MAE 263F HW5*

Deformation of a Clamped Thin Beam Using a Plate Model

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***Abstract*— In this homework, you I simulate the static deformation of a thin, rectangular beam under its own weight using the *discrete plate model* and compare your numerical results against the classical Euler-Bernoulli beam theory prediction.**

I. PROBLEM STATEMENT

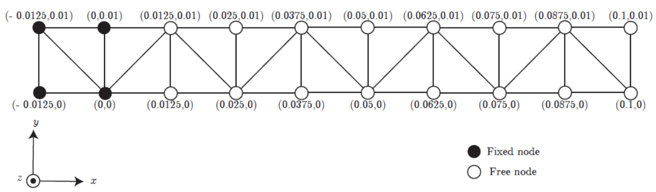


Fig. 1: Beam Mesh

The objective is to simulate the static deformation of the beam across time until the numerical solver converges. I will then report the steady displacement and include a plot of the tip displacement as a function of time.

Beam Geometry

Length	$l = 0.1$ m
Width	$w = 0.01$ m
Thickness	$h = 0.002$ m

Material Properties

Young's Modulus	$Y = 1.07 \times 10^7$ Pa
Density	$\rho = 1000$ kg/m^3
Gravity	$g = 9.81$ m/s^2

Section Properties

Cross-sectional Area	$A = wh$
Second Moment of Area	$I = \frac{wh^3}{12}$
Distributed Load	$q = \rho A g$

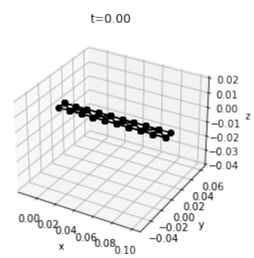
Mesh. Use the mesh shown in the figure above. The length of the free portion of the plate is $l = 0.1$ m. The four leftmost nodes are fixed to enforce the clamped boundary condition. Gravity acts in the negative z-direction.

Boundary Condition. The left edge of the plate (at $x = 0$) is fully clamped: all displacement and rotation components are fixed.

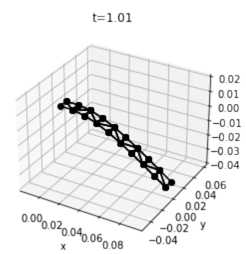
Plate Model Simulation. Model the domain $l \times w$ as a thin plate of thickness h. Compute the static deformation by solving the equilibrium equations until the configuration reaches steady state. Let the tip displacement at the centerline of the free edge $x = l$ be

$$\delta_{\text{plate}}(t) = z_{\text{tip}}(t) - z_{\text{tip}}(t = 0).$$

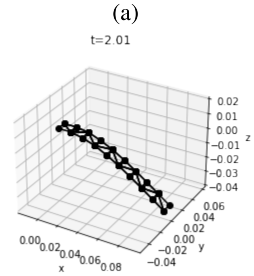
II. COMPARISON WITH EULER–BERNOULLI BEAM THEORY



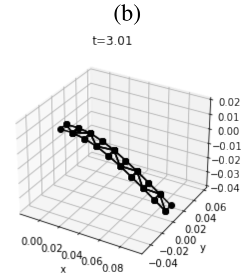
(a)



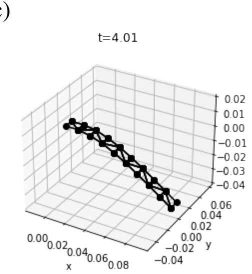
(b)



(c)



(d)



(e) (e)

Fig. 2: Five snapshots of the helix at (a) t = 0 s, (b) t = 1.01 s, (c) t = 2.01 s, (d) t = 3.01 s, (e) t = 4.01 s.

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A. *Theory*

For a cantilever beam under uniform load q (N/m), the Euler–Bernoulli tip displacement is $\delta_{\text{EB}} = \frac{ql^4}{8YI}$. Theoretical prediction δ_{EB}: $\delta_{\text{EB}} = \frac{ql^4}{8YI} = -0.03675m$

B. *Simulation*

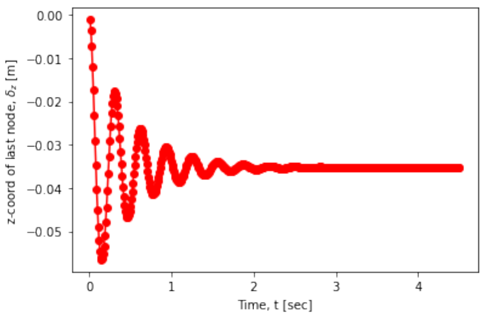

Fig. 3: δ_{plate} vs. time t

Convergence criteria:

```
if ((timeStep*dt) % 0.5 == 0) and (ctime != 0):
    endZ_1secondbefore = endZ[timeStep -int(1.0/dt)]
    endZ_diff = np.abs(endZ[timeStep] - endZ_1secondbefore)
    endZ_percent_diff = (endZ_diff/np.abs(endZ[timeStep])) * 100
    if endZ_percent_diff < 0.1:
      print("End Z position has stabilized.")
      final_timeStep = timeStep
      break
```

The program makes comparisons to a time point 0.5 seconds in the past every 0.5 seconds. If the values are within 0.1% of each other, convergence is reached.

Numerical prediction δ_{plate}: -0.035207m

C. *Comparing Theory and Simulation*

$$\begin{aligned}
\text{Normalized difference} &= \left| \frac{\delta_{\text{plate}} - \delta_{\text{EB}}}{\delta_{\text{EB}}} \right| \\
&= \left| \frac{-0.035207 \text{ m} - (-0.03675 \text{ m})}{-0.03675 \text{ m}} \right| \\
&\approx 0.042 \approx 4.2\%
\end{aligned}$$

The resultant end deflection value produced by numerical simulation appears to be appreciably similar to the theoretical prediction.

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