MAE 263F HW4*

Axial Response of a Helical Spring in Discrete Elastic Rod

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***Abstract*— In this homework, you will model a helical coil as a 3D Discrete Elastic Rod (DER) and quantify its axial stiffness from dynamic relaxations to steady states.**

I. PROBLEM STATEMENT

Helix parameters

Wire diameter	$d = 0.002\,\mathrm{m}$
Mean coil diameter	$D = 0.04\,\mathrm{m}$
Helix radius	$R = D/2$
Pitch per turn	$p = d$
Number of turns	$N = 5$
Helix axis	global z
Axial length	$L_{\text{axial}} = Np$
Arc length per turn	$L_{\text{turn}} = p\sqrt{(2\pi R)^2 + p^2}$
Total contour length	$L = NL_{\text{turn}}$

Material parameters

Young's modulus	$E = 10\,\mathrm{MPa}$
Poisson ratio	$\nu = 0.5$
Shear modulus	$G = \dfrac{E}{2(1+\nu)}$

Section properties (circular wire)

Area	$A = \dfrac{\pi d^2}{4}$
Second moment	$I = \dfrac{\pi d^4}{64}$
Polar moment	$J = \dfrac{\pi d^4}{32}$

Properties that impact convergence

Density	$rho = 1200$
Time step size	$dt = 0.02$
Viscosity	$\nu = 0.00002$

Discretization. Construct the rest configuration of the helix using at least 10 nodes per turn (i.e., $\geq 10N$ nodes).

Boundary Conditions. Clamp the first two nodes and the first edge material angle (first seven DOFs of the discrete elastic rod). These DOFs remain fixed for all time steps.

Loading & Characteristic Force. Apply a constant tensile force F at the last node, aligned with the $\hat{z}$ direction, such that the helix extends axially. The magnitude of this load is related to the characteristic bending force:

$$F_{\text{char}} = \frac{EI}{L^2}.$$

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Tip Displacement. Let the end node position be $(x_{\text{end}}(t), y_{\text{end}}(t), z_{\text{end}}(t))$. Define the axial tip displacement $\delta_z(t) = z_{\text{end}}(t) - z_{\text{end}}(0)$, so that $\delta_z(0) = 0$. Integrate in time long enough for transients to decay and for $\delta_z(t)$ to reach a steady value.

II. SINGLE LOAD LEVEL

The value of $\delta_z(t)$ changes by less than one percent over a one-second interval. Once you determine that the motion has settled according to your chosen rule, mark the corresponding steady value δ_z^* on your time–displacement plot.

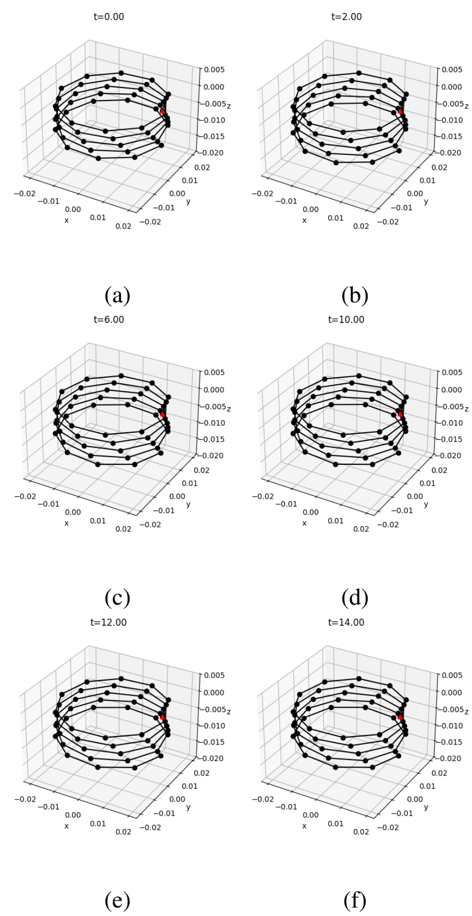

Fig. 1: Five snapshots of the helix at (a) t = 0 s, (b) t = 2 s, (c) t = 6 s, (d) t = 10 s, (e) t = 12 s, (f) t = 14 s").

The convergence of my numerical simulation was determined by comparing the current end position to the end position 1 second past at 1 second intervals. If the end position is 0.1% within each other, convergence is reached. The helix end position stabilized at around $-0.0015\mathrm{m}$.

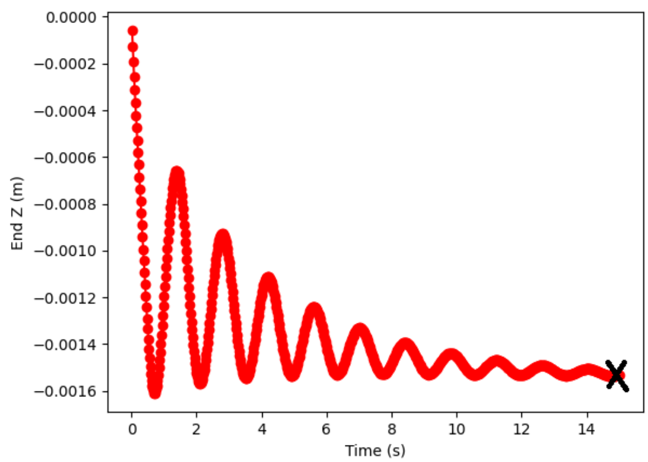

Fig. 2: Plot of end node z-axis displacement versus time. Stability point is marked by an X

III. Force Sweep and Linear Fit

A logarithmic force $0.1F_{char}$ to $10F_{char}$ sweep from was applied to determine the axial stiffness of the helical spring. For each force level, the DER simulation was integrated to steady state, defined by less than 1% change in displacement over one second. The resulting force–displacement data were fit with a zero-intercept linear model $F = k\delta z$ over the small-displacement regime to extract the linear stiffness k.

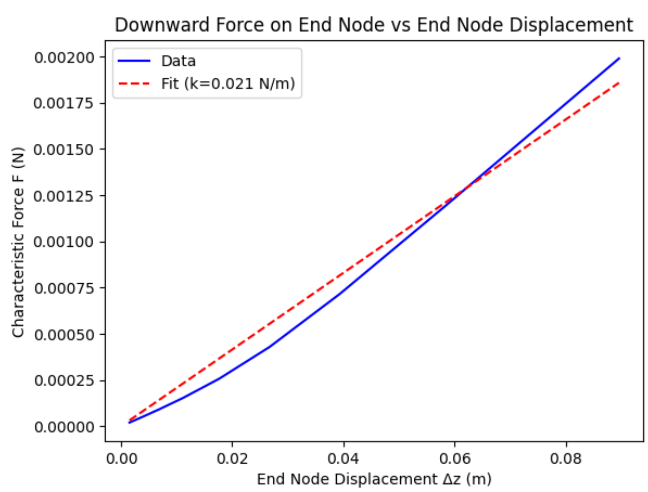

Fig. 3: Plot of F versus δ_z^* with a zero-intercept best-fit line.

Stiffness k (N/m): 0.020733

IV. Diameter Sweep vs. Textbook Trend

The effect of helix diameter on axial stiffness was studied by varying D from 0.01 m to 0.05 m while holding all other parameters constant. For each diameter, the stiffness k was obtained using the same force sweep procedure. The simulated stiffness values were compared against the classical prediction $k_{text} = \frac{Gd^4}{(8ND^3)}$. The results show that stiffness decreases with increasing diameter, consistent with the expected inverse cubic dependence. Although quantitative agreement is not expected due to differing modeling assumptions, the DER simulations capture the correct qualitative stiffness–diameter trend.

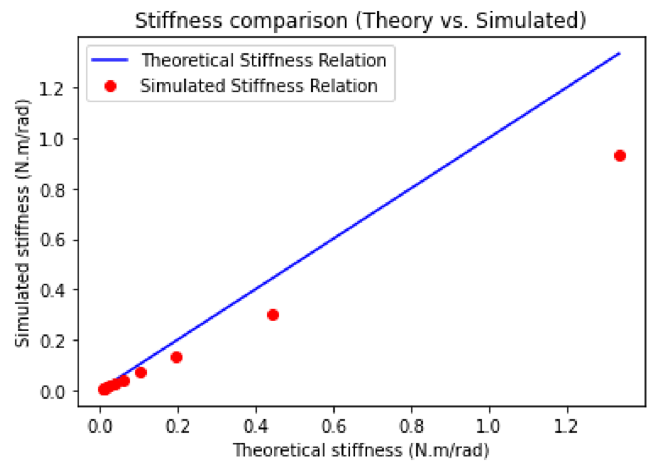

Fig. 4: Plot of k versus $\frac{Gd^4}{8ND^3}$ with a slope-1 reference line.

Diameter(m)	k [N/m]
0.01000	0.93563
0.01444	0.30350
0.01889	0.13394
0.02333	0.07046
0.02778	0.04153
0.03222	0.02652
0.03667	0.01797
0.04111	0.01275
0.04556	0.00939
0.05000	0.00713

TABLE I: Diameter vs. corresponding stiffness parameter k

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