and to develop various algorithms for solving that problem.

The CVRP is the sub-problem of the General Capacitated Vehicle Routing Problems (GVRP). Depending on the fleet composition the GVRP can be classified into two sub-problems, i.e the Homogeneous Fleet (CVRP) and the Heterogeneous ones (HVRP). Depending on the vehicle availability, those sub-problems, each sub-problems can also be classified in to two classes, i.e multi-trips and single-trip (Cruz et al., [3]). In this paper we work for the single-trip CVRP (see Fig.1).

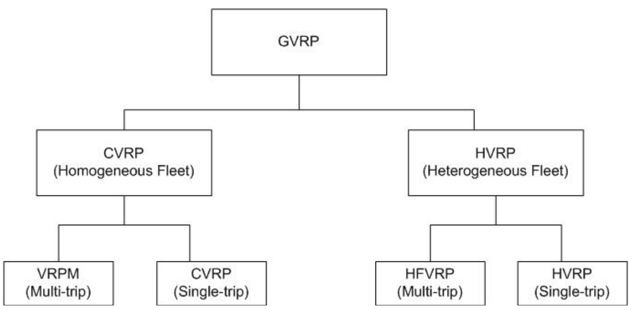

Fig.1 The classification of GVRP [3]

Since it is an NP-hard problem, for the real-world problem, it is very hard to solve the CVRP by exact solution through solving a Linear Programming model. Therefore, the algorithms for solving those problems are needed. In General, those algorithms can be classified into three approaches. They are the branch-and bound approaches: branch and bound, branch and cut, branch and price. Heuristic approaches: saving algorithms, sequential insertion algorithms, cluster first-route second algorithm. Meta Heuristic algorithm: simulated annealing, tabu search, ant colony, genetic algorithm (Pichpibul and Kawtummachai, [4]).

The branch and cut approach need a huge amount of computational time, therefore it is not reasonable to apply this approach for solving a real-world problem. A heuristics approach such as saving algorithm (Clarke and Wright [5]), sequential insertions (Joubert and Claasen [6]), Variable-Neighborhood Search (Hansen and Mladenovic, [7]) can find a feasible solution (near optimal) fast and easily. The meta heuristic approach such as Tabu search (Gendreau et al [8]), Ant Colony (Tan et al. [9]), Genetic (Vaira et al. [10]) is more complex than the heuristic one.

Due to the fast computational time and easiness to apply, in this paper we applied the heuristic approach, i.e. the Clarke and Wright Saving algorithm for solving the distribution problem of single type products from two depots. In this real problem, the company receive order from the customers daily, and every afternoon they arrange the order distribution for each customers for the next day. Therefore, computational time is matter for solving this real-world.

To solve this problem, at first we cluster the data based on the latitude and the longitude of each address. The spirit of this algorithm is similar to Shin and Han [11], in which they constructed the algorithm using three steps, i.,e., clustered first, clustered adjustment and routing. Shin and Han [11] selected the farthest node (customer) among un-clustered nodes as the center to form a cluster. In this work, we first used the K-means algorithm. We then adjust the cluster based on the maximum order in each cluster which constructed via the K-means algorithms. We then routing the distribution based on CWS algorithms. Finally, we measured the distance from the centered of each cluster to each depot. Those clusters will be classified into two groups, based on the closest distance to the selected depot.

II. METHODOLOGY

In this section we summarize the methods that we used to construct the proposed algorithm.

A. Capacitated Vehicle Routing Problems

Capacitated Vehicle Routing Problem (CVRP) is a VRP in which each customer has a deterministic order. Those orders should be completed by several fleets from each selected depot. The customers are spread in several geographical area (Pitchpibul and Kawtummachai, [12]).

The main goal of CVRP is minimizing total cost such that each customer's order is accomplished. Each route is started and ended from the same depot. Each customer is visited once and the total demand cannot exceed the vehicle capacity. Each vehicle has the same capacity Gambardella, *et al.* [13]. The formulation of CVRP is as follows:

Minimize the total cost

$$Minimize \sum_{i\in N}\sum_{j\in N}\sum_{v\in V} c_{ij}\, x_{ij}^{v}$$

Constraints:

Each vehicle visits each customer once.

$$\sum_{v\in V} y_i^v = 1 \text{ for } i \in N$$

Each customer is visited and left by the same vehicle

$$\sum_{i\in N} x_{ij}^v = y_j^v \text{ for } j \in N,\ v \in V$$

$$\sum_{j\in N} x_{ij}^v = y_i^v \text{ for } i \in N \text{ and } v \in V$$

Total delivery demand is not over the vehicle capacity

$$\sum_{i\in N} d_i\, y_i^v \le Q \text{ for } v \in V$$

Vehicle availability should not be exceeded.

$$\sum_{i\in N} x_{i1}^v \le 1 \text{ for } v \in V$$

$$\sum_{j\in N} x_{1j}^v \le 1 \text{ untuk } v \in V$$

where:

N : Number of nodes (customer and depot), $N = \{1,\ldots,n\}$

c_{ij} : Traveling cost from customer i to customer $j (i, j \in N, i \neq j)$

V : Number of available vehicle $(v \in V)$

Q : Vehicle's capacity

d_i : Demand of customer i

x_{ij}^v : Indicator in which $x_{ij}^v = \begin{cases}1\\0\end{cases} (i, j \in N;\ v \in V)$
$x_{ij}^v = 1$ if vehicle v travels from *customer* i to *customer* j and $x_{ij}^v = 0$ if vehicle v does not travel from *customer* i to *customer* j

y_i^v : Indicator in which $y_i^v = \begin{cases}1\\0\end{cases} (i \in N;\ v \in V)$
$y_i^v = 1$ if vehicle v travels to *customer* i and $y_i^v = 0$ if vehicle v does not travel to *customer* i

B. Clarke and Wright's Saving Algorithm

Clarke and Wright's Savings Algorithm (CWSA – [5]) is a heuristic algorithm for solving the CVRP. This algorithm does not guarantee global optimum solution. However the obtain solution basically is good enough. It is a little bit deviate from the global solution (Lysgaard, [14]).

The goal of CWSA is to minimize the total traveling cost. The CWSA saves the cost by joining two routes to be one route (See Fig. 2)

A i

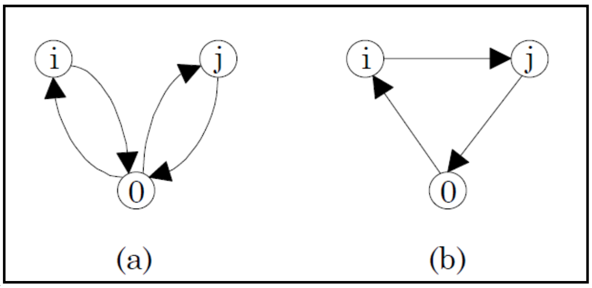

Fig 2. CWSA concepts (Lysgaard, [14])

In Fig.2, customer i is visited from depot O, and the the vehicle left the customer i and back to depot O. From depot O, the customer j is visited (Fig.2a). This route can be saved by directly visiting customer j after the vehicle left customer i. (Fig. 2b). Total traveling cost of Fig.1a (D_a) will be:

$$D_a = c_{0i} + c_{i0} + c_{0j} + c_{j0}$$

Total traveling cost of Fig.1b (D_b) will be:

$$D_b = c_{0i} + c_{ij} + c_{j0}$$

Thus the saving cost (S_{ij}) will be:

$$S_{ij} = D_a - D_b = c_{i0} + c_{0j} - c_{ij}$$

Lysgaard ([14]) stated that, the CWSA computation can be done through three steps. (1) Construct the distance (cost) matrix and saving matrix for each node. (2) Sort each node descending, (3) construct the route based on the sorted saving matrix.

C. Clustering Algorithm

We apply the classical K-means algorithm which is implemented in R-package. The K-means R-packages includes four algorithms: Lloyd, Forgy, MacQueen, and Hartigen Wong (R-package, [15]). We use the R-squared (RS) statistics, which is the ratio of sum square between clusters to sum square within cluster. The value of RS ranges from 0 to 1, with 0 indicating no differences among groups or cluster and 1 indicating maximum differences among groups (Sharma, [16]).

D. Cluster adjustment

We use three schemes for solving this problem. Scheme 1: Without clustering. In this scheme we directly applied the CWSA algorithm to the data. Then we classified those cluster into two groups, based on the closest distance to each depot.

Scheme 2: Depots Clustering - CWSA based on the number of depots. In this scheme, we first clustered the latitude and longitude of delivering addresses into two clusters. Then we measured the distance from the center of each cluster to each depots. Addresses which are close to depot 1, will belong to cluster 1 and which are close to depot 2, will belong to cluster 2. We then applied the

CWSA into those two groups. The un-clustered data will be collected and then we cluster again using the same procedure.

Scheme 3: Clustering - CWSA. The number of clusters (k) is the ceiling of total demand of the day divided by the vehicle capacity. We then clustered the data into k classes. We chose the maximum demand in each cluster as the starting point of the CWSA. The un-clustered data will be collected and then we cluster again using the same procedure. Finally, we classified those cluster into two groups, based on the closest distance to the each depot.

III. RESULTS

We applied the proposed algorithm to routing delivery orders of a company which distributed mineral water in Surabaya – Indonesia. The mineral water is in gallon size (19lt), every vehicle has capacity 130 bottles. For this paper, we will presented a small example with 246 addresses, 1347 total demands. Therefore, we need ceiling (1347/130) = 11 vehicles to distribute those demands. The company has two depots, let say Depot 1 and Depot 2. The distribution of demands is depicted in Fig.3. It can be seen that most of the customers (25%) ordered two bottles mineral water, 17% ordered one bottle of mineral water, and the rest is more than 2 bottles. The distribution of mineral water based on the area is depicted in Fig. 4

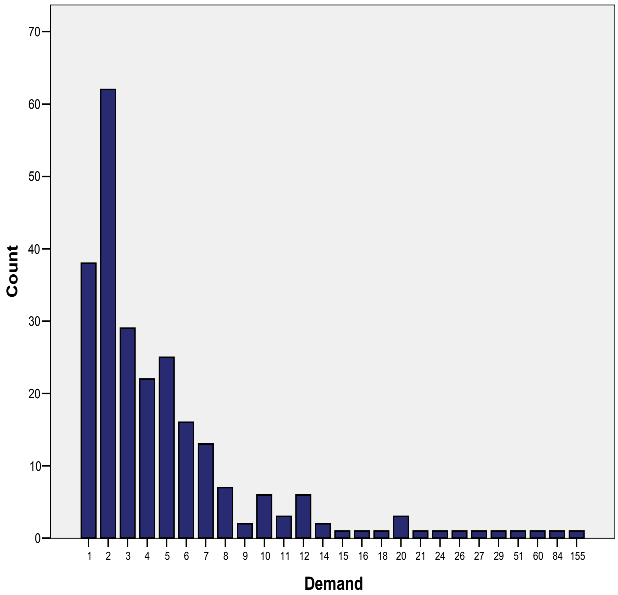

Fig.3 Mineral water's demand distribution

Scheme one: CWAS results is depicted in Fig. 5. In this scheme the traveling route between addresses in a cluster is closed. The total Euclidean distance is 9008.9147Km. Total vehicle used is 11. In this example, 94% of the vehicle capacities is occupied. The route moment is 53645.901. Using notebook with Intel® Core™ i7-2640M CPU@2.80GHZ, RAM 4.0GB, the computational time for running this data is 3070.49 seconds. Number of customers and delivered ordered for every route is tabulated in Table I.

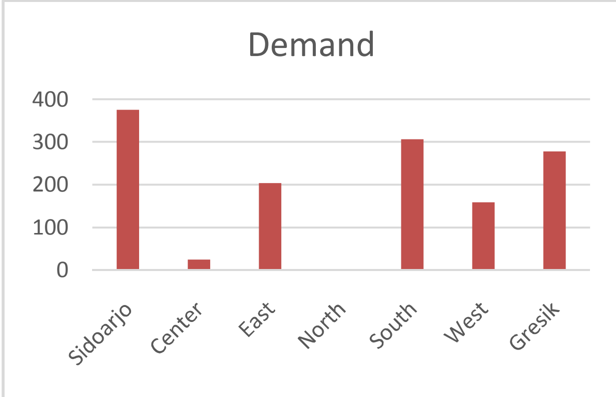

Fig. 4 Mineral Water distribution based on the area

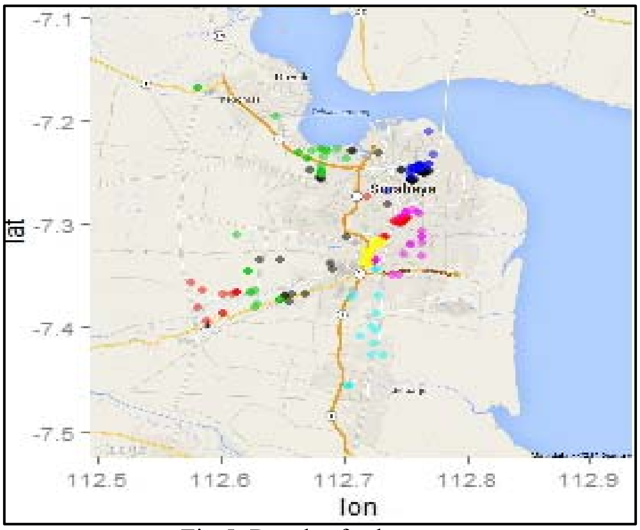

Fig 5. Result of scheme one

TABLE I
Route Distribution of Scheme One

Depot	Route	#Addresses	#Delivering order
1	1	31	129
	2	28	105
	3	26	125
	4	27	125
	5	15	130
2	6	26	126
	7	34	119
	8	20	112
	9	10	121
	10	12	126
	11	16	129

Scheme two: Depots clustering – CWSA. In the first step of this scheme we clustered the addresses into two clusters (see Fig 6), and then on each cluster we applied CWSA. Using the same notebook we got the computational time for running this data is 155.88 seconds. Total of moment is 62044.816 and total of distance is 10752.88 km. The final result of this scheme is depicted in Fig. 7. Number of customers and delivered ordered for every route is tabulated in Table II. In this scheme we need 12 vehicles instead 11, and there are 10 addresses with 43 bottles of mineral water should be delivered from depot 1, 1 address with 15 bottles of mineral water should be delivered from depot 2. Even though the computational time of this scheme is faster than the scheme one, but the result is not optimal. This can be happened since we clustered the addresses into two depots at the first step, so that the CWSA is already restricted at certain areas from the beginning.

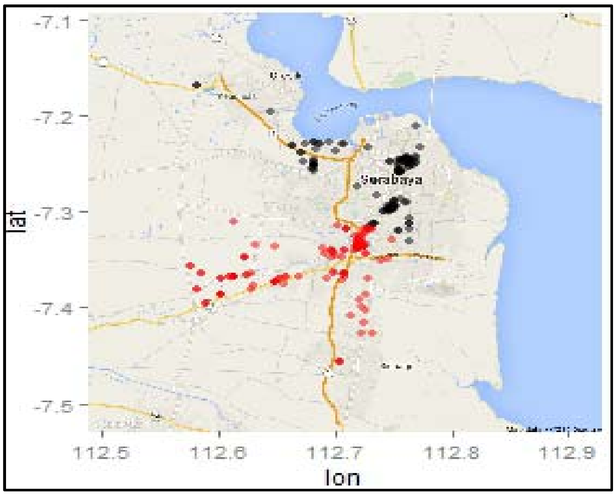

Fig 6. The first step clustering of scheme two

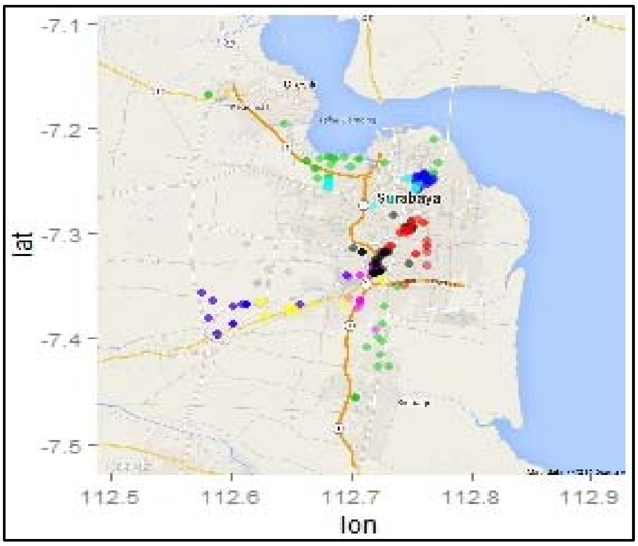

Fig 7. Result of scheme two

Scheme three: clustering – CWSA result is depicted in Fig. 8. The total Euclidean distance is 9013.395 Km, 94% of the vehicle capacities is occupied, and the route moment is 58300.553. Using the same notebook, the computational time for running the data using this scheme is less than 60 seconds. Obviously, we can see that the computational time of the proposed algorithm is faster than the original algorithm. Moreover, we also can see that, 7/11 vehicles delivered the orders in full capacity, i.e. 130 bottles (see Table III). This scheme is much better than the scheme two. The computational time is much faster than the scheme one and scheme two. The result of scheme three is closed to scheme one, which is the original Clarke and Wright Saving Algorithm. So we can deduce that the proposed algorithm that is the third scheme can improve the CWSA in term of computational time as well as the distribution of the delivering order.

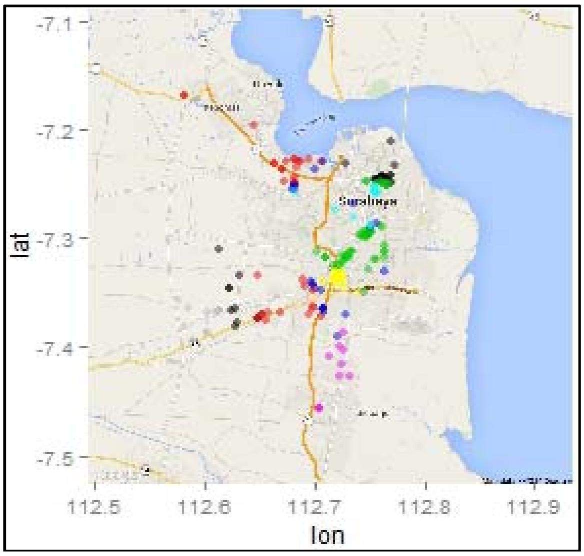

Fig 8. Result of scheme three

TABLE II
Route Distribution of Scheme Two

Depots	Route	#Addresses	#Delivering order
	1	10	43
	2	32	130
1	3	31	130
	4	31	130
	5	22	129
	6	38	120
	7	20	130
	8	12	130
2	9	34	130
	10	1	15
	11	12	130
	12	14	130

TABLE III
Route Distribution of Scheme Three

Depot	Route	#Addresses	#Delivering order
	1	29	130
	2	25	130
1	3	38	130
	4	23	130
	5	16	81
	6	10	112
	7	34	120
2	8	14	130

9	13	130
10	15	130
11	28	124

IV DISCUSSION

This algorithm is proposed for solving a real problem. It is work well for the current situation in the company, i.e. the company only have a single type of fleet, and also the fleets are enough to serve all customers order. However, when the fleets are no longer able to serve the customers, then a single fleet for multi-trips model should be proposed for solving the problem. The algorithm also needs improvement when the company use different types of fleets, so the heterogeneous fleet vehicle routing problems for multi-trips should be modelled to encounter the new tasks.

V. CONCLUSION

Compare to the classical Clarke-Wright Saving Algorithm, the proposed algorithm Adjusted Clustering – CWSA (scheme three) is well performed in term of computational time and the distribution of delivered orders. In term of traveling distance, the proposed algorithm is closed the classical algorithm. So far, the proposed algorithm only works for a single type of products, and the limit of the vehicles capacity is the same for all vehicles. These restrictions should be released in the future research.

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