BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
K. K. BIRLA GOA CAMPUS
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Lab/Practice Sheet-3

Course No. MATH F424 Course title: Applied Stochastic Process

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Use simulation to find the answer of following question. Also, compare the simulation result with theoretical(true) answer.

1. The Canadian forest fire weather index is widely as means of to estimate the risk of wildfire. The Ontario Ministry of Natural Resources uses the index to classify each day's risk of forest fire as either nil, low, moderate, high or extreme. Transition probability matrix for the five state Markov chain for the daily changes in the index is given as:

$$P = \begin{array}{c} \\ Nil \\ Low \\ Moderate \\ High \\ Extreme \end{array} \begin{array}{c} \begin{array}{ccccc} Nil & Low & Moderate & High & Extreme \end{array} \\ \begin{pmatrix} .575 & .118 & .172 & .109 & .026 \\ .453 & .243 & .148 & .123 & .033 \\ .104 & .343 & .367 & .167 & .019 \\ .015 & .066 & .318 & .505 & .096 \\ .000 & .060 & .149 & .567 & .224 \end{pmatrix} \end{array}$$

Using R find the long term likelihood of risk for a typical day in the early summer.

2. University administrators have developed a Markov model to simulate graduation rates at their school. Student might drop out, repeat a year or move on to the next year. Student have a 3% chance of repeating a year. First years and second years have a 6% of dropping out. For third years and fourth years the drop out rate is 4%. Simulate the long term probability that a new student graduates. Also, model it as absorbing chain and find the probability of drop and graduating for each year students.

3. After work, angel goes to the gym and either does aerobics, weights, yoga or goes for jogging. Each day Angle decides her workout routine based on what she did the previous day according to the Markov transition matrix:

$$P = \begin{array}{c} \\ Aerobics \\ Jogging \\ Weights \\ Yoga \end{array} \begin{array}{c} \begin{array}{cccc} Aerobics & Jogging & Weights & Yoga \end{array} \\ \begin{pmatrix} .1 & .2 & .4 & .3 \\ .4 & 0 & .4 & .2 \\ .3 & .3 & 0 & .4 \\ .2 & .1 & .4 & .3 \end{pmatrix} \end{array}$$

Simulate the long term probability that she goes for jogging. Compare this with stationary distribution.

4. Consider a Markov chain with transition probability matrix

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 1/3 & 1/3 & 1/3 \end{pmatrix} \end{array}$$

Simulate μ_{11}.

5. A biased coin comes up head with probability 2/3 and tails with probability 1/3. The coin is repeatedly flipped. Simulate to find the total numbers average number of flips needed, until the pattern HTHTH first appears. Compare it with theoretical result.

6. Some winter days in Minnesota it seems like the snow will never stop. A Minnesotan's view of winter might be described by the following transition matrix for a weather Markov chain, where r, s and c denote rain, snow and clear, respectively.

$$P = \begin{array}{c} \\ r \\ s \\ c \end{array} \begin{array}{c} \begin{array}{ccc} r & s & c \end{array} \\ \begin{pmatrix} 0.2 & 0.6 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.1 & 0.6 & 0.3 \end{pmatrix} \end{array}$$

Find the fraction of days on which it will snow.

7. Danny's daily lunch choices are modeled by a Markov chain with transition probability matrix

$$P = \begin{array}{c} \\ Burrito \\ Falafel \\ Pizza \\ Sushi \end{array} \begin{array}{c} \begin{array}{cccc} Burrito & Falafel & Pizza & Sushi \end{array} \\ \begin{pmatrix} 0.0 & 0.5 & 0.5 & 0.0 \\ 0.5 & 0.0 & 0.5 & 0.0 \\ 0.4 & 0.0 & 0.0 & 0.6 \\ 0.0 & 0.2 & 0.6 & 0.2 \end{pmatrix} \end{array}$$

On Sunday, Danny chooses lunch uniformly at random. Find the probability that he chooses sushi on the following Wednesday and Friday, and pizza on Sunday.

8. Let $X_0, X_1, \cdots$ be a Markov chain with transition matrix P. Let $Y_n = X_{3n}$, for $n = 0, 1, 2, \cdots$. Show that $Y_0, Y_1, \cdots$ is a Markov chain and exhibit its transition matrix.

9. A Markov chain has following transition probability matrix

$$P = \begin{pmatrix} p & 1-p & 0 & 0 \\ (1-p)/2 & p & (1-p)/2 & 0 \\ 0 & (1-p)/2 & p & (1-p)/2 \\ 0 & 0 & 1-p & p \end{pmatrix}$$

$0 < p < 1$, find the stationary distribution.

10. Gambler's ruin: A gambler starts with \$2 and plays a game where the chance of winning each round is 60%. The gambler either wins or loses \$1 on each round. The game stops when the gambler either gains \$5 or goes bust. Model this problem as absorbing Markov chain and find the probability that the gambler is eventually ruined.

11. A coin is flipped repeatedly until three heads in a row appears. What is the expected number of flips needed ? Use Markov chain model to answer this.