

Map Coloring Problem

PROJECT DOCUMENTATION REPORT

PROGRAMMING PROJECT 3

ITCS 6150 - Intelligent Systems

DEPARTMENT OF COMPUTER SCIENCE

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PROBLEM FORMULATION

A k -coloring of a map is an assignment of k colors, one to each country, in such a way that no two countries sharing a border have the same color. This problem can be translated to a constraint graph. A coloring of a graph G assigns a color to each vertex of G , with the restriction that two adjacent vertices never have the same color. The chromatic number of G , written $\chi(G)$, is the smallest number of colors needed to color G . In this project, we will experiment with map coloring techniques and compare the observed results in the context of USA and Australia map.

Below are the algorithms implemented to solve map coloring problem:

1. **Backward Tracking/ Depth First Search** – It is a general algorithmic technique that considers searching every possible combination in order to solve a computational problem. Here, each state is assigned a color from its domain and checked if it is matching with the color of its neighbor. If it matches, then a different color is assigned, and this is done recursively till all the states have colors assigned which is different from its neighbors. In case there are no more colors to assign a state, then we backtrack to the previously assigned state and change its color and algorithm is executed again.
2. **Forward Checking** - In this algorithm, we remove all the conflicting domain from the neighbor of the state to which a color is assigned. Once, an empty domain is encountered we backtrack to the previous step and rerun the algorithm. This helps in early detection of failure.
3. **Forward Checking and propagation through singleton domains**– If during forward checking you reduce a domain to size 1 (singleton domain), then assume the assignment of the singleton domain and repeat forward checking from this variable.

Various heuristics are used to choose the variable to be assigned while implementing the algorithm.

1. **Minimum Remaining Value (MRV)** – This heuristic chooses a state having minimum legal domain values.
2. **Degree Heuristic** – In case, there is a tie between 2 variables/ state while using MRV, we choose a state having maximum neighbors.
3. **Least Constraint Value (LCV)** - Once a domain is chosen, choose its value as the one that rules out the fewest choices for neighboring variables.

Chromatic Number - The Chromatic number of a map is the minimum number of colors required to color it. To find the chromatic number of a map using our code, we initially start with maximum color required and keep reducing it till we encounter a failure.

In case of Australia map, South Australia has 5 neighbors and so we start with 5 colors and keep reducing the number of colors.

The algorithm successfully runs and colors the map, till 3 colors and fails for two colors

From this we conclude that the chromatic number of Australia is 3.

Running same experiment on US map, we find that the chromatic number is 4.

PROGRAM STRUCTURE

2.1 Functions and Procedures

The code implements the various algorithms of Constraint Satisfaction Problems with and without Constraint Satisfaction problems.

forward_checking(failed_index, state_domain_dict, neighbor_state_dict, is_backtrack=False, parent_domain_to_be_chosen=None) : The function implements forward checking algorithm with backtracking using recursion.

read_csv(): The function reads the state names and its neighbors .

forward_checking_lcv(failed_index, state_domain_dict, neighbor_state_dict, is_backtrack=False, parent_domain_to_be_chosen=None): The function implements forward checking algorithm with backtracking using recursion. And the selection of colors for states is by least constraining value.

forward_checking_with_heuristic_mrv(failed_index, state_domain_dict, neighbor_state_dict, is_backtrack=False, parent_domain_to_be_chosen=None):

: The function implements function implements forward checking algorithm with backtracking using recursion. And the selection of states order is by minimum remaining variables and tie breaker algorithm is degree heuristic.

def backtracking(failed_index, state_domain_dict, neighbor_state_dict, is_backtrack=False, parent_domain_to_be_chosen=None): The function implements depth first search algorithm with backtracking using recursion.

def forward_checking_singleton(failed_index, state_domain_dict, neighbor_state_dict, is_backtrack=False, parent_domain_to_be_chosen=None): : The function implements forward checking algorithm with backtracking and singleton algorithm using recursion.

def singleton(state_domain_dict, neighbor_state_dict, key, visited_node): The function checks for neighbours having singleton domain and returns it.

def lcv(key_state, state_domain, neighbour_list, selected_dict): The function calculates the least constraining value from the domain given and returns it.

def minimum_remaining_value(state_domain_dict, neighbor_state_dict, visited_node, key):
: The function checks for the state with minimum values in domain and in case of multiple states with same minimum values, returns the state with the most neighbours.

2.2 Global and Local variables

Global variables:

state_list - list of map states read from csv file
neighbor_state_dict - dictionary mapping states to its neighbours
state_domain_dict – dictionary mapping states to colors
visited_node – list of visited nodes
selected_dict – dictionary mapping states to the assigned colors
selected_backtrack_node– list of states to be backtracked
out- str, where all assignments are stored
back_track_count - int, count of backtracks made
failed - boolean, flag to recognise backtrack

Local variables:

key– str, state explored in a particular loop
failed_index– int, the index where the color domain is empty and failure occurs
parent_domain – color, color chosen for assignment in particular loop
previous_selected_parent_domain – str, the last color chosen for the first state in list
singleton_domain_list – list, list of colors with states of singleton domain
num_neighbour_colors – number of colors in neighbour states to calculate lcv
map_name - str, variable to choose Us map or Australia map
selected_root_domain_values– list, colors already chosen for the first state

Experiment Results

a. Without Heuristic - USA Map

Order of States	Backtracking	Forward Checking	Forward Checking with Singleton
['WA', 'OR', 'CA', 'NV', 'ID', 'MT', 'WY', 'UT', 'AZ', 'NM', 'CO', 'ND', 'SD', 'NE', 'KS', 'OK', 'TX', 'MN', 'IA', 'MO', 'AR', 'LA', 'MS', 'AL', 'FL', 'GA', 'SC', 'NC', 'TN', 'KY', 'VA', 'DE', 'MD', 'WV', 'OH', 'IN', 'IL', 'WI', 'MI', 'PA', 'NJ', 'CT', 'RI', 'NY', 'VT', 'NH', 'MA', 'ME', 'AK', 'HI', 'DC']	Backtrack Count- 8 No of Assignments -163 No of Colors used – 4 Time taken - 1.99 ms	Backtrack Count-10 No of Assignments - 231 No of Colors used -4 Time taken - 3.98 ms	Backtrack Count-0 No of Assignments - 51 No of Colors used – 4 Time taken – 0.99 ms
['HI', 'SC', 'WV', 'OR', 'TN', 'VA', 'KS', 'LA', 'RI', 'MA', 'WY', 'NY', 'IN', 'OH', 'CO', 'MI', 'AL', 'UT', 'MD', 'WA', 'SD', 'AZ', 'TX', 'ND', 'MT', 'NV', 'OK', 'NJ', 'DE', 'VT', 'FL', 'DC', 'PA', 'CA', 'KY', 'AK', 'NM', 'MO', 'IA', 'WI', 'MS', 'AR', 'NC', 'ID', 'ME', 'GA', 'MN', 'IL', 'NH', 'NE', 'CT']	Backtrack Count- 0 No of Assignments - 110 No of Colors used – 4 Time taken - 1.05 ms	Backtrack Count - 0 No of Assignments - 51 No of Colors used – 4 Time taken - 1.00 ms	Backtrack Count - 0 No of Assignments - 51 No of Colors used – 4 Time taken – 0.99 ms
['MA', 'OK', 'SC', 'LA', 'WY', 'WA', 'AR', 'IL', 'FL', 'TX', 'MS', 'AL', 'MN', 'VT', 'IA', 'MT', 'AZ', 'NV', 'UT', 'OH', 'RI', 'KY', 'NM', 'NY', 'WV', 'WI', 'DC', 'ME', 'KS', 'PA', 'MI', 'MD', 'HI', 'NJ', 'NH', 'CT', 'NE', 'NC', 'IN', 'ND', 'GA', 'MO', 'ID', 'VA', 'AK', 'CO', 'TN', 'OR', 'DE', 'SD', 'CA']	Backtrack Count- 60 No of Assignments - 684 No of Colors used – 4 Time taken - 3.98 ms	Backtrack Count- 33 No of Assignments - 385 No of Colors used – 4 Time taken – 4.98 ms	Backtrack Count- 20 No of Assignments - 195 No of Colors used – 4 Time taken – 2.99 ms

b. Without Heuristic - Australia Map

Order of States	Backtracking	Forward Checking	Forward Checking with Singleton
['au-nsw', 'au-vic', 'au-ta', 'au-wa', 'au-nt', 'au-qld', 'au-sa']	Backtrack Count- 0 No of Assignments - 14 No of Colors used – 4 Time taken – 0.0 ms	Backtrack Count- 0 No of Assignments - 7 No of Colors used -4 Time taken – 0.0 ms	Backtrack Count-0 No of Assignments - 7 No of Colors used – 4 Time taken – 0.0 ms
['au-wa', 'au-nt', 'au-qld', 'au-sa', 'au-vic', 'au-nsw', 'au-ta']	Backtrack Count- 0 No of Assignments - 11 No of Colors used – 4 Time taken – 0.0 ms	Backtrack Count - 0 No of Assignments - 7 No of Colors used – 4 Time taken – 0.0 ms	Backtrack Count - 0 No of Assignments - 7 No of Colors used – 4 Time taken – 0.0 ms
['au-qld', 'au-sa', 'au-wa', 'au-ta', 'au-nt', 'au-nsw', 'au-vic']	Backtrack Count- 0 No of Assignments - 12 No of Colors used – 4 Time taken – 0.0 ms	Backtrack Count- 0 No of Assignments - 7 No of Colors used – 4 Time taken – 0.0 ms	Backtrack Count- 0 No of Assignments - 7 No of Colors used – 4 Time taken – 0.0 ms

c. With Heuristic - USA Map

Order of States	Forward Checking with MRV	Forward Checking with LCV	Forward Checking with Singleton - MRV	Forward Checking with Singleton - LCV
['WA', 'OR', 'CA', 'NV', 'ID', 'MT', 'WY', 'UT', 'AZ', 'NM', 'CO', 'ND', 'SD', 'NE', 'KS', 'OK', 'TX', 'MN', 'IA', 'MO', 'AR', 'LA', 'MS', 'AL', 'FL', 'GA', 'SC', 'NC', 'TN', 'KY', 'VA', 'DE', 'MD', 'WV', 'OH', 'IN', 'IL', 'WI', 'MI', 'PA', 'NJ', 'CT', 'RI', 'NY', 'VT', 'NH', 'MA', 'ME', 'AK', 'HI', 'DC']	Backtrack Count-0 No of Assignments - 51 No of Colors used -4 Time taken – 3.75 ms	Backtrack Count-0 No of Assignments - 51 No of Colors used -4 Time taken – 2.12 ms	Backtrack Count-0 No of Assignments - 51 No of Colors used -4 Time taken – 4.22 ms	Backtrack Count-0 No of Assignments - 51 No of Colors used – 4 Time taken – 2.02 ms
['HI', 'SC', 'WV', 'OR', 'TN', 'VA', 'KS', 'LA', 'RI', 'MA', 'WY', 'NY', 'IN', 'OH', 'CO', 'MI', 'AL', 'UT', 'MD', 'WA', 'SD', 'AZ', 'TX', 'ND', 'MT', 'NV', 'OK', 'NJ', 'DE', 'VT', 'FL', 'DC', 'PA', 'CA', 'KY', 'AK', 'NM', 'MO', 'IA', 'WI', 'MS', 'AR', 'NC', 'ID', 'ME', 'GA', 'MN', 'IL', 'NH', 'NE', 'CT']	Backtrack Count-0 No of Assignments - 51 No of Colors used -4 Time taken – 4.06 ms	Backtrack Count - 0 No of Assignments - 51 No of Colors used – 4 Time taken – 2.22 ms	Backtrack Count-0 No of Assignments - 51 No of Colors used -4 Time taken – 3.03 ms	Backtrack Count - 0 No of Assignments - 51 No of Colors used – 4 Time taken – 2.85 ms
['MA', 'OK', 'SC', 'LA', 'WY', 'WA', 'AR', 'IL', 'FL', 'TX', 'MS', 'AL', 'MN', 'VT', 'IA', 'MT', 'AZ', 'NV', 'UT', 'OH', 'RI', 'KY', 'NM', 'NY', 'WV', 'WI', 'DC', 'ME', 'KS', 'PA', 'MI', 'MD', 'HI', 'NJ', 'NH', 'CT', 'NE', 'NC', 'IN', 'ND', 'GA', 'MO', 'ID', 'VA', 'AK', 'CO', 'TN', 'OR', 'DE', 'SD', 'CA']	Backtrack Count-0 No of Assignments - 51 No of Colors used -4 Time taken – 3.57 ms	Backtrack Count-0 No of Assignments - 51 No of Colors used -4 Time taken – 3.06 ms	Backtrack Count-0 No of Assignments - 51 No of Colors used -4 Time taken – 2.92 ms	Backtrack Count-0 No of Assignments - 51 No of Colors used -4 Time taken – 2.06 ms

d. With Heuristic - Australia Map

Order of States	Forward Checking with MRV	Forward Checking with LCV	Forward Checking with Singleton - MRV	Forward Checking with Singleton - LCV
['au-nsw', 'au-vic', 'au-ta', 'au-wa', 'au-nt', 'au-qld', 'au-sa']	Backtrack Count- 0 No of Assignments - 7 No of Colors used -4 Time taken – 0.0 ms	Backtrack Count- 0 No of Assignments - 7 No of Colors used -4 Time taken – 0.0 ms	Backtrack Count- 0 No of Assignments - 7 No of Colors used -4 Time taken – 0.0 ms	Backtrack Count- 0 No of Assignments - 7 No of Colors used -4 Time taken – 0.0 ms
['au-wa', 'au-nt', 'au-qld', 'au-sa', 'au-vic', 'au-nsw', 'au-ta']	Backtrack Count- 0 No of Assignments - 7 No of Colors used -4 Time taken – 0.0 ms	Backtrack Count- 0 No of Assignments - 7 No of Colors used -4 Time taken – 0.0 ms	Backtrack Count- 0 No of Assignments - 7 No of Colors used -4 Time taken – 0.0 ms	Backtrack Count- 0 No of Assignments - 7 No of Colors used -4 Time taken – 0.0 ms
['au-qld', 'au-sa', 'au-wa', 'au-ta', 'au-nt', 'au-nsw', 'au-vic']	Backtrack Count- 0 No of Assignments - 7 No of Colors used -4 Time taken – 0.0 ms	Backtrack Count- 0 No of Assignments - 7 No of Colors used -4 Time taken – 0.0 ms	Backtrack Count- 0 No of Assignments - 7 No of Colors used -4 Time taken – 0.0 ms	Backtrack Count- 0 No of Assignments - 7 No of Colors used -4 Time taken – 0.0 ms

