Private Experimentation and Sender's Commitment*

Wentao Fu †　　Meng Sun‡　　Naibao Zhao§

May 27, 2019

Abstract

This article considers, in a bad news setting, how an uninformed sender persuades a receiver to deliver a high reward by selectively disclosing evidence that is acquired sequentially and privately. We show the roles of two critical values: the participation limit restricts the scope of unraveling and over-experimentation equilibria; and the over-experimentation threshold marks the boundary between them. When the sender can commit to a certain number of disclosed successes to prove his quality, it is not always optimal for him to commit to a smaller number if only the over-experimentation equilibria exist. Instead, he commits to the smallest credible number as long as other types of equilibria exist.

Keywords: Private Experimentation; Learning; Disclosure; Over-Experimentation

JEL Codes: C73; D82; D83

*We thank Antonio Cabrales, Jacob Glazer, Ilan Kremer, Motty Perry, Peter Tsigaris, and Lei Zhang for their constructive feedback. All errors are solely our own responsibility.

†School of Economics, Sichuan University, Chengdu, China. *Email*: wentao.fu@scu.edu.cn.

‡School of Business and Economics, Thompson Rivers University, Kamloops, Canada. *Email*: msun@tru.ca.

§Research Institute of Economics and Management, Southwestern University of Finance and Economics, Chengdu, China. *Email*: nzhao@swufe.edu.cn.

1 Introduction

We study a situation in which an uninformed sender (he) selectively discloses the evidence from experimentation and an uninformed receiver (she) delivers a reward according to the sender's disclosure. Since the experimental stage is the sender's private information and the experiments are sequentially conducted, this situation is called private experimentation.[1] The current literature on private experimentation discusses new features in the equilibria due to the private strategic learning/experimentation, such as Henry (2009), Felgenhauer and Schulte (2014), etc. However, the discussion on the role of the sender's commitment is missing, which becomes the main question in this study.

In our model, the sender's type is initially unknown, which is either good or bad. He learns his type through the costly experiments, in which a good type always succeeds and only a bad type fails with a certain probability. Since the experimental stage is the sender's private information, he sequentially decides whether to conduct more experiments after observing previous results. Once experiments are stopped, he discloses the obtained results strategically and receives a reward from the receiver afterwards. The receiver who has no commitment power delivers the reward based on her belief, whereas the sender always prefers a higher one.

Our work builds upon but differs from the pioneer work by Felgenhauer and Schulte (2014). They propose a simple cut-off rule when the receiver makes a binary acceptance decision and the experiment features a symmetric information structure. Their model fails to answer how the receiver interprets the disclosed evidence and delivers the associated reward. More importantly, the receiver's binary decision leaves little room for the sender's commitment.

To answer the question proposed before, we firstly characterise the equilibria when the commitment is absent. Two critical values determine the existence and the properties of the equilibria are identified (Proposition 1). The first critical value is the participation limit, which equals to the largest number of disclosed successes where the expected total cost of experiments can be covered by the sender's prior expected value. This value determines the set of equilibria with a positive number of disclosed successes. If the total cost is higher than the prior expected

[1] Consider an example in the job market. A candidate learns his ability from sitting in exams, undertaking internships, acquiring professional certificates, etc. The results from learning are also potential evidence listed in his CV, which would affect the salary package offered by the employer. Since the employer does not observe the learning stage directly, the candidate strategically learns and discloses evidence. Thus, the employer is skeptical about the potential hidden evidence and then the informativeness of the CV is undermined.

value, the uninformed sender is better off by deviating not to experiment.

The second critical value is the over-experimentation threshold. It is measured by the largest number of successes where conditional expected value is smaller than the expected cost of acquiring one success by a bad type sender. This value determines the boundary between unraveling equilibria and over-experimentation equilibria. On the equilibrium path, when the number of disclosed successes by a potential good type is lower than the over-experimentation threshold, a bad type sender always find it is too costly to continue experimentation. Thus, the receiver can easily distinguish a bad type from a potential good type once observing different numbers of disclosed successes. However, when it is higher than the threshold, a bad type sender whose failure arrives late continues experimentation as the extra benefit is now higher than the extra expected cost. Thus, on the equilibrium path, the bad type sender who fails early still reveals his true type with less successes, but the bad type sender who fails late now pools with the potential good type by disclosing the same number of successes.

We then introduce the role of the sender's commitment and compare the results relative to no commitment scenario (Proposition 4). We show that the sender's commitment must lie in the set of equilibria that is proposed above due to the concern of credibility, and the optimal commitment is the one which maximises his continuation value. Ideally, the sender prefers to commit to no-experiment strategy, which will give him the highest continuation value. This is the unique equilibrium when the experimental stage is public information (Proposition 3). However, if the restrictions off-the-equilibrium path are not satisfied, he always has incentives to deviate and disclose some successes, which makes such commitment not credible. Thus, the sender comprises to commit to a positive number of disclosed successes. As a result, the sender is always better off with the power of commitment.

More importantly, when only the set of over-experimentation equilibria exists, it is not always optimal for the sender to commit to a smaller number of disclosed successes. This is because the compounded effect of increasing the committed number is ambiguous in over-experimentation equilibria: the expected cost before over-experimentation increases but the expected cost of over-experimentation decreases. Meanwhile, as long as the no-experiment equilibrium or the unraveling equilibria exist, the sender prefers committing to the smallest credible number. This is because, in these two types of equilibria, the compounded effect always raises the cost of experimentation and the fear of failure deters the sender's willingness to experiment.

2 Literature Review

This paper relates to the literature on private experimentation. Henry (2009) shows that the sender tends to conduct more experiments in private information environment. However, in his model, the sender cannot stop experiments at interim stage once the first experiment is conducted. Felgenhauer and Schulte (2014) assume that, when the experiment has a symmetric information structure, the receiver makes a binary acceptance decision and the reward for the sender after being accepted is exogenously given. Also, the discussion of the sender's commitment is absent in their work. Our paper differs in endogenising the agent's reward and focusing on the scenario in which the experiment has an asymmetric information structure. These differences are critical. It not only allows us to identify sender's deviation incentives and examine how the receiver's belief evolves due to her skeptical thinking, but also leaves the room for the discussion on the role of the sender's commitment. Fu (2018) considers the receiver's optimal contract and presents the multi-step reward schemes. Contrast to him, we show that the multi-step scheme collapse to a cut-off reward when the sender has the power of the commitment instead. As far as we know, this is the first paper which discusses the sender's commitment in the sequential private experimentation.

This work relates to literature on strategic experimentation. Bergemann and Hege (2005) show the optimal way to finance an innovative project without full commitment, Henry and Ottaviani (2018) show that the receiver free rides on the sender's experiments when results are public information, and Halac and Kremer (2017) show that inefficiency is increased due to the sender's career concern in a bad news setting. Bhaskar and Mailath (2019) discuss the ratchet effect with learning/experimentation when only the spot contracts can be offered. In their work, the receiver can use the timing of when they observe success to determine the monetary transfer: this is a key difference from our model as well as the models in private experimentation.

This work also relates to literature on information disclosure and persuasion. Rayo and Segal (2010) and Kolotilin (2015) focus on the sender's optimal mechanism. Kamenica and Gentzkow (2011) find the optimal way for the sender to design the structure of the experiment, and Bergemann et al. (2015) consider a monopolist who can design the experiment and set the selling price. They all focus on public experimentation, where results can be publicly observed. In contrast, Our work mainly focuses on the private case, and also compared the difference between public and private case. Glazer and Rubinstein (2004, 2006) and Hart et al. (2017) analyse the commitment in evidence games where the sender's set of hard evidence is exogenously given. Contrast to them, the disclosed evidence is endogenously acquired and the type of the sender is statically learned at the experimental stage in our model.

There is a growing literature on bayesian persuasion, which allows the sender to design the information structure in the experiment, such as Kamenica and Gentzkow (2011), Bergemann et al. (2015), Felgenhauer and Loerke (2017), etc. Contrast to them, the information structure of the experiment is fixed in our model, and thus the concavification approach in their work cannot be applied.

3 Model

There is an uninformed risk-neutral sender (he) whose type m is initially unknown, where $m \in \{M, 0\}$. With probability p_0, the sender's type is good and the associated value is M, where $p_0 \in (0,1)$ and $M > 0$; with probability $1 - p_0$, his type is bad and the associated value is normalised as 0.

The sender learns about his type by sequential private experimentation. The experiment has the properties of a bad new setting: in each experiment, after paying a constant cost c, a good type sender always succeeds but a bad type succeeds only with probability $1-\theta$, where $c > 0$ and $\theta \in (0,1)$.[2] Given n^s successes and n^f failures are acquired, the sender makes a binary decision, $S(n^s, n^f)$, on wether to continue experimentation or to stop and disclose experimental results, where $n^s, n^f \in \mathbb{N}^+$, $S(n^s, n^f) \in \{0, 1\}$ and $S(n^s, n^f) = 1$ denotes stopping experimentation. Since the experiments and results are private information, the sender selectively discloses k^s successes and k^f failures after experimental stage, where $0 \leq k^s \leq n^s$ and $0 \leq k^f \leq n^f$.[3]

The receiver, who has no commitment power, wants to match the reward to the sender's type. Based on the disclosed results, she delivers a reward that equals to the sender's conditional expected value, $a(k^s, k^f) = \mathbb{E}\left[m | k^s, k^f\right]$, where $a : \mathbb{N} \times \mathbb{N} \to \mathbb{R}^+$.[4] Thus, the sender's payoff function is $u(k^s, k^f; n^s, n^f) = a(k^s, k^f) - (n^s + n^f)c$. The timing of the game is summarized as

[2] This information structure helps to identify the over-experimentation incentives explicitly.

[3] Experimental results are hard evidence.

[4] This is a standard result in the evidence game by Hart et al. (2017). This is aslo the same reward as that in a competitive market.

follows:

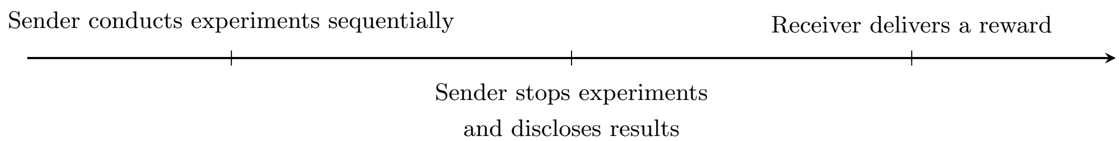

Figure 1: Timing of the Model

History and Equilibrium. The sender's private history, $h^A = (n^s, n^f)$, consists of the number of successes and failures which are acquired. His poster belief $\mathrm{P}^A(n^s, n^f)$ is updated by Bayes' rule:

$$\mathrm{P}^A(n^s, n^f) = \Pr(m = M | n^s, n^f) = \begin{cases} \frac{p_0}{p_0 + (1-p_0)(1-\theta)^{n^s}} \equiv p_{n^s} & \text{for} \quad n^f = 0 \\ 0 & \text{for} \quad n^f > 0 \end{cases} \tag{3.1}$$

Since only a bad type sender can fail in an experiment, he becomes more optimistic on that his type is good if no failures occur, where $p_{n^s} \geq p_0$ in (3.1). For simplicity, the sender's posterior is written as P^A for short. Now consider the sender's continuation value of experimentation,

$$\begin{aligned} \mathrm{U}\left(\mathrm{P}^A\right) = S(n^s, n^f) a(k^s, k^f) + \Big(1 - S(n^s, n^f)\Big) \Big\{ \left(1 - \mathrm{P}^A\right) \theta \, \mathrm{U}\Big(\mathrm{P}^A(n^s, n^f + 1)\Big) \\ + \left[\mathrm{P}^A + \left(1 - \mathrm{P}^A\right)(1 - \theta)\right] \mathrm{U}\Big(\mathrm{P}^A(n^s + 1, n^f)\Big) \Big\} \end{aligned} \tag{3.2}$$

Given n^s successes and n^f failures are acquired, the sender receives $a(k^s, k^f)$ if he stops and selectively discloses the results, in which case $S(n^s, n^f) = 1$. Alternatively, he needs to pay the experimental cost c if he conducts one more experiment. In this case, on the one hand, he fails and gains $\mathrm{U}\left(\mathrm{P}^A(n^s, n^f + 1)\right)$ with probability $\left(1 - \mathrm{P}^A\right)\theta$; on the other hand, he succeeds and gains the value $\mathrm{U}\left(\mathrm{P}^A(n^s + 1, n^f)\right)$ with probability $\mathrm{P}^A + \left(1 - \mathrm{P}^A\right)(1 - \theta)$.

Similarly, the public history, $h^P = (k^s, k^f)$, consists of the number of successes and failures, which are disclosed by the sender. Thus, the receiver's poster belief is $\mathrm{P}^P(k^s, k^f)$. For simplicity, it is written as P^P for short and the associated becomes $a(k^s, k^f) = \mathbb{E}\left[m | k^s, k^f\right] = \mathrm{P}^P M$.

We restrict attention to the set of Perfect Bayesian Equilibria (PBE) in the pure strategy. A candidate equilibrium is described as $\Big\{ \left\{ S^*\left(n^s, n^f\right) \right\}_{n^s \geq 0, n^f \geq 0}, (k^{s*}, k^{f*}), a^*(k^{s*}, k^{f*}), \mathrm{P}^{A*}, \mathrm{P}^{P*} \Big\}$, which satisfies the following conditions:

- *Sequential Rationality*: the sender's strategy $\Big(\left\{ S^*\left(n^s, n^f\right) \right\}_{n^s \geq 0, n^f \geq 0}, (k^{s*}, k^{f*}) \Big)$ maximises $\mathrm{U}\left(\mathrm{P}^A\left(n^s, n^f\right)\right)$ and the receiver's strategy satisfies $a^*(k^{s*}, k^{f*}) = \mathrm{P}^{P*} M$;

- *Belief Consistency*: when $(k^s, k^f) = (k^{s*}, k^{f*})$, (k^s, k^f) is "on the equilibrium" and P^P is determined by Bayes' rule; otherwise, (k^s, k^f) is "off the equilibrium" which requires $\mathrm{P}^P \in [0, 1]$.

4 Equilibrium analysis

Firstly, consider the sender's information disclosure strategy in the disclosure stage. Since a success is the positive evidence for a potential good type, the sender would optimally disclose all of the acquired successes $k^{s*} = n^s$. Moreover, in the bad news setting, the sender learns that his type is bad as long as one failure occurs, regardless the number of successes that he has acquired. Thus he receives zero if the failures are disclosed. Notice that the experimental results are the sender's private information, so the sender can hide the failures and claim that he has not failed. Even if the receiver is skeptical about this claim, the sender can receive a weakly higher reward relative to zero. These are the standard results in information disclosure literature, in which the strategy of disclosing successes and hiding failures is called *sanitization strategy*.[5] This argument is summarized in Claim 1.

Claim 1. *$k^{s*} = n^s$ and $k^{f*} = 0$.*

Proof. Since $\mathrm{P}^P(n^s, k^f) \geq \mathrm{P}^P(k^s, k^f)$ for $k^s \leq n^s$, $a(n^s, k^f) = \mathrm{P}^P(k^s, k^f)M \geq \mathrm{P}^P(k^s, k^f)M = a(k^s, k^f)$. Therefore, $k^{s*} = n^s$. Similarly, for $k^f > 0$, $\mathrm{P}^P(k^s, 0) \geq 0 = \mathrm{P}^P(k^s, k^f)$, so $a(k^s, 0) = \mathrm{P}^P(k^s, 0)M \geq 0 = \mathrm{P}^P(k^s, k^f)M = a(k^s, k^f)$. Thus, $k^{f*} = 0$. □

Based on the sender's optimal *sanitization strategy*, we move backward to analyse the experimental stage. Since the receiver does not observe the experiments or the results before disclosure, a bad type sender, who has already failed, still can continue conducting experiments and collecting more successes and then hide failures in disclosure stage. This *over-experimentation* behaviour would undermine the informativeness of disclosed successes. Taking such behaviour into account, a potential good type sender, who has not failed yet, has to conduct more experiments and disclosure more successes in order to separate himself and achieve a higher reward if it is profitable.

[5] See Shin (2003).

Lemma 1. *In any equilibrium, a potential good type sender has a higher incentive to conduct more experiments than a bad type does.*

Proof. See Appendix A. □

Lemma 1 suggests that a potential good type always has weakly more successes relative to a bad type sender. This is because a bad type sender has a higher expected cost of acquiring one more success. As a result, in general, only three possible scenarios regarding the behaviour "on the equilibrium path" need to be considered:

- *No over-experimentation scenario (unraveling equilibrium)*: the sender stops experimentation and discloses all the successes either when he has acquired n^s successes without failure(s) or when he fails before that.

- *Over-experimentation scenario (over-experimentation equilibrium)* : when the first failure occurs after some early successes, the sender continues experimentation until n^s successes are acquired; otherwise, he stops immediately.

- *No-experiment equilibrium*: the sender does not conducted any experiments.

In the scenario without over-experimentation behaviour, the bad type sender stops experimentation once the first failure occurs and receives zero on the equilibrium path, even if he still have some successes. Consider an equilibrium in which a potential good type sender discloses k successes, where $k \in \mathbb{N}^+$. Thus, the sender would conduct $\tilde{k}$ experiments on expectation, where $\tilde{k} = \sum_{i=1}^{k} \frac{p_0}{p_{i-1}}$. In this scenario, the potential good type sender, who hasn't failed yet, is separated from a bad type. The experimental results are now informative, in which case the public history coincides with the private history, $p^P(n^s, 0) = p^A(n^s, 0) = p_{n^s}$. This sounds similar to the separating equilibrium in the classic signalling game, but they are not the same. On the one hand, the bad type senders with value zero disclose different numbers of successes due to the randomness on the arrival of the first failure; on the second hand, the potential good type sender, who is separated, might still be a lucky bad type sender without failures. These features are caused by the process of costly learning on sender's type, which are missing in the signalling game.

In the scenario with over-experimentation behaviour, the experimental results become less informative. Suppose the first failure occurs in $j + 1_{th}$ experiments, where $0 \leq j < k$. If he stops and discloses the j successes that he has achieved, the receiver's posterior belief is $\mathrm{P}^P(j,0)$ and the reward level would be $\mathrm{P}^P(j,0)M$. Instead, if the sender continues experimentation, the expected experimental cost to achieve another $k-j$ successes on expectation would be $\frac{k-j}{1-\theta}c$, and he can receive $\mathrm{P}^P(k,0)M$. Therefore, the bad type sender is willing to do so as long as the extra benefit is larger than the expected cost:

$$\underbrace{\left(\mathrm{P}^P(k,0)-\mathrm{P}^P(j,0)\right)M}_{\text{Benefit from Over-Experimentation}} > \underbrace{\frac{k-j}{1-\theta}c}_{\text{Expected Cost of Over-Experimentation}} \tag{4.1}$$

Assume now condition (4.1) is violated at $j-1$. Notice that $\mathrm{P}^P(j,0)$ is weakly increasing and the expected cost in (4.1) is decreasing as j increases, so the bad type sender who fails after $j+1_{th}$ experiment would also have the over-experimentation incentive. It is easy to see that a bad type sender, whose first failure occurs in the k_{th} experiment, has the strongest incentive to over-experiment. Furthermore, the bad type sender who fails before the $j+1_{th}$ experiment would stop immediately as the expected net benefit from over-experimenting is negative. As a result, the receiver's belief on the equilibrium path would be

$$\begin{aligned}\mathrm{P}^P(k,0) =& \frac{p_0}{\underbrace{p_0+(1-p_0)(1-\theta)^k}_{\text{sender has not failed}} + \underbrace{\sum_{i=j}^{k-1}(1-p_0)(1-\theta)^i\theta}_{\text{sender fails after } j+1_{th} \text{ experiment}}} \\ =& \frac{p_0}{p_0+(1-p_0)(1-\theta)^j} = p_j < p_k\end{aligned} \tag{4.2}$$

This result implies that the receiver now has a lower posterior belief on that the sender is good when k successes are disclosed, in which case the potential good type receives a lower reward.

In both scenarios above, the potential good type sender always discloses k successes on the equilibrium path. The receiver learns that the sender must be a bad type as long as less successes are disclosed, so her poster belief drops to zero and delivers a reward that equals to the true value of the bad type sender, $a^*(k^s,0)=0$ for $0\le k^s<k$. This is summarized in Lemma 2.

Lemma 2. *In any equilibrium where the potential good type sender discloses $k>0$ successes, $a^*(k^s,0)=0$ for $0\le k^s<k$.*

Proof. In both of the scenarios above, only the bad type sender reports less successes on the equilibrium path. Therefore, for $0 \leq k < k^s$, $\mathrm{P}^P(k^s, 0) = 0$ and $a^*(k^s, 0) = 0 \times M = 0$. $\square$

Therefore, given the potential good type sender discloses k successes on the equilibrium path, the sender's continuation value before experiments are conducted, $\mathrm{U}(p_0|k)$, can be simplified as

$$\mathrm{U}(p_0|k) = \underbrace{\frac{\mathrm{P}^P(k,0)}{p_k} p_0 M - \tilde{k}c}_{\text{without over-experimentation}} + \underbrace{\sum_{i=0}^{k-1} (1-p_0)(1-\theta)^i \theta \max\left\{0, \mathrm{P}^P(k,0)M - \frac{(k-i)c}{1-\theta}\right\}}_{\text{over-experimentation incentive}} \tag{4.3}$$

Notice that the sender would receive $a^*(0,0) = 0$ if he deviates to terminate experimental stage at the very beginning. Thus, he would only start experimentation if $\mathrm{U}(p_0|k)$ is positive. When the expected gain from experimentation cannot afford one experiment, the third scenario occurs, in which no experiments are conducted.

Also, Lemma 2 implies that experimenting forever cannot be an equilibrium. Suppose it is. Thus, the receiver would consider the sender as a bad type on the equilibrium path as long as the sender reports any finite number of successes. Since the costly experiments and results are private information which needs to be disclosed after the sender stops experimentation, the sender would deviate not to conduct any experiments. As a result, a contradiction emerges.

To sustain the equilibria which are described in the three scenarios above, it is worth discussing the off-equilibrium path behaviour in which case the sender discloses more successes. Suppose that the sender has already acquired k successes. If he sticks to the strategy on the equilibrium path, the reward level is $\mathrm{P}^P(k,0)M$. Instead, if he discloses more successes together with some failures, the receiver learns that this sender is a bad type, where $\mathrm{P}^P(k^s, k^f) = 0$ for $k^s > k$ and $k^f > 0$. Moreover, if he reports more successes without failures, the associated reward might be potentially higher. Notice that the expected cost of acquiring one success for a potential good type sender is lower than that for a bad type sender, so a potential good type sender has a higher incentive to continue experimentation relative to a bad type. Therefore, a restriction on the off-equilibrium path must deter such deviation. Under the restriction, the benefit for a potential good type from continuing experimentation after k successes are acquired must be the lower than the cost of doing so. Otherwise, the three scenarios above cannot be equilibria. This restriction on the off-equilibrium path behaviour is summarized in Lemma 3: The left hand side in (4.4) represents the benefit from continuing experimentation for a potential good type; and the right hand side represents the associated experimental cost and opportunity cost.

Lemma 3. *In any equilibrium where the potential good type sender reports* $k \geq 0$ *successes, for* $k^s > k$ *and* $\forall n \in \mathbb{N}^+$, $\mathrm{P}^P(k^s, 0)$ *satisfies that,*

$$\begin{aligned}\sum_{j=0}^{n-1}(1-p_k)(1-\theta)^j\theta \max\left\{\max_{i\in\{0,\dots,j\}} \mathrm{P}^P(k+i,0)M, \max_{i\in\{j+1,\dots,n\}} \mathrm{P}^P(k+i,0)M - \frac{(i-j)c}{1-\theta}\right\} \\ + \frac{p_k}{p_{k+n}} \max_{i\in\{0,\dots,n\}} \mathrm{P}^P(k+i,0)M \quad \leq \quad \sum_{i=1}^{n} \frac{p_k}{p_{k+i-1}} c + \mathrm{P}^P(k,0)M\end{aligned} \tag{4.4}$$

Proof. See Appendix A. □

Given the restriction in (4.4), the existence of equilibria and their properties are now characterised in Proposition 1, in which two critical numbers of successes are introduced. The *participation limit*, $\overline{k}$, determines the largest possible number of disclosed successes on the equilibrium path. It is measured by the largest number of successes whose associated continuation value is still positive.[6] It also illustrates that the sender would conduct experiments only if the expected gain is higher than the value of a bad type.

The second critical number $\hat{k}$ is *over-experimentation threshold*, which determines the boundary between the set of unraveling equilibria and the set of over-experimentation equilibria. It is measured by the largest number of successes whose conditional expected value is smaller than the expected cost of acquiring one success by a bad type sender. It is also the point at which a bad type sender has the highest incentive to deviate from an unraveling equilibria. Consider a potential unraveling equilibrium in which the potential good type sender discloses k successes. For a bad type sender whose first failure arrives in k_{th} experiment, he receives zero after stopping and disclosing $k-1$ successes. Instead, by deviating to continue experimentation, he can acquire one more success and claim $p_k M$ after paying the cost $\frac{c}{1-\theta}$ on expectation. Thus, he would do so as long as the extra gain of over-experimentation is higher than the expected cost, leading that the proposed unraveling equilibrium cannot be sustained.

[6]Ideally, the participation threshold just makes the continuation value equal to zero. However, due to the discreteness, such equality cannot be achieved all the time. Similarly, the over-experimentation threshold ideally makes the conditional expected value equal to the expected cost of acquiring one success by a bad type sender.

Proposition 1. *In private experimentation, given (4.4) is satisfied, there exist a participation limit* $\overline{k} = \begin{cases} \max\{k \in \mathbb{N} : \mathrm{U}(p_0|k) \geq 0\} & p_0 M \geq c \\ 0 & p_0 M < c \end{cases}$ *and an over-experimentation threshold* $\hat{k} = \begin{cases} \max\left\{k \in \mathbb{N} : p_k M \leq \frac{c}{1-\theta}\right\} & p_1 M \leq \frac{c}{1-\theta} \\ 0 & p_1 M > \frac{c}{1-\theta} \end{cases}$ *such that,*

1) No-experiment equilibrium exists in which case $a^*(0,0) = p_0 M$*. Particularly, it is unique when* $\frac{M}{c} \in \left[0, \frac{1}{p_0}\right)$*.*

2) Unraveling equilibria exist at $0 < k \leq \min\left\{\overline{k}, \hat{k}\right\}$ *when* $\frac{M}{c} \in \left[\frac{1}{p_0}, \frac{1}{p_1(1-\theta)}\right]$*, in which case* $a^*(k,0) = p_k M$ *and* $a^*(k^s, 0) = 0$ *for* $0 \leq k^s < k$*. Particularly,* $\hat{k} \to \infty$ *when* $p_0 \geq 1-\theta$ *and* $\frac{M}{c} \in \left[\frac{1}{p_0}, \frac{1}{1-\theta}\right]$*.*

3) Over-experimentation equilibria exist at $\hat{k}+1 < k \leq \overline{k}$ *when* $\frac{M}{c} \in \left(\max\left\{\frac{1}{p_0}, \frac{1}{1-\theta}\right\}, +\infty\right) \bigcap \mathrm{S}$*, in which case* $a^*(k,0) = p_{\hat{k}+l} M$ *and* $a^*(k^s, 0) = 0$*, where* $0 \leq k^s < k$*,* $0 < l \leq k - \hat{k}$ *and* $\mathrm{S} = \left(\frac{k-\hat{k}-l}{p_{k+l}(1-\theta)}, \frac{k-\hat{k}-l+1}{p_{k+l}(1-\theta)}\right]$*.*

Proof. See Appendix A. □

When $p_0 M < c$, the value-cost ratio is very low and the participation limit drops to zero. This means that either the value of a good type sender is too low or the cost of one experiment is too high. Thus, the uninformed sender now prefers not to conduct any experiments. Given the restriction on the off-equilibrium behaviour, the receiver optimally delivers a reward that is equal to the prior expected value $p_0 M$. Thus, the no-experiment equilibrium is unique when the restriction on the off-equilibrium path behaviour is satisfied, which is summarized in Proposition 1.1. Also, it represents the area which is below the red curve in Figure 2.

When $p_0 M \geq c$, the prior expected value $p_0 M$ is higher than the cost of one experiment, in which case the participation threshold is strictly positive. Although the no-experiment equilibrium might still exist, it leaves the room for the equilibria in which experiments are conducted and a positive number of successes is disclosed. Proposition 1.2 shows that the unraveling equilibria exist at a medium level of value-cost ratio, where $\frac{M}{c} \in \left[\frac{1}{p_0}, \frac{1}{p_1(1-\theta)}\right]$, and the number of disclosed successes is constrained at $0 < k \leq \min\{\overline{k}, \hat{k}\}$. Now the value-cost ratio falls in either Regime I or II in Figure 2. In this case, for a bad type sender whose first failure arrives at k_{th} experiment, he has no incentives to deviate since the extra gain of over-experimentation is lower than the expected cost, in which case $p_k M < \frac{c}{1-\theta}$. Similarly, when the first failure arrives even earlier, the bad type sender also has no incentives to deviate. This is because he needs to acquire more than one successes and the expected cost would be higher than $\frac{c}{1-\theta}$. Therefore, the public history coincides with the private history and the unraveling result is achieved, where the potential good type sender receives his conditional expected value $p_k M$ as a reward. Particularly, when the prior belief is higher than the passing rate for a bad type sender, in which case $p_0 \geq 1-\theta$, it is always too costly for a bad type to over-experiment as long as the value-cost ratio is at the lower medium level, where $\frac{M}{c} \in \left[\frac{1}{p_0}, \frac{1}{1-\theta}\right]$ and $\hat{k} \to \infty$. In this case, the value-cost ratio falls in regime I in Figure 2.

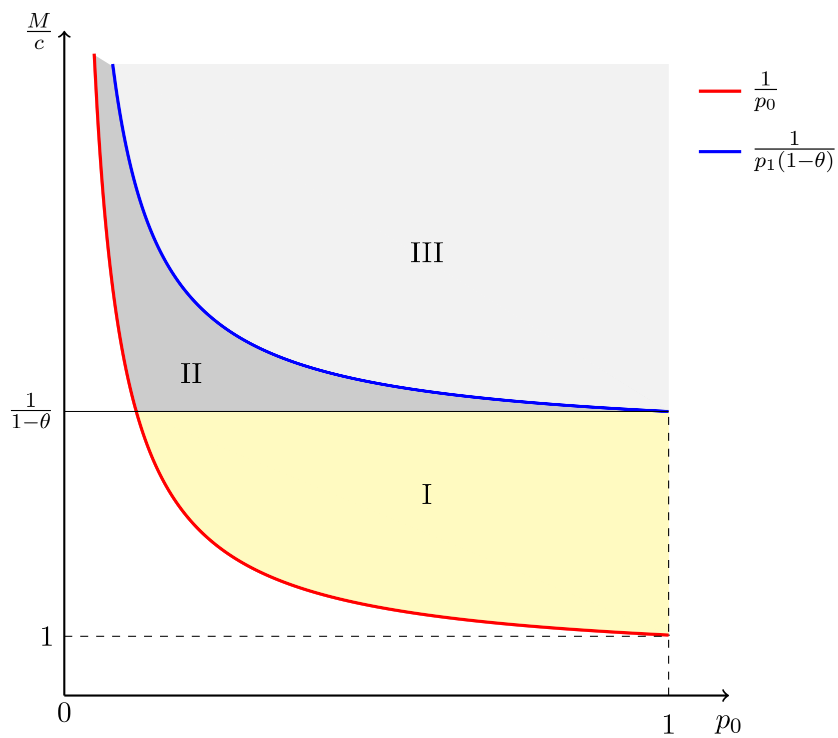

Figure 2: Equilibria in different regimes

The existence of over-experimentation equilibria relies on the over-experimentation threshold $\hat{k}$ and extra constraints. When the value-cost ratio is very high and falls in Regime III in Figure 2, where $\frac{M}{c} \in \left(\frac{1}{p_1(1-\theta)}, +\infty\right)$, the extra gain from over-experimentation is very high and $\hat{k} = 0$. In this case, unraveling equilibria cannot be sustained and only over-experimentation equilibria potentially exist as long as a positive number of successes is disclosed on the equilibrium path. When the value-cost ratio is at the upper medium level, where $\frac{M}{c} \in \left(\max\left\{\frac{1}{p_0}, \frac{1}{1-\theta}\right\}, \frac{1}{p_1(1-\theta)}\right]$, the over-experimentation equilibria exist potentially as long as the disclosed number of successes is high, $\hat{k} < k \leq \overline{k}$, on the equilibrium path. This is shown as regime II in Figure 2. In a representative over-experimentation equilibrium, the bad type sender over-experiments and discloses k successes on the equilibrium path as long as his first failure arrives after $\hat{k} + l + 1_{th}$ experiment, which is guaranteed by the interval S.

Particularly, one interesting observation in Proposition 1.3 is that, at the point $\hat{k} + 1$, only the no-experiment equilibrium potentially exists, even if it is lower than the participation limit $\overline{k}$. Since the reward for a potential good type sender $p_{\hat{k}+1}M$ is higher than the cost of over-experimentation $\frac{c}{1-\theta}$, an unraveling equilibrium cannot be sustained and the bad type sender always over-experiment once his first failure occurs at $\hat{k} + 1_{th}$ experiment. If so, the receiver adjusts her posterior belief to $\mathrm{P}^P(\hat{k}+1, 0) = p_{\hat{k}}M$, which now is lower than the cost of experimentation. Thus, the bad type sender is not willing to over-experiment, in which case such over-experimentation equilibrium is not sustained.

Now we analyse how the change of parameters would affect the two critical values. The related results are then summarized in Proposition 2.

Proposition 2. *In private experimentation, $\overline{k}$ is weakly increasing in $\frac{M}{c}$, p_0 and θ. $\hat{k}$ is weakly decreasing in $\frac{M}{c}$ and p_0.*

Proof. See Appendix A. □

When the value-cost ratio $\frac{M}{c}$ increases, on the one hand, the sender becomes relatively more valuable and his prior expected value can cover a higher level of total expected cost. Thus, the participation limit $\overline{k}$ rises, in which case the sender is willing to conduct more experiments and stop later. On the other hand, the higher value-cost ratio also gives the bad type sender a stronger incentive to over-experiment, in which case the extra benefit of over-experimentation is now higher. As a result, the over-experimentation threshold $\hat{k}$ falls and the informativeness of the reported successes is less.

When the prior belief of the sender being a good type increases, the sender's prior expected value is higher and the participation limit tends to rise. This is a positive effect. However, a higher prior belief also means that the expected total cost for the same amount of experiments is also higher. This negative effect tends to reduce the participation limit at the same time. As a compounded effect, we show that the positive effect always dominates and thus the participation limit increases. Meanwhile, when the sender is very optimistic on that he is a good type, he has a stronger incentive to over-experiment after he fails. This is because his loss from the failure is larger than that with a lower prior belief. As a result, the over-experimentation threshold falls.

The effect of the failure rate θ for a bad type sender varies in different cases. A higher failure rate means that a failure arrives earlier on average, given the sender is a bad type. Thus, at the very beginning, the expected cost of experimentation falls and the uninformed sender is willing to disclose more successes on the equilibrium path. In this case, the participation limit increases. Meanwhile, a higher failure rate also causes a higher expected cost of over-experimentation and thus a bad type sender has lower incentives to over-experimentation. This is a positive effect on the over-experimentation threshold. However, there also exists a negative effect. Since the uninformed sender learns faster with a higher failure rate, the conditional expected value also increases given no failures arrive. Thus, the the extra benefit of over-experimentation is also getting larger, which gives the bad type sender a higher incentive to over-experiment. As a result, the compounded effect of the failure rate on the over-experimentation threshold is ambiguous.

5 Comparisons with Public Experimentation

We now examine the role of the asymmetric information in the experimental stage by comparing with public experimentation. Consider now the experiments and results are publicly observed. In this case, the private history and public history are always the same, so the sender has no reason to continue experimentation once a failure arrives. This implies that the over-experimentation threshold does not play a role and the set of over-experimentation equilibria cannot exist. Notice that the sender is also uninformed before experimentation, so his *ex ante* expected gain from experimentation is always equal to his prior expected value p_0M regardless of the cost. However, if the sender chooses not to conduct any experiments, the receiver would also deliver p_0M to the sender as a reward. As a result, the sender would optimally terminate experimentation at the very beginning, in which case only the no-experiment equilibrium exists in public experimentation. This result and the comparisons with private experimentation are then summarized in Proposition 3. It suggests that the existence of equilibria with a positive number of disclosed successes brings the cost for the uninformed sender. As a result, the sender is worse off in private experimentation.

Proposition 3. *Relative to public experimentation where only the no-experiment equilibrium exists, the sender is weakly worse off in private experimentation.*

Proof. See Appendix A. □

Due to the skeptical thinking of the receiver, the uninformed sender is forced to learn and acquire more successes to prove that he is not a bad type. Our results in Proposition 3 also implies that the learning process only occurs when there is asymmetric information in experimental stage.

6 The sender's commitment

In this section, we consider a scenario in which the sender can commit to disclose a certain number of successes to prove that he is a potential good type before he conducts experiment(s). The timing of the game is changed as follows:

1. The sender commits to disclose $k \geq 0$ successes;
2. The sender conducts experiments;
3. The sender stops experiments and discloses results;
4. The receiver delivers a reward.

With the help of the commitment, the sender is able to his maximise his *ex ante* continuation value, which is constrained by the credibility of his commitment. Intuitively, the sender might tend to commit to no-experiment strategy, in which case he does not conduct any experiments or disclose any results, and hen his net benefit equals to his prior expected value. This is because the sender's *ex ante* expected gain is always equal to his expected value and the experiments reduces the net benefit. However, such commitment might not be credible. This is because the sender is better off by deviating to conduct some experiments and disclose some successes as long as (4.4) is violated. As a result, the sender compromises to commit to a larger number of disclosed successes. As the committed number is getting larger, the participation limits plays a role due to the presence of out-side option. Also, the over-experimentation threshold steps in because of the receiver's skeptical thinking. We summarize the sender's optimal commitment in Proposition 4.

Proposition 4. *In private experimentation, a potential good type sender optimally commits to disclose k^c successes such that $k^c = \arg\max_{k \in \mathrm{A}} \mathrm{U}(p_0|k)$, where* A *is the set of equilibria in Proposition 1. Particularly, $k^c = \min_{0 \leq k \leq \min\left\{\overline{k}, \hat{k}\right\}} \mathrm{A}$ given $\mathrm{A} \neq \varnothing$ at $0 \leq k \leq \min\left\{\overline{k}, \hat{k}\right\}$. When $\mathrm{A} = \varnothing$, the sender's commitment is not credible.*

Proof. See Appendix A. □

Proposition 4 shows that the sender is weakly better off with the power of commitment, in which case the optimal committed number of disclosed successes is the one which maximises his continuation value in the set of equilibria proposed in Proposition 1. Notice that the sender now has the same incentives of deviation as those in the scenario in Section 4, so the credible committed number of disclosed successes coincides with the equilibrium level of disclosed successes in Section 4.

Particularly, when this committed number is smaller than both of the two critical thresholds, given the existence of equilibria, where $0 \leq k \leq \min\left\{\overline{k}, \hat{k}\right\}$, the sender always prefers to commit to the smallest credible number of disclosed successes. In this case, the extra experiment always increases the total expected cost and lowers the expected net benefit. Thus, the no-experiment commitment is the ideal one if (4.4) is not violated. Otherwise, the sender comprises to the next lowest number due to the concern of credibility.

However, when only the over-experimentation equilibria exist, where $\hat{k} + 1 < k \leq \overline{k}$, the optimal committed number is not necessarily to be smallest number of disclosed successes in the set of equilibria. Suppose now the sender commits to $k + 1$ rather than k, in which case he only has an incentive to over-experiment if the first failure arrives after $\hat{k} + l + 2_{nd}$ experiment. Thus, he would stop experimentation if the first failure arrives at $\hat{k}+l+1_{st}$ experiment. Compared with committing to k, the sender now pays the extra cost $[p_0+(1-p_0)(1-\theta)^{\hat{k}+l}]c$. This a negative effect which drives a smaller committed number. Meanwhile, since the over-experimentation incentive occurs later, the associated probability of failure is also lower, and thus the total expected cost of over-experimentation is also lower. This positive effect leads to a higher committed number. Therefore, the compounded effect determines the change of the sender's continuation value and it is ambiguous in general. The specific expression is summarized in (6.1):

$$\mathrm{U}(p_0|k+1)-\mathrm{U}(p_0|k) = \underbrace{-[p_0+(1-p_0)(1-\theta)^{\hat{k}+l}]c}_{\text{Increased expected cost of experimentation}} \quad \underbrace{+(k-\hat{k}-l)(1-p_0)(1-\theta)^{\hat{k}+l-1}\theta c}_{\text{Saved expected cost of over-experimentation}} \tag{6.1}$$

As a result, a smaller committed number of discloses successes might not increase the sender's net benefit, and then the sender comprises to a higher committed number.

7 Conclusion

We characterise the properties of equilibria and discuss the role of the sender's commitment in sequential private experimentation. When the commitment is absent, we show that the participation limit determines the set of equilibria with a positive number of disclosed successes and the over-experimentation threshold determines the boundary between the unraveling and over-experimentation equilibria. In presence of sender's commitment, the sender is weakly better off. However, it is not always optimal for him to credibly commit to a smaller number of disclosed successes if the set of over-experimentation equilibria exists uniquely. Instead, he commits to the smallest credible number as long as no-experiment or unraveling equilibria exist.

A Appendix: Proofs of Lemmas and Propositions

Proof of Lemma 1

We prove this Lemma by using the following claim.

Claim 2. *The expected cost of acquiring a success for a potential good type sender is lower than that for a bad type, and it falls as his posterior belief increases.*

Proof. Suppose a potential good type sender has n successes without failures. Now his posterior belief is $p^A_{(n,0)} = p_n$. If he conducts one more experiment, he can acquire a success with probability $p_n + (1-p_n)(1-\theta)$; if he fails with probability $(1-p_n)\theta$, he knows that he is actually a bad type and the expected cost of acquiring a success becomes $\frac{c}{1-\theta}$. Thus, the expected cost of acquiring a success for the potential good type would be $\frac{(1-p_n\theta)c}{1-\theta}$, which is smaller than $\frac{c}{1-\theta}$. Moreover, when p_n increases, the numerator in the expected cost is lower as the coefficient of p_n is negative. $\square$

This claim suggests that only a bad type might have the incentives to stop before k successes are acquired, and the potential good type sender has a stronger incentive to conduct more experiments.

Proof of Lemma 3

We prove this lemma by using the following claim.

Claim 3. *Given $k \geq 0$ successes have been acquired, a potential good type sender has a stronger incentive to continue experimenting relative to a bad type sender.*

Proof. Suppose now the sender has $k \geq 0$ successes already. If the sender is a potential good type who hasn't failed yet, his posterior belief is $\mathrm{P}^A(k,0) = p_k$. To acquire another $n > 0$ successes, his continuation value $\mathrm{U}_G(p_k|n)$ would be:

$$\begin{aligned}
&\mathrm{U}_G(p_k|n) \\
&\quad = \sum_{j=0}^{n-1}(1-p_k)(1-\theta)^j\theta \max\left\{\max_{i\in\{j+1,\ldots,n\}} \mathrm{P}^P(k+i,0)M - \frac{(i-j)c}{1-\theta}, \max_{i\in\{0,\ldots,j\}} \mathrm{P}^P(k+i,0)M\right\} \\
&\qquad + \frac{p_k}{p_{k+n}} \max_{i\in\{0,\ldots,n\}} \mathrm{P}^P(k+i,0)M - \sum_{i=1}^{n} \frac{p_k}{p_{k+i-1}}c
\end{aligned} \tag{A.1}$$

Similarly, the continuation value of a bad type sender when deviating to acquire n more successes is:

$$\mathrm{U}_B(\mathrm{P}^A = 0|n) = \max_{i\in\{0,\ldots,n\}} \mathrm{P}^P(k+i,0)M - \frac{nc}{1-\theta} \tag{A.2}$$

Notice that the following result always holds:

$$\begin{aligned} &\max\left\{\max_{i\in\{j+1,\ldots,n\}} \mathrm{P}^P(k+i,0)M - \frac{(i-j)c}{1-\theta}, \max_{i\in\{0,\ldots,j\}} \mathrm{P}^P(k+i,0)M\right\} \\ &\qquad \geq \max_{i\in\{0,\ldots,n\}} \mathrm{P}^P(k+i,0)M - \frac{nc}{1-\theta} \end{aligned} \tag{A.3}$$

Therefore, the difference between $U_G^n(p_k)$ and $U_B^n(\mathrm{P}^A = 0)$ can be simplified as:

$$\begin{aligned} \mathrm{U}_G(p_k|n) - &\mathrm{U}_B(\mathrm{P}^A = 0|n) \\ \geq &\frac{p_k}{p_{k+n}} \max_{i\in\{0,\ldots,n\}} \mathrm{P}^P(k+i,0)M - \max_{i\in\{0,\ldots,n\}} \mathrm{P}^P(k+i,0)M + \frac{nc}{1-\theta} \\ &- \sum_{i=1}^{n} \frac{p_k}{p_{k+i-1}} c + \sum_{j=0}^{n-1} (1-p_k)(1-\theta)^j \theta \left[\max_{i\in\{0,\ldots,n\}} \mathrm{P}^P(k+i,0)M - \frac{nc}{1-\theta}\right] \\ = &\frac{nc}{1-\theta} - \sum_{i=1}^{n} \frac{p_k}{p_{k+i-1}} c - \sum_{j=0}^{n-1} (1-p_k)(1-\theta)^j \theta \frac{(n-j)c}{1-\theta} \\ = &\sum_{i=1}^{n} \frac{p_k}{p_{k+i-1}} \frac{c}{1-\theta} - \sum_{i=1}^{n} \frac{p_k}{p_{k+i-1}} c = \sum_{i=1}^{n} \frac{p_k}{p_{k+i-1}} \frac{\theta c}{1-\theta} > 0 \end{aligned} \tag{A.4}$$

As a result, $\mathrm{U}_G(p_k|n) > \mathrm{P}^P(k,0)M$ as long as $\mathrm{U}_B(\mathrm{P}^A = 0|n) \geq \mathrm{P}^P(k,0)M$. □

Claim 3 suggests that, if the receiver's posterior belief provides the potential good type no incentives to continue experimenting, the bad type would also not do so. Thus, the potential good sender would not continue experimenting if the current payoff is larger than the expected payoff of continuing experimenting. This implies that, for $\forall n \in \mathbb{N}^+$, $\mathrm{U}_G(p_k|n) \leq \mathrm{P}^P(k,0)M$ must be satisfied, which is rewritten as:

$$\begin{aligned} &\sum_{j=0}^{n-1} (1-p_k)(1-\theta)^j \theta \max\left\{\max_{i\in\{0,\ldots,j\}} \mathrm{P}^P(k+i,0)M, \max_{i\in\{j+1,\ldots,n\}} \mathrm{P}^P(k+i,0)M - \frac{(i-j)c}{1-\theta}\right\} \\ &\qquad + \frac{p_k}{p_{k+n}} \max_{i\in\{0,\ldots,n\}} \mathrm{P}^P(k+i,0)M \leq \sum_{i=1}^{n} \frac{p_k}{p_{k+i-1}} c + \mathrm{P}^P(k,0)M \end{aligned} \tag{A.5}$$

If the condition above is not satisfied, the potential good type would always continue experimenting, and this contradicts to an equilibrium where the potential good type stops after k successes are acquired.

Proof of Proposition 1

Proof of Proposition 1.1. Given the sender does not conduct any experiments, the receiver's posterior belief becomes $\mathrm{P}^{P}(0,0) = p_0$ and the reward is $a^*(p_0) = p_0 M$. Consider the sender deviates to conduct experiments. As long as (4.4) is satisfied, the extra benefit from the deviation is always lower than the associated expected cost. As a result, the sender has no incentives to conduct any experiments.

The following claim is applied in the proof of uniqueness of no-experiment equilibria at $p_0 M < c$.

Claim 4. *There does not exist an equilibrium with a positive number of disclosed successes at $k > \overline{k}$.*

Proof. Suppose there exists at least one equilibrium in which case the potential good type sender discloses $k > \overline{k}$ successes. Given the definition of $\overline{k}$, the continuation value for the sender at the beginning now is negative, where $\mathrm{U}(p_0|k) < 0$ for $k > \overline{k}$. Instead, if the sender deviates not to conduct any experiments, he does not have any successes to disclose and receives zero. Therefore, the sender is better off by deviating to stop at the beginning, which is a contradiction. □

When $p_0 M < c$, $\overline{k} = 0$. Thus, $k > \overline{k}$ for $\forall k > 0$. From Claim 4, we conclude that the no-experiment equilibrium is unique given (4.4) is satisfied. The rest of proof would focus on the scenario when $p_0 M \geq c$.

Proof of Proposition 1.2. Consider an unraveling equilibrium in which case the sender stops experimentation either when k successes are acquired or a failure arrives. On the equilibrium path, only the potential good type sender discloses k successes. Thus, $\mathrm{P}^{P}(k,0) = \mathrm{P}^{A}(k,0) = p_k$ and $\mathrm{P}^{P}(k^s,0) = \mathrm{P}^{A}(k^s,0) = 0$ for $k^s < k$. As a result, $a^*(k,0) = p_k M$ and $a^*(k^s,0) = 0$.

For a potential good type sender, if deviating to stop experimentation earlier, he receives zero at most. Thus, he is worse off by deviation. Also, he has no incentives to conduct more experiments as long as (4.4) is satisfied.

Similarly, we check the bad type's over-experimentation incentive. Consider the case in which the first failure arrives in k_{th} experiment. Since he only needs one more success to pretend to be a good type, the bad type sender now has the strongest incentive to over-experiment. If he stops, the reward would be zero. Alternatively, if over-experimenting, he receives the extra gain $p_k M$ by paying the cost $\frac{c}{1-\theta}$ on expectation. Thus, The bad type sender will not over-experiment if the extra benefit $p_k M$ is less than the expected cost $\frac{c}{1-\theta}$,

$$p_k M \leq \frac{c}{1-\theta} \quad \Longrightarrow \quad k \leq \hat{k} = \begin{cases} \max\left\{k \in \mathbb{N} : p_k M \leq \frac{c}{1-\theta}\right\} & p_1 M \leq \frac{c}{1-\theta} \\ 0 & p_1 M > \frac{c}{1-\theta} \end{cases} \tag{A.6}$$

Following the similar argument, for $k < \hat{k}$, the bad type sender never over-experiments if the failure arrives before k_{th} experiment. Therefore, the set of unraveling equilibria would must satisfy $\left\{k \in \mathbb{N} : 0 < k \leq \min\{\hat{k}, \overline{k}\}\right\}$.

Moreover, when $\frac{M}{c} > \frac{1}{p_1(1-\theta)}$, $\hat{k} = 0$ and $\left\{k \in \mathbb{N} : 0 < k \leq \min\{\hat{k}, \overline{k}\}\right\} = \varnothing$. As a result, the unraveling equilibria exist only when $\frac{M}{c} \in \left[\frac{1}{p_0}, \frac{1}{p_1(1-\theta)}\right]$. Particularly, when $p_0 \geq 1-\theta$ and $\frac{M}{c} \in \left[\frac{1}{p_0}, \frac{1}{1-\theta}\right]$, $p_k M < M \leq \frac{c}{1-\theta}$ for $\forall k > 0$ and thus $\hat{k} \to \infty$.

Proof of Proposition 1.3. If a bad type sender has incentives to deviate from an unraveling equilibrium, it must be true that $\hat{k} < k \leq \overline{k}$. Otherwise, the extra benefit cannot cover the expected cost of doing so. When $\frac{M}{c} \in \left[\frac{1}{p_0}, \frac{1}{\theta}\right]$, $\hat{k} \to \infty$ and over-experimentation equilibria do not exist. Therefore, to guarantee $\hat{k} < \infty$, the value-cost ratio must satisfy $\frac{M}{c} \left(\max\left\{\frac{1}{p_0}, \frac{1}{1-\theta}\right\}, +\infty\right)$.

Claim 5. *There does not exist an equilibrium with a positive number of disclosed successes at* $k = \hat{k} + 1$.

Proof. Claim 4 has proved this claim in the case where $\overline{k} \leq \hat{k}$. Now consider $\hat{k} < \overline{k}$. In the first place, an unraveling equilibrium cannot exists since $p_{\hat{k}+1} M > \frac{c}{1-\theta}$. Otherwise, the bad type sender whose first failure arrives in $\hat{k} + 1_{th}$ experiment would deviate to over-experiment. In the second place, consider an over-experimentation equilibrium. Suppose only the bad type whose first failure arrives in $\hat{k} + 1_{th}$ experiment would over-experiment on the equilibrium path. Thus, $\mathrm{P}^P\left(\hat{k}+1, 0\right) = p_{\hat{k}}$. However, since $p_{\hat{k}} M \leq \frac{c}{1-\theta}$, the bad type sender would not over-experiment, which is a contradiction. Similarly, for a bad type sender whose first failure arrives before k_{th} experiment, he also has no incentives to over-experiment since the expected cost of doing so is higher than $\frac{c}{1-\theta}$. □

Given Claim 5, the existence of the over-experimentation equilibria can be restricted in $\hat{k}+1 < k \leq \overline{k}$. Furthermore, $\overline{k} = \hat{k}+1$, the over-experimentation equilibria do not exist for $\overline{k} > \hat{k}+1$.

Now focus on the case in which $\overline{k} > \hat{k}+1$. Suppose, on the equilibrium path, a potential good type sender discloses k successes and a bad type sender over-experiments if his failure arrives after $\hat{k}+l+1_{th}$ experiment, where $0 < l \leq \overline{k} - \hat{k}$. Thus, $\mathrm{P}^P(k,0) = p_{\hat{k}+l}$ and $\mathrm{P}^P(k^s,0) = 0$ for $k^s < k$. To sustain this equilibrium, for the bad type sender whose first failure arrives in $\hat{k}+l+1_{th}$ experiment, he would indeed over-experiment as long as the extra benefit is larger than the expected total cost by doing so:

$$p_{k+l}M > \frac{k-\hat{k}-l}{1-\theta}c \quad \Longrightarrow \quad \frac{M}{c} > \frac{k-\hat{k}-l}{p_{\hat{k}+l}(1-\theta)} \tag{A.7}$$

Meanwhile, for the bad type sender whose first failure occurs in $\hat{k}+l_{th}$ experiment, the total expected cost of over-experimentation must be higher than the extra benefit:

$$p_{k+l}M \leq \frac{k-\hat{k}-l+1}{1-\theta}c \quad \Longrightarrow \quad \frac{M}{c} \leq \frac{k-\hat{k}-l+1}{p_{\hat{k}+l}(1-\theta)} \tag{A.8}$$

Thus, this can be summarized as a set $\mathrm{S} = \left(\frac{k-\hat{k}-l}{p_{\hat{k}+l}(1-\theta)}, \frac{k-\hat{k}-l+1}{p_{\hat{k}+l}(1-\theta)}\right]$. Therefore, To sum up, the value-cost ratio must satisfy the following condition:

$$\frac{M}{c} \in \mathrm{S} \bigcap \left(\max\left\{\frac{1}{p_0}, \frac{1}{1-\theta}\right\}, +\infty\right)$$

Proof of Proposition 2

1) Consider $\overline{k}$. When $p_0 M < c$, $\overline{k} = 0$. This condition can be rewritten as $p_0\frac{M}{c} < 1$, and it shows that the change of θ does not affect $\overline{k}$. Notice that $p_0\frac{M}{c}$ increases as p_0 or $\frac{M}{c}$ increases, so it is harder to make the inequality in $p_0\frac{M}{c} < 1$ being satisfied. Once it is violated, $\overline{k}$ becomes strictly positive, which is higher than zero.

When $p_0 M \geq c$, from (4.3), in a representative unraveling equilibrium,

$$\mathrm{U}(p_0|k) = p_0 M - \sum_{i=1}^{k}\left[p_0 + (1-p_0)(1-\theta)^{i-1}\right]c \propto p_0\frac{M}{c} - \sum_{i=1}^{k}\left[p_0 + (1-p_0)(1-\theta)^{i-1}\right] \tag{A.9}$$

The expression above suggests that $\mathrm{U}(p_0|k)$ increases as $\frac{M}{c}$ increases. Thus, $\overline{k}$ also increases in this case. Also, from $\mathrm{U}(p_0|k) \geq 0$, we can conclude that the marginal effect of p_0 on the continuation value in an unraveling equilibrium is positive:

$$\begin{aligned}
\mathrm{U}(p_0|k) \geq 0 &\implies p_0 M - \sum_{i=1}^{k} \left[p_0 + (1-p_0)(1-\theta)^{i-1}\right] c \geq 0 \\
&\implies p_0 \left[M - \sum_{i=1}^{k}[1-(1-\theta)^{i-1}]c\right] \geq \sum_{i=1}^{k}(1-\theta)^{i-1}c > 0 \qquad \text{(A.10)} \\
&\implies \frac{\partial\, \mathrm{U}(p_0|k)}{\partial p_0} = M - \sum_{i=1}^{k}[1-(1-\theta)^{i-1}]c > 0
\end{aligned}$$

Therefore, when p_0 increases, $\mathrm{U}(p_0|k)$ and hence $\overline{k}$ increases in this case. Similarly, consider the marginal effect of θ:

$$\frac{\partial\, \mathrm{U}(p_0|k)}{\partial \theta} = (1-p_0)\sum_{i=1}^{k}(i-1)(1-\theta)^{i-2}c > 0 \qquad \text{(A.11)}$$

As a result, when p_0 increases, $\mathrm{U}(p_0|k)$ increases and then $\overline{k}$ increases.

We use similar argument to discuss the change of $\overline{k}$ in a representative over-experimentation equilibrium. Now the continuation value becomes

$$\begin{aligned}
\mathrm{U}(p_0|k) =& p_0 M - \sum_{i=1}^{\hat{k}+l} \left[p_0 + (1-p_0)(1-\theta)^{i-1}\right] c - \left[p_0 + (1-p_0)(1-\theta)^{\hat{k}+l-1}\right](k-\hat{k}-l)c \\
\propto& p_0 \frac{M}{c} - \sum_{i=1}^{\hat{k}+l} \left[p_0 + (1-p_0)(1-\theta)^{i-1}\right] - \left[p_0 + (1-p_0)(1-\theta)^{\hat{k}+l-1}\right](k-\hat{k}-l)
\end{aligned} \qquad \text{(A.12)}$$

The expression above shows that $\mathrm{U}(p_0|k)$ increases as $\frac{M}{c}$ rises. Thus, $\overline{k}$ would also increases. Moreover, from $\mathrm{U}(p_0|k) \geq 0$, we can show that the marginal effect of p_0 on the continuation value in an over-experimentation equilibrium is also positive:

$$
\begin{aligned}
\mathrm{U}(p_0|k) \geq 0 \implies p_0 & \left[M - \sum_{i=1}^{\hat{k}+l}[1-(1-\theta)^{i-1}]c - \left[1-(1-\theta)^{\hat{k}+l-1}\right](k-\hat{k}-l)c\right] \\
& \geq \sum_{i=1}^{\hat{k}+l}(1-\theta)^{i-1}c + (1-\theta)^{\hat{k}+l-1}(k-\hat{k}-l)c > 0 \\
\implies \frac{\partial\, \mathrm{U}(p_0|k)}{\partial p_0} & = M - \sum_{i=1}^{\hat{k}+l}[1-(1-\theta)^{i-1}]c - \left[1-(1-\theta)^{\hat{k}+l-1}\right](k-\hat{k}-l)c > 0
\end{aligned} \tag{A.13}
$$

Thus, $\mathrm{U}(p_0|k)$ and $\overline{k}$ must increase as p_0 increases. Furthermore, consider the marginal effect of θ:

$$
\frac{\partial\, \mathrm{U}(p_0|k)}{\partial \theta} = (1-p_0)\left[\sum_{i=1}^{\hat{k}+l}(i-1)(1-\theta)^{i-2} + (1-\theta)^{\hat{k}+l-1}(k-\hat{k}-l)\right] c > 0 \tag{A.14}
$$

Therefore, $\mathrm{U}(p_0|k)$ increases and then $\overline{k}$ increases as θ increases. To sum up, $\overline{k}$ increases as $\frac{M}{c}$, p_0 or θ increases.

2) Consider $\hat{k}$. In the first case where $\frac{M}{c} > \frac{1}{p_1(1-\theta)}$, $\hat{k}=0$. When $\frac{M}{c}$ increases, $\hat{k}$ remains unchanged. We now derive the marginal effect of p_0 and θ on $\frac{1}{p_k(1-\theta)}$ for $k>0$:

$$
\begin{aligned}
\frac{\partial \frac{1}{p_k(1-\theta)}}{\partial \theta} & = \frac{p_0 + (1-p_0)(1-\theta)^k(1-k)}{p_0(1-\theta)^2} \\
\frac{\partial \frac{1}{p_k(1-\theta)}}{\partial p_0} & = -\frac{(1-\theta)^{k-1}}{p_0^2} < 0
\end{aligned} \tag{A.15}
$$

Notice that $\frac{1}{p_k(1-\theta)}$ decreases when p_0 increases at $\forall k \geq 1$, so $\frac{M}{c} > \frac{1}{p_1(1-\theta)}$ is easier to be satisfied and $\hat{k}$ remains unchanged. Particularly, from (A.15), $\frac{1}{p_1(1-\theta)}$ increases as θ increases. Therefore, the right hand side in $\frac{M}{c} > \frac{1}{p_1(1-\theta)}$ increases and the inequality is earlier to be violated.

In the case where $\frac{M}{c} \leq \frac{1}{1-\theta}$, $\hat{k} \to \infty$. Now p_0 has no effect on the inequality and $\hat{k}$ remains unchanged. When $\frac{M}{c}$ increases, $\frac{M}{c} \leq \frac{1}{1-\theta}$ is harder to be satisfied as the left hand side increases. Once this inequality is violated, $\hat{k}$ becomes finite. Moreover, when θ increases, $\frac{1}{1-\theta}$ increases and thus $\hat{k}$ remains unchanged.

In the case where $\frac{1}{1-\theta} \leq \frac{M}{c} \leq \frac{c}{p_1(1-\theta)}$, $0 < \hat{k} < \infty$. Since $\frac{1}{p_k(1-\theta)}$ is decreasing as p_0 or $\frac{M}{c}$ increases, $\frac{M}{c} \leq \frac{1}{p_k(1-\theta)}$ is harder to be satisfied. As a result, $\hat{k}$ tends to fall. Moreover, notice that the sign of $\frac{\partial \frac{1}{p_k(1-\theta)}}{\partial \theta}$ varies at different k, so there is not a monotonic effect of θ on $\hat{k}$.

To sum up, $\hat{k}$ decreases as p_0 or $\frac{M}{c}$ increases.

Proof of Proposition 3

Firstly, we prove the no-experiment equilibrium is unique in public experimentation. Because the acquired results cannot be hidden, $a^P(k,1) = 0$ for $\forall k \geq 0$ and the over-experimentation strategy is never profitable. Suppose the sender's optimal strategy is to stop experimentation either when k' successes are acquired or when a failure arrives, where $k' \geq 1$. Thus, the associated rewards are $a^P(k',0) = p_{k'}M$ and $a^P(k^s,1) = 0$ for $0 \leq k^s < k'$. In this case, the sender's continuation value from experimentation at the very beginning is $U^P(p_0|k') = p_0 M - \tilde{k}'c$, where $\tilde{k}' = \sum_{i=1}^{k'} \frac{p_0}{p_{i-1}}$. If the sender deviates to terminate experimentation at the beginning, the receiver can observe such deviation and optimally deliver $a^P(0,0) = p_0 M$ and the sender's continuation value from no experimentation is $U^P(p_0|0) = p_0 M$. Notice that $p_0 M > p_0 M - \tilde{k}'c$ for $\forall k' \geq 1$ as long as $c > 0$. As a result, no-experiment equilibrium uniquely exists.

Secondly, we compare the sender's payoff in pubic and that in private experimentation. In private experimentation, given (4.4) is satisfied, if the no-experiment equilibrium survives, $\mathrm{U}(p_0|0) = \mathrm{U}_P(p_0|0) = p_0 M$. Thus, the sender's continuation payoffs in two difference scenarios are the same.

Consider an unraveling equilibrium, in which case the potential good type sender discloses $0 < k \leq \min\left\{\overline{k}, \hat{k}\right\}$ successes. Thus, $\mathrm{U}(p_0|k) = p_0 M - \tilde{k}c < p_0 M = U^P(p_0|0)$ and the sender is worse off in private experimentation.

Consider an over-experimentation equilibrium, in which case the potential good type discloses $\hat{k} + 1 < k \leq \overline{k}$ successes and the bad type sender whose first failure occurs after $\hat{k} + l + 1_{th}$ experiment would over-experiment, where $0 < l \leq k - \hat{k}$. Thus, $\mathrm{U}(p_0|k) = p_0 M - \sum_{i=1}^{\hat{k}+l} \frac{p_0}{p_{i-1}}c - \frac{p_0(k-\hat{k}-l)}{p_{\hat{k}+l-1}}c < p_0 M = U^P(p_0|0)$ and the sender is also worse off in private experimentation.

Proof of Proposition 4

Firstly, we show that, in a credible commitment, the committed number of disclosed successes must support an equilibrium in Proposition 1. Suppose not. We denote the set of equilibria in Proposition 1 by A. Thus, there exists $k' \notin$ A and committing to k' successes is credible. However, from the proof of Proposition 1, it shows that the sender always has incentives to deviate if $k' \notin \mathrm{A}$, which makes the commitment not credible. As a result, k^c must be the solution to the constrained maximisation problem: $k^c = \arg\max_{k \in \mathrm{A}} \mathrm{U}(p_0|k)$. This set implies that k^c exists only if $\mathrm{A} \neq \varnothing$. when $\mathrm{A} = \varnothing$, we cannot find k^c and then there would be no credible commitment.

Secondly, we prove that, given (4.4) is satisfied, the sender's continuation value decreases as k increases at $0 \leq k \leq \min\left\{\overline{k}, \hat{k}\right\}$. Now the sender's continuation value is $\mathrm{U}(p_0|k) = p_0 M - \sum_{i=1}^{k} \frac{p_0}{p_{i-1}} c$. Thus,

$$\mathrm{U}(p_0|k+1) - \mathrm{U}(p_0|k) = p_0 M - \sum_{i=1}^{k+1} \frac{p_0}{p_{i-1}} c - \left(p_0 M - \sum_{i=1}^{k} \frac{p_0}{p_{i-1}} c\right) = -\frac{p_0}{p_{i-1}} c < 0 \tag{A.16}$$

Therefore, k^c must be the minimum number in $\left\{k \in \mathbb{N} : 0 \leq k \leq \min\left\{\overline{k}, \hat{k}\right\}\right\} \bigcap \mathrm{A}$. Now we can rewrite the maximisation problem as $k^c = \min_{0 \leq k \leq \min\left\{\overline{k}, \hat{k}\right\}} \mathrm{A}$.

Finally, given (4.4) is satisfied, we show that the sender's continuation value does not monotonically change as k increases at $\hat{k} + 1 < k \leq \overline{k}$. In a representative over-experimentation equilibrium, the bad type sender whose first failure arrives after $\hat{k} + l + 1_{th}$ experiment would over-experiment and disclose k successes on the equilibrium path. Now his continuation value is $\mathrm{U}(p_0|k) = p_0 M - \sum_{i=1}^{\hat{k}+l} \frac{p_0}{p_{i-1}} c - \frac{p_0(k-\hat{k}-l)}{p_{\hat{k}+l-1}} c$. This requires $\frac{k-\hat{k}-l}{1-\theta} c < p_{\hat{k}+l} M \leq \frac{k-\hat{k}-l+1}{1-\theta} c$. Consider the equilibrium with $k+1$ disclosed successes on the equilibrium path. In this case the bad type sender whose first failure arrives after $\hat{k} + l + 2_{th}$ experiment would over-experiment, which requires $\frac{k-\hat{k}-l}{1-\theta} c < p_{\hat{k}+l+1} M \leq \frac{k-\hat{k}-l+1}{1-\theta} c$. Now the sender's continuation value is $\mathrm{U}(p_0|k+1) = p_0 M - \sum_{i=1}^{\hat{k}+l+1} \frac{p_0}{p_{i-1}} c - \frac{p_0(k-\hat{k}-l)}{p_{\hat{k}+l}} c$. Thus,

$$\mathrm{U}(p_0|k+1) - \mathrm{U}(p_0|k) \propto -[p_0 + (1-p_0)(1-\theta)^{\hat{k}+l}] + (k - \hat{k} - l)(1-p_0)(1-\theta)^{\hat{k}+l-1}\theta \tag{A.17}$$

The sign of the difference now depends on the parameters. Therefore, it is unclear how the sender's continuation value varies as k increases.

References

BERGEMANN, D., A. BONATTI, AND A. SMOLIN (2015): "Selling Experiments," *Working Paper*.

BERGEMANN, D. AND U. HEGE (2005): "the Financing of Innovation: Learning and Stopping," *The RAND Journal of Economics*, 719–752.

BHASKAR, V. AND G. J. MAILATH (2019): "The Curse of Long Horizons," *Jounal of Mathematical Economics*, 82, 74–89.

FELGENHAUER, M. AND P. LOERKE (2017): "Bayesian Persuasion with Private Experimentation," *International Economic Review*, 58.

FELGENHAUER, M. AND E. SCHULTE (2014): "Strategic Private Experimentation," *American Economic Journal: Microeconomics*, 6, 74–105.

FU, W. (2018): "Optimal Contract to Reward Private Experimentation," *Working Paper*.

GLAZER, J. AND A. RUBINSTEIN (2004): "On Optimal Rules of Persuasion," *Econometrica*, 72, 1715–1736.

——— (2006): "A Study in the Pragmatics of Persuasion: a Game Theoretical Approach," *Theoretical Economics*, 1, 395–410.

HALAC, M. AND I. KREMER (2017): "Experimenting with Career Concerns," *Working Paper*.

HART, S., I. KREMER, AND M. PERRY (2017): "Evidence Games: Truth and Commitment," *The American Economic Review*, 107, 690–713.

HENRY, E. (2009): "Strategic Disclosure of Research Results: the Cost of Proving Your Honesty," *The Economic Journal*, 119, 1036–1064.

HENRY, E. AND M. OTTAVIANI (2018): "Research and the Approval Process: The Organization of Persuasion," *American Economic Review*, Forthcoming.

KAMENICA, E. AND M. GENTZKOW (2011): "Bayesian Persuasion," *The American Economic Review*, 101, 2590–2615.

Kolotilin, A. (2015): "Experimental Design to Persuade," *Games and Economic Behavior*, 90, 215–226.

Rayo, L. and I. Segal (2010): "Optimal Information Disclosure," *Journal of political Economy*, 118, 949–987.

Shin, H. S. (2003): "Disclosures and Asset Returns," *Econometrica*, 71, 105–133.