The Traffic Assignment Problem

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SDS 385: Statistical Models for Big Data

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October 18, 2018

Overview

The Traffic Assignment Problem

- Traffic Assignment (TA) is a process of allocating the given origin-destination (OD) trip to the transportation network under certain rules.
- User Equilibrium (UE) Principle: All of the used paths have equal and minimum travel times; all of the unused paths have equal or higher travel times.

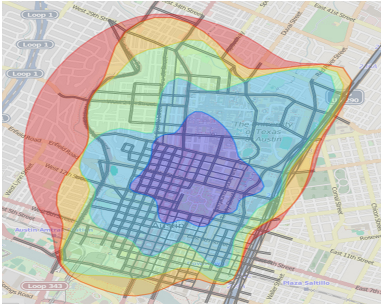

Figure: Travel time during the morning peak in downtown Austin (UT-NMC)

User Equilibrium

A transportation network $G = (N, A)$ is given, where N and A are the sets of nodes and links, and each link is associated with a positive travel time $t(x)$ as a function of link flow x. For origin-destination (O-D) pair (rs), there is a given positive path (π) flow h^π and its corresponding path travel time is C^π. Then, the objective function of the UE principle is [1]:

$$\min_{\boldsymbol{x},\boldsymbol{h}} \sum_{(i,j)\in A} \int_0^{x_{ij}} \boldsymbol{t}_{ij}(x)dx$$

s.t.

$\boldsymbol{h}^\pi \geq 0$	$\forall \pi \in \Pi$	Non negative path flow
$\boldsymbol{C}^\pi \geq \kappa_{rs}$	$\forall (\boldsymbol{r}, \boldsymbol{s}) \in \boldsymbol{Z}^2$	κ_{rs} is the shortest path
$\boldsymbol{h}^\pi (\boldsymbol{C}^\pi - \kappa_{rs}) \geq 0$	$\forall \pi \in \Pi$	If the path is use, its travel time is κ_{rs}

[1] Known as the Beckmann function.

Frank-Wolfe Algorithm

- Frank and Wolfe (1956) designed this conditional gradient algorithm to solve the convex quadratic problem, and LeBlanc et al. (1975) first adopted it for solution of the TA problem.
- Advantages: Memory efficiency (only link variables need to be stored).
- Disadvantage: Slow convergence near the optimal point, take long time to reach high precision.

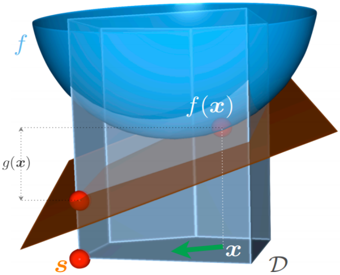

Figure: Frank-Wolfe Algorithm (Jaggi, 2013)

Frank-Wolfe Algorithm: Traffic Assignment Problem

- Define $\boldsymbol{X}' = \{\boldsymbol{x}' : \boldsymbol{x} = \lambda \boldsymbol{x}^* + (1-\lambda)\boldsymbol{x}, \lambda \in [0,1]\}$
- Find $\boldsymbol{x}' \in \boldsymbol{X}'$ such that $\boldsymbol{t}(x') \cdot (\boldsymbol{x}' - \boldsymbol{x}'') \leq 0 \; \forall \boldsymbol{x}'' \in \boldsymbol{X}'$
- We assume that the solution is not in any endpoint ($\lambda = 1$ or $\lambda = 1$).
- Then, $\sum_{ij} t_{ij}(x'_{ij})(x^*_{ij} - x_{ij}) = 0$
- Where, $x' = x_{ij} + \lambda(x^*_{ij} - x_{ij})$ or $x' = \lambda x^*_{ij} + (1-\lambda)x_{ij}$

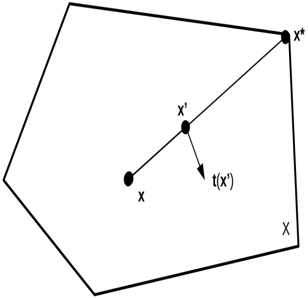

Figure: A solution to the Beckmann function in the FW method (Boyles, 2016)

Frank-Wolfe Algorithm: Large Networks Case Study

- Lee et al. (2002) evaluated the FW computational performance in mid- to large-scale randomly generated grid networks.
- Path-based algorithm:, gradient projection (GP) and disaggregate simplicial decomposition (DSD),
- Link-based algorithms: FrankWolfe (FW), PARTAN (PT), and restricted simplicial decomposition (RSD)

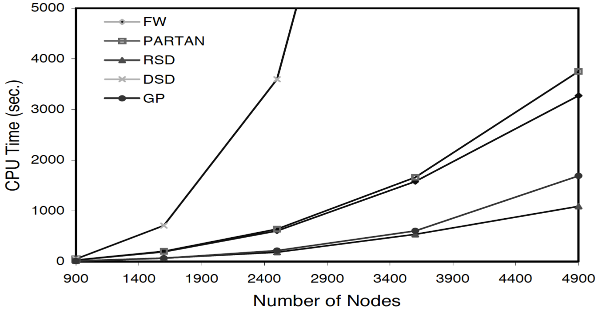

Figure: CPU time versus network size (Lee at al., 2002)

Thank You!

Questions or Comments?

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