The Traffic Assignment Problem

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The Traffic Assignment Problem

- Traffic Assignment (TA) is a process of allocating the given origin-destination (OD) trip to the transportation network under certain rules.
- User Equilibrium (UE) Principle: All of the used paths have equal and minimum travel times; all of the unused paths have equal or higher travel times.

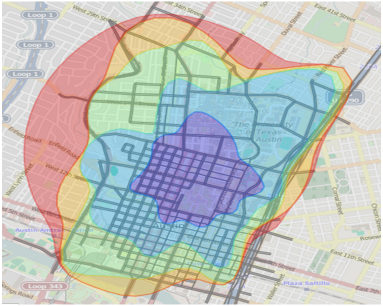

Figure: Travel time during the morning peak in downtown Austin (UT-NMC)

User Equilibrium

A transportation network $G = (N, A)$ is given, where N and A are the sets of nodes and links, and each link is associated with a positive travel time $t(x)$ as a function of link flow x. For origin-destination (O-D) pair $(rs) \in Z^2$ ($Z \subseteq N$), there is a given positive path (π) flow h^π and its corresponding path travel time is C^π. Then, the objective function of the UE principle is [1]:

$$\min_{\boldsymbol{x},\boldsymbol{h}} \sum_{(i,j)\in A} \int_0^{x_{ij}} \boldsymbol{t}_{ij}(x)dx$$

s.t.

$$\begin{aligned} x_{ij} &= \textstyle\sum_{\pi\in\Pi} \delta_{ij}^{\pi} h^{\pi} && \forall (i,j) \in A \\ d^{rs} &= \textstyle\sum_{\pi\in\Pi^{rs}} h^{\pi} && \forall (r,s) \in Z^2 \\ \boldsymbol{h}^{\pi} &\geq 0 && \forall \pi \in \Pi \end{aligned}$$

[1] Known as the Beckmann function.

Frank-Wolfe Algorithm

- Frank and Wolfe (1956) designed this conditional gradient algorithm to solve the convex quadratic problems with linear constraints, and LeBlanc et al. (1975) first adopted it for solution of the TA problem.
- Advantages: Memory efficiency (only link variables need to be stored).
- Disadvantage: Slow convergence near the optimal point, take long time to reach high precision.

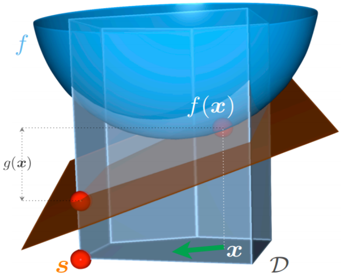

Figure: Frank-Wolfe Algorithm (Jaggi, 2013)

Frank-Wolfe Algorithm: The Traffic Assignment Problem

1. Generate a initial solution $\boldsymbol{x} \in \boldsymbol{X}$.
2. Generate a target solution $\boldsymbol{x}^*$ which minimize the linear approximation $f(\boldsymbol{x}) + \nabla f(\boldsymbol{x})^T(\boldsymbol{x}^* - \boldsymbol{x})$.
3. Update the current solution $\boldsymbol{x'} := \boldsymbol{x} + \lambda(\boldsymbol{x}^* - \boldsymbol{x})$, $\lambda \in [0, 1]$.
4. Calculate new travel times $\boldsymbol{t}(\boldsymbol{x'})$.
5. If the convergence criterion is satisfied, stop; otherwise return to 2.

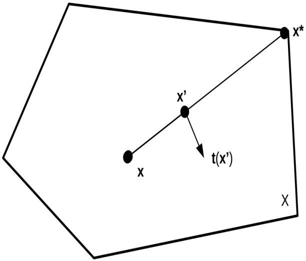

Figure: A solution to the Beckmann function in the FW method (Boyles, 2016)

Frank-Wolfe Algorithm: Large Networks Case Study

- Lee et al. (2002) evaluated the FW computational performance in mid- to large-scale randomly generated grid networks.
- Path-based algorithm:, gradient projection (GP) and disaggregate simplicial decomposition (DSD).
- Link-based algorithms: Frank Wolfe (FW), PARTAN (PT), and restricted simplicial decomposition (RSD).

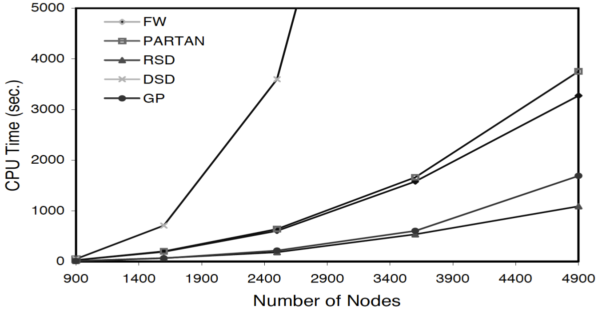

Figure: CPU time versus network size (Lee at al., 2002)

Thank You!

Questions or Comments?

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