Margin with United Profit Function across Horizontal Market and Vertical Market in Airline Industry

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Review on Gayle's model:

1. The ticketing carrier t maximizes profit given the operating cost $\mathbf{p}_j^u$ for product j from the upstream operating carrier u and the it's own marginal cost c_j^d for providing product j .

$$\underset{p_j}{\text{Max}} \left\{ \sum_{j \in \mathcal{J}_t} \left(p_j - p_j^u - c_j^d \right) M \cdot s_j(\mathbf{p}) \right\}$$

2. The pure operating carrier (upstream carriers) maximize profit:

$$\underset{p_j^u}{\text{Max}} \left\{ \sum_{j \in \mathcal{J}_u} \left(p_j^u - c_j^u \right) M \cdot s_j(\mathbf{p}(\mathbf{p^u})) \right\}$$

Incorporating the incentive of raising operating price for other firm, operating profit should be considered into the profit:

Revised model:

1. Downstream (Ticketing carrier)

$$\underset{p_j}{\text{Max}} \left\{ \underbrace{\sum_{j \in \mathcal{J}_t} \left(p_j - p_j^u - c_j^d \right) M \cdot s_j(\mathbf{p})}_{\text{codeshare product as ticketing carrier}} + \underbrace{\sum_{j \in \mathcal{J}_u} \left(p^u - c^u \right) M \cdot s_j(\mathbf{p})}_{\text{codeshare product as operating carrier}} + \underbrace{\sum_{j \in \mathcal{J}} \left(p_j - c_j \right) M \cdot s_j(\mathbf{p})}_{\text{non-codeshare product}} \right\}$$

2. Upstream (pure operating)

$$\underset{p_j^u}{\text{Max}} \left\{ \underbrace{\sum_{j \in \mathcal{J}_t} \left(p_j - p_j^u - c_j^d \right) M \cdot s_j(\mathbf{p})}_{\text{codeshare product as ticketing carrier}} + \underbrace{\sum_{j \in \mathcal{J}_u} \left(p^u - c^u \right) M \cdot s_j(\mathbf{p})}_{\text{codeshare product as operating carrier}} + \underbrace{\sum_{j \in \mathcal{J}} \left(p_j - c_j \right) M \cdot s_j(\mathbf{p})}_{\text{non-codeshare product}} \right\}$$

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Basically upstream and downstream will have the similar profit function, because in codeshare alliance, no carrier is a pure operating carrier or a pure ticketing carrier. The timeline is: upstream carriers set a upstream price p^u anticipating the effect of upstream price on downstream price , then down stream carriers set a downstream price p^d.

Downstream F.O.C:

$$s_m + [\Omega_m^d \circ \frac{\delta s_m(p_m;\theta)}{\delta p_m}]m^d + [\Omega_m^{ud} \circ \frac{\delta s_m(p_m;\theta)}{\delta p_m}]m^u = 0$$

Upstream F.O.C with respect to the codeshare product's upstream operating price p^u:

$$s_m + [\Omega^{du} \circ \frac{\delta p_m}{\delta p^u}]s_m + [\Omega^{du} \circ \frac{s_m(p_m;\theta)}{\delta p_m^u}]m^d + [\Omega^u \circ \frac{\delta s_m(p_m;\theta)}{\delta p_m^u}]m^u = 0$$

The cost pass through term $\frac{\partial p_m}{\partial p_m^u}$ is a J*NC matrix (NC: number of codeshare product) , which is derived by totally differentiating downstream F.O.C equation (4) for product j with respect to all downstream prices and upstream price p_f^u:

$$\sum_{k=1}^{N} \underbrace{\left[\frac{\partial s_j}{\partial p_k} + \sum_{i=1}^{N}\left(\Omega_d(i,j)\frac{\partial^2 s_i}{\partial p_j \partial p_k}\left(p_i - p_i^u - c_i^d\right)\right) + \Omega_d(k,j)\frac{\partial s_k}{\partial p_j}\right]}_{g(j,k)} dp_k$$
$$+ \underbrace{\left[(I(j,f)\Omega_u(j,f) - (II(j,f))\Omega_d(f,j))\frac{\partial s_f}{\partial p_j}\right]}_{h(j,f)} dp_f^u = 0$$

where $I(j,f)$ is the indicator which is 1 if flight f is a codeshare product and not ticketed by carrier j (carrier j will be earning upstream operating profit) and $II(j,f)$ is the indicator which is 1 if flight f is a codeshare product and ticketed by carrier j

In vector form, stacking all downstream equation j together, G denotes the matrix with element $g(j,k)$ and H_f denotes the vector with element $h(j,f)$. We get $Gdp - H_f dp_f^u = 0$. Then the derivative of all downstream prices with respect to the upstream price p_f^u is obtained:

$$\frac{dp}{dp_f^u} = G^{-1}H_f \tag{3}$$

Method to compute the margin: Now downstream markup can not be computed from Downstream FOC directly because this depend on how upstream carrier set the upstream margin according to equation (1). Also from equation (2), upstream margin is connected to the price pass-through effect and the downstream margin. (2) is nonlinear in the downstream margin. From (3), price passthrough effect depends on the dowsntream margin.

1. Take a initial downstream markup $m^{d(0)}$, compute the price pass-through effect $\frac{\partial p_m}{\partial p_m^u}^{(0)}$ from

equation (3).

2. Take $\frac{\partial p_m}{\partial p_m^u}^{(0)}$ as given, compute downstream $m^{d(1)}$ and upstream markup $m^{u(1)}$ from (1) and (2)

3. Repeat step 1 with updated downstream markup $m^{d(1)}$ until markups converge.[1]

[1] Diff threshold is set at $1 * 10^{-10}$