BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
K. K. BIRLA GOA CAMPUS
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Lab/Practice Sheet-2

Course No. MATH F424 Course title: Applied Stochastic Process

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Attempt the following theoretically, also, verify the same using programming (in R you can update library with packages "ConvergenceConcept" for convergence and "tseries" for stationary process.

1. Let U be $U(0,1)$ random variable and define $X_n = I_{[m/2^k,(m+1)/2^k]}(U)$ where $n = 2^k + m$ for $k \geq 1$ and with $0 \leq m < 2^k$. Check if $X_n \xrightarrow{\text{a.s.}} 0$ and $X_n \xrightarrow{p} 0$.

2. Let $X_1, X_2, \cdots$ be i.i.d. continuous random variables with $N(2,9)$ distribution. Define $Y_n = (0.5)^n X_n$, $n = 1, 2, \cdots$. Also define T_n and A_n to be the sum and the average, respectively, of $Y_1, Y_2, \cdots, Y_n$. Check if $Y_n \xrightarrow{p} 0$, $T_n \xrightarrow{p} 2$, $A_n \xrightarrow{p} 0$ and $T_n \xrightarrow{d} N(2,3)$.

3. Let $X_1, X_2, \cdots$ be $U(0,1)$ i.i.d. random variables. Define $M_n = \max\{X_1, \cdots, X_n\}$. Prove that $M_n \xrightarrow{p} 1$, and $M_n \xrightarrow{a.s.} 1$.

4. Suppose we choose at random n numbers from interval $[0,1]$ with uniform distribution. Let X_n be the random variable describing n^{th} choice, then show that X_n obeys WLLN.

5. Generate sample path of Gaussian white noise $GWN(0,1)$, check if it is stationary time series.

6. Generate the sample of $Y_t = \beta_0 + \beta_1 t + \epsilon_t$, $\epsilon_t \sim GWN(0, \sigma^2)$, $t = 0, 1, \cdots$ for $\beta_0 = 0$, $\beta_1 = 0.1$ and $\sigma^2 = 1$. Check if these are stationary time series. Also, choose a simple transformation $Z_t = Y_t - \beta_1 t$, check is resulting time series is stationary.

Solve the following theoretically:

7. Consider a process $\{X_n\}_{n\geq 1}$, where each X_n have Poisson distribution with parameter n, $n = 1, 2, \cdots$. Find the limiting distribution of $Y_n = \frac{X_n - n}{\sqrt{n}}$. (Hint: find mgf of Y_n as $n \to \infty$.)

8. Consider a process $\{X_n\}_{n\geq 1}$, where each X_n having CDF

$$F_{X_n}(x) = \begin{cases} 0 & -\infty < x < 0 \\ 1 - \left(1 - \frac{x}{n}\right)^n & 0 \leq x < n \\ 1 & n \leq x < \infty \end{cases}.$$

Find the limiting distribution of $\{X_n\}_{n\geq 1}$.

9. Let Y is $U(-\frac{1}{2}, \frac{1}{2})$ and $X_n = (-1)^{n+1} Y$, $n = 1, 2, \cdots$. Check if $X_n \xrightarrow{d} Y$ and $X_n \xrightarrow{p} Y$.

10. Let X be a random variable and $X_n = X + Y_n$, where $E(Y_n) = \frac{1}{n}$ and variance of Y_n is $\frac{\sigma^2}{n}$, where $\sigma > 0$ is a constant. Show that $X_n \xrightarrow{p} X$. (Hint: it is easy to show $Y_n \xrightarrow{p} 0$.)

11. Let X_n be $U(-\frac{1}{n}, \frac{1}{n})$ and $X = 0$ be a constant. Check if $X_n \xrightarrow{d} X$ and if $X_n \xrightarrow{p} X$.

12. Prove or disprove, a process with stationary and independent increments is strongly stationary.

13. Prove or disprove, strongly stationary process is always weakly stationary.

14. X_n is an i.i.d. random process with mean $E(X_n) =$ and variance $Var(X_n) = \sigma^2$. Find auto-covariance of the process.

15. Let X_n represent n^{th} Bernoulli trial with probability of success p. Define $Y_n = \sum_{i=1}^{n} X_i$ for $n \in \mathbb{N}$, prove that $\{Y_n\}_{n\geq 1}$ is a second order process with stationary independent increments. Find mean and covariance function of $\{Y_n\}_{n\geq 1}$.

16. On time $[0, \infty)$, let events are occurring according to Poisson process $\{Y_t\}_{t\geq 0}$ with rate λ. If T_n is arrival time for the n^{th} event, then show that the process $\{T_n\}_{n\geq 1}$, is second order process with stationary independent increments. Also find the mean and covariance function.

17. Prove or disprove, strictly stationary process is i.i.d..

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