STAT2371 Assignment

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Question 1

Suppose that two independent binomial random variables X_1 and X_2 are observed where X_1 has a Binomial(n, p) distribution and X_2 has a Binomial($2n, p$) distribution. You may assume that n is known, whereas p is an unknown parameter. Define two possible estimators of p

$$\hat{p}_1 = \frac{1}{3n}(X_1 + X_2) \quad \textbf{and} \quad \hat{p}_2 = \frac{1}{2n}(X_1 + 0.5X_2).$$

(a) Show that both of the estimators $\hat{p}_1$ and $\hat{p}_2$ are unbiased estimators of p.

$$\begin{aligned}
E(\hat{p}_1) &= \frac{1}{3n}(E(X_1) + E(X_2)) \quad \text{applying expected value linearity} \\
&= \frac{1}{3n}(n \cdot p + 2n \cdot p) \quad \text{applying the binomial expectation formula} \\
&= p \\
\text{bias}(\hat{p}_1, p) &= E(\hat{p}_1) - p = 0.
\end{aligned}$$

Similarly,

$$\begin{aligned}
E(\hat{p}_2) &= \frac{1}{2n}(E(X_1) + 0.5E(X_2)) \\
&= \frac{1}{2n}(n \cdot p + 0.5 \cdot 2n \cdot p) \\
&= p \\
\text{bias}(\hat{p}_2, p) &= E(\hat{p}_2) - p = 0.
\end{aligned}$$

Hence $\hat{p}_1$ and $\hat{p}_2$ are unbiased estimators of p.

(b) Find Var($\hat{p}_1$) and Var($\hat{p}_2$).

$$\begin{aligned}
Var(\hat{p}_1) &= \frac{1}{9n^2}(Var(X_1) + Var(X_2)) \quad \text{applying the formula for independent case} \\
&= \frac{1}{9n^2}(np(1-p) + 2np(1-p)) \quad \text{applying the binomial variance formula}
\end{aligned}$$

$$= \frac{p(1-p)}{3n}.$$

Similarly,

$$\begin{aligned} Var(\hat{p}_2) &= \frac{1}{4n^2}(Var(X_1) + 0.5^2 Var(X_2)) \\ &= \frac{1}{4n^2}(np(1-p) + 0.25 \cdot 2np(1-p)) \\ &= \frac{3p(1-p)}{8n}. \end{aligned}$$

(c) Show that both estimators are consistent estimators of p.