Uncertainty in Procurement Contracting with Time Incentives*

Wenzheng Gao† Daiqiang Zhang‡ Naibao Zhao§

Abstract

This paper studies A+B (cost-plus-time) procurement contracting with time incentives in the highway construction industry. In the presence of construction uncertainty, the contractor's actual completion time may deviate from the bid completion time, and the A+B contract design is not ex post efficient. Using data from highway procurement contracts in California, we show through a counterfactual analysis that the ex post efficient lane rental contract would reduce the social cost by $44.73 million (43.66%) on average. In particular, the average commuter cost would decrease by $60.27 million (85.47%), suggesting a substantial reduction in the construction externality to commuters from lane rental contracts.

Keywords: Procurement Contracts, Scoring Auctions, Incentives, Uncertainty.
JEL: C14, C57, D44.

*We would like to thank Yonghong An, Qi Li, Li Gan, Silvana Krasteva, and Zheng Fang for providing invaluable guidance and support. We are grateful to Badi Baltagi, Stephane Bonhomme, Xavier d'Haultfoeuille, Ali Hortasu, Hiroaki Kaido, Tong Li, Fernando Luco, Yao Luo, Robert Miller, Thierry Magnac, Steve Puller, Xun Tang, Steven Wiggins, and Yuanyuan Wan for their insightful comments. Financial support from the Fundamental Research Funds for the Central Universities is gratefully acknowledged by Wenzheng Gao. All errors are our own.

†School of Economics, Nankai University, Tianjin, China 300071. Email: wenzhenggao@nankai.edu.cn.

‡Department of Economics, University at Albany, State University of New York, Albany, NY 12222. Email: dzhang6@albany.edu.

§Research Institute of Economics and Management, Southwestern University of Finance and Economics, Chengdu, China 610074. Email: nzhao@swufe.edu.cn.

1 Introduction

In many procurement contracts, the buyer cares about multiple attributes and uses scoring auctions to select the seller. Moreover, post-auction uncertainty is an inherent component in many auctions such as auctions for oil tracts, timber, and construction (Esö and White, 2004; Luo, Perrigne, and Vuong, 2018a). One prominent example is that in highway construction procurement, buyers often care about both construction cost and commuter cost, where the commuter cost is represented by the negative externality to commuters during construction. In these situations, buyers use scoring auctions to choose the contractor. The bid score in scoring auctions is a weighted sum of bids of construction cost and completion time, and the bidder with the lowest score wins the contract. During the construction stage, however, the contractor usually faces unexpected shocks, including technical and logistical shocks (Perry and Hayes, 1985). These unexpected shocks may lead to late completion of the project. Since accelerating construction is costly for contractors, the procurer provides time incentives to reward early completion and punish late completion to induce the contractor to shorten the actual completion time. Therefore, designing efficient multi-attribute mechanisms in the presence of uncertainty has attracted considerable attention from theorists and practitioners in recent decades.

This paper investigates the efficiency of multi-attribute mechanisms in the presence of uncertainty by using highway procurement contracts from the California Department of Transportation (Caltrans). The contract awarded through scoring auctions is a cost-plus-time contract, the so-called A+B contract, in which A refers to the bid cost and B refers to the bid completion time. Motivated by the empirical relevance of uncertainty in highway construction, we theoretically analyze how the presence of construction uncertainty affects the efficiency of A+B contracts with time incentives. Then, we identify and estimate the model primitives. Using data on highway procurement contracts, we conduct counterfactuals to quantify the differences in efficiency between A+B and alternative contracts.

Our model builds on the literature on scoring auctions by incorporating time incentives and ex post construction uncertainty. In equilibrium, the actual number of working days can be smaller or larger than the number of days bid. This prediction is consistent with the pattern in our data, where more than half of contracts are completed earlier or later than the bid completion time. Since the winning contractor does not necessarily maximize social welfare regardless of whoever wins the contract, the A+B contract is not ex post efficient. However, if there were no uncertainty, the project would always be completed on time, and the A+B contract design could be ex post efficient.

We show that the model primitives, including the contractor's cost function, the distribution of the contractor's private cost type, and the uncertainty distribution, can be identified from the joint distribution of observed bids of cost and completion time, actual working days, and the equilibrium implications of the model. Our identification strategy takes several steps. First, we follow Guerre, Perrigne, and Vuong (2000) and explore the one-to-one mapping between the bid score and bidder's pseudo type to back out the pseudo type. This pseudo type is simply an increasing transformation of the bidder's cost type. Then, we parameterize the cost function because we cannot use bid days to identify both cost type and a nonparametric cost function of working days and because the other observables are used for identification elsewhere. Specifically, the bid score was used to recover the pseudo type in the first step, and the bid cost has no additional identification power because it is simply a known linear transformation of the score and bid days, and the number of early or late working days will be used to back out the corresponding uncertainty.

Next, we exploit the quantile relationship between bid days and pseudo type to identify the parameters of the cost function. This key relationship is implied by the correlation between actual working days and bid days with two one-to-one structural links among bid days, pseudo type and cost type. As a result, we are able to recover bidders' types. Moreover, we back out those uncertainties associated with contracts that are completed earlier or later than the number of days bid. However, uncertainties associated with contracts with on-time completion cannot be recovered, as the number of bid days does not depend on uncertainty. Hence, we use the recovered truncated pseudo sample of uncertainty to achieve the parametric identification of the uncertainty distribution. Following the identification strategy, we propose a multi-step semi-parametric estimation procedure.

We apply our model to evaluate the social welfare of highway A+B procurement contracts in California. We find that a contractor's capacity has significantly negative effects on the actual working days. This implies that contractors with larger capacities tend to complete the project earlier. Using our estimates, we compare welfare between A+B and lane rental contracts. A lane rental contract, in which the contractor pays a fixed fee for each day the lanes are occupied, is designed to reduce completion time and commuter costs in heavily populated areas or on busy roads (Srinivasan and Harris, 1991; Herbsman and Glagola, 1998). Although both lane rental and A+B contracts are designed primarily to reduce construction time, there is no consensus among researchers and practitioners regrading which contract mechanism is preferred (Strong, 2006).

To alleviate the discrepancy in contract design preferences, we study the differences in efficiency between A+B and lane rental contracts. First, we prove that the lane rental contract is ex post efficient in the presence of uncertainty when the disincentive for late completion is equal to the daily commuter cost. In the counterfactual analysis, we find that the average social cost under ex post efficient lane rental contracts would be \$44.73 million (43.66%) lower than that under A+B contracts, where the social cost is the sum of the construction cost and commuter cost. In particular, the average commuter cost under lane rental contracts would decrease by \$60.27 million (85.47%). This suggests a substantial reduction in the construction externality to commuters. However, this substantial commuter gain entails higher construction costs due to the lower number of working days in lane rental contracts.

Our work is closely related to the theoretical and empirical literature on auctions with ex post uncertainty. Esö and White (2004) show that the presence of ex post uncertainty can affect bidding strategy with risk-averse bidders. The empirical literature studies the effects of ex post uncertainty in gas lease auctions (Hendricks and Porter, 1988) and timber auctions with resale markets (Haile, 2001). In particular, Bajari, Houghton, and Tadelis (2014) use highway procurement contracts with first-price sealed-bid auctions in California to evaluate the adaption cost caused by unexpected construction shocks. Lewis and Bajari (2014) examine how the ex post shocks affect social welfare by influencing contractor effort in Minnesota's highway construction industry. Furthermore, Luo, Perrigne, and Vuong (2018a) demonstrate the identification of first-price sealed-bid auctions under ex post uncertainty. In comparison, this paper theoretically analyzes how ex post uncertainty affects the equilibrium and efficiency of scoring auctions with time incentives and then uses highway procurement contracts to evaluate their welfare performance.

Moreover, as multidimensional mechanisms, scoring auctions have been used to study many procurement situations in which the buyers care about multiple attributes (Che, 1993; Branco, 1997; Fang and Morris, 2006; Asker and Cantillon, 2008; Wang and Liu, 2014). Our model builds on Che (1993) and Asker and Cantillon (2008) by incorporating ex post uncertainty and time incentives to compare the efficiency between A+B and lane rental contracts.

This paper is related to a broader literature on the identification of auctions and contracts (Guerre, Perrigne, and Vuong, 2000; Jofre-Bonet and Pesendorfer, 2003; Krasnokutskaya, 2011; Hu, McAdams, and Shum, 2013; Gentry and Li, 2014; Li, Lu, and Zhao, 2015; Luo, Perrigne, and Vuong, 2018a,b; An and Tang, 2019). The identification arguments in these papers rely crucially on the mappings between the unobserved types of agents and the contract characteristics in the data. In comparison, in addition to the one-to-one mappings, we exploit the correlation between bid days and actual working days to construct a quantile relationship between the identified pseudo type and the observed bid days to identify our model.

Our work also contributes to a growing empirical literature on auctions with multi-dimensional contract attributes (e.g., Levin and Athey, 2001; Lewis and Bajari, 2011; Bajari, Houghton, and Tadelis, 2014; Koning and Van De Meerendonk, 2014; Krasnokutskaya, Song, and Tang, 2018). In contrast to these works, our model considers a scoring auction with time incentives followed by construction shocks in the construction stage. The presence of uncertainty may lead the actual number of working days to deviate from the number of bid days and, therefore, may affect the bidding strategy and efficiency of A+B contracts.

While we use the same source of data as Lewis and Bajari (2011), there are major qualitative differences between our model and theirs. Lewis and Bajari (2011) offer the first attempt to analyze A+B contracting with time incentives in California in the absence of construction uncertainty. In another paper, Lewis and Bajari (2014) use highway procurement contracts in Minnesota to examine how construction uncertainty affects the construction effort exerted by the contractor. Motivated by the deviation of the actual working days from the bid days, our model complements these two papers by incorporating construction uncertainty to explain the deviation pattern in the data. Moreover, the deviation feature suggests that an A+B contract is no longer ex post efficient in the presence of uncertainty. However, it could be ex post efficient if uncertainty were absent.

In addition, our work is also motivated by different policy issues from Lewis and Bajari (2011, 2014). These two papers investigate the welfare gains from switching the standard contract with bid-cost-only auctions to the A+B contract with scoring auctions and to the lane rental contract, respectively. They provide empirical evidence for the superiority of both A+B and lane contracts over standard contracts in terms of welfare gains. In contrast, we compare the welfare difference between A+B and lane rental contracts. We find that lane rental contracts perform better in terms of reducing commuter cost and social cost than A+B contracts; while A+B contracts are preferred in terms of reducing construction cost.

In Section 2, we analyze the equilibrium and efficiency of our model. Section 3 provides the main identification results. Section 4 introduces the background of highway A+B procurement contracts and describes the data. Section 5 defines our estimation method and reports the estimation results. In Section 6, we compare the welfare under lane rental contracts with that in the data. Section 7 concludes the paper. Proofs, figures, tables, and other details are collected in the Appendix.

2 The Model

2.1 Setup

A risk-neutral buyer (or procurer) seeks to procure a highway project from among $N \geq 2$ potential risk-neutral bidders (or contractors). In the scoring auction, each bidder submits a bid, which is a combination of cost $p^B \in \mathcal{P} \subset \mathbb{R}_+$ and working days $x^B \in \mathcal{X} \subset \mathbb{R}_+$. Prior to the auction stage, the procurer announces three messages: (i) an engineer's estimate of the pair (p^E, x^E) for the project, where p^E and x^E are the estimates of project cost and working days, respectively; (ii) a scoring rule $s : \mathcal{P} \times \mathcal{X} \mapsto s(\mathcal{P}, \mathcal{X})$ that associates a score with a bid pair (p^B, x^B) and represents a continuous preference of the procurer over (p^B, x^B); and (iii) an incentive/disincentive scheme (r, d), where $r \in \mathbb{R}_+$ and $d \in \mathbb{R}_+$ are the daily cash bonus for early completion and the daily cash penalty for late completion, respectively. We maintain $r < d$ to follow the practice used by Caltrans and because it is consistent with our data (e.g., Lewis and Bajari, 2011).

There is an ex ante private cost type θ_i for bidder $i \in \mathcal{N} = \{1, \cdots, N\}$, which is drawn independently from a distribution $F_\Theta(\cdot)$ with density $f_\Theta(\cdot)$ and support $\mathcal{S}_\Theta \subset \mathbb{R}_+$. Type θ_i reflects contractor i's innate cost, such as managerial capacity, expertise in working on a tight schedule, or relationships with input suppliers or subcontractors. The uncertainty ε_i ex post auction for bidder $i \in \mathcal{N}$ is distributed as $F(\cdot)$ with density $f(\cdot)$ and support $\mathcal{S}_\varepsilon \subset \mathbb{R}_+$. Uncertainty captures the unexpected shocks (such as productivity shocks and input delays) encountered by the contractor during the construction stage. Empirical evidence for the relevance of construction uncertainty in highway procurement contracts can be found in Bajari, Houghton, and Tadelis (2014) and Lewis and Bajari (2014), among others. Following the related literature (e.g., Esö and White, 2004; Luo, Perrigne, and Vuong, 2018a), the ex post uncertainty ε_i is assumed to be independent of all θ_j, including θ_i.[1] However, we allow ε_i to be correlated among bidders or even identical in the case of macroeconomic or other exogenous shocks common to all bidders. Upon drawing the private cost type θ_i, bidder i quotes a sealed bid pair (p_i^B, x_i^B). The contract is awarded to

[1] This independence of uncertainty from cost type can be interpreted as conditional on the observed characteristics of the project, contractor, or economic environment (e.g., Lewis and Bajari, 2014). Therefore, in the empirics, we allow for correlation between uncertainty and cost type through observed heterogeneous characteristics across bidders.

the bidder with the lowest score, where the score is calculated according to the announced scoring rule.

In the spirit of Esö and White (2004), the *deterministic* construction cost for bidder i is $c(x_i, \theta_i)$, which represents the cost of completing the project in days x_i for a contractor of type θ_i, and the *actual* construction cost is the deterministic cost times the construction uncertainty, i.e., $\varepsilon_i \cdot c(x_i, \theta_i)$. This multiplicative structure is similar to that in Bajari, Houghton, and Tadelis (2014), who evaluate the adaptation cost caused by ex post construction shocks in California's highway procurement contracts. Note that if ε_i is constant with $\varepsilon_i = 1$ for any $i \in \mathcal{N}$, our cost specification reduces to the case without uncertainty as in Lewis and Bajari (2011). As will be shown below, in this multiplicative specification, the actual working days can deviate from the bid days in equilibrium.

An alternative specification for the *actual* construction cost could be the *deterministic* construction cost plus the ex post uncertainty. However, in equilibrium, the actual working days would always equal the bid days, as the marginal *actual* construction cost of working days does not depend on uncertainty. This is not consistent with the pattern in our data, where actual working days can deviate from bid days. This echoes Luo, Perrigne, and Vuong (2018a), who note that, under the above additive structure, the equilibrium bid in the first-price sealed-bid auction with ex post uncertainty is the same as that without ex post uncertainty. Therefore, we use the multiplicative structure to rationalize the deviation pattern in our data.[2]

After winning the contract, the winning bidder's *total* cost of completing the project is the sum of the *actual* construction cost $\varepsilon \cdot c(x^A, \theta)$ and the incentive cost $K(x^A, x^B, r, d)$, where x^A represents the actual working days. We suppress the dependence of variables on the winner's identity for simplicity. The actual working days x^A may differ from the bid days x^B. This is because under time incentives, the contractor may adjust his initial implementation plan in response to the realized uncertainty during the construction stage. The incentive cost $K(x^A, x^B, r, d)$ represents the incentive scheme determined by the procurer. In that used by Caltrans, the contractor faces a punishment $d \cdot (x^A - x^B)$ if the actual working days x^A exceed the bid days x^B or receives a reward $r \cdot (x^B - x^A)$ if the project is completed in less than the bid days. The model primitives $\mathcal{M} = [F_\Theta(\cdot), F(\cdot), c(\cdot, \cdot)]$ are common knowledge for all players but unknown to the econometrician. Finally, our model can be viewed as the second stage of a game with entry in the first stage in the spirit of Levin and Smith (1994) and Luo, Perrigne, and Vuong (2018a), where N is the

[2] Under the additive structure of the *actual* construction cost, if bidders are risk-averse, the actual working days might deviate from the bid days. However, this is beyond the scope of this paper and left to future research.

number of entrants.

2.2 Equilibrium

We maintain the standard assumptions to analyze the equilibrium of A+B contracting with time incentives in the presence of uncertainty. We focus on the symmetric pure strategy Bayesian Nash Equilibrium (psBNE), where a psBNE consists of a bidding strategy $(p^{B^*}(\theta), x^{B^*}(\theta))$ and an actual working days strategy $x^{A^*}(\theta, \varepsilon)$ for any (θ, ε). For a generic function $g(\cdot)$ with more than one argument, we use $g_i(\cdot)$ to denote the first-order derivative with respect to its i-th argument and $g_{ij}(\cdot)$ to denote the second-order derivative with respect to its i-th and j-th arguments.

Assumption 1. *The deterministic cost function satisfies* $c(\cdot, \cdot) \geq 0$, $c_1(\cdot, \cdot) < 0$, $c_2(\cdot, \cdot) > 0$, $c_{11}(\cdot, \cdot) > 0$, *and* $c_{12}(\cdot, \cdot) < 0$.

Assumption 1 imposes standard conditions on the cost function $c(\cdot, \cdot)$ (e.g., Laffont and Tirole, 1993; Lewis and Bajari, 2011). The decreasing monotonicity of the cost function implies that construction acceleration is costly. The condition that the marginal cost of working days is decreasing in type implies that a less efficient (larger θ) contractor enjoys a larger cost-reduction induced by one additional working day. Therefore, it may provide higher-powered incentives for less efficient contractors to bid more working days. As shown below, the equilibrium number of bid days is increasing in type.

Next, we specify a linear-in-price scoring rule, which is widely used in highway procurement contracts by the Department of Transportation (DoT) in the United States.[3] The scoring rule is given by

$$s(p^B, x^B) = p^B + c_u \cdot x^B, \tag{1}$$

where the user cost $c_u \geq 0$ measures the time value of the externality entailed by construction.

We use backward induction to analyze the model equilibrium. In the second stage of construction, we analyze the winning contractor's optimal actual working days given his type θ, bid pair (p^B, x^B), and uncertainty ε. Because the payoff in the construction stage is the bid cost p^B minus the total cost, the contractor chooses the optimal actual working

[3]These states include California, Delaware, Idaho, Massachusetts, Oregon, Texas, Utah, and Virginia. In addition, other scoring rules are used, such as the price-over-quality ratio rule in Alaska and Colorado. We restrict our analysis to the linear-in-price rule since it is consistent with the data in hand.

days by minimizing his total cost,

$$\widetilde{x}^{A^*}(x^B, \theta, \varepsilon) = \underset{x^A}{\operatorname{argmin}} \{\varepsilon \cdot c(x^A, \theta) + K(x^A, x^B, r, d)\}. \tag{2}$$

Intuitively, $\widetilde{x}^{A^*}(x^B, \theta, \varepsilon)$ may not equal the bid days x^B due to the presence of uncertainty. If the uncertainty is small (large) relative to incentive r (disincentive d), contractor tends to complete the project before (after) the bid days; if the uncertainty is mild relative to the incentive/disincentive, the contractor tends to complete the project on time. Therefore, the optimal actual working days is a trade-off between the cost reduction and the penalty from late completion or between the cost increase and the reward for early completion. The lemma below formalizes these results.

Lemma 1. *Under Assumption 1, the second-stage optimal actual working days given $(x^B, \theta, \varepsilon)$ satisfies*

$$\widetilde{x}^{A^*}(x^B, \theta, \varepsilon) = \begin{cases} x^r(\theta, \varepsilon), & \text{if } \varepsilon \leq \varepsilon^r(\theta, x^B) \\ x^B, & \text{if } \varepsilon \in [\varepsilon^r(\theta, x^B), \varepsilon^d(\theta, x^B)] \\ x^d(\theta, \varepsilon), & \text{if } \varepsilon \geq \varepsilon^d(\theta, x^B) \end{cases} \tag{3}$$

where the two cutoff levels of uncertainty are given by

$$\varepsilon^r(\theta, x^B) = \frac{-r}{c_1(x^B, \theta)} \quad \text{and} \quad \varepsilon^d(\theta, x^B) = \frac{-d}{c_1(x^B, \theta)}, \tag{4}$$

the early working days $x^r(\theta, \varepsilon)$ and late working days $x^d(\theta, \varepsilon)$ are given by

$$-\varepsilon \cdot c_1(x^r(\theta, \varepsilon), \theta) = r \quad \text{and} \quad -\varepsilon \cdot c_1(x^d(\theta, \varepsilon), \theta) = d. \tag{5}$$

Moreover, $\varepsilon^r(\theta, x^B) < \varepsilon^d(\theta, x^B)$ and $x^r(\theta, \varepsilon) \leq x^B \leq x^d(\theta, \varepsilon)$.

Proof: See the Appendix.

Next, we use Lemma 1 to analyze the equilibrium bids of cost and completion time. Since we focus on the symmetric equilibrium, we drop the index of a bidder i for expositional simplicity. In the bidding stage, a bidder with type θ quotes a pair of cost and working days $(p^{B^*}(\theta), x^{B^*}(\theta))$ to maximize his expected payoff

$$\begin{aligned}\big(p^{B^*}(\theta), x^{B^*}(\theta)\big) &= \underset{p^B, x^B}{\operatorname{argmax}} \left\{ \left(p^B - \mathbb{E}_\varepsilon \Big[\varepsilon \cdot c(\widetilde{x}^{A^*}(x^B, \theta, \varepsilon), \theta) + K(\widetilde{x}^{A^*}(x^B, \theta, \varepsilon), x^B, r, d) \Big] \right) \right. \\ &\quad \times \left. \Pr\Big(\text{win} \;\Big|\; s = p^B + c_u x^B \Big) \right\}, \end{aligned} \tag{6}$$

where $\Pr(\text{win}|s = p^B + c_u x^B)$ is the conditional probability of winning the auction given bid score $s = p^B + c_u x^B$, and $\mathbb{E}_\varepsilon$ denotes the expectation with respect to ε. Following the literature on scoring auctions (e.g., Che, 1993; Asker and Cantillon, 2008), the contractor's optimization problem in (6) is equivalent to choosing the optimal score $s(v(\theta))$

$$s(v(\theta)) = \underset{b}{\operatorname{argmax}} \Big\{ \big(b - v(\theta)\big) \times \Pr(\text{win} \mid b) \Big\}, \tag{7}$$

where the contractor's pseudo type $v(\theta)$ is given by

$$v(\theta) = \min_{x^B} \Big\{ c_u x^B + \mathbb{E}_\varepsilon \Big[\varepsilon \cdot c(\widetilde{x}^{A^*}(x^B, \theta, \varepsilon), \theta) + K(\widetilde{x}^{A^*}(x^B, \theta, \varepsilon), x^B, r, d) \Big] \Big\}. \tag{8}$$

The interior minimizer for $v(\theta)$ is the equilibrium bid days $x^{B^*}(\theta) = \underset{x^B}{\operatorname{argmax}}\, v(\theta)$.[4] As a result, the equilibrium payment is $p^{B^*}(\theta) = s(v(\theta)) - c_u x^{B^*}(\theta)$, and the equilibrium actual working days is $x^{A^*}(\theta, \varepsilon) = \widetilde{x}^{A^*}(x^{B^*}(\theta), \theta, \varepsilon)$.

In addition, the winner is the contractor with the lowest type because

$$\frac{d}{d\theta} s(v(\theta)) = s'(v(\theta)) v'(\theta) > 0, \tag{9}$$

where $v'(\theta) > 0$ and $s'(v(\theta)) > 0$, as shown in the proof of Proposition 1, are typical properties in the auction literature (e.g., Asker and Cantillon, 2008; Krishna, 2009).

Proposition 1. *Under Assumption 1, there exists a unique symmetric psBNE* $\big(p^{B^*}(\theta), x^{B^*}(\theta), x^{A^*}(\theta, \varepsilon)\big)$ *for the A+B contract such that for any* θ*, we have the following:*
(i) The equilibrium bid for working days is

$$x^{B^*}(\theta) = \arg\min_{x^B} \Big\{ c_u x^B + \mathbb{E}_\varepsilon \Big[\varepsilon \cdot c(\widetilde{x}^{A^*}(x^B, \theta, \varepsilon), \theta) + K(\widetilde{x}^{A^*}(x^B, \theta, \varepsilon), x^B, r, d) \Big] \Big\}. \tag{10}$$

[4] The interior minimizer $x^{B^*}(\theta)$, if it exists, must be unique because it can be shown that the objective function (8) is globally convex in x^B by using the proofs in Proposition 1.

Moreover, $dx^{B^*}(\theta)/d\theta > 0$.

(ii) The equilibrium actual number of working days is

$$x^{A^*}(\theta,\varepsilon) = \widetilde{x}^{A^*}(x^{B^*}(\theta),\theta,\varepsilon) = \begin{cases} x^r(\theta,\varepsilon), & \text{if } \ \varepsilon \le e^r \\ x^{B^*}(\theta), & \text{if } \ \varepsilon \in [e^r, e^d] \\ x^d(\theta,\varepsilon), & \text{if } \ \varepsilon \ge e^d \end{cases} \tag{11}$$

where in equilibrium the two cutoff levels of uncertainty are constant for any θ

$$\varepsilon^r(\theta, x^{B^*}(\theta)) = e^r \quad \textit{and} \quad \varepsilon^d(\theta, x^{B^*}(\theta)) = e^d, \tag{12}$$

with $e^r < e^d$.

(iii) The equilibrium bid for cost is

$$\begin{aligned} p^{B^*}(\theta) \;&=\; \mathbb{E}_\varepsilon\Big[\varepsilon \cdot c(x^{A^*}(\theta,\varepsilon),\theta) + K(x^{A^*}(\theta,\varepsilon), x^{B^*}(\theta), r, d)\Big] \\ &+\; \int_\theta^{\overline{\theta}} \mathbb{E}_\varepsilon[\varepsilon \cdot c_2(x^{A^*}(\theta,\varepsilon),\theta)] \left[\frac{1-F_\Theta(t)}{1-F_\Theta(\theta)}\right]^{N-1} dt \end{aligned} \tag{13}$$

Proof: See the Appendix.

In Part (i), bid days $x^{B^*}(\theta)$ is increasing in type θ, which implies that for a less efficient (larger type) contractor, more working days are required to complete the project. In Part (ii), the actual working days may deviate from the bid days, depending on the level of uncertainty. However, as shown in Lewis and Bajari (2011), if there were no uncertainty, the actual working days would always equal the bid days. In addition, the two cutoff levels of uncertainty in equilibrium are constant for any type. This is because the cutoff level of uncertainty depends on type through the marginal cost of bid days, while the marginal cost of bid days in equilibrium is constant for any type. This constancy of the marginal cost can be explained by the direct effect of type on the marginal cost and the indirect effect through the equilibrium bid days $x^{B^*}(\theta)$. As shown in the proof of Proposition 1, these two opposite effects cancel out when type changes, leading to the constancy of the marginal cost and hence the cutoff levels of uncertainty.

2.3 Efficiency

In a similar spirit to Lewis and Bajari (2011), the social welfare in the presence of uncertainty is given by

$$W(\theta, \varepsilon, x^A) = V_c - \varepsilon \cdot c(x^A, \theta) - c_s x^A = V_c - S_c, \tag{14}$$

where V_c is the social value of the project, $\varepsilon \cdot c(x^A, \theta)$ is the construction cost, $c_s x^A$ is the commuter cost induced by the construction externality to commuters with daily commuter cost $c_s \geq 0$, and the social cost S_c is the sum of the construction cost and commuter cost.

We say that a contract design is ex post efficient if the completion time x^A is welfare maximizing for all types θ and all uncertainties ε. In other words, ex post efficiency implies that regardless of which contractor wins the contract, the winner always maximizes social welfare. A contract design is ex ante efficient if the contract is always awarded to the bidder who generates the highest social welfare in equilibrium for all uncertainties ε. These two notions distinguish regulating the winning contractor (ex post efficiency) from choosing that contractor (ex ante efficiency).

First, as suggested by the deviation of actual working days from the bid days in Proposition 1, the A+B contract is not ex post efficient in the presence of uncertainty. Formally, the ex post efficient completion time $x_o^A(\theta, \varepsilon)$ must satisfy

$$-\varepsilon \cdot c_1(x_o^A(\theta, \varepsilon), \theta) = c_s. \tag{15}$$

However, due to $d > r$, it cannot be that $d = r = c_s$. Consequently, as implied by (5), both $x^r(\theta, \varepsilon) = x_o^A(\theta, \varepsilon)$ for early completion and $x^d(\theta, \varepsilon) = x_o^A(\theta, \varepsilon)$ for late completion cannot hold simultaneously for all types θ and all uncertainties ε. If the uncertainty were absent, however, the actual working days would equal the bid days. That is, the A+B contract could be ex post efficient (see Lewis and Bajari, 2011).

Second, the A+B contract with uncertainty can be ex ante efficient. The social welfare in equilibrium is given by

$$W^*(\theta, \varepsilon) = V_c - \varepsilon \cdot c(x^{A^*}(\theta, \varepsilon), \theta) - c_s x^{A^*}(\theta, \varepsilon) \tag{16}$$

As shown in the proof of Proposition 2, if $r < d \leq c_s$, then $\partial W^*(\theta, \varepsilon)/\partial \theta < 0$ for any (θ, ε), which implies the ex ante efficiency of A+B contracts. Therefore, this may explain the fact that the procurer specifies $r < d \leq c_s$ in practice, because it can guarantee the ex ante efficiency of A+B contracts in the presence of uncertainty.

Proposition 2. *Under Assumption 1, the A+B contract in the presence of uncertainty is ex ante efficient if $r < d \leq c_s$, but it cannot be ex post efficient.*

Proof: See the Appendix.

3 Identification

We consider model identification in an environment for which the data report the bid pair of cost and working days (P^B, X^B), the bid score S, and the actual working days X^A, where $X^A = X^B$ for on-time completion, $X^A = X^r$ for early completion, and $X^A = X^d$ for late completion. We explain how to use these observables and the equilibrium conditions to recover the model primitives $\mathcal{M} = [F_\Theta(\cdot), F(\cdot), c(\cdot, \cdot)]$. Our identification arguments apply conditional on contract or bidder characteristics observed in the data. For expositional simplicity, we suppress this dependence whenever there is no ambiguity. A random variable is denoted by an upper-case letter, while realized values are denoted by lower-case letters.

First, using the observed score can directly back out the bidder's pseudo type. This is ascribed to the exploration of the one-to-one mapping between bid score and pseudo type, which is a standard identification argument from Guerre, Perrigne, and Vuong (2000). As shown in the proof of Proposition 1, the first-order condition of (7) with respect to score implies

$$s'(v) = (N-1)(s(v) - v)\frac{f_V(v)}{1 - F_V(v)} > 0 \tag{17}$$

with boundary condition $s(\overline{v}) = \overline{v}$, where $\overline{v}$ is the upper bound of pseudo type. Denote by $F_S(\cdot)$ and $f_S(\cdot)$ the score's cumulative distribution function and density function, respectively. Since $s'(v) > 0$ implies $F_S(b) = F_V(s^{-1}(b)) = F_V(v)$, we can recover the pseudo type as

$$v = s - \frac{1}{N-1}\frac{1 - F_S(s)}{f_S(s)}. \tag{18}$$

Next, we assume that the cost function is multiplicative in type

$$c(x, \theta) = \theta c_o(x), \tag{19}$$

where $c_o(\cdot)$ is the base cost, which is a function of working days. This specification follows the related identification literature. For example, Krasnokutskaya (2011) derives bidder's cost as the product of an individual cost component and an auction-common component in the highway procurement auction. Similar specifications are used in the identification of various economic theories (e.g., Ekeland, Heckman, and Nesheim, 2004; Perrigne and Vuong, 2011).

Even with the multiplicative specification of the cost function and the recovered pseudo type, the model is still not identified without further information. This is because it seems impossible to use only one observed value of bid days to identify both the cost function $c_o(\cdot)$ and type θ and because the identification power of the other observables is exploited elsewhere. Specifically, the bid score is used to recover the pseudo type, the early or late working days are used to back out the uncertainty because each of them depends on both type and uncertainty, and the bid cost has no additional identification power because it is simply a known linear transformation of score and bid days.[5] As shown in the following lemma, an observationally equivalent structure can be obtained by multiplying type θ by some positive constant and dividing $c_o(\cdot)$ by this constant in the cost function (19).

Lemma 2. *Consider a structure $\mathcal{M} = [c_o(\cdot), F_\Theta(\cdot), F(\cdot)]$. Define another structure $\widetilde{\mathcal{M}} = [\widetilde{c}_o(\cdot), \widetilde{F}_\Theta(\cdot), F(\cdot)]$, where $\widetilde{c}_o(\cdot) = c_o(\cdot)/\delta$, $\widetilde{F}_{\widetilde{\Theta}}(\cdot) = F_\Theta(\cdot/\delta)$ for some $\delta > 0$. Then, the two structures are observationally equivalent.*

To address the difficulty of limited observables, we parameterize the cost function and exploit the relationship between bid days and pseudo type for identification.

Assumption 2.
(a) The cost function is $c(x, \theta) = \theta(\alpha_2 x^2 + \alpha_1 x + \alpha_0)$ with $\alpha_2 > 0$, $\alpha_1 < 0$, and $\alpha_0 \neq 0$.
(b) The lower bound of type support is $\underline{\theta} = 1$.

Under the specification of the cost function in Part (a) of Assumption 2, Assumption 1 implies the restrictions on cost parameters $\alpha = (\alpha_2, \alpha_1, \alpha_0)$. The parameterization of the cost function is also used for identification in wage contracts and nonlinear pricing with adverse selection, such as in D'Haultfoeuille and Février (2016) and Luo, Perrigne, and Vuong (2018b). Part (b) is a scale normalization implied by Lemma 2. More generally, the normalization of any $\tau \in [0, 1]$-th quantile of the type distribution can be used for identification. In addition, an alternative normalization of the cost function could also be useful for identification (e.g, Luo, Perrigne, and Vuong, 2018b).

The key identification argument arises from the exploitation of the quantile relationship between bid days X^B and pseudo type V, which is implied by the correlation between

[5] As shown in the proof of Proposition 3, the bid cost can be written as a known function of the observables and primitives already identified.

X^B and X^A, as well as two one-to-one mappings between V and θ and between X^B and θ. Variations in quantiles are explored for the identification of standard auctions (e.g., Guerre, Perrigne, and Vuong, 2009). We rewrite (8) as

$$V = c_u \cdot X^B + \mathbb{E}_\varepsilon \Big[\varepsilon \cdot c\left(X^A(X^B, \theta, \varepsilon), \theta\right) + K\left(X^A(X^B, \theta, \varepsilon), X^B, r, d\right) \Big]. \tag{20}$$

Then, we will show how to explore the equilibrium conditions and the parametric specification of the cost function to obtain the quantile relationship between X^B and V based on (20). First, since the two cutoff levels of uncertainty are constant for any type, we can identify the probabilities of early and late completion, respectively, through

$$F(e^r) = \Pr(\varepsilon < e^r) = \Pr(X^A < X^B) \text{ and } 1 - F(e^d) = \Pr(\varepsilon > e^d) = \Pr(X^A > X^B). \tag{21}$$

Combining this result with the first-order condition in (10), we obtain the expression for the constant marginal cost of bid days

$$c_1(X^B, \theta) = \frac{rF(e^r) + d - dF(e^d) - c_u}{\int_{e^r}^{e^d} \varepsilon dF(\varepsilon)} = \frac{\kappa_0}{\int_{e^r}^{e^d} \varepsilon dF(\varepsilon)} = \kappa_1, \tag{22}$$

where $\kappa_0 = rF(e^r) + d - dF(e^d) - c_u$ is an identified constant, κ_1 is a unknown constant, and the constancy of the marginal cost of bid days was explained in Section 2.

Next, using the parametric specification of the cost function in Assumption 2, we obtain closed-form expressions for the early and late working days according to their equilibrium conditions in (5) and obtain the expression for the bid days:

$$X^r = \frac{-r\varepsilon^{-1}\theta^{-1} - \alpha_1}{2\alpha_2}, \quad X^d = \frac{-d\varepsilon^{-1}\theta^{-1} - \alpha_1}{2\alpha_2}, \quad \text{and } X^B = \frac{\kappa_1}{2\alpha_2\theta} - \frac{\alpha_1}{2\alpha_2}. \tag{23}$$

Then, we can rewrite the expressions for X^r and X^d in terms of (X^B, ε) instead of (θ, ε). As a result, combining the rewritten expressions for X^r and X^d with the independence between θ and ε gives rise to

$$r(\alpha_1 + 2\alpha_2\mathbb{E}(X^B))\mathbb{E}(\varepsilon^{-1} | \varepsilon \le e^r) = -\kappa_1(\alpha_1 + 2\alpha_2\mathbb{E}(X^r)) \tag{24}$$

$$d(\alpha_1 + 2\alpha_2\mathbb{E}(X^B))\mathbb{E}(\varepsilon^{-1} | \varepsilon \ge e^d) = -\kappa_1(\alpha_1 + 2\alpha_2\mathbb{E}(X^d)) \tag{25}$$

Denote by m_ε the mean of uncertainty. Using (22) leads to

$$m_\varepsilon = E(\varepsilon) = \int_{\varepsilon \le e^r} \varepsilon dF(\varepsilon) + \int_{\varepsilon \ge e^d} \varepsilon dF(\varepsilon) + \frac{\kappa_0}{\kappa_1}. \tag{26}$$

Therefore, we use the results in (23)-(26) to rewrite the expression for pseudo type V in (20) as a parametric functional form of X^B only instead of $(X^B, X^A, \theta, \varepsilon)$. Since there are two one-to-one structural links from $dv(\theta)/d\theta > 0$ and $dX^B(\theta)/d\theta > 0$ in equilibrium, we can further replace X^B and V with their respective quantiles $Q_{X^B}(\tau)$ and $Q_V(\tau)$, where $Q_{X^B}(\tau)$ and $Q_V(\tau)$ are the quantiles of X^B and V, respectively, for any $\tau \in [0,1]$. After rearrangement, we obtain the key reduced-form relationship between $Q_V(\tau)$ and $Q_{X^B}(\tau)$ as

$$0 = \beta_0 + \beta_1 Q_V(\tau) + \beta_2 Q_V(\tau) Q_{X^B}(\tau) + \beta_3 Q_{X^B}(\tau) + \beta_4 (Q_{X^B}(\tau))^2 \tag{27}$$

where $\beta = (\beta_0, \beta_1, \beta_2, \beta_3, \beta_4)$, defined in the proof of Proposition 3, is a system of equations of the primitives $(\alpha_0, \alpha_1, \alpha_2, m_\varepsilon)$.

Due to the nonlinear relationship between X^B and V in (20), in general, the full rank property holds in (27). Therefore, $\beta = (\beta_0, \beta_1, \beta_2, \beta_3, \beta_4)$ is identified by choosing any five different values of $\tau \in (0,1)$ to construct five linearly independent equations. Then, combining the normalization in Assumption 2 recovers the slope parameters (α_1, α_2). Moreover, we use bid days X^B to back out the bidder's type θ and recover the type distribution $F_\Theta(\cdot)$ on its support $\mathcal{S}_\Theta$. Since the intercept parameter α_0 is involved only in β_0 and β_0 includes the mean uncertainty m_ε, α_0 is identified if m_ε is known.

Finally, we use the early or late working days to recover the corresponding uncertainty. For those contracts with on-time completion, the actual working days equals bid days, which does not rely on uncertainty, and we cannot recover the uncertainty on its entire support. Therefore, we can identify the truncated distribution of ε denoted by $G(\cdot)$ on the support $\widetilde{\mathcal{S}}_\varepsilon = \mathcal{S}_r \cup \mathcal{S}_d \subset \mathcal{S}_\varepsilon$ with $\mathcal{S}_r = \{\varepsilon : \varepsilon \le \varepsilon^r\}$ and $\mathcal{S}_d = \{\varepsilon : \varepsilon \ge \varepsilon^d\}$. Then, the uncertainty distribution $F(\cdot)$ is identified on $\widetilde{\mathcal{S}}_\varepsilon$ as

$$F(\varepsilon) = G(\varepsilon) F(\varepsilon^r) \text{ if } \varepsilon \in \mathcal{S}_r, \text{ and } F(\varepsilon) = G(\varepsilon)(1 - F(\varepsilon^d)) \text{ if } \varepsilon \in \mathcal{S}_d. \tag{28}$$

Proposition 3. *Suppose that Assumptions 1-2 hold and the mean uncertainty is known. Then, the cost parameters $(\alpha_2, \alpha_1, \alpha_0)$ are identified, and the type distribution $F_\Theta(\cdot)$ and the uncertainty distribution $F(\cdot)$ are identified on the supports $\mathcal{S}_\Theta$ and $\widetilde{\mathcal{S}}_\varepsilon$, respectively.*

An immediate result of Proposition 3 is that if the uncertainty distribution is parameterized, with, say, the commonly used log normal distribution, then using the above-recovered pseudo sample of uncertainty can identify the parameters of the uncertainty distribution.

Corollary 1. *Suppose that Assumptions 1-2 hold and the uncertainty distribution is parameterized. Then, the cost parameters $\alpha = (\alpha_2, \alpha_1, \alpha_0)$ and parameters of the uncertainty distribution are identified, and the type distribution $F_\Theta(\cdot)$ is identified on the support $\mathcal{S}_\Theta$.*

4 A+B Contracts: Background and Data

Caltrans is a government department of the state of California that is responsible for the planning, construction and maintenance of public transportation facilities such as highways, bridges, and railways. The A+B contract design was introduced by Caltrans in the 1990s as an experiment for emergency-type projects following the criticism that highway construction took too much time, and it was extended to non-emergency-type projects in 2000. First, the engineer estimates the project's cost and a target number of working days for project completion. The maximum number of lanes that can be closed during each phase of the project and their closure times are also specified by engineers. Once informed of the engineer's estimates, scoring rule, and time incentives, contractors draw private costs for completing the project and quote their costs and completion time in the bidding stage.[6] The contract is awarded to the contractor with the lowest score according to the announced scoring rule. In the construction stage, faced with realized uncertainty, contractor may have actual working days that differs from its bid days. This strategic deviation is contractor's trade-off between a cost increase from early completion and the corresponding bonus or between a cost reduction from late completion and the corresponding penalty.

4.1 Data

We use the same source of data on A+B contracts as Lewis and Bajari (2011). The data include 424 bids submitted by contractors in 80 A+B contracts from 2003 to 2008. These contracts include barrier construction, bridge repair or resurfacing, new lane and ramp construction, road rehabilitation, slope work and widening/realignment. For each contract, the data report the bid pair of cost and completion time submitted by each

[6] Caltrans sets the user cost in the scoring rule based on a standardized statewide formula for liquidated damages, which depends on the engineer's estimates, type of work, and the expenses of the resident engineer's office, among other factors.

bidder, the number of bidders, the engineer's estimated project cost and working days, daily incentive and disincentive, user cost specified in the scoring rule, and the actual working days.

We also observe additional characteristics in the data, including each contractor's capacity, measured as the total value of all contracts held by a particular contractor during our sample period, the distance between a contractor's location and the project work site, and an indicator for whether the contract is federally funded. In addition, we have information on the daily commuter cost, which is a measure of the negative externality to commuters during project construction.[7]

[Insert Table 1 here]

Table 1 presents the summary statistics of the data at the contract level.[8] In the upper panel, the average estimated cost and completion time are approximately $22.4 million and 322 days, respectively. The user cost has an average value $14,800 lies between the daily incentive, at an average of $8900, and the daily disincentive, at an average of $16,100. The user cost is approximately one-third of the daily commuter cost, which is $50,500, on average. The average winning bid for the cost is $20.4 million, which is slightly smaller than the engineer's average estimated cost. The number of bidders ranges from 2 to 14, with an average of 6. Most contracts are funded by the federal government. On average, the firm's capacity is $71.3, and its distance to the work site is 65.9 miles.

In the lower panel, we report the difference and the ratio between actual working days and bid days for each contract. The average bid days and actual working days are 250 and 263, respectively. On average, an A+B contract is delayed by approximately 13 days, and the ratio of actual working days to bid days is 1.1, both of which suggest the relevance of uncertainty in A+B contracts.

Table 2 reports the summary statistics of the data at the bid level. The average bid cost and bid days are $19.1 million and 207 days, respectively. On average, the bid cost is slightly smaller than engineer's estimated cost, and the number of bid days is smaller than or equal to the engineer's estimate of working days. Both capacity and distance at the bidder level in Table 2 are very close to those at the contract level in Table 1. Since

[7]The commuter cost is constructed by Lewis and Bajari (2011). Using the information on traffic volumes, the percentage of trucks at each of the contract locations, time value and delay, they calculate the daily commuter cost by multiplying the traffic, delay, and time value.

[8]We drop 3 contracts that have only one bidder and thus have 77 contracts and 421 bidders in the data.

the bid score is a weighted sum of bid cost and days, the average bid score of $22.2 million in Table 1 is smaller than the average engineer's estimated score of $28.1 million in Table 2.

[Insert Table 2 here]

4.2 Motivating evidence

We present some preliminary evidence that motivates our model. Figure 1 presents histograms of the difference and ratio between bid days and actual working days. We find that two-thirds of contracts are not completed on time. Among the 77 A+B contracts, 40% (31 cases) were completed on time, 30% (23 cases) were completed earlier than bid, and 30% (23 cases) were completed late. This provides preliminary evidence that contractors may adjust the completion time by deviating from the bid days in response to unexpected construction uncertainty.

[Insert Figure 1 here]

Moreover, Figure 2 plots the empirical distributions of the normalized deviation days for contracts that are completed earlier or later than bid, where the normalized early (late) deviation days is the difference between the early (late) working days and bid days divided by the engineer's estimated days to capture contract heterogeneity. As implied by Figure 2, for an A+B contract with an average engineer's completion time of 322 days, if the contract is early (late) completion, there is a 50% chance that the saved (delayed) working time is greater than 10 days (71 days). These results suggest that uncertainty, to some extent, leads contractor to strategically deviate the actual working days from bid days.

[Insert Figure 2 here]

5 Empirical Analysis

In this section, we follow the identification strategy to estimate a semiparametric model that accounts for the heterogeneity of bidders and contracts in the data.

5.1 Estimation strategy

First, due to the limited sample size for early or late working days, we use linear regressions to analyze the effects of various characteristics on the early or late working days

$$\mu_r(z) = \mathbb{E}(X^r|Z = z) = z'\chi^r \quad \text{and} \quad \mu_d(z) = \mathbb{E}(X^d|Z = z) = z'\chi^d, \tag{29}$$

where χ^r and χ^d are unknown parameters, Z is the logarithm of contractor's capacity, which is normalized by dividing it by the engineer's estimate of working days to capture project heterogeneity. We also consider various linear regressions of early and late working days on different characteristics. As indicated in Table 3 below, only the contractor's capacity has a statistically significant effect on the actual working days in most specifications. Therefore, we use the normalized logarithm of contractor's capacity as the characteristic variable in the remaining estimations. This is more relevant for the conditional nonparametric estimation of model primitives in terms of avoiding the curse of dimensionality and our limited sample size.

Given the linear regression specifications for early or late working days presented above, however, there are no clear implications for the relationship between bid days and early or late working days. To circumvent potential conflicts in the specifications and due to the larger sample size available for bid days than that for early or late working days, we use nonparametric procedures to estimate the conditional mean of bid days,

$$\mu_B(z) = \mathbb{E}(X^B|Z = z) = g(z), \tag{30}$$

where $g(\cdot)$ is an unknown function. The corresponding estimators are $\widehat{\chi}^r$, $\widehat{\chi}^d$, and $\widehat{g}(\cdot)$.

Then, we follow (18) to estimate the pseudo type V. Using the traditional nonparametric approach, we estimate the conditional distribution of score $F_{S|Z}(s|z)$ and its density $f_{S|Z}(s|z)$ given $Z = z$ (see, e.g., Li and Racine, 2007). Hence, the estimate of pseudo type is given by

$$\widehat{v}_{ji} = s_{ji} - \frac{1}{n_j - 1} \frac{1 - \widehat{F}_{S|Z}(s_{ji}|z_{ji})}{\widehat{f}_{S|Z}(s_{ji}|z_{ji})}, \tag{31}$$

where n_j is the number of bidders for project j, s_{ji} and z_{ji} are the score and characteristics of contractor i in project j with the corresponding estimates $\widehat{v}_{ji}$, $\widehat{F}_{S|Z}(s_{ji}|z_{ji})$, and $\widehat{f}_{S|Z}(s_{ji}|z_{ji})$.[9] Similarly, score s_{ji} is also normalized by dividing it by the engineer's score

[9] Due to the limited sample size, we divide the bidding data into two sub-samples: in the first subsample, the number of bidders is less than 6, and we choose $n_j = 4$; the second subsample contains the

estimate to capture project heterogeneity.

Next, we consider the parametric estimation of the uncertainty distribution. Suppose that the conditional distribution of ε given $Z = z$ is lognormal $(\mu, \sigma^2(z))$ with $\sigma(z) = z'\psi_\sigma$, where (μ, ψ_σ) are unknown parameters. To construct a tractable likelihood function for (μ, ψ_σ), we specify the relationship between the cutoff levels of uncertainty and characteristics as

$$e^d(z) = exp(z'\psi_d), \tag{32}$$

where ψ_d is an unknown parameter. Let I^R be the indicator for early completion, $I^R = 1$ for early completion, and $I^R = 0$ otherwise. Let I^D be the indicator for late completion, $I^D = 1$ for late completion, and $I^D = 0$ otherwise. Then, $I^B = 1 - I^R - I^D$ is the indicator for on-time completion. Therefore, we have

$$\mathbb{E}(I^R|Z = z) = F(e^r(z)|z) \text{ and } \mathbb{E}(I^D|Z = z) = 1 - F(e^d(z)|z), \tag{33}$$

which implies $\mathbb{E}(I^B|Z = z) = F(e^d(z)|z) - F(e^r(z)|z)$. Then, the likelihood function of $(\mu, \psi_\sigma, \psi_d)$ is given by

$$\mathcal{L}(\mu, \psi_\sigma, \psi_d) = \prod_{j=1}^{J} [F(e^r(z_j)|z_j)]^{I_j^R} \left[1 - F(e^d(z_j)|z_j)\right]^{I_j^D} \left[F(e^d(z_j)|z_j) - F(e^r(z_j)|z_j)\right]^{I_j^B} \tag{34}$$

where

$$\begin{aligned} F(e^r(z_j)|z_j) &= \Pr(\varepsilon \leq e^r(z_j)|z_j) = \Pr(log(\varepsilon) \leq log(e^r(z_j))|z_j) = \Phi\left(\frac{log(r_j/d_j) + z_j\psi_d - \mu}{z_j\psi_\sigma}\right), \\ F(e^d(z_j)|z_j) &= \Pr(\varepsilon \leq e^d(z_j)|z_j) = \Pr(log(\varepsilon) \leq log(e^d(z_j))|z_j) = \Phi\left(\frac{z_j\psi_d - \mu}{z_j\psi_\sigma}\right). \end{aligned}$$

Using MLE allows us to obtain the corresponding estimators $(\widehat{\mu}, \widehat{\psi}_d, \widehat{\psi}_\sigma)$.

Finally, based on a heterogeneous quantile relationship implied by (27), we construct the set of moments to estimate cost parameters by using an extremum estimator,

$$(\widehat{\alpha}_1, \widehat{\alpha}_2) = \underset{(\alpha_1,\alpha_2)}{\text{argmin}}\, \mathcal{M}_J(\alpha_1, \alpha_2) \tag{35}$$

remaining observations, and we choose $n_j = 10$. Our estimates for the primitive cost parameters are robust to different choices of the number of bidders in these two subsamples.

where

$$\mathcal{M}_J(\alpha_1, \alpha_2) = \frac{1}{J}\sum_{j=1}^{J}\left\{\frac{1}{n_j}\sum_{i=1}^{n_j}\left[\beta_0(z_{ji}) + \beta_1(z_{ji})\widehat{v}_{ji} + \beta_2(z_{ji})\widehat{v}_{ji}x_{ji}^B + \beta_3(z_{ji})x_{ji}^B + \beta_4(z_{ji})(x_{ji}^B)^2\right]^2\right\},$$

and the definitions of $\{\beta_k(z_{ji})\}_{k=0}^4$ are stated in the Appendix. For the identification of (α_1, α_2) in the presence of heterogeneity, we normalize the cost intercept $\alpha_0 = 1$. This normalization also follows Lemma 2; as stated in Section 3, Lemma 2 implies that we can alternatively normalize the cost function for the identification of the model. Combining the above estimates, we obtain the type estimate for bidder i in contract j as $\widehat{\theta}_{ji} = \widehat{\kappa}_1(z_{ji})(\widehat{\alpha}_1 + 2\widehat{\alpha}_2 X_{ji}^B)^{-1}$, where $\widehat{\kappa}_1(z_{ji})$ is defined in the Appendix. Using the estimates of $\{(\widehat{\theta}_{ji})_{i=1}^{n_j}\}_{j=1}^J$ can obtain the estimator of conditional type distribution $F_{\Theta|Z}(\cdot|z)$ given $Z = z$.

5.2 Empirical results

Table 3 reports the estimation results for separate regressions of early and late working days on different characteristics. These characteristics include the indicator for whether the project is federally funded, the logarithm of the contractor's capacity and the logarithm of the contractor's distance to the working site, both of which are normalized by the engineer's estimate of working days to capture project heterogeneity.

[Insert Table 3 here]

We find that a contractor's capacity has a significant negative effect on the actual working days in all specifications except the last specification for late completion. These results imply that contractors with greater capacities tend to have fewer working days. Specifically, the average effect of one standard deviation ($78 Million) of capacity from the mean ($71.3 million) on the early and delayed working days is -4.03 and -8.96, respectively.[10] However, for early or late completion, the distance to the work site and whether the contract is funded by the federal government have no significant effects on working days. As explained above, we use the normalized logarithm of contractor capacity as the characteristic variable in the remaining estimations.

[10] The average effect is computed as $\widehat{\chi}_{capacity} \cdot \{\log[mean(capacity) + std(capacity)] - \log[mean(capacity)\}/mean(engest)$.

To examine the goodness of fit of our estimates, we compare the fitted bid days with the fitted early and late working days. Figure 3 indicates a clear pattern whereby the fitted number of bid days is larger than the fitted number of early working days and smaller than the fitted number of late working days. These results are consistent with the model prediction that the number of bid days is larger (smaller) than the number of early (late) working days.

[Insert Figure 3 here]

Table 4 reports the estimates of the parameters of the uncertainty distribution, the cutoff uncertainty regression and the cost parameters. All estimates are statistically significant. The estimates of the mean and variance for $\log(\varepsilon)$ are -0.089 and 10.073, respectively. The estimate of parameter in the cutoff uncertainty regression is 2.958. The cost parameters α_1 and α_2 are significantly negative and positive, respectively, which are consistent with Assumption 2 in the parametric specification of the cost function.[11]

[Insert Table 4 here]

6 Lane Rental Contracts

In this section, we conduct counterfactuals to compare the welfare performance of A+B and lane rental contracts. In practice, lane rental contracts, which were introduced by the United Kingdom, perform well in reducing completion time. One common practical motivation for A+B and lane rental contracts is to induce contractors to internalize the negative construction externality and reduce construction time in heavily populated areas or on busy roads (Srinivasan and Harris, 1991; Herbsman and Glagola, 1998). For example, Caltrans tends to use the A+B contract if the estimated daily commuter cost is over $5,000.

Although A+B and lane rental contracts are designed to reduce construction time, there have been heated disputes over which contract mechanism should be preferred

[11] These results are qualitatively identical for different magnitudes of normalizations of the cost intercept α_0, ranging from 10^{-6} to 10^{6}.

(Strong, 2006). To resolve these policy disputes, we first study the theoretical differences in efficiency between A+B and lane rental contracts in the presence of uncertainty. Then, we use the estimates from Section 5 to quantify the differences in construction cost, commuter cost and social cost between A+B and lane rental contracts.

6.1 Efficiency

In the lane rental contract, each bidder quotes a cost bid, and the bidder with the lowest bid wins the contract. There is no required completion date. The winning contractor needs to pay a daily fixed amount $d_L > 0$ for the lanes occupied throughout the construction stage. The lane rental is essentially a Pigouvian tax. Therefore, in contrast to A+B contracts, lane rental contracts can be ex post efficient in the presence of uncertainty.

Under lane rental contracts, the contractor's incentive cost with working days x^A is given by

$$K_L(x^A, d_L) = d_L \cdot x^A. \tag{36}$$

Then, the equilibrium number of working days under uncertainty ε is $x_L^{A^*}(\theta, \varepsilon) = x^{d_L}(\theta, \varepsilon)$ with

$$-\varepsilon \cdot c_1(x^{d_L}(\theta, \varepsilon), \theta) = d_L. \tag{37}$$

As a result, lane rental contracts are ex post efficient in the presence of uncertainty when $d_L = c_s$.

Moreover, lane rental contracts are ex ante efficient if $d_L \leq c_s$. The social welfare of lane rental contracts in equilibrium for any (θ, ε) is given by

$$W_L^*(\theta, \varepsilon) = V_c - \varepsilon \cdot c(x^{d_L}(\theta, \varepsilon), \theta) - c_s x^{d_L}(\theta, \varepsilon).$$

with

$$\partial W_L^*(\theta, \varepsilon)/\partial \theta = (d_L - c_s)\partial x^{d_L}(\theta, \varepsilon)/\partial \theta - \varepsilon \cdot c_2(x^{d_L}(\theta, \varepsilon), \theta),$$

It can be shown that $\partial x^{d_L}(\theta, \varepsilon)/\partial \theta > 0$ by using (37). When $d_L \leq c_s$, it follows that $\partial W_L^*(\theta, \varepsilon)/\partial \theta < 0$ by combining $c_2(\cdot, \cdot) > 0$. Therefore, lane rental contracts with uncertainty are ex ante efficient if $d_L \leq c_s$.

In addition, the pseudo type in the lane rental contract is

$$v_L(\theta) = \mathbb{E}_\varepsilon \Big[\varepsilon \cdot c(x^{d_L}(\theta, \varepsilon), \theta) + d_L \cdot x^{d_L}(\theta, \varepsilon) \Big] \tag{38}$$

and

$$v_L'(\theta) = \mathbb{E}_\varepsilon \left[\varepsilon \cdot c_2(x^{d_L}(\theta, \varepsilon), \theta)\right] > 0.$$

Using similar arguments to those for A+B contracts, we can obtain the equilibrium bid for the lane rental contract. The following proposition summarizes the results above.

Proposition 4. *Under Assumption 1, for the lane rental contract in the presence of uncertainty, we obtain the following results.*
(i) The equilibrium number of working days is

$$x_L^{A^*}(\theta, \varepsilon) = x^{d_L}(\theta, \varepsilon) \tag{39}$$

with

$$-\varepsilon \cdot c_1(x^{d_L}(\theta, \varepsilon), \theta) = d_L \tag{40}$$

(ii) The equilibrium cost bid is

$$p_L^*(\theta) = \mathbb{E}_\varepsilon\Big[\varepsilon \cdot c(x^{d_L}(\theta, \varepsilon), \theta) + d_L \cdot x^{d_L}(\theta, \varepsilon)\Big] + \int_\theta^{\overline{\theta}} \mathbb{E}_\varepsilon[\varepsilon \cdot c_2(x^{d_L}(\theta, \varepsilon), \theta)] \left[\frac{1 - F_\Theta(t)}{1 - F_\Theta(\theta)}\right]^{N-1} dt. \tag{41}$$

(iii) The lane rental contract is ex post efficient if $d_L = c_s$ and ex ante efficient if $d_L \leq c_s$.

According to Propositions 2 and 4, the sufficient conditions for ex ante efficiency for A+B and lane rental contracts are very similar in the sense that the daily disincentive is smaller than or equal to the daily commuter cost. However, Proposition 4 suggests that lane rental contracts should be preferred since A+B contracts are not ex post efficient in the presence of uncertainty. For lane rental contracts, the daily penalty d_L is usually equal to the daily commuter cost c_s in practice (Herbsman and Glagola, 1998). This is consistent with our sufficient conditions for ex ante and ex post efficiency in Proposition 4.

6.2 Counterfactuals

We are now able to conduct counterfactual analysis to compare the welfare differences between A+B and lane rental contracts by using the estimated model primitives. The counterfactual procedure consists of three steps. First, we use the observed bid days in all contracts to obtain the estimator $\widehat{F}_{B|Z}(b|z)$ of the conditional distribution of bid days given $Z = z$, where the number of bid days is normalized by dividing it by the engineer's estimated days to capture project heterogeneity. Second, for each observation of A+B contracts, we draw the bid days according to $\widehat{F}_{B|Z}(b|z)$, with z being the characteristics of this contract observation, and then obtain the simulated cost types by using the estimated cost parameters.[12] We draw uncertainties from the estimated conditional distribution of uncertainty lognormal $(\widehat{\mu}, \widehat{\sigma}^2(z))$ with $\widehat{\sigma}(z) = z'\widehat{\psi}_\sigma$. Next, we use the simulated types and uncertainties to calculate the actual working days for each A+B and lane rental contracts. Then, we calculate the construction cost, commuter cost and social cost for each contract. By repeating this process 1000 times, we obtain average cost outcomes for that contract observation. Third, we conduct the second step for each observation of A+B contracts and obtain the average welfare outcomes across all observations.

Table 5 reports the counterfactual results for the social cost (and its two components, construction cost and commuter cost, which are defined in (14)) for A+B and lane rental contracts. First, the average social cost for A+B contracts is \$102.44 million, which is larger than the average social cost of \$57.51 million for lane rental contracts. Hence, on average, lane rental contracts can reduce the social cost by \$44.73 million (43.66%). This result is consistent with our theoretical results that lane rental contracts are ex post efficient while A+B contracts are not ex post efficient.

[Insert Table 5 here]

Second, the average commuter cost for lane rental contracts is \$10.25 million, which is only 14.53% of the average commuter cost of \$70.52 million for A+B contracts. This is also consistent with the theoretical conclusion that the number of working days (and hence the commuter cost) under lane rental contracts is smaller than that under A+B contracts, as indicated by Figure 4. Consequently, the average construction cost under lane rental contracts is larger than that under A+B contracts. From Table 5, we find that the average construction cost for A+B contracts is \$31.92 million, which is \$15.54 million (32.74%) smaller than the \$47.46 million for lane rental contracts. In addition, the advantage commuter cost in lane rental contracts must outweigh its disadvantage in construction costs in magnitude such that its social cost is smaller than that of A+B contracts.

[Insert Figure 4 here]

[12] An asymptotically equivalent method is to directly draw cost types from the estimated conditional distribution of cost type $\widehat{F}_{\Theta|Z}(\theta|z)$. Given the finite sample size of the data, however, this alternative method would incur more finite-sample errors. This is because the conditional distribution of cost type is estimated from the *estimated* cost types, while the conditional distribution of bid days is estimated from the *observed* bid days.

Based on Figure 4, an important policy implication is that if the incentives and disincentives were both equal to the daily commuter cost, i.e., $r = d = c_s$, the A+B contract design would also be ex post efficient because its actual working days would be the same as that in the ex post efficient lane rental contract. However, doing so would substantially increase construction costs, which would be passed on to Caltrans, which faces budget constraints (e.g., Lewis and Bajari, 2011). Therefore, facing its budget constraint, in practice, the DoT might strike an appropriate balance between reducing commuter costs and alleviating budget pressures by setting $r < d$ in A+B contracts.[13]

7 Concluding Remarks

This paper studies how the uncertainty affects the efficiency of A+B procurement contracting with time incentives. We introduce a model with ex post uncertainty to rationalize the deviation of actual working days from ex ante bid days in the highway construction industry. The A+B contract design with uncertainty is not ex post efficient. We show that the model components are identified from the bid days, actual working days and bid score. We use highway procurement contracts in California to estimate the model. Compared to A+B contracts, the counterfactual lane rental contract can substantially reduce the social cost and commuter cost in the presence of construction uncertainty. One future research is to extent the model by considering risk-averse bidders. Our econometric method might be extended to other contract mechanisms with ex post uncertainty.

[13]The budget constraint is often an important concern for highway construction procurement in other states, such as Michigan, Minnesota, and Oklahoma (see, e.g., De Silva, Kosmopoulou, and Lamarche, 2009; Marion, 2009; Lewis and Bajari, 2014; Groeger, 2014).

References

An, Y., and X. Tang (2019): "Specification and negotiation in incomplete contracts," *The RAND Journal of Economics*, 50(1), 57–92.

Asker, J., and E. Cantillon (2008): "Properties of scoring auctions," *The RAND Journal of Economics*, 39(1), 69–85.

Bajari, P., S. Houghton, and S. Tadelis (2014): "Bidding for incomplete contracts: an empirical analysis of adaptation costs," *The American Economic Review*, 104(4), 1288–1319.

Branco, F. (1997): "The design of multidimensional auctions," *The RAND Journal of Economics*, pp. 63–81.

Che, Y.-K. (1993): "Design competition through multidimensional auctions," *The RAND Journal of Economics*, pp. 668–680.

De Silva, D. G., G. Kosmopoulou, and C. Lamarche (2009): "The effect of information on the bidding and survival of entrants in procurement auctions," *Journal of Public Economics*, 93(1-2), 56–72.

D'Haultfoeuille, X., and P. Février (2016): "The provision of wage incentives: A structural estimation using contracts variation," Discussion paper, Center Res. Econ. and Statis., Paris.

Ekeland, I., J. J. Heckman, and L. Nesheim (2004): "Identification and estimation of hedonic models," *Journal of political economy*, 112(S1), S60–S109.

Esö, P., and L. White (2004): "Precautionary bidding in auctions," *Econometrica*, 72(1), 77–92.

Fang, H., and S. Morris (2006): "Multidimensional private value auctions," *Journal of Economic Theory*, 126(1), 1–30.

Gentry, M., and T. Li (2014): "Identification in auctions with selective entry," *Econometrica*, 82(1), 315–344.

Groeger, J. R. (2014): "A study of participation in dynamic auctions," *International Economic Review*, 55(4), 1129–1154.

Guerre, E., I. Perrigne, and Q. Vuong (2000): "Optimal Nonparametric Estimation of First-Price Auctions," *Econometrica*, 68(3), 525–574.

——— (2009): "Nonparametric identification of risk aversion in first-price auctions under exclusion restrictions," *Econometrica*, 77(4), 1193–1227.

Haile, P. A. (2001): "Auctions with resale markets: an application to US forest service timber sales," *American Economic Review*, 91(3), 399–427.

Hendricks, K., and R. H. Porter (1988): "An empirical study of an auction with asymmetric information," *The American Economic Review*, pp. 865–883.

Herbsman, Z. J., and C. R. Glagola (1998): "Lane rental-Innovative way to reduce road construction time," *Journal of Construction Engineering and Management*, 124(5), 411–417.

Hu, Y., D. McAdams, and M. Shum (2013): "Nonparametric identifcation of auction models with non-separable unobserved heterogeneity," *Journal of Econometrics*, 174(2), 186–193.

Jofre-Bonet, M., and M. Pesendorfer (2003): "Estimation of a dynamic auction game," *Econometrica*, 71(5), 1443–1489.

Koning, P., and A. Van De Meerendonk (2014): "The impact of scoring weights on price and quality outcomes: An application to the procurement of Welfare-to-Work contracts," *European Economic Review*, 71, 1–14.

Krasnokutskaya, E. (2011): "Identification and estimation of auction models with unobserved heterogeneity," *The Review of Economic Studies*, 78(1), 293–327.

Krasnokutskaya, E., K. Song, and X. Tang (2018): "The Role of Quality in Internet Service Markets," *Journal of Political Economy, forthcoming*.

Krishna, V. (2009): *Auction theory*. Academic press.

Laffont, J.-J., and J. Tirole (1993): *A theory of incentives in procurement and regulation*. MIT press.

Levin, D., and J. L. Smith (1994): "Equilibrium in auctions with entry," *The American Economic Review*, pp. 585–599.

Levin, J. D., and S. Athey (2001): "Information and Competition in US Forest Service Timber Auctions," *Journal of Political Economy*, 109(2), 375–417.

Lewis, G., and P. Bajari (2011): "Procurement Contracting With Time Incentives: Theory and Evidence," *Quarterly Journal of Economics*, 126(3).

——— (2014): "Moral Hazard, Incentive Contracts, and Risk: Evidence from Procurement," *The Review of Economic Studies*, p. rdu002.

Li, Q., and J. S. Racine (2007): *Nonparametric econometrics: theory and practice*. Princeton University Press.

Li, T., J. Lu, and L. Zhao (2015): "Auctions with selective entry and risk averse bidders: theory and evidence," *The RAND Journal of Economics*, 46(3), 524–545.

Luo, Y., I. Perrigne, and Q. Vuong (2018a): "Auctions with ex post uncertainty," *The RAND Journal of Economics*, 49(3), 574–593.

——— (2018b): "Structural analysis of nonlinear pricing," *Journal of Political Economy*, 126(6), 2523–2568.

Marion, J. (2009): "How costly is affirmative action? Government contracting and California's Proposition 209," *The Review of Economics and Statistics*, 91(3), 503–522.

Perrigne, I., and Q. Vuong (2011): "Nonparametric identification of a contract model with adverse selection and moral hazard," *Econometrica*, 79(5), 1499–1539.

Perry, J., and R. Hayes (1985): "Risk and its management in construction projects.," *Proceedings of the Institution of Civil Engineers*, 78(3), 499–521.

Srinivasan, R., and F. Harris (1991): "Lane rental contracting," *Construction Management and economics*, 9(2), 151–155.

Strong, K. (2006): "Performance effectiveness of design-build, lane rental, and A+ B contracting techniques," Discussion paper, Iowa State University.

Wang, M., and S. Liu (2014): "Equilibrium bids in practical multi-attribute auctions," *Economics Letters*, 123(3), 352–355.

Appendix

A Proofs

Proof of Lemma 1.

We use the backward induction to analyze the equilibrium. First, we show the contractor's optimal actual working days in the second construction stage, given his bid pair (p^B, x^B) in the auction stage, contractor's type θ, and realization of uncertainty ε. In the second stage of construction, the winning contractor chooses the optimal actual working days to maximize the following payoff

$$\pi^{II}\left(x^A \,|\, p^B, x^B, \theta, \varepsilon\right) = p^B - TC = p^B - \varepsilon \cdot c(x^A, \theta) - K(x^A, x^B, r, d), \tag{A.1}$$

where the total cost

$$TC = \varepsilon \cdot c(x^A, \theta) + K(x^A, x^B, r, d),$$

and the incentive cost

$$K(x^A, x^B, r, d) = \mathbb{1}(x^A < x^B) \cdot r \cdot (x^A - x^B) + \mathbb{1}(x^A > x^B) \cdot d \cdot (x^A - x^B).$$

Since p^B is additively separable in (A.1), the optimal actual working days, which is a function of $(x^B, \theta, \varepsilon)$, is given by

$$\widetilde{x}^{A^*}(x^B, \theta, \varepsilon) = \underset{x^A}{\operatorname{argmin}} \{\varepsilon \cdot c(x^A, \theta) + K(x^A, x^B, r, d)\}. \tag{A.2}$$

Due to the discontinuity in the incentive cost $K(x^A, x^B, r, d)$ at x^B, we need to define two cutoff levels of uncertainty

$$\varepsilon^r(\theta, x^B) = \frac{-r}{c_1(x^B, \theta)} \quad \text{and} \quad \varepsilon^d(\theta, x^B) = \frac{-d}{c_1(x^B, \theta)} \tag{A.3}$$

Due to $r < d$ and $c_1(\cdot, \cdot) < 0$ in Assumption 1, we have

$$0 < \varepsilon^r(\theta, x^B) < \varepsilon^d(\theta, x^B). \tag{A.4}$$

Next, we show the main results: (i) if $\varepsilon \leq \varepsilon^r(\theta, x^B)$, $\widetilde{x}^{A^*}(x^B, \theta, \varepsilon) = x^r(\theta, \varepsilon)$ with

$$-\varepsilon \cdot c_1(x^r(\theta, \varepsilon), \theta) = r, \tag{A.5}$$

(ii) if $\varepsilon \geq \varepsilon^d(\theta, x^B)$, $\widetilde{x}^{A^*}(x^B, \theta, \varepsilon) = x^d(\theta, \varepsilon)$ such that

$$-\varepsilon \cdot c_1(x^d(\theta, \varepsilon), \theta) = d, \tag{A.6}$$

(iii) if $\varepsilon \in [\varepsilon^r(\theta, x^B), \varepsilon^d(\theta, x^B)]$, $\widetilde{x}^{A^*}(x^B, \theta, \varepsilon) = x^B$.

Let us consider case (i): $\varepsilon \leq \varepsilon^r(\theta, x^B)$. If the contractor completes the construction on time, i.e., $x^A = x^B$, the total cost is

$$TC^0 = \varepsilon \cdot c(x^B, \theta).$$

If the project completion is delayed, i.e., $x^A > x^B$, the total cost is

$$TC^+ = \varepsilon \cdot c(x^A, \theta) + d \cdot (x^A - x^B).$$

Then, we will prove that when $\varepsilon \leq \varepsilon^r(\theta, x^B)$, $TC^0 < TC^+$ for any $x^A > x^B$, which implies that late completion is worse than on-time completion. Specifically,

$$\begin{aligned} TC^0 - TC^+ &= \varepsilon \cdot c(x^B, \theta) - \left[\varepsilon \cdot c(x^A, \theta) + d \cdot (x^A - x^B)\right] \\ &= \varepsilon \left[c(x^B, \theta) - c(x^A, \theta)\right] - \varepsilon^d(\theta, x^B) \cdot c_1(x^B, \theta) \cdot (x^B - x^A) \\ &= \varepsilon \cdot c_1(y, \theta) \cdot (x^B - x^A) - \varepsilon^d(\theta, x^B) \cdot c_1(x^B, \theta) \cdot (x^B - x^A) \\ &= \left[\varepsilon \cdot c_1(y, \theta) - \varepsilon \cdot c_1(x^B, \theta) + \varepsilon \cdot c_1(x^B, \theta) - \varepsilon^d(\theta, x^B) \cdot c_1(x^B, \theta)\right] \cdot (x^B - x^A) \\ &= \Big(\underbrace{\varepsilon \left[c_1(y, \theta) - c_1(x^B, \theta)\right]}_{+} + \underbrace{(\varepsilon - \varepsilon^d(\theta, x^B))c_1(x^B, \theta)}_{+}\Big) \cdot \underbrace{(x^B - x^A)}_{-} \\ &< 0, \end{aligned} \tag{A.7}$$

where the second equality is from (A.3), the third equality follows the mean-value theorem with $x^B < y < x^A$, the first part in the fifth equality is positive due to the convexity of cost function in Assumption 1, the second part is positive due to $\varepsilon < \varepsilon^r(\theta, x^B) < \varepsilon^d(\theta, x^B)$ and assumption $c_1(\cdot, \cdot) < 0$, and the last part is negative due to $x^A > x^B$.

Accordingly, we just need to show that early completion at the actual working days $\widetilde{x}^{A^*}(x^B, \theta, \varepsilon) = x^r(\theta, \varepsilon)$ is better than on-time completion, that is, $TC^0 > TC^r$, where

$$TC^r = \varepsilon \cdot c(x^r(\theta, \varepsilon), \theta) + r \cdot (x^r(\theta, \varepsilon) - x^B).$$

Specifically,

$$
\begin{aligned}
TC^r - TC^0 &= \varepsilon \cdot c(x^r(\theta, \varepsilon), \theta) + r \cdot (x^r(\theta, \varepsilon) - x^B) - \varepsilon \cdot c(x^B, \theta) \\
&= \varepsilon \cdot c(x^r(\theta, \varepsilon), \theta) - \varepsilon \cdot c_1(x^r(\theta, \varepsilon), \theta) \cdot (x^r(\theta, \varepsilon) - x^B) - \varepsilon \cdot c(x^B, \theta) \\
&= \varepsilon \Big[c(x^r(\theta, \varepsilon), \theta) - c(x^B, \theta) - c_1(x^r(\theta, \varepsilon), \theta) \cdot (x^r(\theta, \varepsilon) - x^B) \Big] \\
&= \varepsilon \Big[c_1(y, \theta) \cdot (x^r(\theta, \varepsilon) - x^B) - c_1(x^r(\theta, \varepsilon), \theta) \cdot (x^r(\theta, \varepsilon) - x^B) \Big] \\
&= \underbrace{\varepsilon}_{+} \underbrace{\Big[c_1(\tilde{y}, \theta) - c_1(x^r(\theta, \varepsilon), \theta) \Big]}_{+} \cdot \underbrace{(x^r(\theta, \varepsilon) - x^B)}_{-} \\
&< 0,
\end{aligned}
$$

where the second equality is from (A.5), the fourth equality follows the mean-value theorem with $x^r(\theta, \varepsilon) < \tilde{y} < y < x^B$, the second part in the fifth equality is positive due to the convexity of cost function, and the last part

$$
x^r(\theta, \varepsilon) < x^B \tag{A.8}
$$

is implied by the defintion of $\widetilde{\varepsilon}^r(\theta, x^B)$ in (A.3), convexity of cost function, and $\varepsilon < \varepsilon^r(\theta, x^B)$.

Using the similar arguments for case (i), we can show $\widetilde{x}^{A^*} = x^d(\theta, \varepsilon)$ for case (ii): $\varepsilon > \varepsilon^d$. If the contractor chooses early completion, i.e, $x^A < x^B$, the total cost is

$$
TC^- = \varepsilon \cdot c(x^A, \theta) + r \cdot (x^A - x^B).
$$

Similarly, we can prove that when $\varepsilon > \widetilde{\varepsilon}^d(\theta, x^B)$, $TC^0 < TC^-$ for any $x^A < x^B$, which implies that early completion is worse than on-time completion. Specifically,

$$
\begin{aligned}
TC^0 - TC^- \;&=\; \varepsilon \cdot c(x^B, \theta) - \left[\varepsilon \cdot c(x^A, \theta) + r \cdot (x^A - x^B) \right] \\
&=\; \varepsilon \left[c(x^B, \theta) - c(x^A, \theta) \right] - \varepsilon^r(\theta, x^B) \cdot c_1(x^B, \theta) \cdot (x^B - x^A) \\
&=\; \varepsilon \cdot c_1(y, \theta) \cdot (x^B - x^A) - \varepsilon^r(\theta, x^B) \cdot c_1(x^B, \theta) \cdot (x^B - x^A) \\
&=\; \left[\varepsilon \cdot c_1(y, \theta) - \varepsilon \cdot c_1(x^B, \theta) + \varepsilon \cdot c_1(x^B, \theta) - \varepsilon^r(\theta, x^B) \cdot c_1(x^B, \theta) \right] \cdot (x^B - x^A) \\
&=\; \Big(\underbrace{\varepsilon \left[c_1(y, \theta) - c_1(x^B, \theta) \right]}_{-} + \underbrace{(\varepsilon - \varepsilon^r(\theta, x^B)) c_1(x^B, \theta)}_{-} \Big) \cdot \underbrace{(x^B - x^A)}_{+} \\
&<\; 0,
\end{aligned} \tag{A.9}
$$

where $x^A < y < x^B$. Accordingly, we just need to show that late completion at the actual working days $\widetilde{x}^{A^*}(x^B, \theta, \varepsilon) = x^d(\theta, \varepsilon)$ is better than on-time completion, that is, $TC^0 > TC^d$, where

$$TC^d = \varepsilon \cdot c(x^d(\theta, \varepsilon), \theta) + d \cdot (x^d(\theta, \varepsilon) - x^B).$$

Using similar arguments, we obtain

$$\begin{aligned}
TC^d - TC^0 &= \varepsilon \cdot c(x^d(\theta, \varepsilon), \theta) + r \cdot (x^d(\theta, \varepsilon) - x^B) - \varepsilon \cdot c(x^B, \theta) \\
&= \varepsilon \cdot c(x^d(\theta, \varepsilon), \theta) - \varepsilon \cdot c_1(x^d(\theta, \varepsilon), \theta) \cdot (x^d(\theta, \varepsilon) - x^B) - \varepsilon \cdot c(x^B, \theta) \\
&= \varepsilon \Big[c(x^d(\theta, \varepsilon), \theta) - c(x^B, \theta) - c_1(x^d(\theta, \varepsilon), \theta) \cdot (x^d(\theta, \varepsilon) - x^B) \Big] \\
&= \varepsilon \Big[c_1(y, \theta) \cdot (x^d(\theta, \varepsilon) - x^B) - c_1(x^d(\theta, \varepsilon), \theta) \cdot (x^d(\theta, \varepsilon) - x^B) \Big] \\
&= \underbrace{\varepsilon}_{+} \underbrace{\Big[c_1(\tilde{y}, \theta) - c_1(x^d(\theta, \varepsilon), \theta) \Big]}_{-} \cdot \underbrace{(x^d(\theta, \varepsilon) - x^B)}_{+} \\
&< 0,
\end{aligned}$$

where $x^B < y < \tilde{y} < x^d$, and the last part

$$x^d(\theta, \varepsilon) > x^B \tag{A.10}$$

is implied by the defintion of $\varepsilon^d(\theta, x^B)$ in (A.3), convexity of cost function, and $\varepsilon > \varepsilon^d(\theta, x^B)$.

Next, we prove $\widetilde{x}^{A^*}(x^B, \theta, \varepsilon) = x^B$ for case (iii): $\varepsilon^r(\theta, x^B) \leq \varepsilon \leq \varepsilon^d(\theta, x^B)$. On the one hand, for any $x^A > x^B$, as (A.7) shows, $TC^0 - TC^+ < 0$. On the other hand, for any $x^A < x^B$, as (A.9) shows, $TC^0 - TC^- < 0$. Therefore, on-time completion is optimal for the contractor, i.e, $\widetilde{x}^{A^*}(x^B, \theta, \varepsilon) = x^B$.

Last, combing (A.8) with (A.10) leads to $x^r(\theta, \varepsilon) \leq x^B \leq x^d(\theta, \varepsilon)$. The proof is complete.

Proof of Proposition 1.

Using the results of optimal actual working days in Lemma 1, we derive the equilibrium of our model by analyzing the first-stage optimal decision on the bid pair of cost and working days. In the first stage, the bidder with type θ quotes the optimal pair of cost and working days to maximize his expected payoff

$$\begin{aligned}\pi(p^B, x^B \mid \theta) &= \Big\{p^B - \mathbb{E}_\varepsilon\Big[\varepsilon \cdot c(\widetilde{x}^{A^*}(x^B, \theta, \varepsilon), \theta) + K(\widetilde{x}^{A^*}(x^B, \theta, \varepsilon), x^B, r, d)\Big]\Big\} \\ &\times \; \Pr\Big(\text{win} \;\Big|\; s = p^B + c_u x^B\Big),\end{aligned}$$

that is

$$\left(p^{B^*}(\theta), x^{B^*}(\theta)\right) = \underset{p^B, x^B}{\operatorname{argmax}} \; \pi\left(p^B, x^B \,|\, \theta\right),$$

where $\mathbb{E}_\varepsilon$ is the expectation with respect to ε, $\Pr(\text{win}|s = p^B + c_u x^B)$ is the probability of winning the contract given his bid score $s = p^B + c_u x^B$, and $\widetilde{x}^{A^*}(x^B, \theta, \varepsilon)$ is defined in Lemma 1.

First, we follow Che (1993) to show that for any type θ, the equilibrium bid days $x^{B^*}(\theta)$ can be determined separately from the choice of score through

$$x^{B^*}(\theta) = arg\min_{x^B} \Big\{c_u x^B + \mathbb{E}_\varepsilon\Big[\varepsilon \cdot c(\widetilde{x}^{A^*}(x^B, \theta, \varepsilon), \theta) + K(\widetilde{x}^{A^*}(x^B, \theta, \varepsilon), x^B, r, d)\Big]\Big\}. \quad \text{(A.11)}$$

Suppose the contractor with type θ bids $(\tilde{p}^B, \tilde{x}^B)$ where $\tilde{x}^B \neq x^{B^*}(\theta)$. Then we only need to show that the contractor is better-off if he chooses bid days $x^{B^*}(\theta)$ and bid cost $p^{B^*}(\theta)$ with

$$p^{B^*}(\theta) = \tilde{p}^B + c_u(\tilde{x}^B - x^{B^*}(\theta)).$$

Note that the scores are identical in both choices due to

$$s^*(\theta) \equiv p^{B^*}(\theta) + c_u x^{B^*}(\theta) = \tilde{p}^B + c_u(\tilde{x}^B - x^{B^*}(\theta)) + c_u x^{B^*}(\theta) = \tilde{p}^B + c_u \tilde{x}^B.$$

The difference of contractor's expected payoff between $(\tilde{p}^B, \tilde{x}^B)$ and $(p^{B^*}(\theta), x^{B^*}(\theta))$ is

$$\begin{aligned}&\pi(p^{B^*}(\theta), x^{B^*}(\theta) \mid \theta) - \pi(\tilde{p}^B, \tilde{x}^B \mid \theta) \\ =&\Big\{\tilde{p}^B + c_u(\tilde{x}^B - x^{B^*}(\theta)) - \mathbb{E}_\varepsilon\big[\varepsilon \cdot c(\widetilde{x}^{A^*}(x^{B^*}(\theta), \theta, \varepsilon), \theta) + K(\widetilde{x}^{A^*}(x^{B^*}(\theta), \theta, \varepsilon), x^B, r, d)\big] \\ &- \tilde{p}^B + \mathbb{E}_\varepsilon\big[\varepsilon \cdot c(\widetilde{x}^{A^*}(\tilde{x}^B, \theta, \varepsilon), \theta) + K(\widetilde{x}^{A^*}(\tilde{x}^B, \theta, \varepsilon), x^B, r, d)\big]\Big\} \times \Pr(\text{win} \mid s^*(\theta)) \\ =&\Big\{\Big(c_u\tilde{x}^B + \mathbb{E}_\varepsilon\big[\varepsilon \cdot c(\widetilde{x}^{A^*}(\tilde{x}^B, \theta, \varepsilon), \theta) + K(\widetilde{x}^{A^*}(\tilde{x}^B, \theta, \varepsilon), x^B, r, d)\big]\Big) \\ &- \Big(c_u x^{B^*}(\theta) + \mathbb{E}_\varepsilon\big[\varepsilon \cdot c(\widetilde{x}^{A^*}(x^{B^*}(\theta), \theta, \varepsilon), \theta) + K(\widetilde{x}^{A^*}(x^{B^*}(\theta), \theta, \varepsilon), x^B, r, d)\big]\Big)\Big\} \times \Pr(\text{win} \mid s^*(\theta)) \\ >&0,\end{aligned}$$

where the last inequality comes from the fact that $x^{B^*}(\theta)$ is the minimizer in (A.11) and that the winning probability is positive as shown in Che (1993).

Second, we prove $dx^{B^*}(\theta)/d\theta > 0$. The first-order condition with respect to x^B in the objective function (A.11) implies

$$
\begin{aligned}
0 =& c_u + \frac{\partial}{\partial x^B}\left[\int_{\varepsilon \leq \varepsilon^r(\theta, x^B)} \left[\varepsilon \cdot c(x^r(\theta,\varepsilon),\theta) + r\cdot (x^r(\theta,\varepsilon) - x^B)\right] dF(\varepsilon)\right] \\
&+ \frac{\partial}{\partial x^B}\left[\int_{\varepsilon^r(\theta,x^B)}^{\varepsilon^d(\theta,x^B)} \varepsilon \cdot c(x^B,\theta) dF(\varepsilon)\right] \\
&+ \frac{\partial}{\partial x^B}\left[\int_{\varepsilon \geq \varepsilon^d(\theta, x^B)} \left[\varepsilon \cdot c(x^d(\theta,\varepsilon),\theta) + d\cdot (x^d(\theta,\varepsilon) - x^B)\right] dF(\varepsilon)\right] \\
=& c_u + \left\{\varepsilon^r(\theta,x^B) c(x^r(\theta,\varepsilon^r(\theta,x^B)),\theta) + r\left[x^r(\theta,\varepsilon^r(\theta,x^B)) - x^B\right]\right\} f(\varepsilon^r(\theta,x^B)) \frac{\partial \varepsilon^r(\theta,x^B)}{\partial x^B} \\
&+ \int_{\varepsilon \leq \varepsilon^r(\theta,x^B)} \frac{\partial}{\partial x^B}\left[\varepsilon\cdot c(x^r(\theta,\varepsilon),\theta) + r\cdot(x^r(\theta,\varepsilon) - x^B)\right] f(\varepsilon) d\varepsilon \\
&+ \left[\varepsilon^d(\theta,x^B) c(x^B,\theta) f(\varepsilon^d(\theta,x^B))\right] \partial\varepsilon^d(\theta,x^B)/\partial x^B \\
&- \left[\varepsilon^r(\theta,x^B) c(x^B,\theta) f(\varepsilon^r(\theta,x^B))\right] \partial\varepsilon^r(\theta,x^B)/\partial x^B + \int_{\varepsilon^r(\theta,x^B)}^{\varepsilon^d(\theta,x^B)} \varepsilon \cdot \frac{\partial c(x^B,\theta)}{\partial x^B} f(\varepsilon) d\varepsilon \\
&- \left\{\varepsilon^d(\theta,x^B) c(x^d(\theta,\varepsilon^d(\theta,x^B)),\theta) + d\left[x^d(\theta,\varepsilon^d(\theta,x^B)) - x^B\right]\right\} f(\varepsilon^d(\theta,x^B)) \frac{\partial \varepsilon^d(\theta,x^B)}{\partial x^B} \\
&+ \int_{\varepsilon \geq \varepsilon^d(\theta,x^B)} \frac{\partial}{\partial x^B}\left[\varepsilon\cdot c(x^d(\theta,\varepsilon),\theta) + d\cdot(x^d(\theta,\varepsilon) - x^B)\right] f(\varepsilon) d\varepsilon \\
=& c_u + \int_{\varepsilon \leq \varepsilon^r(\theta,x^B)} \frac{\partial}{\partial x^B}\left[\varepsilon\cdot c(x^r(\theta,\varepsilon),\theta) + r\cdot(x^r(\theta,\varepsilon) - x^B)\right] f(\varepsilon) d\varepsilon \\
&+ \int_{\varepsilon^r(\theta,x^B)}^{\varepsilon^d(\theta,x^B)} \varepsilon \cdot \frac{\partial c(x^B,\theta)}{\partial x^B} f(\varepsilon) d\varepsilon \\
&+ \int_{\varepsilon \geq \varepsilon^d(\theta,x^B)} \frac{\partial}{\partial x^B}\left[\varepsilon\cdot c(x^d(\theta,\varepsilon),\theta) + d\cdot(x^d(\theta,\varepsilon) - x^B)\right] f(\varepsilon) d\varepsilon \\
=& c_u - \int_{\varepsilon \leq \varepsilon^r(\theta,x^B)} r f(\varepsilon) d\varepsilon + \int_{\varepsilon^r(\theta,x^B)}^{\varepsilon^d(\theta,x^B)} \varepsilon \cdot c_1(x^B,\theta) f(\varepsilon) d\varepsilon - \int_{\varepsilon \geq \varepsilon^d(\theta,x^B)} d f(\varepsilon) d\varepsilon \\
=& c_u + \int_{\varepsilon^r(\theta,x^B)}^{\varepsilon^d(\theta,x^B)} \varepsilon \cdot c_1(x^B,\theta) f(\varepsilon) d\varepsilon - rF(\varepsilon^r(\theta,x^B)) - d\left[1 - F(\varepsilon^d(\theta,x^B))\right], \qquad \text{(A.12)}
\end{aligned}
$$

where the third equality follows $x^r(\theta,\varepsilon^r(\theta,x^B)) = x^B$ and $x^d(\theta,\varepsilon^d(\theta,x^B)) = x^B$. Hence, the equilibrium bid days $x^{B^*}(\theta)$ satisfies

$$-c_u = \int_{\varepsilon^r(\theta, x^{B^*}(\theta))}^{\varepsilon^d(\theta, x^{B^*}(\theta))} \varepsilon \cdot c_1(x^{B^*}(\theta), \theta) f(\varepsilon) d\varepsilon - rF(\varepsilon^r(\theta, x^{B^*}(\theta))) - d\big[1 - F(\varepsilon^d(\theta, x^{B^*}(\theta)))\big]. \tag{A.13}$$

Denote $x^{B^*} = x^{B^*}(\theta)$ for simplicity of exposition whenever there is no ambiguity. Taking the first derivative of both sides of (A.13) with respect to θ implies

$$\begin{aligned}
0 \;&=\; \varepsilon^d(\theta, x^{B^*}) c_1(x^{B^*}, \theta) f(\varepsilon^d(\theta, x^{B^*})) \left(\frac{\partial \varepsilon^d(\theta, x^{B^*})}{\partial \theta} + \frac{\partial \varepsilon^d(\theta, x^{B^*})}{\partial x^{B^*}} \frac{dx^{B^*}}{d\theta} \right) \\
&-\; \varepsilon^r(\theta, x^{B^*}) c_1(x^{B^*}, \theta) f(\varepsilon^r(\theta, x^{B^*})) \left(\frac{\partial \varepsilon^r(\theta, x^{B^*})}{\partial \theta} + \frac{\partial \varepsilon^r(\theta, x^{B^*})}{\partial x^{B^*}} \frac{dx^{B^*}}{d\theta} \right) \\
&+\; \int_{\varepsilon^r(\theta, x^{B^*})}^{\varepsilon^d(\theta, x^{B^*})} \varepsilon \cdot \left[c_{11}(x^{B^*}, \theta) \frac{dx^{B^*}}{d\theta} + c_{12}(x^{B^*}, \theta) \right] f(\varepsilon) d\varepsilon \\
&-\; r f(\varepsilon^r(\theta, x^{B^*})) \left(\frac{\partial \varepsilon^r(\theta, x^{B^*})}{\partial \theta} + \frac{\partial \varepsilon^r(\theta, x^{B^*})}{\partial x^{B^*}} \frac{dx^{B^*}}{d\theta} \right) \\
&+\; d f(\varepsilon^d(\theta, x^{B^*})) \left(\frac{\partial \varepsilon^d(\theta, x^{B^*})}{\partial \theta} + \frac{\partial \varepsilon^d(\theta, x^{B^*})}{\partial x^{B^*}} \frac{dx^{B^*}}{d\theta} \right) \\
&=\; \int_{\varepsilon^r(\theta, x^{B^*})}^{\varepsilon^d(\theta, x^{B^*})} \varepsilon \cdot \left[c_{11}(x^{B^*}, \theta) \frac{dx^{B^*}}{d\theta} + c_{12}(x^{B^*}, \theta) \right] f(\varepsilon) d\varepsilon,
\end{aligned}$$

where the second equality follows the definitions of $\varepsilon^r(\theta, x^{B^*}(\theta))$ and $\varepsilon^d(\theta, x^{B^*}(\theta))$. Since $\varepsilon > 0$, it follows

$$c_{11}(x^{B^*}(\theta), \theta) \frac{dx^{B^*}(\theta)}{d\theta} + c_{12}(x^{B^*}(\theta), \theta) = 0 \tag{A.14}$$

Due to $c_{11}(\cdot, \cdot) > 0$ and $c_{12}(\cdot, \cdot) < 0$, we obtain

$$\frac{dx^{B^*}(\theta)}{d\theta} = -\frac{c_{12}(x^{B^*}(\theta), \theta)}{c_{11}(x^{B^*}(\theta), \theta)} > 0. \tag{A.15}$$

In addition, we prove that for any type θ, in equilibrium the two cutoff levels of uncertainty are constant. Based on the equilibrium bid days $x^{B^*}(\theta)$, the two cutoff levels in equilibrium depend only on type θ in the way

$$\varepsilon^r(\theta) \equiv \varepsilon^r(\theta, x^{B^*}(\theta)) \;\text{ and }\; \varepsilon^d(\theta) \equiv \varepsilon^d(\theta, x^{B^*}(\theta)).$$

According to definitions of $\varepsilon^r(\theta)$ and $\varepsilon^d(\theta)$, in equilibrium it satisfies

$$\varepsilon^r(\theta)c_1(x^{B^*}(\theta),\theta) = -r \quad \text{and} \quad \varepsilon^d(\theta)c_1(x^{B^*}(\theta),\theta) = -d.$$

Then, taking first derivatives with respect to θ on both sides leads to

$$\frac{d\varepsilon^r(\theta)}{d\theta}c_1(x^{B^*}(\theta),\theta) + \varepsilon^r(\theta)\left[c_{11}(x^{B^*}(\theta),\theta)\frac{dx^{B^*}(\theta)}{d\theta} + c_{12}(x^{B^*}(\theta),\theta)\right] = 0$$

and

$$\frac{d\varepsilon^d(\theta)}{d\theta}c_1(x^{B^*}(\theta),\theta) + \varepsilon^d(\theta)\left[c_{11}(x^{B^*}(\theta),\theta)\frac{dx^{B^*}(\theta)}{d\theta} + c_{12}(x^{B^*}(\theta),\theta)\right] = 0$$

Therefore, combinaing (A.14) with $c_1(\cdot,\cdot) < 0$ implies

$$\frac{d\varepsilon^d(\theta)}{d\theta} = 0 \quad \text{and} \quad \frac{d\varepsilon^d(\theta)}{d\theta} = 0,$$

that is,

$$\varepsilon^d(\theta) \equiv e^r \quad \text{and} \quad \varepsilon^d(\theta) \equiv e^d \quad \text{for any } \theta, \tag{A.16}$$

where both e^r and e^d are constant with $e^r < e^d$ implied by (A.4). As a result, using the optimal actual working days in Lemma 1 we obtain the equilibrium actual working days

$$x^{A^*}(\theta,\varepsilon) = \widetilde{x}^{A^*}(x^{B^*}(\theta),\theta,\varepsilon) = \begin{cases} x^r(\theta,\varepsilon), & \textit{if} \ \ \varepsilon \le e^r \\ x^{B^*}(\theta), & \textit{if} \ \ \varepsilon \in [e^r, e^d] \\ x^d(\theta,\varepsilon), & \textit{if} \ \ \varepsilon \ge e^d \end{cases} \tag{A.17}$$

Third, we derive the equilibrium bid of cost $p^{B^*}(\theta)$ by using the result on the unique symmetric monotone Bayesian Nash Equilibrium (psBNE) in the standard literature (see, Krishna (2009)). To do so, we start with the symmetric and increasing bidding strategy for optimal score $s_i^* = s(v_i)$ for the pseudo type v_i of contractor $i \in \{1,\cdots,N\}$ such that

$$s_i^* = \underset{b}{\operatorname{argmax}}\left\{\big(b - v_i\big) \times \Pr(\text{win}_i \mid b)\right\}, \tag{A.18}$$

where $\Pr(\text{win}_i \mid b)$ denotes the winning probability of contractor i when his score is b, and

$$\Pr(\text{win}_i \mid b) = \Pi_{j\neq i}\Pr(b_j \ge b) = \Pi_{j\neq i}\Pr(v_j \ge s^{-1}(b)) = [1 - F_V(s^{-1}(b))]^{N-1},$$

where $F_V(\cdot)$ is the cumulative distribution function of pseudo type V. With a slight abuse of notation, we define s_i^* as $s(v_i)$ rather than $s(v(\theta_i))$ to emphasize that here the optimal score is a function of pseudo type. Then the first-order condition of (A.18) with respect to b yields

$$-(N-1)(s_i^* - v_i)[1 - F_V(s^{-1}(s_i^*))]^{N-2} \frac{f_V(s^{-1}(s_i^*))}{s'(s^{-1}(s_i^*))} + [1 - F_V(s^{-1}(s_i^*))]^{N-1} = 0. \quad \text{(A.19)}$$

Due to the symmetry of equilibrium, we drop the subscript i for simplicity. As a result,

$$\begin{aligned} \frac{d}{dv}\big\{[1-F_V(v)]^{N-1}s(v)\big\} &= -(N-1)s(v)[1-F_V(v)]^{N-2}f_V(v) + [1-F_V(v)]^{N-1}s'(v) \\ &= -(N-1)v[1-F_V(v)]^{N-2}f_V(v) \\ &= v\frac{d}{dv}\big\{[1-F_V(v)]^{N-1}\big\}, \end{aligned} \quad \text{(A.20)}$$

where the second equality follows (A.19). Then, integrating by part on both sides of (A.20) with the boundary condition $s(\overline{v}) = \overline{v}$ yields

$$s(v) = v + \int_v^{\overline{v}} \left[\frac{1-F_V(t)}{1-F_V(v)}\right]^{N-1} dt, \quad \text{(A.21)}$$

where $\overline{v}$ is the upper bound of pseudo type. Furthermore, the first-order derivative of (A.21) implies

$$s'(v) = (N-1)(s(v)-v)\frac{f_V(v)}{1-F_V(v)} > 0. \quad \text{(A.22)}$$

Next, we prove $v'(\theta) > 0$ and in turn obtain $F_V(v) = F_\Theta(\theta)$, which is critical for the equilibrium bid of cost $p^{B^*}(\theta)$. Recall the definition

$$v(\theta) = \min_{x^B}\Big\{c_u x^B + \mathbb{E}_\varepsilon\Big[\varepsilon \cdot c(\widetilde{x}^{A^*}(x^B,\theta,\varepsilon),\theta) + K(\widetilde{x}^{A^*}(x^B,\theta,\varepsilon), x^B, r, d)\Big]\Big\}. \quad \text{(A.23)}$$

Then, using the envelope theorem leads to

$$\begin{aligned} v'(\theta) &= \partial\Big\{c_u x^{B^*} + \mathbb{E}_\varepsilon\big[\varepsilon \cdot c(\widetilde{x}^{A^*}(x^{B^*},\theta,\varepsilon),\theta) + K(\widetilde{x}^{A^*}(x^{B^*},\theta,\varepsilon), x^{B^*}, r, d)\big]\Big\}/\partial\theta \\ &= \partial\Big\{\int_{\varepsilon \le \varepsilon^r(\theta, x^{B^*})} \big[\varepsilon \cdot c(x^r(\theta,\varepsilon),\theta) + r(x^r(\theta,\varepsilon) - x^{B^*})\big] f(\varepsilon)d\varepsilon\Big\}/\partial\theta \\ &+ \partial\Big\{\int_{\varepsilon^r(\theta,x^{B^*})}^{\varepsilon^d(\theta,x^{B^*})} \varepsilon \cdot c(x^{B^*},\theta) f(\varepsilon)d\varepsilon\Big\}/\partial\theta \end{aligned}$$

$$
\begin{aligned}
&+ \quad \partial\Big\{\int_{\varepsilon\geq\varepsilon^d(\theta,x^{B^*})}\big[\varepsilon\cdot c(x^d(\theta,\varepsilon),\theta)+d(x^d(\theta,\varepsilon)-x^{B^*})\big]f(\varepsilon)d\varepsilon\Big\}/\partial\theta \\
&= \quad \varepsilon^r\cdot c(x^r(\theta,\varepsilon^r),\theta)f(\varepsilon^r)\partial\varepsilon^r(\theta,x^{B^*})/\partial\theta \\
&+ \quad \int_{\varepsilon\leq\varepsilon^r(\theta,x^{B^*})}\Big\{\varepsilon\big[c_1(x^r(\theta,\varepsilon),\theta)\partial x^r(\theta,\varepsilon)/\partial\theta+c_2(x^r(\theta,\varepsilon),\theta)\big]+r\partial x^r(\theta,\varepsilon)/\partial\theta\Big\}dF(\varepsilon) \\
&+ \quad \varepsilon^d\cdot c(x^{B^*},\theta)f(\varepsilon^d)\partial\varepsilon^d(\theta,x^{B^*})/\partial\theta-\varepsilon^r\cdot c(x^{B^*},\theta)f(\varepsilon^r)\partial\varepsilon^r(\theta,x^{B^*})/\partial\theta \\
&+ \quad \int_{\varepsilon^r(\theta,x^{B^*})}^{\varepsilon^d(\theta,x^{B^*})}\Big\{\varepsilon\cdot c_2(x^{B^*},\theta)\Big\}dF(\varepsilon)-\varepsilon^d\cdot c(x^d(\theta,\varepsilon^d),\theta)f(\varepsilon^d)\partial\varepsilon^d(\theta,x^{B^*})/\partial\theta \\
&+ \quad \int_{\varepsilon\geq\varepsilon^d(\theta,x^{B^*})}\Big\{\varepsilon\big[c_1(x^d(\theta,\varepsilon),\theta)\partial x^d(\theta,\varepsilon)/\partial\theta+c_2(x^d(\theta,\varepsilon),\theta)\big]+d\partial x^d(\theta,\varepsilon)/\partial\theta\Big\}dF(\varepsilon) \\
&= \quad \int_{\varepsilon\leq\varepsilon^r(\theta,x^{B^*})}\Big\{\varepsilon\big[c_1(x^r(\theta,\varepsilon),\theta)\partial x^r(\theta,\varepsilon)/\partial\theta+c_2(x^r(\theta,\varepsilon),\theta)\big]+r\partial x^r(\theta,\varepsilon)/\partial\theta\Big\}dF(\varepsilon) \\
&+ \quad \int_{\varepsilon^r(\theta,x^{B^*})}^{\varepsilon^d(\theta,x^{B^*})}\Big\{\varepsilon\cdot c_2(x^{B^*},\theta)\Big\}dF(\varepsilon) \\
&+ \quad \int_{\varepsilon\geq\varepsilon^d(\theta,x^{B^*})}\Big\{\varepsilon\big[c_1(x^d(\theta,\varepsilon),\theta)\partial x^d(\theta,\varepsilon)/\partial\theta+c_2(x^d(\theta,\varepsilon),\theta)\big]+d\partial x^d(\theta,\varepsilon)/\partial\theta\Big\}dF(\varepsilon) \\
&= \quad \int_{\varepsilon\leq\varepsilon^r(\theta,x^{B^*})}\varepsilon\cdot c_2(x^r(\theta,\varepsilon),\theta)dF(\varepsilon)+\int_{\varepsilon^r(\theta,x^{B^*})}^{\varepsilon^d(\theta,x^{B^*})}\varepsilon\cdot c_2(x^{B^*},\theta)dF(\varepsilon) \\
&+ \quad \int_{\varepsilon\geq\varepsilon^d(\theta,x^{B^*})}\varepsilon\cdot c_2(x^d(\theta,\varepsilon),\theta)dF(\varepsilon)>0, \qquad \text{(A.24)}
\end{aligned}
$$

where the third equality and fourth equality follow $x^r(\theta,\varepsilon^r)=x^{B^*}$ and $x^d(\theta,\varepsilon^r)=x^{B^*}$, and the fifth equality is obtained as follows. For expositional simplicity we denote $x^r=x^r(\theta,\varepsilon)$ and $x^d=x^d(\theta,\varepsilon)$ whenever there is no ambiguity. Note that

$$
\varepsilon\cdot c_1(x^r(\theta,\varepsilon),\theta)=r \ \text{ and } \ \frac{\partial x^r(\theta,\varepsilon)}{\partial\theta}=-\frac{c_{12}(x^r,\theta)}{c_{11}(x^r,\theta)}
$$

Then, the integrand in the first integral of the fourth equality equals

$$
\varepsilon\left[-c_1(x^r,\theta)\frac{c_{12}(x^r,\theta)}{c_{11}(x^r,\theta)}+c_2(x^r,\theta)\right]+\varepsilon\cdot c_1(x^r,\theta)\frac{c_{12}(x^r,\theta)}{c_{11}(x^r,\theta)}=\varepsilon\cdot c_2(x^r,\theta)>0,
$$

where the positiveness follows $\varepsilon>0$ and $c_2(\cdot,\cdot)>0$ in Assumption 1. Similarly, the integrand in the third integral of the fourth equality equals $\varepsilon\cdot c_2(x^d,\theta)>0$.

Using the property $F_V(v)=F_\Theta(\theta)$ implied by $v'(\theta)>0$, the integral with respect to θ

in (A.21) and $v'(\theta)$ in (A.24) yields the equilibrium bid of cost $p^{B^*}(\theta)$. Specifically, since

$$\begin{aligned} s(v(\theta)) &= c_u x^{B^*}(\theta) + \mathbb{E}_\varepsilon\Big[\varepsilon \cdot c(x^{A^*}(\theta,\varepsilon),\theta) + K(x^{A^*}(\theta,\varepsilon), x^{B^*}(\theta), r, d)\Big] \\ &+ \int_\theta^{\overline{\theta}} \mathbb{E}_\varepsilon[\varepsilon \cdot c_2(x^{A^*}(\theta,\varepsilon),\theta)]\left[\frac{1-F_\Theta(t)}{1-F_\Theta(\theta)}\right]^{N-1} dt, \end{aligned}$$

according to the scoring rule, we obtain

$$\begin{aligned} p^{B^*}(\theta) &= s(v(\theta)) - c_u x^{B^*}(\theta) = \mathbb{E}_\varepsilon\Big[\varepsilon \cdot c(x^{A^*}(\theta,\varepsilon),\theta) + K(x^{A^*}(\theta,\varepsilon), x^{B^*}(\theta), r, d)\Big] \\ &+ \int_\theta^{\overline{\theta}} \mathbb{E}_\varepsilon[\varepsilon \cdot c_2(x^{A^*}(\theta,\varepsilon),\theta)]\left[\frac{1-F_\Theta(t)}{1-F_\Theta(\theta)}\right]^{N-1} dt. \end{aligned} \tag{A.25}$$

The proof is complete.

Proof of Proposition 2

In the text we have shown that the A+B contract with uncertainty is not ex post efficient. Now we prove that the A+B contract with uncertainty may be ex ante efficient. According to Proposition 1, the social welfare in equilibrium is given by

$$W^*(\theta,\varepsilon) = \begin{cases} V_c - \varepsilon \cdot c(x^r(\theta,\varepsilon),\theta) - c_s x^r(\theta,\varepsilon), & \textit{if } \ \varepsilon \le e^r \\ V_c - \varepsilon \cdot c(x^{B^*}(\theta),\theta) - c_s x^{B^*}(\theta), & \textit{if } \ e^r \le \varepsilon \le e^d \\ V_c - \varepsilon \cdot c(x^d(\theta,\varepsilon),\theta) - c_s x^d(\theta,\varepsilon), & \textit{if } \ \varepsilon \ge e^d \end{cases} \tag{A.26}$$

Then, using (A.5) and (A.6) leads to

$$\frac{\partial W^*(\theta,\varepsilon)}{\partial\theta} = \begin{cases} (r-c_s)\frac{\partial x^r(\theta,\varepsilon)}{\partial\theta} - \varepsilon \cdot c_2(x^r(\theta,\varepsilon),\theta), & \textit{if } \ \varepsilon \le e^r \\ -\left(\varepsilon \cdot c_1(x^{B^*}(\theta),\theta) + c_s\right)\frac{\partial x^{B^*}(\theta)}{\partial\theta} - \varepsilon \cdot c_2(x^{B^*}(\theta),\theta), & \textit{if } \ e^r \le \varepsilon \le e^d \\ (d-c_s)\frac{\partial x^d(\theta,\varepsilon)}{\partial\theta} - \varepsilon \cdot c_2(x^d(\theta,\varepsilon),\theta), & \textit{if } \ \varepsilon \ge e^d \end{cases} \tag{A.27}$$

Therefore, the ex ante efficiency is equivalent to $\partial W^*(\theta,\varepsilon)/\partial\theta < 0$ for any (θ,ε). Due to the properties that $\partial x^r(\theta,\varepsilon)/\partial\theta = -c_{12}(x^r(\theta,\varepsilon),\theta)/c_{11}(x^r(\theta,\varepsilon),\theta) > 0$ and similarly $\partial x^d(\theta,\varepsilon)/\partial\theta > 0$, one set of sufficient conditions for ex ante efficiency is that the daily cost c_s is sufficiently large with $r < d \le c_s$ and $c_s \ge -e^d \cdot c_1(x^{B^*}(\theta),\theta)$ for any θ. As implied by (A.14), $c_1(x^{B^*}(\theta),\theta) \equiv \kappa_1$ is constant for any θ. Since the constant cutoff uncertainty is $e^d = -d/\kappa_1$, which is implied by (A.3) and (A.16), then $c_s \geq -e^d \cdot c_1(x^{B^*}(\theta), \theta)$ reduces to $c_s \geq d$. Therefore, one sufficient condition for ex ante efficiency is $r < d \leq c_s$.

Proof of Lemma 2

Let $\widetilde{\theta} = \delta\theta$, which is distributed as $\widetilde{F}_{\widetilde{\Theta}}(\cdot)$ on $[\underline{\widetilde{\theta}}, \overline{\widetilde{\theta}}] = [\delta\underline{\theta}, \delta\overline{\theta}]$. Denote by $(\widetilde{X}^B, \widetilde{P}^B, \widetilde{I}^R, \widetilde{I}^D, \widetilde{X}^r, \widetilde{X}^d)$ the endogenous variables generated by the structure $\widetilde{\mathcal{M}}$. Due to the fact that the pseudo type $V = v(\theta)$ can be directly identified by

$$v = s - \frac{1}{N-1}\frac{1-F_S(s)}{f_S(s)},$$

we will show $(\widetilde{X}^B, \widetilde{P}^B, \widetilde{I}^R, \widetilde{I}^D, \widetilde{X}^r, \widetilde{X}^d, \widetilde{V}) = (X^B, P^B, I^R, I^D, X^r, X^d, V)$, which implies the observational equivalency.

Recall that in equilibrium the bid days is strictly increasing in cost type. Let $\widetilde{X}^B(\cdot) = X^B(\cdot/\delta)$. As shown by (A.14), in equilibrium both $c_1(X^B(\theta), \theta) = \kappa_1$ and $\widetilde{c}_1(\widetilde{X}^B, \widetilde{\theta}) = \widetilde{\kappa}_1$ are constant for any θ and any $\widetilde{\theta}$, respectively. Then

$$\widetilde{\kappa}_1 = \widetilde{c}_1(\widetilde{X}^B, \widetilde{\theta}) = \widetilde{\theta}\widetilde{c}_{o,1}(\widetilde{X}^B(\widetilde{\theta})) = \theta c_{o,1}(\widetilde{X}^B(\widetilde{\theta})) = \theta c_{o,1}(X^B(\theta)) = c_1(X^B(\theta), \theta) = \kappa_1.$$

Denote by $\widetilde{e}^r$ and $\widetilde{e}^d$ the two cutoff levels of uncertainty for early completion and delay completion, respectively. Therefore, from (A.3) we obtain $\widetilde{e}^r = e^r$ and $\widetilde{e}^d = e^d$, which implies

$$\widetilde{I}^R = I^R \text{ and } \widetilde{I}^D = I^D.$$

As a result,

$$r = -\varepsilon\widetilde{c}_1(\widetilde{X}^r(\widetilde{\theta}, \varepsilon), \widetilde{\theta}) = -\varepsilon\widetilde{\theta}\widetilde{c}_{o,1}(\widetilde{X}^r(\widetilde{\theta}, \varepsilon)) = -\varepsilon\theta c_{o,1}(\widetilde{X}^r(\widetilde{\theta}, \varepsilon))$$

$$d = -\varepsilon\widetilde{c}_1(\widetilde{X}^d(\widetilde{\theta}, \varepsilon), \widetilde{\theta}) = -\varepsilon\widetilde{\theta}\widetilde{c}_{o,1}(\widetilde{X}^d(\widetilde{\theta}, \varepsilon)) = -\varepsilon\theta c_{o,1}(\widetilde{X}^d(\widetilde{\theta}, \varepsilon)).$$

Due to $c_{o,1}(\cdot) < 0$ implied by the assumption $0 > c_1(\cdot, \theta) = \theta c_{o,1}(\cdot)$, it follows

$$\widetilde{X}^r(\widetilde{\theta}, \varepsilon) = X^r(\theta, \varepsilon) \text{ and } \widetilde{X}^d(\widetilde{\theta}, \varepsilon) = X^d(\theta, \varepsilon).$$

In addition, according to the definition of pseudo type in (A.23), combining all the above results implies

$$\widetilde{V} = \widetilde{v}(\tilde{\theta}) \quad = \quad c_u\widetilde{X}^B(\widetilde{\theta}) + \mathbb{E}_\varepsilon\left[\varepsilon\widetilde{c}(\widetilde{X}^A(\widetilde{X}^B(\widetilde{\theta}), \widetilde{\theta}, \varepsilon) + K(\widetilde{X}^A(\widetilde{X}^B(\widetilde{\theta}), \widetilde{\theta}, \varepsilon), \widetilde{X}^B(\widetilde{\theta}), r, d)\right]$$

$$
\begin{aligned}
&= c_u X^B(\theta) + \mathbb{E}_\varepsilon\left[\varepsilon c(X^A(X^B(\theta),\theta,\varepsilon) + K(X^A(X^B(\theta),\theta,\varepsilon), X^B(\theta), r, d)\right] \\
&= v(\theta) = V
\end{aligned}
$$

Last, we need to show $\widetilde{P}(\widetilde{\theta}) = P(\theta)$. Note that $\widetilde{F}_{\widetilde{\Theta}}(\widetilde{\theta}) = F_\Theta(\theta)$ due to $\widetilde{F}_{\widetilde{\Theta}}(\cdot) = F_\Theta(\cdot/\delta)$. According to the equilibrium bid cost (A.25), we obtain $\widetilde{P}^B(\widetilde{\theta}) = P^B(\theta)$ since

$$
\begin{aligned}
\widetilde{P}^B(\widetilde{\theta}) &= \mathbb{E}_\varepsilon\left[\varepsilon \cdot \widetilde{c}\left(\widetilde{X}^A(\widetilde{X}^B(\widetilde{\theta}),\widetilde{\theta},\varepsilon\right) + K\left(\widetilde{X}^A(\widetilde{X}^B(\widetilde{\theta}),\widetilde{\theta},\varepsilon), \widetilde{X}^B(\widetilde{\theta}), r, d\right)\right] \\
&+ \int_{\widetilde{\theta}}^{\overline{\widetilde{\theta}}} \mathbb{E}_\varepsilon\left[\varepsilon \cdot \widetilde{c}_o\left(\widetilde{X}^A(\widetilde{X}^B(\widetilde{\theta}),\widetilde{\theta},\varepsilon),\theta\right)\right]\left[\frac{1-F_{\widetilde{\Theta}}(\widetilde{t})}{1-F_{\widetilde{\Theta}}(\widetilde{\theta})}\right]^{N-1} d\widetilde{t} \\
&= \mathbb{E}_\varepsilon\left[\varepsilon \cdot c\left(X^A(X^B(\theta),\theta,\varepsilon\right) + K\left(X^A(X^B(\theta),\theta,\varepsilon), X^B(\theta), r, d\right)\right] \\
&+ \int_{\theta}^{\overline{\theta}} \mathbb{E}_\varepsilon\left[\varepsilon \cdot c_o\left(X^A(\theta,\varepsilon),\theta\right)/\delta\right]\left[\frac{1-F_\Theta(t)}{1-F_\Theta(\theta)}\right]^{N-1} d(\delta t) \\
&= P^B(\theta).
\end{aligned}
$$

The proof is complete.

Proof of Proposition 3

Recall in equilibrium the two cutoff levels of uncertainty are constant for any type. Then we can identify the probabilities of early and delay completion, respectively, through

$$
F(e^r) = \Pr(\varepsilon < e^r) = \Pr(X^A < X^B) \quad \text{and} \quad 1 - F(e^d) = \Pr(\varepsilon > e^d) = \Pr(X^A > X^B).
$$

The first-order condition in (A.11) implies

$$
c_1(X^B,\theta) = \frac{rF(e^r) + d - dF(e^d) - c_u}{\int_{e^r}^{e^d} \varepsilon dF(\varepsilon)} = \frac{\kappa_0}{\int_{e^r}^{e^d} \varepsilon dF(\varepsilon)} = \kappa_1, \tag{A.28}
$$

where $\kappa_0 = rF(e^r) + d - dF(e^d) - c_u$ is a known constant and κ_1 is a unknown constant. Under the parametric cost function, we use (A.5) and (A.6) to obtain the bid day, early working days, and late working days, repsectively, by

$$
\begin{aligned}
X^B &= \frac{\kappa_1}{2\alpha_2\theta} - \frac{\alpha_1}{2\alpha_2} && \text{for on-time completion} && \text{(A.29)} \\
X^r &= \frac{-r\varepsilon^{-1}\theta^{-1} - \alpha_1}{2\alpha_2} && \text{for early completion} && \text{(A.30)} \\
X^d &= \frac{-d\varepsilon^{-1}\theta^{-1} - \alpha_1}{2\alpha_2} && \text{for late completion} && \text{(A.31)}
\end{aligned}
$$

Under the independence between θ and ε, we substitute (A.29) into (A.30) and (A.31) to obtain

$$r(\alpha_1 + 2\alpha_2\mathbb{E}(X^B))\mathbb{E}(\varepsilon^{-1}|\varepsilon \leq e^r) = -\kappa_1(\alpha_1 + 2\alpha_2\mathbb{E}(X^r)) \quad (A.32)$$

$$d(\alpha_1 + 2\alpha_2\mathbb{E}(X^B))\mathbb{E}(\varepsilon^{-1}|\varepsilon \geq e^d) = -\kappa_1(\alpha_1 + 2\alpha_2\mathbb{E}(X^d)) \quad (A.33)$$

Define the following notations:

$$\mu_r = \mathbb{E}(X^r), \;\; \mu_d = \mathbb{E}(X^d), \;\; \mu_B = \mathbb{E}(X^B),$$

$$\kappa_2 = \int_{\varepsilon \leq e^r} \varepsilon dF(\varepsilon), \;\; \kappa_3 = \int_{\varepsilon \geq e^d} \varepsilon dF(\varepsilon), \;\; \kappa_4 = \mathbb{E}(\varepsilon^{-1}|\varepsilon \leq e^r), \quad \kappa_5 = \mathbb{E}(\varepsilon^{-1}|\varepsilon \geq e^d).$$

Then

$$m_\varepsilon = E(\varepsilon) = \kappa_2 + \kappa_3 + \frac{\kappa_0}{\kappa_1}. \quad (A.34)$$

Note that (A.23) can be rewritten as

$$V = c_u \cdot X^B + \mathbb{E}_\varepsilon\Big[\varepsilon \cdot c\left(X^A(X^B,\theta,\varepsilon),\theta\right) + K\left(X^A(X^B,\theta,\varepsilon), X^B, r, d\right)\Big]. \quad (A.35)$$

Let $Q_V(\tau)$ and $Q_{X^B}(\tau)$ be the quantiles of V and X^B, respectively, for any $\tau \in [0,1]$. Since $dv(\theta)/d\theta > 0$ and $dX^B(\theta)/d\theta > 0$ in equilibrium, combining these two one-to-one structural links and substituting the parametric cost function into (A.35), we can obtain a key reduced-form relationship

$$0 = \beta_0 + \beta_1 Q_V(\tau) + \beta_2 Q_V(\tau)Q_{X^B}(\tau) + \beta_3 Q_{X^B}(\tau) + \beta_4(Q_{X^B}(\tau))^2 \quad (A.36)$$

where

$$\beta_0 = \alpha_0 \frac{m_\varepsilon \beta_1^3}{2\beta_2\alpha_1^3} + \frac{\beta_1^2}{\beta_2} - \frac{\beta_1^2(m_\varepsilon\beta_1^2 + 2\kappa_0\alpha_1^2\beta_2)}{4\alpha_1^2\beta_2^2} - \kappa_6 \quad (A.37)$$

with

$$\kappa_6 = r\frac{\beta_1^2}{2\beta_2}\frac{\beta_1 + \beta_2\mu_r}{\beta_1 + \beta_2\mu_B}F(e^r) + d\frac{\beta_1^2}{2\beta_2}\frac{\beta_1 + \beta_2\mu_d}{\beta_1 + \beta_2\mu_B}(1 - F(e^d)), \quad (A.38)$$

$$\beta_1 = -4\kappa_1\alpha_1\alpha_2 \quad (A.39)$$

$$\beta_2 = -8\kappa_1\alpha_2^2 \quad (A.40)$$

$$\beta_3 = -\beta_1(c_u + \kappa_0) + 2\beta_1\left[rF(e^r) + d(1 - F(e^d))\right] - \frac{2\alpha_1^2\beta_2}{\beta_1}\left[r^2\kappa_4 F(e^r) + d^2\kappa_5(1 - F(e^d))\right]$$

$$\beta_4 \;=\; -\beta_2\left\{c_u - \left[rF(e^r) + d(1-F(e^d))\right]\right\} - \kappa_0\beta_2/2 - \frac{\alpha_1^2\beta_2^2}{\beta_1^2}\left[r^2\kappa_4 F(e^r) + d^2\kappa_5(1-F(e^d))\right]$$

Due to the nonlinear relationship between X^B and V in (A.35), the support of

$$\left[1, Q_V(\tau), Q_V(\tau)Q_{X^B}(\tau), (Q_{X^B}(\tau))^2\right]$$

has full rank in general. Hence, $\beta = (\beta_0, \beta_1, \beta_2, \beta_3, \beta_4)$ is identified by choosing any five different values of $\tau \in (0,1)$ to construct five linearly independent equations. As a result,

$$\alpha_2 \;=\; \frac{\alpha_1\beta_2}{2\beta_1} \tag{A.41}$$

$$\kappa_1\alpha_1^2 \;=\; -\frac{\beta_1^2}{2\beta_2} \tag{A.42}$$

Due to the one-to-one mapping between θ and X^B, (A.29) implies

$$\underline{X}^B = \frac{\kappa_1}{2\alpha_2\underline{\theta}} - \frac{\alpha_1}{2\alpha_2} \tag{A.43}$$

where $\underline{X}^B$ is the lower bound of the bid days. Combining (A.41), (A.42), (A.43) with Part (b) in Assumption 2 can identify (α_1, α_2)

$$\alpha_1 \;=\; -\beta_1\left[2\underline{\theta}\beta_2\left(\beta_1 + \beta_2\underline{X}^B\right)\right]^{-1/3} \tag{A.44}$$

$$\alpha_2 \;=\; \frac{\alpha_1\beta_2}{2\beta_1} \tag{A.45}$$

As a result, we can recover the corresponding contractor's type and the uncertainty

$$\theta \;=\; -\frac{\beta_1^2}{2\beta_2\alpha_1^2\left(\alpha_1 + 2\alpha X^B\right)} \quad \text{for any contract} \tag{A.46}$$

$$\varepsilon \;=\; -\frac{r}{\theta(\alpha_1 + 2\alpha_2 X^r)} \quad \text{for early completion contract} \tag{A.47}$$

$$\varepsilon \;=\; -\frac{d}{\theta(\alpha_1 + 2\alpha_2 X^d)} \quad \text{for delay completion contract} \tag{A.48}$$

Then, using the identified types θ identify the type distribution $F_\Theta(\cdot)$ on its support $\mathcal{S}_\Theta$.

For those contracts which are completed on time, the actual working days equals the bid days, which does not depend on uncertainty. Consequently, one cannot recover the uncertainty associated with on-time completion contracts, which implies that the uncertainty distribution $F(\cdot)$ cannot be identifed on its entire support $\mathcal{S}_\varepsilon$. In other words, using X^r and X^d we can back out the corresponding uncertainties, respectively, and thus identify the truncated cumulative distribution function denoted by $G(\cdot)$ of ε on its partial support $\widetilde{\mathcal{S}}_\varepsilon = \mathcal{S}_r \cup \mathcal{S}_d$ with $\mathcal{S}_r = \{\varepsilon : \varepsilon \leq \varepsilon^r\}$ and $\mathcal{S}_d = \{\varepsilon : \varepsilon \geq \varepsilon^d\}$. As a result, the uncertainty distribution $F(\cdot)$ is identified on $\widetilde{\mathcal{S}}_\varepsilon$ as

$$F(\varepsilon) = G(\varepsilon)F(\varepsilon^r) \text{ if } \varepsilon \in \mathcal{S}_r, \text{ and } F(\varepsilon) = G(\varepsilon)(1 - F(\varepsilon^d)) \text{ if } \varepsilon \in \mathcal{S}_d.$$

Under the assumption that the mean of uncertainty m_ε is known, we use (A.37) to identify α_0 as

$$\alpha_0 = \frac{2\beta_2\alpha_1^3}{m_\varepsilon\beta_1^3}\left(\beta_0 + \kappa_6 + \frac{\beta_1^2(m_\varepsilon\beta_1^2 + 2\kappa_0\alpha_1^2\beta_2)}{4\alpha_1^2\beta_2^2} - \frac{\beta_1^2}{\beta_2}\right), \tag{A.49}$$

where κ_6 in (A.38) is obviously identified.

As explained in the text, the equilibrium bid cost (A.25) provides no additional identification power. Formally, the bid cost can be written as a function of all identified objects as below

$$\begin{aligned} P^B &= V - c_u X^B \\ &+ \int_{y \geq X^B} \left[\frac{2\beta_2}{\beta_1^3}\left\{\kappa_6 + \frac{\beta_1^2\kappa_0}{2\beta_2} - \frac{\beta_1^2}{\beta_2} + \beta_0\right\} + \left\{\frac{2\beta_2\kappa_6}{\beta_1^5}(\beta_1 + \beta_2 y)^2 - \frac{\kappa_0}{\beta_1} - \frac{\kappa_0\beta_2\left[2\beta_1 y + \beta_2 y^2\right]}{\beta_1^3}\right\}\right] \\ &\quad \left[\frac{1 - F_{X^B}(y)}{1 - F_{X^B}(X^B)}\right]^{N-1} \frac{\beta_1^3}{2(\beta_1 + \beta_2 y)^2} dy. \end{aligned}$$

The proof is complete.

B Further Details for Empirical Application

We derive the expressions of $\{\beta_k(z_{ji})\}_{k=0}^4$ in (35). Using the prior estimates, we have

$$\widehat{e}^d(z_{ji}) = exp(z_{ji}\widehat{\psi}_d). \tag{B.1}$$

Combining (A.3) with (A.28) implies

$$\frac{e^r(z_{ji})}{e^d(z_{ji})} = \frac{-r_j/c_1(x_{ji}^B, \theta_i)}{-d_j/c_1(x_{ji}^B, \theta_i)} = \frac{-r_j/\kappa_1(z_{ji})}{-d_j/\kappa_1(z_{ji})} = \frac{r_j}{d_j}.$$

Then

$$\widehat{e}^r(z_{ji}) = \frac{r_j}{d_j}\widehat{e}^d(z_{ji}) = \frac{r_j}{d_j}exp(z_{ji}\widehat{\psi}_d). \tag{B.2}$$

As a result,

$$\widehat{\kappa}_0(z_{ji}) = r_j\widehat{F}(\widehat{e}^r(z_{ji})|z_{ji}) + d_j\left[1 - \widehat{F}(\widehat{e}^d(z_{ji})|z_{ji})\right] - c_{u_j}, \tag{B.3}$$

$$\widehat{\kappa}_1(z_{ji}) = \frac{\widehat{\kappa}_0(z_{ji})}{\int_{\widehat{e}^r(z_{ji})}^{\widehat{e}^d(z_{ji})} \varepsilon d\widehat{F}(\varepsilon|z_{ji})}. \tag{B.4}$$

Then

$$\widehat{\kappa}_4(z_{ji}) = -r_j^{-1}\widehat{\kappa}_1(z_{ji})\frac{\alpha_1 + 2\alpha_2\widehat{\mu}_r(z_{ji})}{\alpha_1 + 2\alpha_2\widehat{\mu}_B(z_{ji})}, \tag{B.5}$$

$$\widehat{\kappa}_5(z_{ji}) = -d_j^{-1}\widehat{\kappa}_1(z_{ji})\frac{\alpha_1 + 2\alpha_2\widehat{\mu}_d(z_{ji})}{\alpha_1 + 2\alpha_2\widehat{\mu}_B(z_{ji})}. \tag{B.6}$$

Let

$$\widehat{m}_\varepsilon(z_{ji}) = exp(\widehat{\mu} + \widehat{\sigma}(z_{ji})^2/2). \tag{B.7}$$

Note that (A.34) suggests

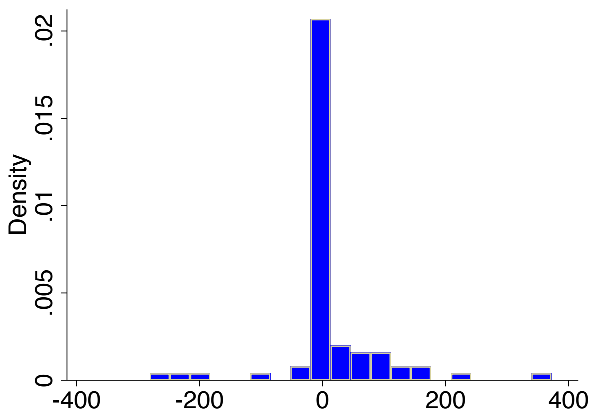
$$\widehat{\kappa_2(z_{ji}) + \kappa_3(z_{ji})} = \widehat{\kappa}_2(z_{ji}) + \widehat{\kappa}_3(z_{ji}) = \widehat{m}_\varepsilon(z_{ji}) - \frac{\widehat{\kappa}_0(z_{ji})}{\widehat{\kappa}_1(z_{ji})}. \tag{B.8}$$

Therefore, (A.36) implies

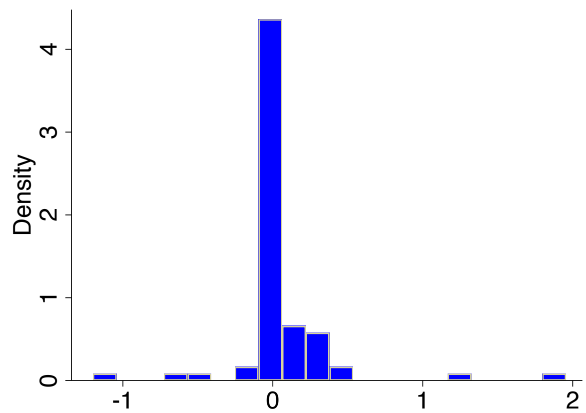
$$\begin{aligned}
\beta_0(z_{ji}) &= -\alpha_1^2\Big\{r_j^2\widehat{\kappa}_4(z_{ji})\widehat{F}(\widehat{e}^r(z_{ji})|z_{ji}) + d_j^2\widehat{\kappa}_5(z_{ji})(1 - \widehat{F}(\widehat{e}^d(z_{ji})|z_{ji})) + 2\widehat{\kappa}_1(z_{ji}) \\
&\qquad + \widehat{\kappa}_1^2(z_{ji})[\widehat{\kappa}_2(z_{ji}) + \widehat{\kappa}_3(z_{ji})]\Big\} + 4\alpha_0\alpha_2\Big\{\widehat{\kappa}_1^2(z_{ji})[\widehat{\kappa}_2(z_{ji}) + \widehat{\kappa}_3(z_{ji})] + \widehat{\kappa}_0(z_{ji})\widehat{\kappa}_1(z_{ji})\Big\}, \\
\beta_1(z_{ji}) &= -4\widehat{\kappa}_1(z_{ji})\alpha_1\alpha_2, \\
\beta_2(z_{ji}) &= -8\widehat{\kappa}_1(z_{ji})\alpha_2^2, \\
\beta_3(z_{ji}) &= \alpha_1\alpha_2\Big\{4\widehat{\kappa}_1(z_{ji})(c_{u_j} + \widehat{\kappa}_0(z_{ji})) - 8\widehat{\kappa}_1(z_{ji})\left[r_j\widehat{F}(\widehat{e}^r(z_{ji})|z_{ji}) + d_j(1 - \widehat{F}(\widehat{e}^d(z_{ji})|z_{ji}))\right] \\
&\quad - 4\left[r_j^2\widehat{\kappa}_4(z_{ji})\widehat{F}(\widehat{e}^r(z_{ji})|z_{ji}) + d_j^2\widehat{\kappa}_5(z_{ji})(1 - \widehat{F}(\widehat{e}^d(z_{ji})|z_{ji}))\right]\Big\}, \\
\beta_4(z_{ji}) &= \alpha_2^2\Big\{8\widehat{\kappa}_1(z_{ji})\left\{c_{u_j} - \left[r_j\widehat{F}(\widehat{e}^r(z_{ji})|z_{ji}) + d_j(1 - \widehat{F}(\widehat{e}^d(z_{ji})|z_{ji}))\right]\right\} + 4\widehat{\kappa}_1(z_{ji})\widehat{\kappa}_0(z_{ji}) \\
&\quad - 4\left[r_j^2\widehat{\kappa}_4(z_{ji})\widehat{F}(\widehat{e}^r(z_{ji})|z_{ji}) + d_j^2\widehat{\kappa}_5(z_{ji})(1 - \widehat{F}(\widehat{e}^d(z_{ji})|z_{ji}))\right]\Big\}.
\end{aligned}$$

C Figures

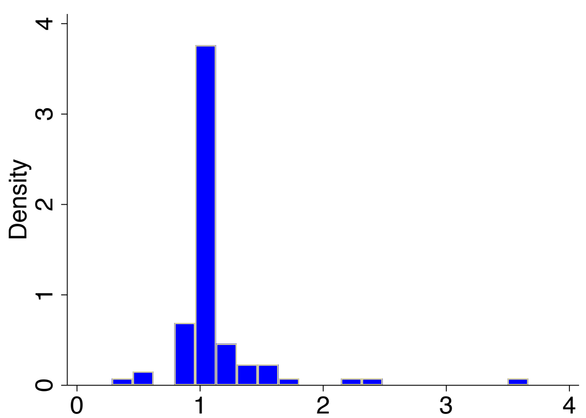
Figure 1: Comparisons between working days and bid days



(1) Working days-Bid days



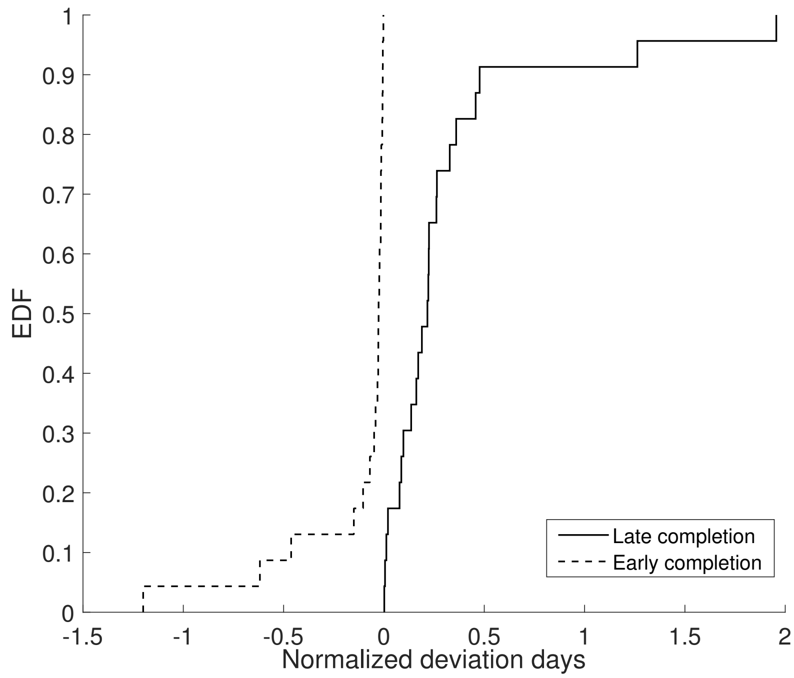
(2) (Working days-Bid days)/Engineer days



(3) Working days/Bid days

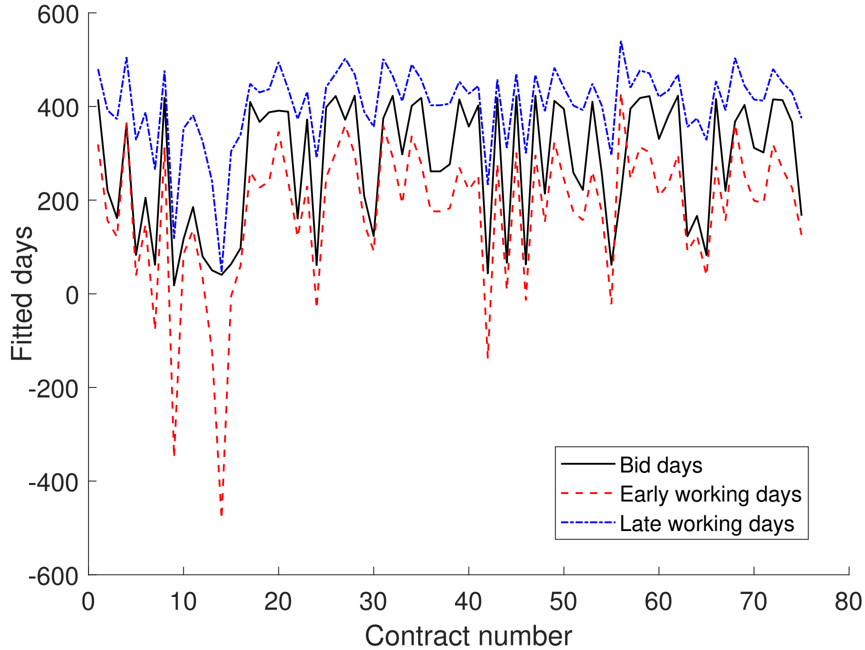
Notes: Figure 1 shows the histograms of the difference and ratio between actual working days and bid days for each contract in the data. In panel (2), the difference is normalized by the engineer's estimated days to capture the contract heterogeneity.

Figure 2: Empirical distribution functions of normalized deviation days



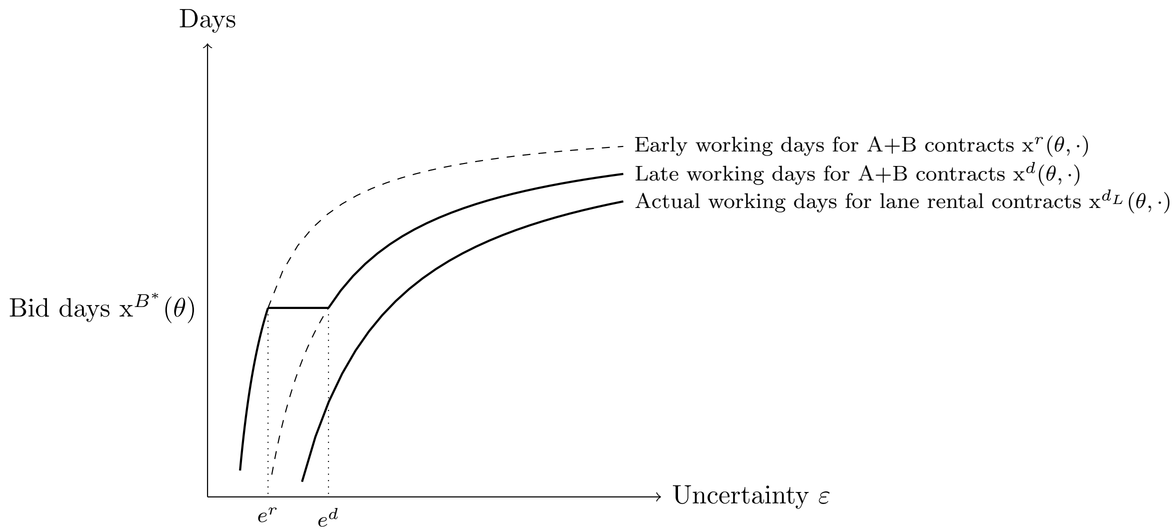
Notes: Figure 2 plots the empirical distribution functions of the normalized deviation days for early and late completions, where the normalized deviation days $= \frac{\text{early (late) working days - bid days}}{\text{engineer's estimated days}}$. Hence, the upper (lower) bound of the support for the normalized early (late) deviation days is zero.

Figure 3: Fitted bid days and actual working days



Notes: Figure 3 plots the fitted bid days, early working days and late working days based on $\widehat{\mu}_B(z)$, $\widehat{\mu}_r(z)$, and $\widehat{\mu}_d(z)$, respectively.

Figure 4: Comparison of actual working days between A+B and lane rental contracts



Notes: Figure 4 plots the equilibrium relationship between actual working days and uncertainty in A+B and lane rental contracts, respectively, with $r < d < c_s = d_L$. The upper solid curve depicts the actual working days in A+B contracts; the lower solid curve depicts the actual working days in lane rental contracts.

D Tables

Table 1: Summary Statistics for A+B Highway Construction Contracts in California
(Contract Level, 77 Contracts)

Variable	Mean	Std. Dev.	Min	10-th pctile	Median	90-th pctile	Max
Engineer Cost ($M)	22.4	29.6	0.86	4.6	12.5	48.6	198
Engineer Days	322.29	201.72	45	130	250	600	1000
Usercost ($K)	14.83	15.62	1.8	4.2	11.5	25.1	93.99
Incentive Payments ($K)	8.9	9.37	1.08	2.52	6.9	15.06	56.39
Liquidated Damages ($K)	16.11	18.43	1.8	4.1	11.6	27.3	111.5
Engineer Score ($M)	28.1	38.2	1	6.1	15.4	56.8	26.6
Winning Bid ($M)	20.4	28.6	0.7	4.23	10.6	43.9	178
Winning Bid/Engineer Cost	0.91	0.2	0.59	0.65	0.89	1.19	1.38
Number of Bidders	5.64	2.45	2	3	5	8	14
Federal Contract	0.81	0.4	0	0	1	1	1
Firm Capacity ($M)	71.3	78	0	4.9	52.4	252	285
Distance (miles)	65.88	129.38	1.91	7.04	24.93	255.94	802.14
Commuter Cost ($K)	50.54	46.83	0.25	5.31	37.16	129.35	185.15
Contract Days	249.96	209.63	25	70	167	515	950
Working Days	262.73	232.98	42	75	171	602	1120
Working-Contract Days	12.77	82.02	-281	-13	0	88	372
Working/Contract Days	1.10	0.42	0.28	0.92	1	1.38	3.67

Table 2: Summary Statistics for A+B Highway Construction Contracts in California
(Bid Level, 421 Bids)

Variable	Mean	Std. Dev.	Min	10-th pctile	Median	90-th pctile	Max
Bid Cost ($M)	19.1	21.8	0.93	4.47	10.8	44.5	106
Bid Cost/Engineer Cost	0.97	0.23	0.61	0.71	0.94	1.26	1.65
Bid Days	207.24	138.24	30	80	172.5	360	750
Bid Days/Engineer Days	0.66	0.19	0.27	0.41	0.66	0.92	1
Firm Capacity ($M)	72.2	76.5	0	4.90	52.4	192	285
Distance (miles)	69.82	121.37	1.75	9.68	29.29	149.05	669.68
Bid Score ($M)	22.2	25	1.02	5.43	12.6	49.8	121

Table 3: Early and Late Working Days Regression

	Early Working Days			Late Working Days		
Log(Capacity)/Engineer Days	-3610**	-3605.1**	-2702.3*	-2079.9***	-2208.9***	538.9
	(1321.6)	(1387.0)	(1357.0)	(534.5)	(502.2)	(2322.6)
Federal Contract		-5.999	-2.923		-164.3	-34.70
		(46.17)	(48.75)		(160.6)	(137.8)
Log(Distance)/Engineer Days			-6773.3			-9630.0
			(4662.8)			(8404.0)
Constant	454.8***	459.3***	472.5***			553.8***
	(106.8)	(88.26)	(86.74)			(145.1)
Observations	23	23	23	23	23	23
Adjusted R^2	0.384	0.351	0.364	0.152	0.154	0.059

Notes: Robust standard errors in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table 4: Estimates of Other Parameters

	Parameters/Variables	Estimates
Distribution of Uncertainty	Mean of log(Uncertainty)	-0.089*
		(0.049)
	SD of log(Uncertainty)	10.073***
		(2.048)
Cutoff Uncertainty	Log (Capacity)/Engineer Days	2.958**
		(1.287)
Cost Parameters	Working Days	$-3.599 \times 10^{-4***}$
		(6.994×10^{-7})
	Working Days^2	$3.246 \times 10^{-8***}$
		(1.752×10^{-10})

Notes: Bootstrap standard errors in parentheses are calculated using 500 bootstrap samples. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table 5: Counterfactual Results for A+B and Lane Rental Contracts

	Construction Cost	Commuter Cost	Social Cost
A+B ($M)	31.92	70.52	102.44
Lane Rental ($M)	47.46	10.25	57.71
Absolute Change ($M)	15.54	60.27	44.73
Percentage Change	32.74%	85.47%	43.66%

Notes: Counterfactual welfare results under A+B and lane rental contracts. The counterfactual results are averaged across 1000 simulations and 77 A+B contracts. *Construction Cost* equals realized uncertainty ε times deterministic cost $c(x^A, \theta)$. *Commuter Cost* equals to daily cost c_s times actual working days x^A. *Social Cost* is the sum of construction cost and commuter cost.