MAE 263F HW4*

Axial Response of a Helical Spring in Discrete Elastic Rod

Nathan Ge[1]

***Abstract*— In this homework, you will model a helical coil as a 3D Discrete Elastic Rod (DER) and quantify its axial stiffness from dynamic relaxations to steady states.**

I. PROBLEM STATEMENT

Helix parameters

Wire diameter	$d = 0.002\,\text{m}$
Mean coil diameter	$D = 0.04\,\text{m}$
Helix radius	$R = D/2$
Pitch per turn	$p = d$
Number of turns	$N = 5$
Helix axis	global z
Axial length	$L_{\text{axial}} = Np$
Arc length per turn	$L_{\text{turn}} = p\sqrt{(2\pi R)^2 + p^2}$
Total contour length	$L = NL_{\text{turn}}$

Material parameters

Young's modulus	$E = 10\,\text{MPa}$
Poisson ratio	$\nu = 0.5$
Shear modulus	$G = \dfrac{E}{2(1+\nu)}$

Section properties (circular wire)

Area	$A = \dfrac{\pi d^2}{4}$
Second moment	$I = \dfrac{\pi d^4}{64}$
Polar moment	$J = \dfrac{\pi d^4}{32}$

Properties that impact convergence

Density	$rho = 1200$
Time step size	$dt = 0.02$
Viscosity	$\nu = 0.00002$

Discretization. Construct the rest configuration of the helix using at least 10 nodes per turn (i.e., $\geq 10N$ nodes).

Boundary Conditions. Clamp the first two nodes and the first edge material angle (first seven DOFs of the discrete elastic rod). These DOFs remain fixed for all time steps.

Loading & Characteristic Force. Apply a constant tensile force F at the last node, aligned with the $\hat{z}$ direction, such that the helix extends axially. The magnitude of this load is related to the characteristic bending force:

$$F_{\text{char}} = \frac{EI}{L^2}.$$

*This work was not supported by any organization

[1]Nathan Ge is a student in the Department of Mechanical and Aerospace Engineering, University of California, Los Angeles, 405 Hilgard Avenue, Los Angeles, CA 90095 nzge@g.ucla.edu

Tip Displacement. Let the end node position be $(x_{\text{end}}(t), y_{\text{end}}(t), z_{\text{end}}(t))$. Define the axial tip displacement $\delta_z(t) = z_{\text{end}}(t) - z_{\text{end}}(0)$, so that $\delta_z(0) = 0$. Integrate in time long enough for transients to decay and for $\delta_z(t)$ to reach a steady value.

II. SINGLE LOAD LEVEL

The value of $\delta_z(t)$ changes by less than one percent over a one-second interval. Once you determine that the motion has settled according to your chosen rule, mark the corresponding steady value δ_z^* on your time–displacement plot.

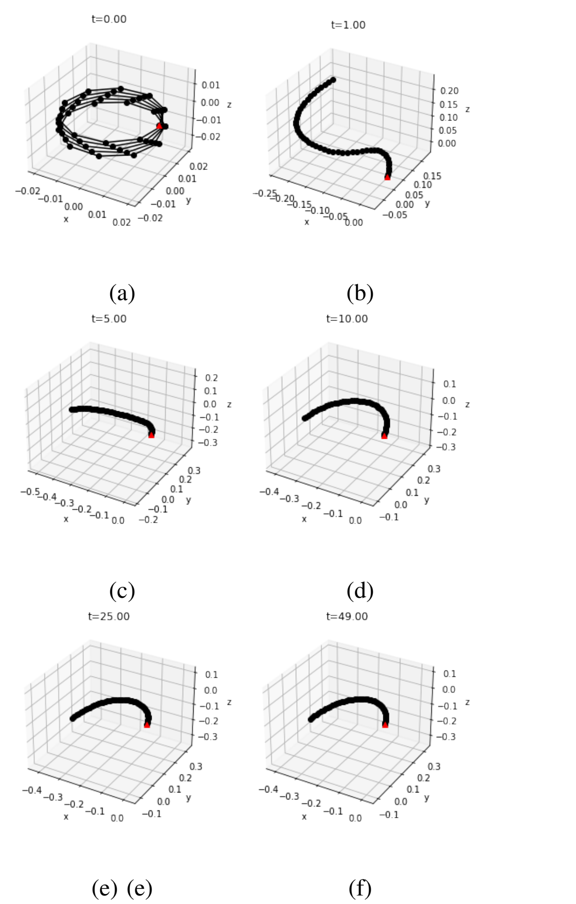

Fig. 1: Five snapshots of the helix at (a) t = 0 s, (b) t = 1 s, (c) t = 5 s, (d) t = 10 s, (e) t = 25 s, (f) t = 49 s").

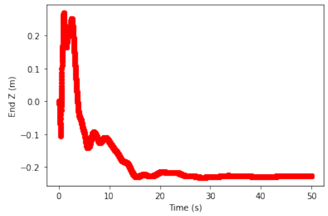

Fig. 2: Plot of end node z axis displacement versus time.

III. Force Sweep and Linear Fit

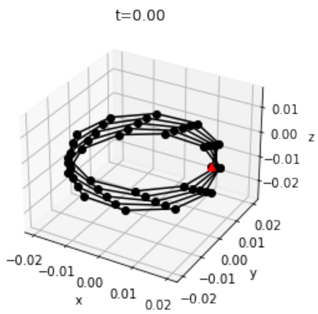

Fig. 3: Plot of F versus δ_z^* with a zero-intercept best-fit line.

IV. Diameter Sweep vs. Textbook Trend

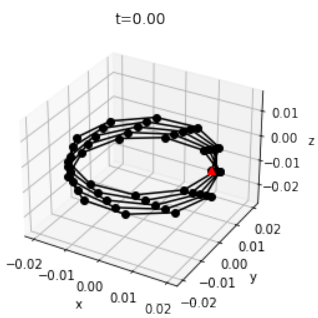

Fig. 4: Plot of k versus $\frac{Gd^4}{8ND^3}$ with a slope-1 reference line.

Acknowledgment

The author thanks Professor Khalid Jawed for guidance and instruction throughout the MAE 263F Soft Robotics course.