BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
K. K. BIRLA GOA CAMPUS
First Semester 2019-2020
Lab/Practice Sheet-1

Course No. MATH F424 Course title: Applied Stochastic Process

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Use simulation to find the answer of following question. Also, compare the simulation result with theoretical(true) answer.

1. Simulate flipping three fair coins and counting the number of heads $X$. (a) Use simulation to estimate $P(X = 1)$ and $E(X)$. (b) Modify the experiment to allow for a biased coin where $P(Heads) = 3/4$.

2. Cards are drawn from a standard deck, with replacement, until an ace appears. Find the mean and variance of number of cards required.

3. The time until a bus arrives has an exponential distribution with mean 30 minutes. (a) Use command rexp() to simulate probability that the bus arrives in the first 20 minutes. (b) Use the command pexp() to compare the exact probability.

4. Gambler's ruin: On each day a fair coin is tossed and the gambler wins \$1 if head occurs, or loose \$1 if tails occurs. The gambler stops when he reaches $\$n (n > k)$ or losses all his money. Use Monte carlo simulation to find the probability that the gambler will eventually loose.

5. Every day Bob goes to the pizza shop and picks a topping- pepper, pepperoni, pineapple, or chicken- uniformly at random. On the day that Bob first pics pineapple, using simulation find the expected number of prior days in which he picked pepperoni.

6. Ellen's insurance will pay for a medical expense subject to a \$100 deductible. Assume that the amount of expense is exponentially distributed with mean \$500. Use simulation to find the expectation and standard deviation of the payout.

7. A number $X$ is uniformly distributed on $(0, 1)$. If $X = x$, then $Y$ is picked uniformly on $(0, x)$. Find the variance of $Y$.

8. On any day, the number of accidents on the highway has the Poisson distribution with parameter $\Lambda$. The parameter $\Lambda$ varies from day to day and is itself a random variable. Find mean and variance of the number of accidents per day when $\Lambda$ is uniformly distributed on $(0, 3)$.

Solve the following theoretically:

9. A restaurant receives $N$ customers per day, where $N$ is a random variable with mean 200 and standard deviation 40. The amount spent by each customer is normally distributed with mean \$15 and standard deviation \$3. The amounts that customers spend are independent of each other and independent of $N$. Find the mean and standard deviation of the total amount spent at the restaurant per day.

10. Let $X$ be a Poisson random variable with $\lambda = 3$. Find $E(X/X > 2)$.

11. From the definition of conditional expectation given an event, show that $E(I_B/A) = P(B/A)$.

12. Let $X_1, X_2, \cdots$ be an i.i.d. sequence of random variables with common mean $\mu$. Let $S_n = X_1 + \cdots + X_n$, for $n \geq 1$. (a) Find $E(S_m/S_n)$ for $m \leq n$. (b) Find $E(S_m/S_n)$ for $m > n$.

13. Find the PGF of Poisson random variable with parameter $\lambda$. Also use the same to find mean and variance for the distribution.

14. If a random variable $X$ has Poisson distribution with parameter $\lambda$, show that

$$E(X(X-1)(X-2)\cdots(X-k)) = \lambda^{k+1}.$$

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