

# Bayes classifier

In statistical classification the **Bayes classifier** minimizes the probability of misclassification<sup>[1]</sup>

## Definition

Suppose a pair  $(X, Y)$  takes values in  $\mathbb{R}^d \times \{1, 2, \dots, K\}$ , where  $Y$  is the class label of  $X$ . This means that the conditional distribution of  $X$ , given that the label  $Y$  takes the value  $r$  is given by

$$X \mid Y = r \sim P_r \text{ for } r = 1, 2, \dots, K$$

where " $\sim$ " means "is distributed as", and where  $P_r$  denotes a probability distribution.

A classifier is a rule that assigns to an observation  $X=x$  a guess or estimate of what the unobserved label  $Y=r$  actually was. In theoretical terms, a classifier is a measurable function  $C : \mathbb{R}^d \rightarrow \{1, 2, \dots, K\}$ , with the interpretation that  $C$  classifies the point  $x$  to the class  $C(x)$ . The probability of misclassification, or risk, of a classifier  $C$  is defined as

$$\mathcal{R}(C) = P\{C(X) \neq Y\}.$$

The Bayes classifier is

$$C^{\text{Bayes}}(x) = \operatorname{argmax}_{r \in \{1, 2, \dots, K\}} P(Y = r \mid X = x).$$

In practice, as in most of statistics, the difficulties and subtleties are associated with modeling the probability distributions effectively—in this case,  $P(Y = r \mid X = x)$ . The Bayes classifier is a useful benchmark in statistical classification

The excess risk of a general classifier  $C$  (possibly depending on some training data) is defined as  $\mathcal{R}(C) - \mathcal{R}(C^{\text{Bayes}})$ . Thus this non-negative quantity is important for assessing the performance of different classification techniques. A classifier is said to be consistent if the excess risk converges to zero as the size of the training data set tends to infinity

## See also

- Naive Bayes classifier

## References

1. Devroye, L.; Györfi, L. & Lugosi, G. (1996) *A probabilistic theory of pattern recognition* Springer. ISBN 0-3879-4618-7.

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