Classic Model Replication & Critical Review
Self-Organized Critical Forest-Fire Model

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Complex Systems
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Summary

The Drossel & Schwabl (1992) forest fire model demonstrates a self-organized critical (SOC) system, meaning that the system naturally progresses to a critical state in which small disruption events can trigger events of any size. The model uses a d-dimensional lattice in which trees grow (probability p) and lightning triggers burning trees that ignite neighboring trees (probability f). Criticality arises when $f/p \rightarrow 0$, resulting in a power-law distribution of fire sizes. The replication was done in Python using AgentPy with various f and f/p values in a finite grid on finite timescales. While not a realistic forest fire management tool, the model highlights SOC dynamics that emphasize the importance of resilience-based policy over control-based policy.

Methods, Motivation, and Model Purpose

Self-organized criticality (SOC) was first introduced by Bak et al. (1987). They proposed that SOC systems are drawn into a critical point. Drossel & Schwabl (1992) built on their research by further exploring SOC mechanisms in their forest fire model. Their SOC forest fire model demonstrates how a set of rules can generate SOC dynamics. Their model consists of a d-dimensional lattice where cells can be one of three states: i) empty ii) burning tree iii) tree. At each discrete time-step, a set of update rules apply:

- a burning tree becomes an empty site
- a tree with at least one burning tree neighbor becomes a burning tree
- empty sites change to trees with probability p
- a tree without a burning neighbor becomes a burning tree with probability f

In this system, criticality refers to the balance of two extremes. First, if lightning occurs too frequently (large f), only small fires develop due to a lack of fuel. Second, if tree growth is too fast (large p), fires can burn the entire system. As such, the system reaches criticality when $(f/p)^{-1} \ll p^{-1} \ll f^{-1}$ or $f/p \rightarrow 0$. As long as the timescale set by these parameters holds, tree growth and fire spread balance the system into a critical state. Appendix A shows the system's causal structure.

Design & Purpose

The model is a spatial cellular automaton with rules that are applied stochastically (through p and f). The model's spatial component is important because forest cluster connectivity governs fire spread. With this set-up, the model's purpose is to explore system dynamics under SOC assumptions. It approaches that exploration through a stylized mechanistic model of systems with slow accumulation, rare triggers, and fast cascades.

Use Context & Policy Angle

Being a stylized model, it abstracts away from realism. Thus, the model is not designed for forest-fire management, but for conceptual insight. This generalizes the model. In an SOC regime, many small, few medium, and rare catastrophic events are inevitable. This power-law distribution implies that policy cannot eliminate catastrophic events, but should plan for resilience and recovery. Using this insight, the SOC model emphasizes resilience-based management (permitting small events to prevent fuel build-up) over controlling every disruption event.

Core Methods

Our replication model was recreated with Python 3.12, utilizing AgentPy 0.1.5. The skeleton for the code was adapted from a regular forest fire simulation by Foramitti (2021), using the rules outlined by Drossel & Schwabl (1992). Parameters used as an input can be found in Appendix B.1. At each time step, trees have a probability f of being ignited. Fire spreads to 4-connected neighbourhoods, burning trees become empty, and empty lots have a probability $p = f \times p$_over_f. Then the simulation records metrics such as time step, amount of trees and fire clusters. These were used to examine the model behaviour across different combinations of f and p_over_f. Demonstration of approximate behaviour of the simulation with smaller grid and timescale is included in Appendix B.2.

Results

Drossel & Schwabl (1992) demonstrate that at a tree density of ρ ≈ 0.39 in a two-dimensional space, the forest reaches a point where it balances the tree growth and burning in a way that allows the development of connected forest clusters, but does not overcrowd the forest. In our replication, we used this as an indicator of a steady state. Table 2 shows the values calculated across different values of f and p_over_f. These indicate that with f set at 10^{-5} and p_over_f set at 1,000, the model is closest to the density proposed by Drossel & Schwabl (1992) at ρ ≈ 0.34, although the densities still fall short of the density in the original model. This is most likely due to the limited time steps of the model. Full list of forest densities can be found at Appendix B.3

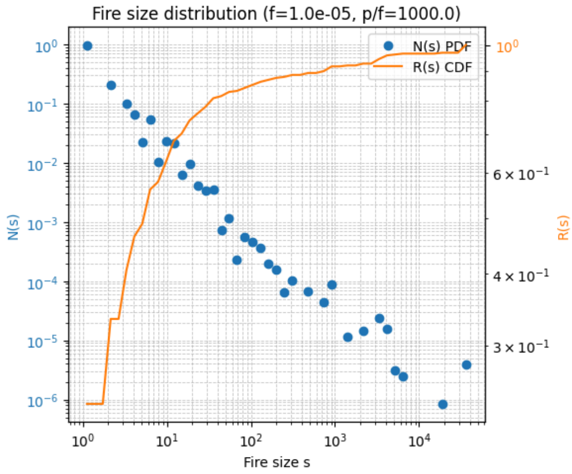

Figure 1: Fire size when near critical regime

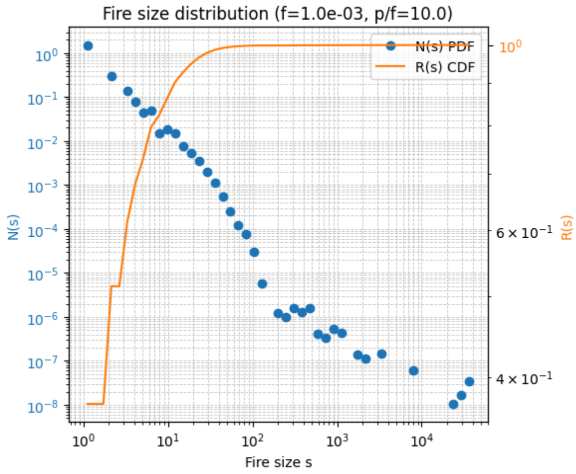

Figure 2: Fire size when regime is not critical

In the original Drossel-Schwabl forest fire model, fire spreads dynamically. A fire continues to spread to connected healthy trees until either i) no connected trees remain or ii) another fire burns parts of the cluster before the initial fire reaches them.

In our implementation, we were unable to simulate these exact spatiotemporal dynamics. In particular, we encountered that once a site burns and becomes empty, the information concerning which fire cluster it belonged to is lost. Therefore, we estimated fire sizes by counting the number of healthy, connected trees at the location of each lightning strike. To do this, we introduced a fourth cell state called ignited, which marked the starting point of a fire. This aligned with the fact that, in the original model, all trees connected to the ignition point would eventually burn.

This approximation worked well when $f << p$ and f/p -> 0. Under these conditions, lightning strikes were extremely rare, which meant that overlap between fires (which would overestimate cluster sizes) was unlikely. Furthermore, under these conditions, the forest had enough time to develop large, connected clusters which were exactly the circumstances under which the original model exhibited self-organized criticality. This means that fire sizes follow a power-law distribution.

We plotted the probability density function (PDF), $N(s)$, and cumulative density function (CDF), $R(s)$, for different values of f and different f and p ratios, in order to see if we observe such a power-law behaviour where $N(s) \sim s^{-\tau}$. This behavior was reproduced in our model for low f and f/p as shown in Figure 1. The PDF followed the anticipated scaling trend reported by Drossel-Schwabl (1992), while the CDF revealed a strong dominance of small fires, suggesting that the system had not yet reached the fully critical regime.

However, at higher values of f/p we also observed distributions resembling a power law (Figure 2). This result is misleading because our simulation used a small 2D lattice (200x200) and only 2000 time steps due to a lack of computation power and time. A small surface limits the possibility for rare large fires to show a distinguishing result in the true scaling behavior of these dynamics from finite-size effects. We expect that, as time approaches infinity and if the 2D lattice is of infinite size, low values of f/p would exhibit more genuine SOC behavior, including a clearer power-law scaling of fire sizes with a heavy-tailed distribution. At higher f/p the system is expected to show an exponential cutoff which makes large scale fires almost impossible due to the lack of large-scale connectivity.

Finally, in the original model, higher values of f “stabilize” the density of the forest because they prevent large clusters from forming. This means that fires act as a regulatory mechanism and that their sizes are mostly low with large forest fires being almost impossible. Our model, unfortunately, did not capture this effect in the power-law scaling of sizes, since it assumed that lightning events consume the entire associated tree cluster, regardless of possible disruptions of other fire events. As a result, as f increased, the cluster sizes of forest fires were overestimated and our model failed to reproduce the transition from critical to non-critical behavior as a power law of event sizes, which was assumed by the original model.

Insights & generalisation

Generalization

The forest fire model offers valuable insights in how small local interactions could emerge into large-scale events. Self-organized criticality helps us to understand real systems like forest fires, where rare massive events occur without fine tuning parameters to specific values: the system’s dynamics will drive it near criticality over a wide range of values. Many real world systems like earthquakes (Sornette & Sornette, 1989) and disease spreading (Johansen, 1994) exhibit similar self organizing behavior that is characterised by a power-law distribution.

Boundaries

In this model, self-organized criticality is expected in infinite large systems over infinite time. However, a computer cannot run simulations of infinite size and over infinite time. In smaller simulations, finite-size effects could dominate and distribution of fire sizes might not follow a clear power-law distribution. In the real world forests are also limited in size, suggesting that in some forests finite-size effects could also dominate. Furthermore, the model requires strict timescale separation between lightning strikes (f), fire spread (f/p) and tree growth (p). When this separation breaks down (high p, high f) the system no longer self-organizes into a critical state. In the real world, this separation might not always hold up in areas where ignition occurs more regularly. Finally, the model simplifies reality by assuming homogeneous spread of fire. In doing so, it ignores variations in spread due to external factors like wind or vegetation type that could impact the fire spread.

This means that the model is assumed to display genuine SOC behavior under idealized abstract settings in infinite time and size with strict timescale seperation. Without these conditions, the predictions of the dynamics of the model could be misleading.

Strengths & weaknesses

Simplicity is one of the model's strengths. With only a few parameters, it is easy to see how changing these parameters affects the model, and which parameters are most sensitive to changes. Because of this, it is easy to observe how the dynamics of the model change based on different values of the parameters.

The model also fits nicely with the theory behind SOC. The model highlights slow accumulation and fast release, which happens under SOC, shown in the timescale seperation $(f/p)^{-1} \ll p^{-1} \ll f^{-1}$. The model also exhibits self-organized criticality under these conditions, with fire sizes following a power-law distribution. As previously mentioned, this behavior was also reproduced in our model for low f and f/p.

Regarding validation, later work has shown that some of the findings do not hold up under very large lattices. Grassberger (2002) ran a simulation with a lattice size of 65536 × 65536, and a p/f of 256000. He found that under these large lattice sizes, the previously seen power law does not hold up, and that such scaling laws in this model are spurious. Grassberger (2002, p.1) argues that the steady state tree density consists of large, uniform patches which are either far above or below the critical density for spreading. Partly because of this broad range of patch sizes, the fires can have a broad range of sizes, which causes power laws to still be observable. This is in line with earlier remarks from Grassberger (1993), where the scaling claims were already scrutinised under smaller lattice sizes.

These findings suggest that, as the lattice size increases, we might also not observe power law scaling in our model when $f/p \to 0$.

Policy relevance

Insights from Drossel & Schwabl (1992) provide some clear policy interventions for SOC regimes. Firstly, there is an importance to letting smaller, controlled events happen. As seen in the forest-fire model, smaller events can help keep the buildup of fuel in check. If all fires were to be suppressed, this build-up of fuel would increase the risk of larger, more catastrophic events happening. Additionally, as the power-law distribution highlights that large-scale events can still happen, policy measures that prepare for such events, such as through resilience and recovery-based measures, are of great importance.

Contribution statement

Binti Dekker coded parts of the model and wrote sections about the insights and generalizations and some of the results

Martijn Schotte wrote the sections on the strengths and weaknesses and on the policy relevance.

Noah Zuijderwijk wrote the summary and methods, motivation, and model purpose section (except for the core methods part, which Rosa wrote) and created figure 1 (CLD).

Rosa Muilu coded parts of the model and wrote sections about the core methods and some of the results.

Acknowledgements

We acknowledge that AI assistance (specifically OpenAI's ChatGPT) was used to support coding syntax and to improve the flow of the program. All final decisions were made by the authors.

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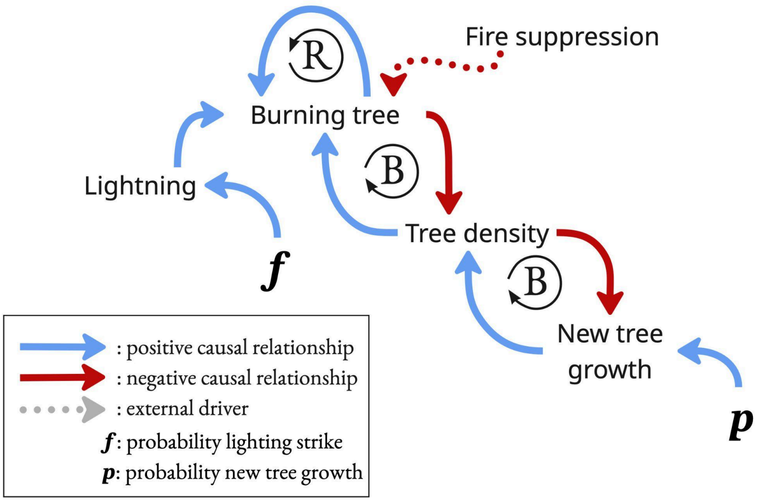
Appendix A: Causal loop diagram of the system

A.1 Causal loop diagram of forest fire model (with spatial boundary)



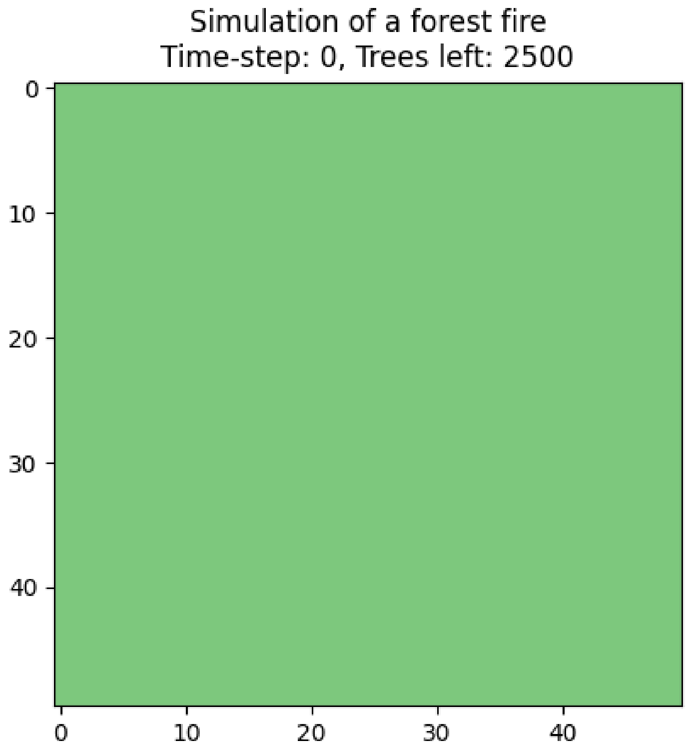
Appendix B: Replication

Full code can be found at https://github.com/rosamuilu/replicatingforestfires

B.1 Simulation parameters

Parameter	**Symbol / Variable**	**Value**	**Units / Description**
Grid size	size	200	cells per dimension
Time Steps	steps	2000	length of simulation
Tree density	Tree density	1	tree coverage at the initial step
Probability of lightning	f	[1e-3, 1e-5, 1e-6]	probability of a random fire
Ratio p/f	p_over_f	[100, 1000, 10000]	defines regrowth relative to lightning
Growth probability	growth	f * p_over_f	probability of a new tree in an empty site
Random seed	–	fixed at 42	ensures reproducibility

B.2 Demonstration of the replication on a smaller lattice and shorter timestep



B.3 Mean forest densities across parametres

f (lightning probability)	p_over_f (growth / lightning ratio)	Mean forest density ρ
10^{-6}	10	0.066
10^{-6}	1 000	0.297
10^{-6}	10 000	0.523
10^{-5}	10	0.111
10^{-5}	1 000	0.339
10^{-5}	10 000	0.282
10^{-3}	10	0.228
10^{-3}	1 000	0.778
10^{-3}	10 000	0.780