MAE 263F HW2

Euler Beam Theory

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I. PROBLEM STATEMENT

Task: Write a solver that simulates the beam as a function of time for $0 \leq t \leq 1$ s using an **implicit method**. Use $\Delta t = 10^{-2}$ s and $N = 50$ for the simulation.

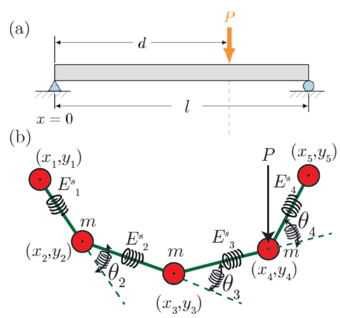
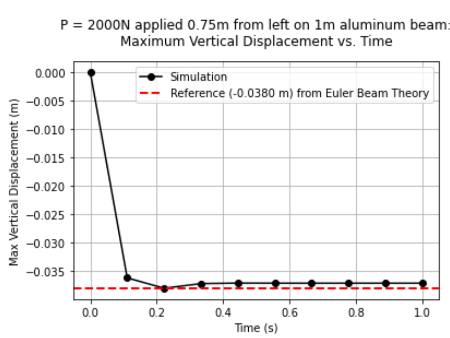


Fig. 1: (a) Elastic beam and (b) its discrete representation.

II. TASKS

1) *Plot the maximum vertical displacement, y_{max}, of the beam as a function of time. Depending on your coordinate system, y_{max} may be negative. Determine if y_{max} eventually reaches a steady value. Compare the accuracy of your simulation against the theoretical prediction from Euler beam theory:*

$$y_{\max} = \frac{Pc(l^2 - c^2)^{3/2}}{9\sqrt{3}EIl}, \quad \text{where } c = \min(d, l - d) \tag{1}$$



The beam's maximum vertical displacement, $y_{\max}$, eventually reaches a steady-state value in the simulation.

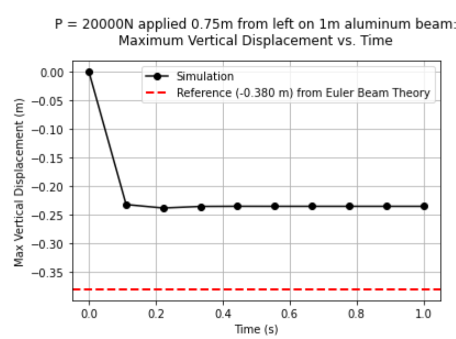
- Theoretical prediction from Euler–Bernoulli beam theory: $y_{\max} \simeq -0.038045$ m
- Simulation steady-state val: $y_{\max} \simeq -0.0371$ m
- Relative steady-state error:

$$E_{\text{rel}} = 100 \times \frac{y_{\text{sim, ss}} - y_{\text{theory}}}{y_{\text{theory}}} \simeq 2.46\%$$

The simulation results show that the beam's maximum deflection eventually reaches a steady-state value of -0.03711 m, while the theoretical deflection predicted by Euler–Bernoulli beam theory is -0.038045 m. The relative steady-state error of approximately 2.46% indicates that the simulation accurately captures the static response of the beam.

2) *Discuss the benefit of your simulation over the Euler beam theory prediction. Consider a higher load P = 20,000 N such that the beam undergoes large deformation. Compare the simulated result against the theoretical prediction in Eq. (1). Euler beam theory is only valid for small deformations, whereas the implicit simulation should be able to handle large deformation. Create a plot of P (20 N $\leq P \leq$ 20,000 N) versus y_{max} using data from both simulation and beam theory, and quantify the value of P where the two solutions begin to diverge.*

To investigate large deformations, consider a higher load P = 20,000 N.



- Theoretical prediction from Euler–Bernoulli beam theory: $y_{\max} \simeq -0.3805$ m
- Simulation steady-state val: $y_{\max} \simeq -0.2352$ m
- Relative steady-state error:

$$E_{\text{rel}} = 100 \times \frac{y_{\text{sim, ss}} - y_{\text{theory}}}{y_{\text{theory}}} \simeq 38.19\%$$

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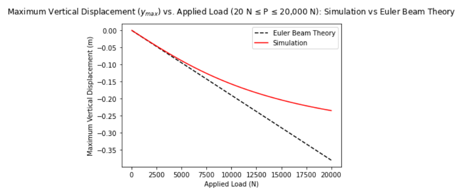
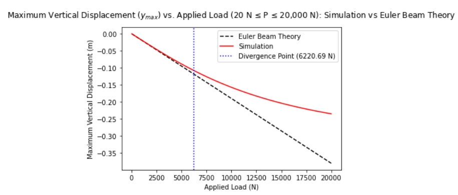
The large relative steady-state error of approximately 38.19% indicates a significant deviation from the theoretical value, showing that the assumptions of the Euler–Bernoulli beam model are no longer valid for large deformations.



Point of Significant Departure: The simulation predicts a displacement of -0.107887 m at $P = 6220.69$ N, deviating from the theoretical value by more than 1 cm. This marks the point where the simulation begins to depart from Euler–Bernoulli beam theory predictions significantly.



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