International Journal of Industrial Organization 86 (2023) 102893

Contents lists available at ScienceDirect

International Journal of Industrial Organization

journal homepage: www.elsevier.com/locate/ijio

A sufficient statistics approach for welfare analysis of oligopolistic third-degree price discrimination ☆

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ARTICLE INFO

Article history:
Received 6 December 2021
Revised 9 November 2022
Accepted 13 November 2022
Available online 28 November 2022

JEL classification:
D43
L11
L13

Keywords:
Third-degree price discrimination
Oligopoly
Sufficient statistics

ABSTRACT

This paper proposes a sufficient statistics approach to studying the welfare effects of third-degree price discrimination in differentiated oligopoly. Specifically, our sufficient conditions for price discrimination to increase or decrease social welfare simply entail a cross-market comparison of multiplications of such sufficient statistics as *pass-through, conduct*, and *profit margin* that are functions of first-order and second-order elasticities of the firm's demand. Notably, these results are derived under a general class of market demand, and can be readily extended to accommodate heterogeneous firms. These features suggest that our approach has potential for conducting welfare analysis without a full specification of an oligopoly model.

1. Introduction

This paper explores the welfare effects of third-degree price discrimination in oligopoly. Specifically, we consider a fairly general setting, and present sufficient conditions under which oligopolistic third-degree price discrimination increases or decreases Marshallian social welfare (i.e., the sum of consumer and producer surplus) when all discriminatory markets are served even in the absence of price discrimination. To do this task, we employ the *sufficient statistics approach* as a unifying methodology: a technique often used in public economics (Chetty, 2009; Kleven, 2021; Adachi and Fabinger, 2022) as well as macroeconomics (Barnichon and Mesters, 2022). Our analysis is mainly developed under firm symmetry; however, it can readily be extended to accommodate heterogeneous firms (see Online Appendix C). Moreover, our analysis permits a moderate degree of cost differences across separate markets.

Under third-degree price discrimination, consumers are segmented into separate markets and charged different unit prices in accordance with their identifiable characteristics (e.g., age, occupation, location, or time of purchase). In contrast, all consumers are charged the same price if third-degree price discrimination is not practiced (i.e., "uniform pricing"). With-

☆ I am grateful to Giacomo Calzolari and Armin Schmutzler (Editors-in-charge) as well as an anonymous referee for insightful comments and suggestions. I also thank Iñaki Aguirre, Jacob Burgdorf, Susumu Cato, Yong Chao, Jacob Gramlich, Makoto Hanazono, Hiroaki Ino, Noritaka Kudoh, Toshifumi Kuroda, Jeanine Miklós-Thal, Ignacio Palacios-Huerta, Dan Sasaki, Susumu Sato, Nicolas Schutz, Shohei Tamura, Tsuyoshi Toshimitsu, Yusuke Zennyo, and conference and seminar participants at numerous places for helpful comments and discussions. An earlier version of the paper was circulated under the title of "Output and Welfare Implications of Oligopolistic Third-Degree Price Discrimination" as a coauthored paper with Michal Fabinger, who contributed to the initial stage of the paper but has unfortunately retreated afterwards. I acknowledge a Grant-in-Aid for Scientific Research (C) (15K03425, 18K01567, 21K01440) from the Japan Society for the Promotion of Science. All remaining errors are my own.

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https://doi.org/10.1016/j.ijindorg.2022.102893
0167-7187/

out loss of generality, the case of two markets can be considered to understand how price discrimination might change welfare in each market. If all firms are symmetric, the prevailing equilibrium price is common in either market whether price discrimination or uniform pricing is implemented. In this situation, if a discriminatory price becomes greater than the uniform price in one market, and the unit price decreases in the other market, the former market is traditionally called a "strong" market (s), and the latter a "weak" market (w) in the literature since Robinson (1933).[1] More formally, this situation is expressed by $p_s^* > \overline{p} > p_w^*$, where p_s^* and p_w^* are the equilibrium prices under price discrimination in the strong and weak markets, respectively, and $\overline{p}$ is the equilibrium uniform price.[2] Given such a price change, price discrimination increases output and social welfare in the weak market, but decreases them in the strong market. What are the overall effects of the price change?

In the analysis below, we follow Leontief (1940), Silberberg (1970), Schmalensee (1981), Holmes (1989), and Aguirre et al. (2010) to add the constraint $p_s - p_w = t$, where $t \geq 0$ is interpreted as an artificial constraint on the profit maximization problem for oligopolistic firms under symmetry. Then, the regime change, which is discrete in its nature, is now measured by t and is continuously connected between $t = 0$ as uniform pricing and $t^* \equiv p_s^* - p_w^*$ as price discrimination in equilibrium. This formulation enables us to describe social welfare as a function of t, $W(t)$, and characterize $W'(t)$ in terms of economic concepts based on elasticity terms of market demand. In this way, whether social welfare improves or deteriorates by this global change of the regime can be determined. This methodology shares the central idea of the sufficient statistics approach where welfare consequences of policy changes are derived "in terms of estimable elasticities" (Kleven, 2021, p. 516). One benefit of focusing on sufficient statistics "rather than deep primitives" (Chetty, 2009, p. 452) in conducting welfare analysis is that one can focus on the deeper *structure* that is "robust across a broad class of underlying models" (Kleven, 2021, p. 535) without a particular specification of market demand. If we instead start with a specific class of demand, it remains unclear to what extent the welfare analysis is valid under another class of market demand.[3]

Our sufficient conditions for oligopolistic price discrimination to increase or decrease social welfare are provided by means of a cross-market comparison of the multiplications of three of the following economic concepts: (i) *profit margin*, which is the difference between price and marginal cost ($\mu \geq 0$); (ii) *pass-through*, i.e., how the price responds to a small change in marginal cost ($\rho > 0$); and (iii) *conduct*, which measures the degree of market monopolization ($\theta \in [0, 1]$). These three sufficient statistics are determined by the following two first-order and two second-order elasticities: (a) the own price elasticity of the firm's demand (ϵ^{own}), (b) the cross price elasticity of the firm's demand (ϵ^{cross}), (c) the curvature of the firm's demand (α^{own}), and (d) the elasticity of the cross-price effect of the firm's demand (α^{cross}).

Specifically, this paper demonstrates that the product of all three concepts, $\mu\theta\rho$, provides the sufficient condition for the change in welfare. As explained in Section 3.2, the product of conduct and pass-through evaluated at the discriminatory prices $\theta_m^*\rho_m^*$, $m = s, w$, in Fig. 1(A) is interpreted as *quantity pass-through*, measuring how output in each individual market changes in response to a marginal change in price. To evaluate a marginal change in welfare, profit margin μ_m^* should be considered because it measures the welfare gain or loss that results from a marginal change in quantity under imperfect competition in which the price exceeds marginal cost. In this way, the welfare implications can be obtained by means of a cross-market comparison of the quantity change multiplied by the profit margin.

Existing literature on third-degree price discrimination has a centennial tradition, pioneered by Pigou (1920) and Robinson (1933), with their main focus on whether price discrimination increases or decreases social welfare (see Varian, 1989; Armstrong, 2006; Armstrong, 2008; and Stole, 2007 for comprehensive surveys of this literature). Among others, Schmalensee (1981) and Aguirre et al. (hereafter, ACV) (2010) study how *demand curvatures* relate to output and welfare effects. Third-degree price discrimination necessarily entails allocative inefficiency because some consumers exist who have the same marginal utility but face different prices simply because they belong to different markets. Thus, for third-degree price discrimination to increase social welfare, it must sufficiently expand aggregate output to offset such misallocation across markets. Schmalensee (1981) shows that an increase in aggregate output is a necessary condition for third-degree price discrimination to increase social welfare—a conclusion that is generalized by Varian (1985) and Schwartz (1990)—and ACV (2010) identify a sufficient condition for price discrimination to raise social welfare: inverse demand in the weak market is more convex than that in the strong market at the discriminatory prices. Fig. 1(B) provides a graphical illustration of ACV's (2010) argument: if uniform pricing is implemented instead, welfare loss in the weak market due to the output

[1] To be precise, Robinson (1933, p. 189) originally states "stronger" and "weaker" markets.

[2] In this paper, price discrimination is present when $p_s > p_w$, i.e., when prices between markets are not uniform. As Clerides (2004, p. 402) states, once cost differentials are allowed, "there is no single, widely accepted definition of price discrimination." To understand this, consider symmetric firms and let mc_s and mc_w be the marginal cost at equilibrium output in markets s and w, respectively (they do not necessarily have to be constants for any output levels). Then, two alternative definitions can be considered. One is the *margin* definition: price discrimination occurs when $p_s - mc_s > p_w - mc_w$. The other one is the *markup* definition as per Stigler (1987): price discrimination occurs when $p_s/mc_s > p_w/mc_w$. Our simpler definition is aligned with the former definition, and employed for its tractability and connectivity to the existing literature on third-degree price discrimination with no cost differentials. Moreover, our definition of price discrimination coincides with what Chen and Schwartz (2015) and Chen et al. (2021) call "differential pricing." As long as cost differentials are sufficiently small, these differences will not significantly alter the results because if $mc_s = mc_w$, these three definitions are equivalent.

[3] One may criticize that sufficient statistics are only endogenous variables by holding that a sufficient condition is meaningful only when it consists of exogenous parameters. However, in equilibrium, our sufficient conditions are functions of exogenous parameters for the same reason that in equilibrium, endogenous variables are functions of exogenous variables, as demonstrated in Section 4. However, deep parameters themselves do not always allow economic interpretations in a direct manner; for example, in the case of linear demand, the slope coefficient is not directly to related to demand elasticity. In contrast, sufficient statistics such as elasticities almost always have economic interpretations. This *is* the benefit from the sufficient statistics approach because welfare analysis can be conducted based on economic concepts one-level higher that underlie a plausible class of model specification.

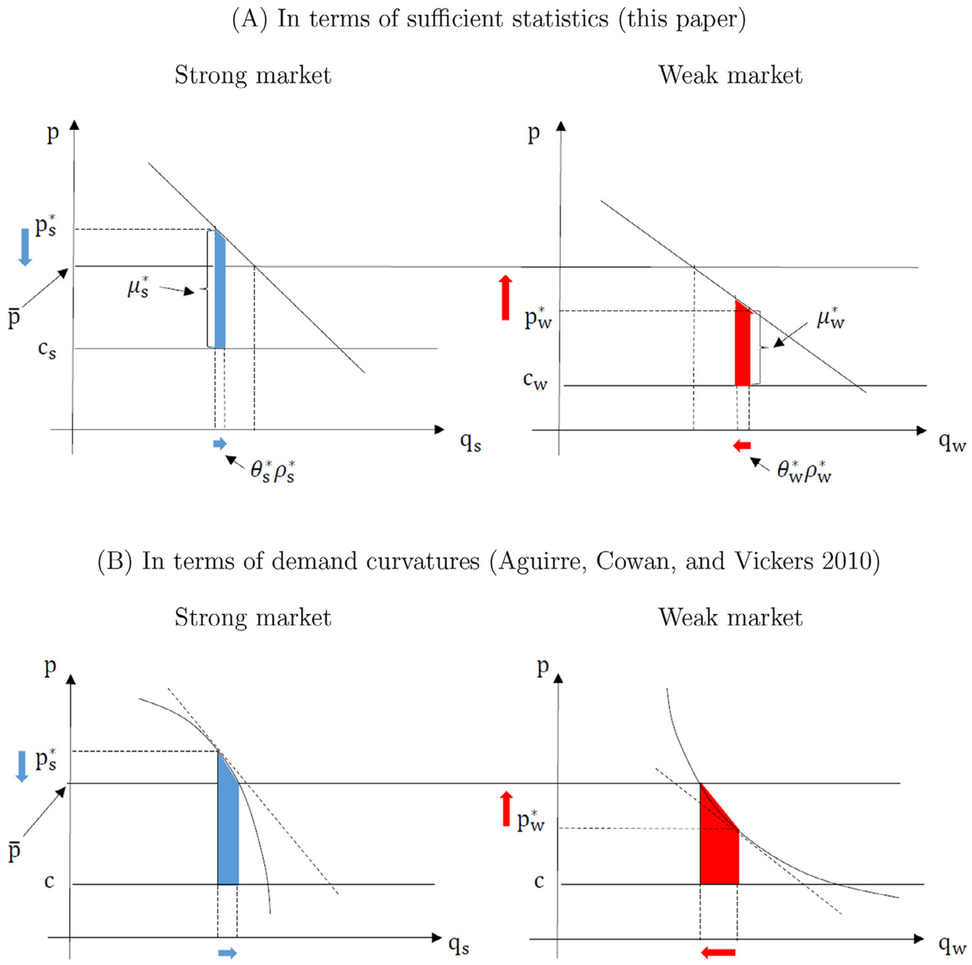

Fig. 1. A graphical illustration of welfare changes in strong and weak markets.

reduction that has arisen under price discrimination is sufficiently large (the right panel) as compared to the welfare gain in the strong market (the left panel), provided that the inverse demand in the weak market is sufficiently convex as compared to that in the strong market.

However, these studies are limited to *monopolistic* third-degree discrimination: to date, "there are *virtually no predictions* as to how discrimination impacts welfare" (Hendel and Nevo, 2013, p. 2723; emphasis added) when *oligopolistic* competition is considered. For example, Holmes (1989) employs the same technique used by Schmalensee (1981) and ACV (2010) to examine the output effects of third-degree price discrimination in a symmetric oligopoly (see Section 3 for details). However, Holmes (1989) provides no welfare predictions (see also Dastidar, 2006).[4] In this paper, we contribute to the literature by providing fairly general conditions regarding whether oligopolistic price discrimination increases or decreases social welfare.[5]

Notably, our analysis does not necessitate the assumption of no cost differentials between discriminatory markets. In almost all theoretical studies on price discrimination, this assumption is made mainly to focus on demand differences. However, in many real-world cases of price discrimination, cost differentials are quite often observed, such as in the typical example of freight charges across regional markets with different transportation and storage costs (Phlips, 1983, pp. 5–7). In the narrowest definition of price discrimination, this might not be considered price discrimination because they can be

[4] In a similar vein, Armstrong and Vickers (2001) consider a model of symmetric duopoly with product differentiation à la Hotelling (1929), and study the consequences of third-degree price discrimination in the competitive limit around zero transportation costs wherein the equilibrium prices are almost equal to marginal cost. Under this setting, Armstrong and Vickers (2001) show that price discrimination decreases social welfare if the weak market has a lower value of price elasticity of demand Adachi and Matsushima (2014) also derive a similar result by assuming linear demand in a standard model of symmetrically differentiated duopoly). Our paper aims to fill the gap between monopoly, such as in Schmalensee (1981) and ACV (2010), and Armstrong and Vickers' (2001) competitive limit with respect to welfare implications. I thank Susumu Sato for suggesting this interpretation.

[5] Rhodes and Zhou (2022) incorporate oligopolistic competition into a model of personalized pricing or first-degree price discrimination as the limit case of third-degree price discrimination.

regarded as distinct products. However, airlines can be arguably motivated to offer different types of seats because they seek to exploit heterogeneity among consumers. In light of this observation, this study permits a moderate amount of cost differentials to exist across discriminatory markets. Specifically, our analysis below needs not employ an explicit assumption regarding constant marginal costs in strong and weak market, c_s and c_w, as long as the second-order conditions for profit maximization are satisfied and a sufficiently large discrepancy between c_s and c_w does not change the order of discriminatory prices from the one with no cost differentials.[6] Fig. 1 also reflects this generalization: in (A), c_s and c_w are different, whereas in (B), marginal cost, c, is common for both strong and weak markets.

In a closely related study, Chen et al. (2021) extend Chen and Schwartz' (2015) analysis of monopoly to investigate the welfare effects of cost-based price discrimination ("differential pricing") in oligopoly.[7] In their setting, demand in market $m = 1, 2, \ldots, M$ with N symmetric firms is given by (using the notation of Chen et al. (2021)) $\lambda_m \cdot \widetilde{D}(\mathbf{p}_m)$, where $\mathbf{p}_m = (p_{m1}, p_{m2}, \ldots, p_{mN})$ is the price vector in market m and $\lambda_m \in (0, 1)$ is the weight for market m satisfying $\sum_{m=1}^{M} \lambda_m = 1$. As such, market heterogeneity arises only from the supply side—firms' marginal costs are different across markets—because own and cross elasticities are *identical across markets* that result from the common demand component, $\widetilde{D}(\cdot)$. Under this setting, Chen et al. (2021) are able to identify demand conditions to determine aggregate social welfare—expressed in terms of the weights $(\lambda_1, \lambda_2, \ldots, \lambda_M)$—is concave or convex as a function of price: because the uniform price lies in between the discriminatory prices, social welfare is higher (corr. lower) under differential pricing if the welfare function is convex (corr. concave).

However, aggregate social welfare is no longer expressed in this simple manner once *demand heterogeneity across markets* is allowed. A typical situation when cost-based price discrimination can be at issue comes from the universal service requirement and fairness concerns (Okada, 2014; Geruso, 2017; DellaVigna and Gentzkow, 2019). In these cases, markets segmented by, e.g., geographical areas would differ in terms of price elasticities of market demand, and if so, the methodology of Chen et al. (2021) is no longer valid. In contrast, our analysis provides welfare implications more directly. As pointed out by Chen and Schwartz (2015, p. 103), our methodology "neither implies nor is implied" by the conditions in the analysis of Chen and Schwartz (2015) for monopoly and Chen et al. (2021) for oligopoly. In this sense, the analysis by Chen et al. (2021) and mine are not mutually exclusive but are complementary.

Our study is also in line with Mrázová and Neary (2017) who show the usefulness of *demand manifold*—the relationship between demand elasticity and convexity which is not ascribed to a function or a correspondence—in comparative statics by suggesting the linkage between these first- and second-order elasticities and sufficient statistics such as markup and pass-through as shown in an empirical study by De Loecker et al. (2016).[8] Mrázová and Neary (2017) point out that one of the advantages of working with the demand manifold instead of the demand function per se is that it is clearer to understand results from comparative statics and counterfactual experiments because demand elasticity and curvature are more closely related to them than demand primitives themselves.[9] However, Mrázová and Neary (2017) mainly focus on perfect and monopolistic competition: when firm heterogeneity is taken into account, only cost/productivity heterogeneity à la Melitz (2003) is considered. In other words, *neither* ϵ^{cross} *nor* α^{cross} *appears* in Mrázová and Neary's (2017) analysis because product differentiation in a strategic context is not taken into account. Therefore, Mrázová and Neary (2017) are only able to focus on two parameters, ϵ^{own} and α^{own}. While we do not make use of their method directly, we explicitly consider imperfect competition based on product differentiation: further research would be promising to investigate how Mrázová and Neary's (2017) methodology can be more utilized for welfare analysis of imperfectly competitive behavior with the use of sufficient statistics.

Our methodology has the following policy implications. Admittedly, our welfare predictions are not "perfect" in that they are stated only as *sufficient* conditions that justify the current regime: for example, the first part of Proposition 1 below provides one sufficient condition for when price discrimination is justified from a standpoint of social welfare. Hence, one may still miss some other parametric cases of market demand that can also support price discrimination simply because our sufficient condition does not hold. However, our results enable one to conclude that once our sufficient condition holds, a regime change that bans price discrimination*definitely* decreases social welfare. In this sense, our sufficient conditions are "conservative" but "secure" in line with the "*in dubio pro reo*" principle behind juridical decisions: it is important to prevent the "innocent" from being mistakenly judged as "guilty".

The remainder of this paper is organized as follows. Section 2 presents our base model of oligopolistic pricing with symmetric firms and constant marginal costs. Then, we derive the sufficient statistics implications of welfare effects of price discrimination in Section 3. Subsequently, Section 4 provide parametric examples of three representative classes of market demand with product differentiation that are often employed in applies studies: linear, CES (constant elasticity of substi-

[6] In the context of reduced-fare parking as a form of third-degree price discrimination with cost differentials, Flores and Kalashnikov (2017) characterize a sufficient condition for free parking (drivers receive a price discount in the form of complementary parking while pedestrians do not) to be welfare improving.

[7] See also Galera and Zaratiegui (2006) and Bertoletti (2009) as studies of conditions under which price discrimination increases social welfare when cost differentials between markets are allowed.

[8] In our context, (i) *profit margin* is determined by the firm-level price elasticity (or the own price elasticity), (ii) *pass-through* is determined mainly by the demand curvature, and (iii) *conduct* is determined by the ratio of the industry-level elasticity to the firm-level elasticity. See Expression (17) below for the case of price discrimination when market-wise elasticities are defined.

[9] Beggs (2021) derives a necessary and sufficient condition for two demand functions to have the same demand manifold: one is derived from the other by a change in market size and a change in quality.

tution), and multinomial logit with outside option. Section 5 concludes.[10] Implications of aggregate output and consumer surplus are provided in Online Appendix B, and we argue in Online Appendix C that our methodology can be readily extended when firm heterogeneity is introduced.

2. The model of oligopolistic pricing

For ease of exposition, this section follows Holmes (1989) and ACV (2010) to consider the case of two symmetric firms and two separate markets or consumer groups (hereafter, simply called "markets"). It is straightforward to extend the following analysis to the case of more than two symmetric firms and more than two separate markets: in Section 4 below, where we consider parametric examples of market demand, the number of symmetric firms is assumed to be $N \geq 2$.[11] As explained in Introduction, we call one market s (strong), where the equilibrium discriminatory price is higher than the equilibrium uniform price, and the other w (weak), where the opposite is true.

Two firms, A and B, have an identical cost structure in each market. Specifically, each firm has an identical cost function, $c_m(q_{im})$, in market $m = s, w$, where q_{im} is firm i's output ($i = A, B$). For simplicity of exposition, we assume, with a slight abuse of notation, that firms have a constant marginal cost in each market m, $c_m \geq 0$; here, c_s and c_w can be different. However, as mentioned again in Section 2.3 below, it is assumed that the strong market either has a higher marginal cost or only slightly lower marginal cost so that its price still increases with price discrimination. In this sense, this paper does not consider the role of cost differences in differential pricing (see also Footnote 2 above).

2.1. Consumers

In market $m = s, w$, given firms A and B's prices p_{Am} and p_{Bm}, the representative consumer purchases $x_{Am} > 0$ and $x_{Bm} > 0$ to maximize her net utility (i.e., surplus)

$$U_m(\mathbf{x}_m) - p_{Am}x_{Am} - p_{Bm}x_{Bm},$$

where $\mathbf{x}_m = (x_{Am}, x_{Bm})$, U_m is three-times continuously differentiable, $\partial U_m/\partial x_{im} > 0$ and $\partial^2 U_m/\partial x_{im}^2 < 0$ for firm $i = A, B$, and $\partial^2 U_m/(\partial x_{Am}\partial x_{Bm}) < 0$ (i.e., firms A and B produce substitutable products). Here, it is assumed that the representative consumer has a large amount of income so that this maximization problem is valid.

Inverse demands in market m, $p_{im} = P_{im}(x_{im}, x_{-i,m})$, are derived from the representative consumer's utility maximization (where $-i = A, B$, $-i \neq i$): $\partial U_m(x_{im}, x_{-i,m})/\partial x_{im} - p_{im} = 0$, which also implicitly defines firm i's *direct demand* in market m, $x_{im} = x_{im}(p_{im}, p_{-i,m})$. We assume that $x_{im}(\cdot)$ is twice continuously differentiable. Because of the assumptions regarding the utility, firm i's demand in market m decreases as its own price increases ($\partial x_{im}/\partial p_{im} < 0$), and it rises as the rival's price increases ($\partial x_{im}/\partial p_{-i,m} > 0$; the firms' products are substitutes).

We also assume that from a viewpoint of consumers, firms are symmetric: $U_m(x', x'') = U_m(x'', x')$ for any $x' > 0$ and $x'' > 0$. Then, the firms' demands in market m are also symmetric: $x_{Am}(p', p'') = x_{Bm}(p', p'')$ for any $p' > 0$ and $p'' > 0$. Because the firms' technologies are also identical, we focus on symmetric Nash equilibrium until we allow firm heterogeneity in Section 5.[12]

We define the demand in symmetric pricing by $q_m(p) \equiv x_{Am}(p, p)$. Note here that

$$q_m'(p) = \underbrace{\frac{\partial x_{Am}}{\partial p_A}(p_A, p)\bigg|_{p_A = p}}_{<0 \text{ (ACV's } q_m')} + \underbrace{\frac{\partial x_{Am}}{\partial p_B}(p, p_B)\bigg|_{p_B = p}}_{>0 \text{ (strategic)}}. \tag{1}$$

Thus, for $q_m'(p)$ to be negative, we assume that $|\partial x_{Am}(p, p)/\partial p_A| > \partial x_{Am}(p, p)/\partial p_B$. Note also that by symmetry, the following relationship also holds (this corresponds to Holmes' (1989) Eq. (4)):

$$\underbrace{\frac{\partial x_{Am}}{\partial p_A}(p, p)}_{\text{own}} = \underbrace{q_m'(p)}_{\text{industry}} - \underbrace{\frac{\partial x_{Bm}}{\partial p_A}(p, p)}_{\text{strategic}}.$$

This exchangeability is key in Holmes' (1989) derivation below. Intuitively, each firm, under symmetry, treats the industry demand $q_m(p)$ as if it is its own demand. Thus, how a firm's pricing behavior affects its own demand as an industry demand

[10] In this paper, the only policy instrument is an enforcement of uniform pricing. Cowan (2018) studies a model of monopoly to consider a more moderate instrument by which a government regulates the monopolist's profit margins or price-marginal cost ratios across different markets.

[11] See Online Appendix A for the case of a general number of markets. We assume that resale between markets is impossible to prevent consumers in the strong market from being better off buying the good at a lower price in the week market (see Boik, 2017 for an empirical analysis of oligopolistic third-degree price discrimination when arbitrage may matter).

[12] Here, $\partial^2 x_{im}(p, p)/\partial p_i^2$ can be positive, zero or negative. Following Dastidar's (2006, p.234) Assumption 2 (iv), we assume that

$$\frac{\partial^2 x_{im}}{\partial p_i^2}(p, p) + \frac{\partial^2 x_{im}}{\partial p_i \partial p_{-i}}(p, p) \leq 0.$$

has the following two effects: a small decrease in p_A by firm A by deviating from the industry price p (i) not only raises its own demand by $\partial x_{Am}/\partial p_A$ as the *residual* monopolist (*industry effects*), (ii) firm A can now also obtain some of the consumers originally attached to firm B, and this amount is $\partial x_{Bm}/\partial p_A$ (*strategic effects*).

2.1.1. The (first-order) price elasticities of market demand

Under symmetric pricing, we are able to define, following Holmes (1989, p. 245), the *price elasticity of the industry's demand* by

$$\epsilon_m^I(p) \equiv -\frac{pq_m'(p)}{q_m(p)} > 0. \tag{2}$$

As Weyl and Fabinger (2013, p. 542) state, this should not "be confused with the elasticity of the residual demand that any of the firms faces."[13] Similarly, the *own* and the *cross price elasticities of the firm's demand* are defined by

$$\epsilon_m^{own}(p) \equiv -\frac{p}{q_m(p)}\frac{\partial x_{Am}}{\partial p_A}(p, p) > 0$$

and by

$$\epsilon_m^{cross}(p) \equiv \frac{p}{q_m(p)}\frac{\partial x_{Bm}}{\partial p_A}(p, p) > 0,$$

respectively. Then, Holmes (1989) shows that under symmetric pricing,

$$\epsilon_m^{own}(p) = \epsilon_m^I(p) + \epsilon_m^{cross}(p) \tag{3}$$

holds.[14] This implies that the own-price elasticity must be equal to or greater than the industry's elasticity and greater than the cross-price elasticity (i.e., $\epsilon_m^{own}(p) \geq \epsilon_m^I(p)$ and $\epsilon_m^{own}(p) > \epsilon_m^{cross}(p)$).

2.1.2. The second-order price elasticities of market demand

We also consider two second-order elasticities. First, the *curvature of the firm's (direct) demand* in market m is defined by

$$\alpha_m^{own}(p) \equiv -\frac{p}{\partial x_{Am}(p, p)/\partial p_A}\frac{\partial^2 x_{Am}}{\partial p_A^2}(p, p),$$

which measures the convexity/concavity of the firm's direct demand, and corresponds to $\alpha_m(p)$ in Aguirre et al. (2010, p. 1603). Second, we define the *elasticity of the cross-price effect* of the firm's direct demand in market m by

$$\alpha_m^{cross}(p) \equiv -\frac{p}{\partial x_{Am}(p, p)/\partial p_A}\frac{\partial^2 x_{Am}}{\partial p_B \partial p_A}(p, p),$$

which never appears in monopoly. Here, α_m^{own} and α_m^{cross} are positive (resp. negative) if and only if $\partial^2 x_{Am}/\partial p_A^2$ and $\partial^2 x_{Am}/(\partial p_B \partial p_A)$ are positive (resp. negative), respectively. Note also that the sign of α_m^{own} indicates whether the firm's own part of the demand slope under symmetric pricing given the rival's price p, $\partial x_{Am}(\cdot, p)/\partial p_A$, is *convex* ($\alpha_m^{own}$ is positive) or *concave* (α_m^{own} is negative). On the other hand, α_m^{cross} measures to what extent the rival's price level matters to how many of the firm's customers switch to the rival's product when the firm raises its own price ($\partial x_{Am}/\partial p_A$). Thus, a large α_m^{cross} implies that $\partial x_{Am}/\partial p_A$ is very responsive to a change in p_B, and vice versa.

2.2. Firms

Firm i's profit in market m is written as

$$\pi_{im}(\mathbf{p}_m) = (p_{im} - c_m)x_{im}(\mathbf{p}_m), \tag{4}$$

where $\mathbf{p}_m = (p_{im}, p_{-i,m})$. As in Dastidar's (2006, pp. 235–6) Assumptions 3 and 4, for the existence and the global uniqueness of pricing equilibrium under either uniform pricing or price discrimination, we assume that for each firm $i = A, B$, $\partial^2 \pi_{im}/\partial p_{im}^2 < 0$, $\partial^2 \pi_{im}/(\partial p_{im}\partial p_{-i,m}) > 0$, and

$$-\frac{\partial^2 \pi_{im}/(\partial p_{im}\partial p_{-i,m})}{\partial^2 \pi_{im}/\partial p_{im}^2} < 1$$

(see Dastidar's (2006) Lemmas 1 and 2 for the existence and the uniqueness).

[13] Note that ϵ_m^I here is conceptually identical to η in Aguirre et al. (2010, p. 1603) and ϵ_D in Weyl and Fabinger (2013, p. 542).

[14] In general, when there are $N \geq 2$ symmetric firms as in Section 4 below, this identity still holds if the cross price elasticity is defined by $\epsilon_m^{cross}(p) \equiv (N-1)[p/q_m(p)][\partial x_{Bm}(p, p)/\partial p_A]$.

We then define the first-order partial derivative of the profit in market m, evaluated at a symmetric price p, by

$$\begin{aligned}\partial_p \pi_m(p) &\equiv \left.\frac{\partial \pi_{im}(p_{im}, p_{-i,m})}{\partial p_{im}}\right|_{p_{im}=p_{-i,m}=p} \\ &= q_m(p) + (p - c_m)\frac{\partial x_{Am}}{\partial p_A}(p, p). \end{aligned} \tag{5}$$

Under symmetric discriminatory pricing, p_m^* satisfies $\partial_p \pi_m(p_m^*) = 0$ for $m = s, w$, whereas under symmetric uniform pricing, $\overline{p}$ is a (unique) solution of $\partial_p \pi_s(\overline{p}) + \partial_p \pi_w(\overline{p}) = 0$. Throughout this paper, we consider the situation where the weak market is open under uniform pricing (for which $q_w(p_s^*) > 0$ is a sufficient condition).[15]

2.2.1. The second-order derivative of the profit function under symmetry

As a measure of concavity of the market-wise profit function in symmetric equilibrium, we define:

$$\begin{aligned}\pi_m''(p) &\equiv q_m'(p) + \frac{\partial x_{Am}}{\partial p_A}(p, p) + (p - c_m)\frac{d}{dp}\left(\frac{\partial x_{Am}}{\partial p_A}(p, p)\right) \\ &= \underbrace{\partial_p^2 \pi_m(p)}_{\text{ACV's}\pi_m''} + \underbrace{\frac{\partial x_{Am}}{\partial p_B}(p, p) + (p - c_m)\frac{\partial^2 x_{Am}}{\partial p_B \partial p_A}(p, p)}_{\text{strategic}},\end{aligned} \tag{6}$$

where $\partial_p^2 \pi_m(p)$ is given by

$$\partial_p^2 \pi_m(p) \equiv \left[2 + (p - c_m)\frac{\partial^2 x_{Am}(p, p)/\partial p_A^2}{\partial x_{Am}(p, p)/\partial p_A}\right]\frac{\partial x_{Am}}{\partial p_A}(p, p), \tag{7}$$

which corresponds to Aguirre et al. (2010, p. 1603) $\pi_m''(p)$. The second and third terms in Eq. (6) arise due to oligopoly. Here, in each m, $\pi_m''(p)$ is assumed to be negative for all $p \geq 0$.[16]

We now argue how π_m'' is expressed in terms of the first- and second-order price elasticities of demand. Note first that Eq. (7) implies that

$$\begin{aligned}\partial_p^2 \pi_m(p) &= -\{2 - \underbrace{\frac{p - c_m}{p}}_{=L_m(p)}\underbrace{[-\frac{p}{\frac{\partial x_{Am}}{\partial p_A}(p, p)}\frac{\partial^2 x_{Am}}{\partial p_A^2}(p, p)]}_{=\alpha_m^{own}(p)}\}\underbrace{[-\frac{p}{q_m(p)}\frac{\partial x_{Am}}{\partial p_A}(p, p)]}_{=\epsilon_m^{own}(p)}\frac{q_m(p)}{p} \\ &= -[2 - L_m(p)\alpha_m^{own}(p)]\epsilon_m^{own}(p)\frac{q_m(p)}{p},\end{aligned}$$

where

$$L_m(p) \equiv \frac{p - c_m}{p} \tag{8}$$

is the *markup rate* (i.e., the Lerner index). Then, from Eq. (6), it can be verified that $\pi_m''(p)$ is expressed in terms of the four elasticities (ϵ_m^{own}, ϵ_m^{cross}, α_m^{own}, and α_m^{cross}) as well as $q_m(p)$ and p itself:

$$\begin{aligned}\pi_m''(p) &= -[2 - L_m(p)\alpha_m^{own}(p)]\epsilon_m^{own}(p)\frac{q_m(p)}{p} + \underbrace{\left[\frac{p}{q_m(p)}\frac{\partial x_{Am}}{\partial p_B}(p, p)\right]}_{=\epsilon_m^{cross}(p)}\frac{q_m(p)}{p} \\ &\quad - \underbrace{\left(\frac{p - c_m}{p}\right)}_{=L_m(p)}\underbrace{\left[-\frac{p}{\frac{\partial x_{Am}}{\partial p_A}(p, p)}\frac{\partial^2 x_{Am}}{\partial p_B \partial p_A}(p, p)\right]}_{=\alpha_m^{cross}(p)}\underbrace{\left[\frac{p}{q_m(p)}\frac{\partial x_{Am}}{\partial p_A}(p, p)\right]}_{=-\epsilon_m^{own}(p)}\frac{q_m(p)}{p} \\ &= -\{[2 - (\alpha_m^{own} + \alpha_m^{cross})L_m]\epsilon_m^{own} - \epsilon_m^{cross}\}\frac{q_m}{p}.\end{aligned} \tag{9}$$

[15] Note that $q_w(\overline{p}) > q_w(p_s^*)$ because $q_w(\cdot)$ is strictly decreasing and $p_s^* > \overline{p}$. Thus, if $q_w(p_s^*) > 0$, then the weak market is open under uniform pricing, i.e., $q_w(\overline{p}) > 0$. Alternatively, we would be able to show that there exist $\underline{c}_s$ and $\overline{c}_s$, $\underline{c}_s < \overline{c}_s$, such that $p_s^* > p_w^*$ and $q_w(\overline{p}) > 0$ for $c_s \in (\underline{c}_s, \overline{c}_s)$ in a similar spirit of Adachi and Matsushima (2014).

[16] ACV's (2010) Appendix A discusses the concavity of the profit function.

2.2.2. Conduct

Now, we are able to define the *conduct parameter*[17] in market m by $\theta_m(p) \equiv 1 - ADR_m(p)$, where $ADR_m(p)$ is the *aggregate diversion ratio* (Shapiro, 1996) in market m, defined by

$$ADR_m(p) \equiv -\frac{\partial x_{Bm}(p,p)/\partial p_A}{\partial x_{Am}(p,p)/\partial p_A} = \frac{\epsilon_m^{cross}(p)}{\epsilon_m^{own}(p)} \geq 0.$$

This concept will be utilized in Section 3. Here, $ADR_m(p)$ measures the intensity of *rivalness*: if $ADR_m(p)$ is close to one, consumers who leave a firm as a response to an increase in its price are mostly switching to its rival's product.[18]

As Weyl and Fabinger (2013, p. 544) argue, $\theta_m(p)$ captures the degree of *industry-level brand loyalty or stickiness*[19] in market m. To see this, note that the conduct parameter is also expressed by

$$\theta_m(p) = \frac{\epsilon_m^I(p)}{\epsilon_m^{own}(p)}, \tag{10}$$

where $\epsilon_m^{own}(p) \geq \epsilon_m^I(p)$. If $\epsilon_m^{own}(p) \to \infty$ as in the case of the price-taking assumption, $\theta_m(p)$ is zero. On the other hand, if $\epsilon_m^{own}(p)$ is equal to $\epsilon_m^I(p)$, that is, the own elasticity is nothing but the industry's elasticity, then it is monopoly and $\theta_m(p) = 1$.[20] Note here that by using this the Holmes decomposition (Eq. (3)), we can rewrite Eq. (9) as

$$\pi_m''(p) = -\{1 + \theta_m - (\alpha_m^{own} + \alpha_m^{cross})L_m\}\frac{q_m\epsilon_m^{own}}{p}. \tag{11}$$

Note also here that $\theta_m(p)$, which ranges between 0 and 1, better captures the brand stickiness than $L_m(p)$ does: the markup rate, L_m, alone is not appropriate to measure the rivalness within market m because it can be the case that p_m is close to c_m (the markup rate is close to zero) simply because the price elasticity of the industry's demand $\epsilon_m^I(p_m)$ is very large, whereas the brand rivalness is so weak that the cross-price elasticity, ϵ_m^{cross}, remains very small (as a result, in total, ϵ_m^{own} is very large, which is actually the reason for the low markup rate). However, if ϵ_m^{cross} is close to ϵ_m^{own} (i.e., almost of all consumers who leave a firm as a response to its price increase are switching to other rivals' products), then θ_m becomes close to zero *irrespective of the value of the markup rate*.

2.3. Equilibrium

The equilibrium discriminatory price in market $m = s, w$, p_m^*, satisfies the following Lerner formula:

$$\epsilon_m^{own}(p_m^*)L_m(p_m^*) = 1. \tag{12}$$

This shows that the discriminatory price in market m approaches to the marginal cost as the own-price elasticity for the firm, $\epsilon_m^{own}(p_m^*)$, becomes large. Because of Holmes' (1989) elasticity formula explained above, $\epsilon_m^{own}(p_m^*)$ can be large (i) when $\epsilon_m^I(p_m^*)$ is very large even if $\epsilon_m^{cross}(p_m^*)$ is close to zero, or (ii) when $\epsilon_m^{cross}(p_m^*)$ is very large even if $\epsilon_m^I(p_m^*)$ is close to zero. Evidently, if there are no cost differentials between markets, which market is strong or weak is solely determined by the difference in the own-price elasticity. As mentioned above, we assume that the marginal cost in the strong market is not sufficiently low to assure that $p_s^* > \overline{p} > p_w^*$ indeed holds.[21,22]

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[17] This term originates from the empirical literature where conduct itself is a target of estimation ("parameter") without an exact specification of strategic interaction (see, e.g., Bresnahan, 1989; Genesove and Mullin, 1998 and Corts, 1999). Here, strategic interaction is explicitly modeled (i.e., price competition), and thus the degree of conduct is solely based on product differentiation with no possibility of collusive pricing.

[18] Alternatively, Weyl and Fabinger (2013, p. 531) and Adachi and Fabinger (2022) define the conduct parameter in a market (which, in our interest in price discrimination, can be indexed by m) by $\theta_m \equiv \epsilon_m^I L_m$ (their mc and ϵ_D are replaced by our c_m and ϵ_m^I, respectively) as the Lerner index adjusted by the elasticity of the *industry's* demand. If the first-order condition is given for each market (that is, if full price discrimination is allowed), then $\theta_m(p)$ defined as in Weyl and Fabinger (2013) coincides with $1 - ADR_m(p)$ because $[(p_m - c_m)/p_m]\epsilon_m^{own} = 1$ and thus

$$\begin{aligned}
\epsilon_m^I(p)L_m(p) &= \frac{1}{\epsilon_m^F(p)}\left(-\frac{p}{q_m(p)}\right)q_m'(p) \\
&= -\frac{q_m(p)}{p}\frac{1}{\partial x_{Am}(p,p)/\partial p_A}\left(-\frac{p}{q_m(p)}\right)\left(\frac{\partial x_{Am}}{\partial p_A}(p,p) + \frac{\partial x_{Am}}{\partial p_B}(p,p)\right) \\
&= \frac{\partial x_{Am}(p,p)/\partial p_A + \partial x_{Bm}(p,p)/\partial p_A}{\partial x_{Am}(p,p)/\partial p_A} \text{ (by symmetry)} \\
&= 1 - ADR_m(p) \equiv \theta_m(p)
\end{aligned}$$

is established. It turns out that this alternative definition is more tractable when firm heterogeneity is introduced in Online Appendix C.

[19] Even if firms' products have the same characteristics across different markets (with no product differentiation), brand loyalty may differ across markets, reflecting the differences in market characteristics (as summarized in demand functions).

[20] Because $[(p_m - c_m)/p_m]\epsilon_m^{own} = 1$ and $\epsilon_m^{own} = \epsilon_m^I + \epsilon_m^{cross}$, it is verified that $\theta_m + [(p_m - c_m)/p_m]\epsilon_m^{cross} = 1$. Thus, as long as the products are substitutes ($\epsilon_m^{cross} > 0$), θ_m is less than one.

[21] See Nahata et al. (1990) for an example of all discriminatory prices being lower than the uniform price with a plausible demand structure under monopoly. In the case of oligopoly, Corts (1998) shows that best-response asymmetry, in which firms differ in ranking strong and weak markets, is necessary for all discriminatory prices to be lower than the uniform price ("all-out price competition"). As long as symmetric firms are considered, this case never arises.

[22] When price discrimination is allowed, each firm may not price discriminate even if it is allowed to do so because it is still able to set a uniform price (i.e., it is not forced to price discriminate). We assume that $\pi_{im}(\cdot, p_{-i,m}^*)$ is strictly increasing (decreasing) at $p_{im} = \overline{p}$ in market $m = s$ (m) and thus firm i

Lastly, let y_m be per-firm (symmetric) *market share* of output in market m, that is, $y_m(p_s, p_w) \equiv q_m(p_m)/[q_s(p_s) + q_w(p_w)]$. Then, the equilibrium uniform price, $\overline{p} \equiv \overline{p}(c_s, c_w)$, satisfies:

$$\sum\nolimits_{m=s,w} \overline{y}_m \epsilon_m^{own}(\overline{p}) L_m(\overline{p}) = 1, \tag{13}$$

where $\overline{y}_m \equiv y_m(\overline{p}(c_s, c_w), \overline{p}(c_s, c_w))$ for $m = s, w$.[23] In this way, the equilibrium level of uniform price is determined by the market-share weighted average of the own price elasticities, whereas the equilibrium level of discriminatory price solely depends on the firm's own price elasticity in that market. In the rest of the paper, the dependence of the equilibrium price is often implicit when there are no confusions. In particular, the superscript star (the upper bar) denotes price discrimination (uniform pricing). For example, we write $(\epsilon_m^I)^* \equiv \epsilon_m^I(p_m^*)$ and $\bar{\epsilon}_m^I \equiv \epsilon_m^I(\overline{p})$ as the industry's elasticities in equilibrium.

3. Welfare analysis

As mentioned in Introduction, we add the constraint $p_s - p_w = t$, where $t \geq 0$, to the firms' profit maximization problem.[24] Then, we express social welfare (as well as aggregate output and consumer surplus) as a function of t in $[0, t^*]$, where $t = 0$ corresponds to uniform pricing, and $t = t^* \equiv p_s^* - p_w^*$ to price discrimination. Note that under this constrained problem of profit maximization, p_w satisfies $\partial_p \pi_s(p_w + t) + \partial_p \pi_w(p_w) = 0$. Thus, we write the solution by $p_w(t)$. Then, we define $p_s(t) \equiv p_w(t) + t$. Applying the implicit function theorem to this equation yields to

$$\begin{cases} p_w'(t) = -\dfrac{1}{1 + \pi_w''/\pi_s''} < 0 \\ p_s'(t) = \dfrac{1}{1 + \pi_s''/\pi_w''} > 0. \end{cases} \tag{14}$$

They show the natural relationship between p_m' and π_m'': as the π_m'' becomes smaller around the equilibrium price, i.e., the profit function in market m becomes flatter at the peak point, the price becomes more responsive in that market. i.e., $|p_m'|$ is larger.

3.1. Preliminaries

We now define the representative consumer's utility in symmetric pricing by $\widetilde{U}_m(q) = U_m(q, q)$. Then, social welfare under symmetric pricing as a function of t is written as

$$\begin{aligned} W(t) &\equiv \widetilde{U}_s(q_s[p_s(t)]) + \widetilde{U}_w(q_w[p_w(t)]) - 2c_s \cdot q_s[p_s(t)] - 2c_w \cdot q_w[p_w(t)] \\ &= \left(\widetilde{U}_s' - 2c_s\right) \cdot q_s' \cdot p_s'(t) + \left(\widetilde{U}_w' - 2c_w\right) \cdot q_w' \cdot p_w'(t), \end{aligned}$$

which implies (using $\widetilde{U}_m' = \partial U_m/\partial q_A + \partial U_m/\partial q_B = 2(\partial U_m/\partial q_A)$ by symmetry) that

$$\begin{aligned} \frac{W'(t)}{2} &= [p_s(t) - c_s] \cdot q_s' \cdot p_s'(t) + [p_w(t) - c_w] \cdot q_w' \cdot p_w'(t) \\ &= \underbrace{\left(-\frac{\pi_s'' \pi_w''}{\pi_s'' + \pi_w''}\right)}_{>0} \{z_w[p_w(t)] - z_s[p_s(t)]\}, \end{aligned}$$

where, as in Aguirre et al. (2010, p. 1605),

$$z_m(p) \equiv \frac{\mu_m(p) q_m'(p)}{\pi_m''(p)}$$

is "the ratio of the marginal effect of a price increase on social welfare to the second derivative of the profit function," and

$$\mu_m(p) \equiv p - c_m \tag{15}$$

has an incentive to deviate from the equilibrium uniform price if the other firm chooses $p_{-i,s}^*$ and $p_{-i,w}^*$, and that $\pi_{im}(\cdot, p_{-i,m}^*)$ attains the global optimum at $p_{im} = p_{im}^*$.

[23] If there are no cost differentials, i.e., $c_s = c_w$ ($\equiv c$), then the formula is simpler:

$$\frac{\overline{p} - c}{\overline{p}} = \frac{1}{\sum_{m=s,w} \overline{y}_m \epsilon_m^{own}(\overline{p})}$$

as shown by Holmes (1989, p. 247): the markup rate (common to all markets) is equal to the inverse of the average of own-price elasticities weighted by the output shares.

[24] Alternatively, Vickers (2020) analyzes properties of social welfare and consumer surplus as a scalar argument to make a comparison between price discrimination and uniform pricing in monopoly. Vickers (2020) especially focuses on the case where quantity elasticity or inverse demand curvature is constant for all markets. See also Cowan (2017) for an analysis of the role of price elasticity and demand curvature in determining the effects of monopolistic third-degree price discrimination.

is the *profit margin* in market m.[25] In contrast to ACV (2010), our q'_m and π''_m have *strategic effects* as Eqs. (1) and (6) above show.

Now, if we assume z_m is *increasing* in p (the increasing ratio condition for welfare; IRCW),[26] then as in ACV's (2010) Lemma, it is verified that if there exists $\widehat{t}$ such that $W'(\widehat{t}) = 0$, then $W''(\widehat{t})/2 < 0$. This is because

$$\frac{W''(t)}{2} = \left(-\frac{\pi_s''\pi_w''}{\pi_s''+\pi_w''}\right)(z'_w p'_w - z'_s p'_s) + (z_w - z_s)\frac{d}{dt}\left(-\frac{\pi_s''\pi_w''}{\pi_s''+\pi_w''}\right),$$

and thus $\text{sign}[W''(\widehat{t})/2] = \text{sign}[z'_s p'_s - z'_w p'_w]$ is negative ($\because z'_w p'_w < 0$, $z'_s p'_s > 0$, and $z_w = z_s$ for $t = \widehat{t}$). Hence, $(1/2)W(t)$ is strictly quasi-concave on $[0, t^*]$, and behaves in either manner:

1. If $W'(0) \le 0$, then $(1/2)W(t)$ is *monotonically decreasing* in r, and as a result $\Delta W/2 \equiv [W(t^*) - W(0)]/2 < 0$; *price discrimination decreases social welfare*.
2. If $W'(0) > 0$, then $(1/2)W(t)$ either
   (a) is *monotonically increasing* (if $W'(t^*) > 0$, this is true), and as a result, $\Delta W/2 > 0$; *price discrimination increases social welfare*.
   (b) *first increases, and then after the reaching the maximum* (where $W'(t) = 0$), *decreases until* $t = t^*$. In this case, *price discrimination may increase or decrease social welfare*.

Below, we focus on the first two cases that provide sufficient conditions for determining the welfare effects of price discrimination. All the three parametric examples in Section 4 below satisfy the IRCW.

3.2. Sufficient conditions using pass-through

To provide a formal statement that permits the graphical interpretation explained in Introduction, we must define the remaining sufficient statistic—*pass-through* in market m by $\rho_m \equiv \partial p_m/\partial c_m$. It is a function of $t \in [0, t^*]$ of the constrained problem considered above. In particular,

$$\rho_m[p_m(t)] = \begin{cases} \dfrac{\partial x_{Am}/\partial p_A}{\pi_s''+\pi_w''} & \text{for } t < t^* \\[2ex] \dfrac{\partial x_{Am}/\partial p_A}{\pi_m''} \ (\equiv \rho_m^*) & \text{for } t = t^* \end{cases}$$

is obtained by applying the implicit function theorem to $\partial_p\pi_s(p_w + t) + \partial_p\pi_w(p_w) = 0$ for $t < t^*$ and $\partial_p\pi_m(p_m) = 0$ for $t = t^*$ (i.e., under price discrimination).

From Eq. (9) it is observed that ρ_m^* as defined above can be expressed as

$$\rho_m^* = \frac{\dfrac{\partial x_{Am}}{\partial p_A}}{\left\{2 - [(\alpha_m^{own})^* + (\alpha_m^{cross})^*](L_m)^* - \dfrac{(\epsilon_m^{cross})^*}{(\epsilon_m^{own})^*}\right\}\dfrac{\partial x_{Am}}{\partial p_A}} = \frac{1}{2 - \dfrac{(\epsilon_m^{cross})^* + (\alpha_m^{own})^* + (\alpha_m^{cross})^*}{(\epsilon_m^{own})^*}}$$

because $(L_m)^* = 1/(\epsilon_m^{own})^*$. Note here that in the case of monopoly (i.e., $(\epsilon_m^{cross})^* = 0$ and $(\alpha_m^{cross})^* = 0$),

$$\rho_m^* = \frac{1}{2 - \dfrac{(\alpha_m^{own})^*}{(\epsilon_m^{own})^*}} \tag{16}$$

[25] Here, $\mu_m(p)q'_m(p)$ can be interpreted as the marginal effect of a price increase on social welfare in market m because:

$$\frac{d[\overbrace{\tfrac{1}{2}U_m[q_m(p)] - c_m q_m(p)}^{\text{per-firm (normalized)}}]}{dp} = \mu_m(p)q'_m(p).$$

[26] Note that

$$z'_m(p) = \frac{[\mu_m(p)q''_m(p) + q'_m(p)]\pi''_m(p) - \mu_m(p)q'_m(p)\pi'''_m(p)}{[\pi''_m(p)]^2}$$

and thus, the IRCW is equivalent to

$$[\mu_m(p)q''_m(p) + q'_m(p)]\pi''_m(p) > \mu_m(p)q'_m(p)\pi'''_m(p).$$

Appendix B of ACV (2010) discusses sufficient conditions for the IRCW (IRC in their abbreviation) to hold in the case of monopoly.

and $(\alpha_m^{own})^*/(\epsilon_m^{own})^*$ corresponds to the Aguirre et al. (2010, p. 1603) curvature of the inverse demand, σ_m^*.

Now, using conduct, profit margin, and pass-through, we obtain the following sufficient conditions for price discrimination to increase or decrease social welfare.

Proposition 1. *Given the IRCW, price discrimination increases social welfare if*

$$\mu_s^*\theta_s^*\rho_s^* < \mu_w^*\theta_w^*\rho_w^*$$

holds, and it decreases social welfare if

$$\frac{\overline{\mu}_s\overline{\theta}_s\overline{\rho}_s}{\bar{\pi}_s''} \geq \frac{\overline{\mu}_w\overline{\theta}_w\overline{\rho}_w}{\bar{\pi}_w''}$$

holds, where

$$\bar{\pi}_m'' \equiv \pi_m''(\overline{p}) = -\{[2 - (\overline{\alpha}_m^{own} + \bar{\alpha}_m^{cross})\overline{L}_m]\bar{\epsilon}_m^{own} - \bar{\epsilon}_m^{cross}\}\frac{\overline{q}_m}{\overline{p}},$$

for $m = s, w$.

Proof. See Appendix A. □

In plain words, if either (i) *conduct* (θ), (ii) *profit margin* (μ), or (iii) *pass-through* (ρ) is sufficiently *small* in the *strong* market, then social welfare is likely to be higher under price discrimination. In particular, if these three measures are calculated (or estimated) in each separate market, then it would assist one to judge whether price discrimination is desirable from a society's viewpoint. Specifically, suppose that price discrimination is being conducted. Then, to evaluate it from a viewpoint of social welfare, one only needs the local information: first, θ_m^*, μ_m^* and ρ_m^* for each $m = s, w$, are computed, and if the sufficient condition above is satisfied, then the ongoing price discrimination is justified. In addition, to compute θ_m^*, μ_m^* and ρ_m^* in equilibrium, *information on marginal cost is unnecessary*: once a specific form of demand function, $q_{im} = x_{im}(p_{im}, p_{-i,m})$, is provided (and if the IRCW is satisfied), then the three variables are computed in the following manner:[27]

$$\begin{cases} \theta_m^* = 1 - \dfrac{(\epsilon_m^{cross})^*}{(\epsilon_m^{own})^*} \\ \rho_m^* = \dfrac{1}{2 - \dfrac{(\epsilon_m^{cross})^* + (\alpha_m^{own})^* + (\alpha_m^{cross})^*}{(\epsilon_m^{own})^*}} \\ \mu_m^* = \dfrac{p_m^*}{(\epsilon_m^{own})^*}. \end{cases} \tag{17}$$

Thus, if the firm's demand for each market m is estimated and the discriminatory price p_m^* is observed, then one can easily compute θ_m^*, μ_m^*, and ρ_m^*, using up to second-order demand elasticities.[28]

To provide an intuitive understanding as explained in Introduction, note that $\theta_m^*\rho_m^*$ is interpreted as *quantity* pass-through in market m under price discrimination if the marginal costs are constant: it is defined as $dq_m^*/d\widetilde{q}$, where $\widetilde{q}$ is an exogenous amount of output with $\pi_{jm}(p_{jm}, p_{-j,m}) = (p_{jm} - c_m)[x_{jm}(p_{jm}, p_{-j,m}) - \widetilde{q}]$, which can be expressed by

$$\begin{aligned} \frac{dq_m^*}{d\widetilde{q}} &= q_m'(p_m^*) \cdot \frac{dp_m^*}{d\widetilde{q}} \\ &= \frac{q_m'}{\partial x_{Am}/\partial p_A} \cdot \frac{\partial x_{Am}}{\partial p_A} \cdot \frac{dp_m^*}{d\widetilde{q}} \\ &= \left(\frac{q_m'}{\partial x_{Am}/\partial p_A}\right) \cdot \left(\frac{\partial x_{Am}/\partial p_A}{\pi_m''}\right) \\ &= \theta_m^* \cdot \rho_m^* \end{aligned}$$

because the first-order condition with $\widetilde{q}$ indicates $dp_m^*/d\widetilde{q} = 1/\pi_m''$.[29] Now, $\mu_m^* \times \theta_m^*\rho_m^*$ approximates the trapezoid generated by a small deviation from (perfect) price discrimination that captures the marginal welfare gain in the strong market and the marginal welfare loss in the weak market (see Fig. 1, A). If the latter is larger than the former, such a deviation lowers social welfare, and owing to the IRCW, this argument extends globally so that the regime switch to uniform pricing definitely decreases social welfare.

[27] An alternative expression for μ_m^* is $\mu_m^* = c_m/[(\epsilon_m^{own})^* - 1]$ if the cost information is used.

[28] It should be emphasized that the second-order supply property, i.e., the derivative of marginal cost, would be necessary if non-constant marginal cost is allowed, as suggested by Adachi and Fabinger (2022) in the context of general "taxation" (pure taxation and other additional costs from external changes).

[29] Note that this is the case where $dq_m^*/d\widetilde{q}$ is evaluated at $\widetilde{q} = 0$: Miklós-Thal and Shaffer (2021a) derive a general formula for $\widetilde{q} > 0$, correcting Weyl and Fabinger's (2013) arguments. If marginal costs are non-constant (see Online Appendix D), then $\pi_{im}(p_{im}, p_{-i,m}) = p_{im} \cdot [x_{im}(p_{im}, p_{-i,m}) - \widetilde{q}] - c_m[x_{im}(p_{im}, p_{-i,m}) - \widetilde{q}]$ should be considered, where $c_m(\cdot)$ is the cost function, and thus $\theta_m^*\rho_m^*$ is no longer the quantity pass-through under price discrimination (that is, when $\widetilde{q} = 0$). See Weyl and Fabinger (2013, p. 572) for a precise expression of quantity pass-through with non-constant marginal costs.

Note that this comparison is not straightforward when starting at uniform pricing (see the latter part of Proposition 1): why is the adjustment term, $\bar{\pi}_m''$, necessary for the deviation from uniform pricing? This is because pass-through is not defined market-wise unless the pricing regime is "perfect" or "full" price discrimination (i.e., $t = t^*$), where the first-order conditions are given market-wise. Note that if $|\pi_m''|$ is small, then π_m is "flat," and thus the price shift $|\Delta p_m|$ in response to some change would be large (see Expression 14). Hence, the role of π_w''/π_s'' is to adjust measurement units for ρ_w/ρ_s. For example, if $|\pi_w''|$ is very small, then ρ_w is "over represented," and thus it should be "penalized" so that the right hand side of the inequality in the proposition becomes small.

Proposition 1 cannot be further simplified even if no cost differentials (i.e., $c_s = c_w$) are additionally assumed. In other words, this expression is already robust to the inclusion of cost differentials. Now, if we further assume that there are no strategic effects (i.e., $\theta_m = 1$), then the condition $\mu_w^*\theta_w^*\rho_w^* \geq \mu_s^*\theta_s^*\rho_s^*$ becomes $(p_s^* - c)/(p_w^* - c) \leq (1/\rho_s^*)/(1/\rho_w^*)$, which coincides with

$$\frac{p_w^* - c}{2 - \sigma_w^*} \geq \frac{p_s^* - c}{2 - \sigma_s^*}$$

in Proposition 2 of Aguirre et al. (2010, p. 1606), where σ_m^* is what they call the curvature of the inverse demand function (under price discrimination), because of $\sigma_m^* = (\alpha_m^{own})^*/(\epsilon_m^{own})^*$ and Eq. (16). Thus, price discrimination increases social welfare "if the discriminatory prices are not far apart and the inverse demand function in the weak market is locally more convex than that in the strong market" (Aguirre et al., 2010, p. 1602). As compared to Fig. 1(B), Fig. 1(A) shows the usefulness of the sufficient statistics in welfare evaluation. In Online Appendix B, we extend our arguments to aggregate output and consumer surplus. Online Appendix C argues that our methodology is readily extended to accommodate heterogeneous firms.

3.3. An alternative expression

The next result shows another expression that can be readily verified to be equivalent to Proposition 1.

Corollary 1. *Given the IRCW, price discrimination increases social welfare if*

$$\frac{\mu_s^*\theta_s^*}{1 + \theta_s^* - [(\alpha_s^{own})^* + (\alpha_s^{cross})^*]L_s^*} < \frac{\mu_w^*\theta_w^*}{1 + \theta_w^* - [(\alpha_w^{own})^* + (\alpha_w^{cross})^*]L_w^*}$$

holds, and it decreases social welfare if

$$\frac{\bar{\mu}_s\bar{\theta}_s}{1 + \bar{\theta}_s - (\bar{\alpha}_s^{own} + \bar{\alpha}_s^{cross})\bar{L}_s} \geq \frac{\bar{\mu}_w\bar{\theta}_w}{1 + \bar{\theta}_w - (\bar{\alpha}_w^{own} + \bar{\alpha}_w^{cross})\bar{L}_w}$$

holds.

Proof. Using Eqs. (2), (10), and (11), we can rewrite z_m so that

$$\frac{W'(t)}{2} = \left(-\frac{\pi_s''\pi_w''}{\pi_s'' + \pi_w''}\right)\left(\frac{\mu_w\theta_w}{1 + \theta_w - (\alpha_w^{own} + \alpha_w^{cross})L_w} - \frac{\mu_s\theta_s}{1 + \theta_s - (\alpha_s^{own} + \alpha_s^{cross})L_s}\right)$$

for $t \in [0, t^*]$. □

This corollary indicates that pass-through is, although it facilitates an intuitive interpretation as shown in Proposition 1, not necessary. Instead, the own and cross curvatures are utilized, and the second inequality in Corollary 1 is computationally simpler than the second inequality in Proposition 1 because the second-order derivative for the profit function, π_m'', is not involved. For this reason, this corollary's result is used for numerical exercises in the next section.

Note that our expression for

$$z_m = \frac{(p_m - c_m)\theta_m}{1 + \theta_m - (\alpha_m^{own} + \alpha_m^{cross})L_m}$$

is a generalization of ACV's (2010) Eq. (4),

$$z_m = \frac{p_m - c}{2 - \alpha_m^{own}L_m}$$

if there are *no strategic effects* (i.e., $\theta_m = 1$ and $\alpha_m^{cross} = 0$). Additionally, if there are *no cost differentials* (i.e., $c_s = c_w \equiv c$), then the second part of the corollary reduces to ACV's (2010, p. 1605) Proposition 1 ($\overline{\alpha}_s^{own} \geq \overline{\alpha}_w^{own}$ in our notation; in their notation, $\alpha_s(\overline{p}) \geq \alpha_w(\overline{p})$) because $L_s(\overline{p}) = L_w(\overline{p})$. That is, the firm's "direct demand function in the strong market is at least as convex as that in the weak market at the nondiscriminatory price" (Aguirre et al. 2010, p. 1602).

4. Parametric examples of market demand

To consider the following three examples of parametric market demand, we consider $N \geq 2$ symmetric firms, assuming that there are still two separate markets (strong and weak): let $\mathbf{x}_m = (x_{1m}, x_{2m}, \ldots, x_{Nm})$ be the representative consumer's

Table 1
The four elasticities and the conduct 'parameter' under symmetric price p in market m (with N symmetric firms). See the main text for the notations. Note that for the CES demand, the four elasticities and the conduct parameter are constant for any p. For the linear demand, the two second-order elasticites are zero, and the conduct parameter is constant.

(i) Linear	
First-order	
Own: $\epsilon_m^{own}(p)$	$\frac{[1+(N-2)\delta_m]p}{(1-\delta_m)(\omega_m-p)}$
Cross: $\epsilon_m^{cross}(p)$	$\frac{\delta_m p}{(1-\delta_m)(\omega_m-p)}$
Second-order	
Own: α_m^{own}	0
Cross: α_m^{cross}	0
Conduct	
θ_m	$\frac{1+(N-3)\delta_m}{1+(N-2)\delta_m}$
(ii) CES	
First-order	
Own: ϵ_m^{own}	$\frac{1+(N-1)\sigma_m}{N}$
Cross: ϵ_m^{cross}	$\frac{\sigma_m-1}{N}$
Second-order	
Own: α_m^{own}	$\frac{N\sigma_m[N+1+(N-1)\sigma_m]-2(\sigma_m-1)[1+(N-1)\sigma_m]}{N[1+(N-1)\sigma_m]}$
Cross: α_m^{cross}	$\frac{N\sigma_m(\sigma_m-1)-2(\sigma_m-1)[1+(N-1)\sigma_m]}{N[1+(N-1)\sigma_m]}$
Conduct	
θ_m	$\frac{2+(N-2)\sigma_m}{1+(N-1)\sigma_m}$
(iii) Logit	
First-order	
Own: $\epsilon_m^{own}(p)$	$\beta_m p\cdot[1-q_m(p;N)]$
Cross: $\epsilon_m^{cross}(p)$	$\beta_m p\cdot q_m(p;N)$
Second-order	
Own: $\alpha_m^{own}(p)$	$\beta_m p\cdot[1-2q_m(p;N)]$
Cross: $\alpha_m^{cross}(p)$	$\frac{\beta_m p\cdot q_m(p;N)[1-2q_m(p;N)]}{1-q_m(p;N)}$
Conduct	
$\theta_m(p)$	$\frac{1-2q_m(p;N)}{1-q_m(p;N)}$

consumption bundle in market $m=s,w$, and $\mathbf{p}_m=(p_{1m},p_{2m},\dots,p_{Nm})$ be the prices in that market. We focus on (i) linear, (ii) CES (constant elasticity of substitution), and (iii) multinomial logit demands: these demand functions are among the commonly-used demand systems (Quint, 2014; Choné and Linnemer, 2020), and Online Appendix B verifies that the IRCW holds for these three demands. Note that to save notation, the same β_m is repeatedly used in the following three examples, but with different meanings (similarly, ω_m appears twice: in Sections 4.1 and 4.3).

Let the set of related parameters be denoted by Θ. If

$$G(\Theta,N)\equiv\frac{\bar{\mu}_s\bar{\theta}_s}{1+\bar{\theta}_s-(\bar{\alpha}_s^{own}+\bar{\alpha}_s^{cross})\bar{L}_s}-\frac{\bar{\mu}_w\bar{\theta}_w}{1+\bar{\theta}_w-(\bar{\alpha}_w^{own}+\bar{\alpha}_w^{cross})\bar{L}_w}$$

and

$$H(\Theta,N)\equiv\frac{\mu_s^*\theta_s^*}{1+\theta_s^*-[(\alpha_s^{own})^*+(\alpha_s^{cross})^*]L_s^*}-\frac{\mu_w^*\theta_w^*}{1+\theta_w^*-[(\alpha_w^{own})^*+(\alpha_w^{cross})^*]L_w^*}$$

are defined, then, according to Corollary 1, $G(\Theta,N)\ge 0$ implies $\Delta W<0$ and $H(\Theta,N)<0$ implies $\Delta W>0$. Table 1 shows the first- and second-order elasticities as well as the conduct parameter θ_m (from Eq. (10)) as a function of p: $\overline{p}$ (in the case of uniform pricing) or p_m^* (in the case of price discrimination) is imputed. Interestingly, in the case of CES demand, not only $(\epsilon_m^{own},\epsilon_m^{cross})$ is constant but so are $(\alpha_m^{own},\alpha_m^{cross})$ and θ_m. Given any price p, L_m, and μ_m are obtained from Eqs. (8) and (15), respectively. Then, $G(\Theta,N)$ and $H(\Theta,N)$ are parametrically expressed for each of the demands.

We below focus on cross-market differences in demand in line with the literature on third-degree price discrimination where market differences arise from the demand side. In particular, we focus on one parameter that is closely related to the first-order elasticities, (δ_s,δ_w) for linear demand, (σ_s,σ_w) for CES demand, and (β_s,β_w) for multinomial demand (see below for the definitions).

However, our methodology does not preclude cost differences: in all these examples, we consider both cases of common and different marginal costs across strong and weak markets. To ensure that the strong market is indeed strong when cost differentials are allowed but demand heterogeneity is not allowed, it is sufficient to assume that the marginal cost in the strong market is higher than in that in the weak market, c_w: $c_s > c_w$ (although c_s should not be too much higher than c_w). Hence, we consider c_s that is slightly higher than c_w when cost differentials are allowed. However, this inequality will not be sufficient when demand differentials are also allowed: we exclude the parameter region where $p_s^* \geq p_w^*$ does not hold in each of the three examples.

4.1. Linear demand

Linear demand is derived from the quadratic utility of the representative consumer in market $m = s, w$ under symmetric product differentiation (Shubik and Levitan, 1980):

$$U_m(\mathbf{x}_m) = \omega_m \cdot \sum_{i=1}^{N} x_{im} - \frac{1}{2}\left(\beta_m \sum_{i=1}^{N} x_{im}^2 + 2\gamma_m \sum_{j\neq i} x_{im}x_{jm}\right).$$

This yields linear inverse demand, $P_{im}(x_{im}, \mathbf{x}_{-i,m}) = \omega_m - \beta_m x_{im} - \gamma_m \sum_{j\neq i} x_{jm}$, where $\mathbf{x}_{-i,m} = (x_{jm})_{j=1,2,\ldots,N;\, j\neq i}$, and the corresponding direct demand in market m is

$$x_{im}(p_{im}, \mathbf{p}_{-i,m}; \omega_m, \beta_m, \gamma_m) = \frac{1}{[1+(N-1)\delta_m](1-\delta_m)\beta_m}\left\{\omega_m(1-\delta_m) - [1+(N-2)\delta_m]p_{im} + \delta_m \sum_{j\neq i} p_{jm}\right\}$$

for firm i, where $\mathbf{p}_{-i,m} = (p_{jm})_{j=1,2,\ldots,N;\, j\neq i}$, and $\delta_m \equiv \gamma_m/\beta_m \in [0, 1)$ is the *strength of substitutability*: if δ_m is close to one, market m is approximated by perfect competition, whereas if δ_m is equal to zero, each firm behaves as a monopolist.

In symmetric equilibrium with $\mathbf{p}_m = (p, p, \ldots, p)$, the firm's demand in market m is given by

$$q_m(p) = \frac{\omega_m - p}{[1+(N-1)\delta_m]\beta_m},$$

and thus the own and the cross price elasticities can be obtained as shown in Table 1, which imply that the conduct parameter is given as a constant by

$$\theta_m(p) = \frac{1+(N-3)\delta_m}{1+(N-2)\delta_m} \equiv \widetilde{\theta}_m.$$

Then, the discriminatory price in market m satisfies Eq. (12):

$$\begin{aligned}
&\underbrace{\frac{p_m^* - c_m}{p_m^*}}_{=L_m(p_m^*)} = \underbrace{\frac{(1-\delta_m)(\omega_m - p_m^*)}{[1+(N-2)\delta_m]p_m^*}}_{=1/\epsilon_m^{own}(p_m^*)} \\
\Leftrightarrow\ & p_m^* = p_m^*(c_m, \omega_m, \delta_m, N) \equiv \frac{(1-\delta_m)\omega_m + [1+(N-2)\delta_m]c_m}{2+(N-3)\delta_m},
\end{aligned}$$

whereas the equilibrium uniform price, $\overline{p}$, is derived by solving Eq. (13):

$$\sum_{m=s,w} \frac{[1+(N-2)\delta_m](\overline{p} - c_m)}{[1+(N-1)\delta_m](1-\delta_m)\beta_m} = \sum_{m=s,w} \frac{\omega_m - \overline{p}}{[1+(N-1)\delta_m]\beta_m},$$

because

$$\begin{aligned}
\overline{y}_m \overline{\epsilon}_m^{own} \overline{L}_m &= \frac{\dfrac{\omega_m - \overline{p}}{[1+(N-1)\delta_m]\beta_m}}{\displaystyle\sum_{m=s,w} \frac{\omega_m - \overline{p}}{[1+(N-1)\delta_m]\beta_m}} \cdot \frac{[1+(N-2)\delta_m]\overline{p}}{(1-\delta_m)(\omega_m - \overline{p})} \cdot \frac{\overline{p} - c_m}{\overline{p}} \\
&= \frac{\dfrac{[1+(N-2)\delta_m](\overline{p} - c_m)}{[1+(N-1)\delta_m](1-\delta_m)\beta_m}}{\displaystyle\sum_{m=s,w} \frac{\omega_m - \overline{p}}{[1+(N-1)\delta_m]\beta_m}}
\end{aligned}$$

for $m = s, w$, leading to an explicit solution:

$$\overline{p} = \overline{p}(\mathbf{c}, \boldsymbol{\omega}, \boldsymbol{\delta}, \boldsymbol{\beta}, N) \equiv \frac{\displaystyle\sum_{m=s,w} \frac{(1-\delta_m)\omega_m + [1+(N-2)\delta_m]c_m}{[1+(N-1)\delta_m](1-\delta_m)\beta_m}}{\displaystyle\sum_{m=s,w} \frac{2+(N-3)\delta_m}{[1+(N-1)\delta_m](1-\delta_m)\beta_m}},$$

where $\mathbf{c} = (c_s, c_w)$, $\boldsymbol{\omega} = (\omega_s, \omega_w)$, $\boldsymbol{\delta} = (\delta_s, \delta_w)$, and $\boldsymbol{\beta} = (\beta_s, \beta_w)$.

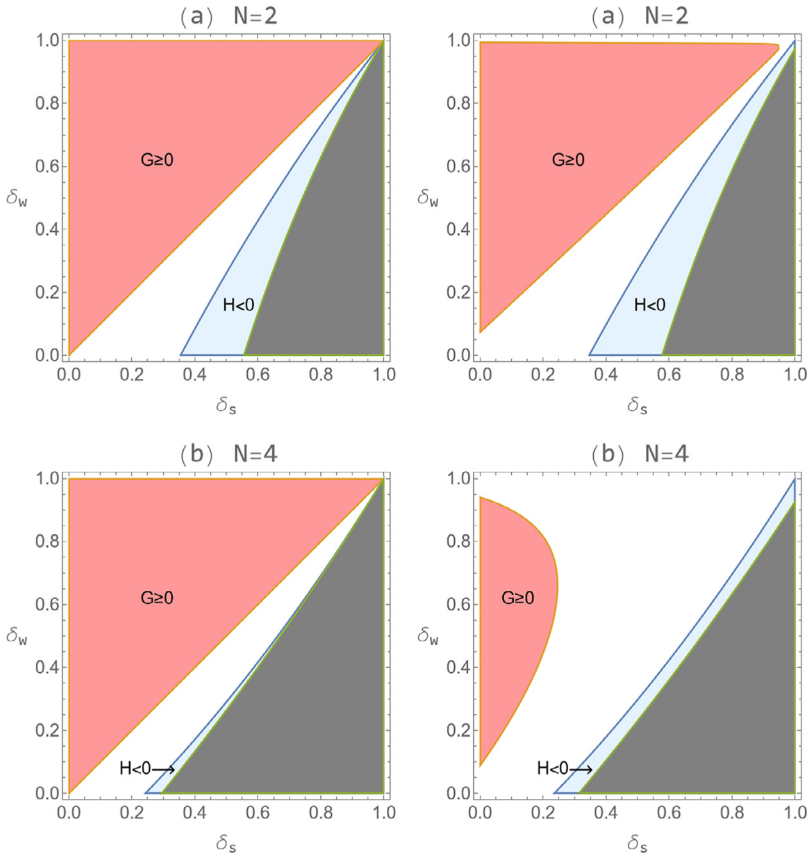

Fig. 2. Linear demand with $\omega_s = 1.5$, $\omega_w = 1.0$, and $\beta_s = \beta_w = 1.0$. Note that for p_w^* to be actually lower than p_s^*, δ_s, relative to δ_w, must not be sufficiently large. Specifically, the lower-right shaded region of (δ_s, δ_w) must be excluded. The regions for $H < 0$ and for $G \geq 0$ are colored when $N = 2$ (top) and when $N = 4$ (bottom), depending on whether the two marginal costs are common (left) or different (right). Note also that $H < 0$ is *only a part* of the region where price discrimination improves social welfare and $G \geq 0$ is also *only a part* of the region where it reduces social welfare.

Finally, noting the two curvatures, α_m^{own} and α_m^{cross}, are necessarily zero and the conduct parameter is constant in each market, it is verified that

$$
\begin{aligned}
G(\boldsymbol{\delta}, \boldsymbol{\omega}, \boldsymbol{\beta}, \mathbf{c}, N) &= \frac{\bar{\mu}_s \widetilde{\theta}_s}{1 + \bar{\theta}_s} - \frac{\bar{\mu}_w \widetilde{\theta}_w}{1 + \bar{\theta}_w} \\
&= \frac{(\overline{p} - c_s)[1 + (N-3)\delta_s]}{2 + (2N-5)\delta_s} - \frac{(\overline{p} - c_w)[1 + (N-3)\delta_w]}{2 + (2N-5)\delta_w}
\end{aligned}
$$

and

$$
\begin{aligned}
H(\boldsymbol{\delta}, \boldsymbol{\omega}, \boldsymbol{\beta}, \mathbf{c}, N) &= \frac{\mu_s^* \widetilde{\theta}_s}{1 + \theta_s^*} - \frac{\mu_w^* \widetilde{\theta}_w}{1 + \theta_w^*} \\
&= \frac{(1-\delta_s)(\omega_s - c_s)[1 + (N-3)\delta_s]}{[2 + (N-3)\delta_s][2 + (2N-5)\delta_s]} - \frac{(1-\delta_w)(\omega_w - c_w)[[1 + (N-3)\delta_w]}{[2 + (N-3)\delta_w][2 + (2N-5)\delta_w]}.
\end{aligned}
$$

In Fig. 2 with $\omega_s = 1.50$, $\omega_w = 1.00$, and $\beta_s = \beta_w = 1.00$, we consider the two cases of identical marginal costs ($c_s = c_w = 0.20$) on the left panel, and of different marginal costs ($c_s = 0.22$ and $c_w = 0.20$) on the right panel. The top panel assumes $N = 2$, whereas the bottle panel has $N = 4$ firms. In each of the four graphs, the dark-shaded are of the (δ_s, δ_w) region is excluded because $p_s^* \geq p_w^*$ does not holds. It is observed that the region for sufficiency for $\Delta W < 0$ when the current regime of uniform pricing is relaxed, i.e, $G(\delta_s, \delta_w; \boldsymbol{\omega}, \boldsymbol{\beta}, \mathbf{c}, N) \geq 0$, appears in the north-west side where the degree of substitutability in the weak market is sufficiently large. Here, a marginal increase in welfare gain due to the lower price in the weak market will be relatively low because the intensity of competition in the weak market is already high. Hence, welfare gain in the weak market due to the price reduction is limited and thus price discrimination is likely to reduce social welfare (Adachi and Matsushima, 2014 also derive a similar finding from their Figures 4 and 5). This effect from competition is also prominent when the number of firms increases to $N = 4$: the region for $H(\delta_s, \delta_w; \boldsymbol{\omega}, \boldsymbol{\beta}, \mathbf{c}, N) < 0$ where sufficiency for $\Delta W > 0$, i.e, social welfare decreases if price discrimination is banned, shrinks.

4.2. CES (Constant elasticity of substitution) demand

Suppose that the representative consumer's utility in market m is given by:

$$U_m(\mathbf{x}_m) = \left(\sum_{i=1}^{N} x_{im}^{\frac{\sigma_m - 1}{\sigma_m}} \right)^{\frac{\sigma_m}{\sigma_m - 1}},$$

where $\sigma_m > 1$ is the *constant elasticity of substitution* across products/firms (Vives, 1999, pp. 147-8).[30] Then, the direct demand function for good/firm i is given by

$$x_{im}(p_{im}, \mathbf{p}_{-i,m}; \sigma_m) = \frac{p_{im}^{-\sigma_m}}{\sum_{j=1}^{N} p_{jm}^{1-\sigma_m}},$$

and, in symmetric equilibrium, the firm's demand in market m is

$$q_m(p) = \frac{1}{Np},$$

and thus the own and the cross price elasticities are obtained as constants as indicated in Table 1 and thus the conduct parameter is also given as a constant by

$$\theta_m(p) = \frac{2 + (N-2)\sigma_m}{1 + (N-1)\sigma_m} \equiv \widehat{\theta}_m.$$

The own and the cross curvatures are also obtained as constants as shown in Table 1. Hence, let the sum of these two curvatures be denoted by

$$\begin{aligned} \hat{\alpha}_m &\equiv \alpha_m^{own}(p) + \alpha_m^{cross}(p) \\ &= \frac{N^2\sigma_m(1+\sigma_m) - 4(\sigma_m - 1)[1 + (N-1)\sigma_m]}{N[1 + (N-1)\sigma_m]}. \end{aligned}$$

Now, the discriminatory price in market m is obtained explicitly by solving Eq. (12):

$$\begin{aligned} &\frac{p_m^* - c_m}{p_m^*} = \frac{N}{1 + (N-1)\sigma_m} \\ \Leftrightarrow \quad & p_m^* = p_m^*(c_m, \sigma_m, N) \equiv \frac{1 + (N-1)\sigma_m}{(N-1)(\sigma_m - 1)} c_m, \end{aligned}$$

which indicate constant markup, whereas the equilibrium uniform price satisfies Eq. (13):

$$\sum_{m=s,w} \frac{[1 + (N-1)\sigma_m](\overline{p} - c_m)}{2N} = \overline{p}$$

because

$$\begin{aligned} \overline{y}_m \overline{\epsilon}_m^{own} \overline{L}_m &= \frac{q_m(\overline{p})}{q_s(\overline{p}) + q_w(\overline{p})} \cdot \frac{1 + (N-1)\sigma_m}{N} \cdot \frac{\overline{p} - c_m}{\overline{p}} \\ &= \frac{[1 + (N-1)\sigma_m](\overline{p} - c_m)}{2N\overline{p}}, \end{aligned}$$

which leads to the following explicit solution:

$$\overline{p} = \overline{p}(\mathbf{c}, \boldsymbol{\sigma}, N) \equiv \frac{[1 + (N-1)\sigma_s]c_s + [1 + (N-1)\sigma_w]c_w}{(N-1)(\sigma_s + \sigma_w - 2)}.$$

Then, it is also verified that

$$\begin{aligned} G(\boldsymbol{\sigma}, \mathbf{c}, N) &= \frac{\overline{\mu}_s \hat{\theta}_s}{1 + \hat{\theta}_s - \hat{\alpha}_s \overline{L}_s} - \frac{\overline{\mu}_w \hat{\theta}_w}{1 + \hat{\theta}_w - \hat{\alpha}_w \overline{L}_w} \\ &= \frac{1}{(N-1)(\sigma_s + \sigma_w - 2)} \end{aligned}$$

[30] Anderson et al. (1992, pp. 85-90), discuss how this demand system can be microfounded by discrete choice modeling. The elasticity of substitution between the two goods is *constant*, $1/(1 - \beta_m)$

$$\times\left[\frac{[2+(N-2)\sigma_s][-(N-1)\sigma_w(c_s-c_w)+(2N-1)c_s+c_w]}{[1+(N-1)\sigma_s](\Sigma_s+1)}\right.$$

$$\left.-\frac{[2+(N-2)\sigma_w][(N-1)\sigma_s(c_s-c_w)+(2N-1)c_w+c_s]}{[1+(N-1)\sigma_w](\Sigma_w+1)}\right],$$

where

$$\begin{cases}\Sigma_s \equiv \dfrac{\left\{\sigma_s\left[(N-2)^2\sigma_s+N(N+4)-8\right]+4\right\}[(N-1)\sigma_w(c_s-c_w)-(2N-1)c_s-c_w]}{N((N-1)\sigma_s+1)(c_s((N-1)\sigma_s+1)+c_w((N-1)\sigma_w+1))}+\dfrac{2+(N-2)\sigma_s}{1+(N-1)\sigma_s}\\ \Sigma_w \equiv -\dfrac{\left\{\sigma_w\left[(N-2)^2\sigma_w+N(N+4)-8\right]+4\right\}[(N-1)\sigma_s(c_s-c_w)+(2N-1)c_w+c_s]}{N((N-1)\sigma_w+1)(c_s((N-1)\sigma_s+1)+c_w((N-1)\sigma_w+1))}+\dfrac{2+(N-2)\sigma_w}{1+(N-1)\sigma_w},\end{cases}$$

and

$$\begin{aligned}H(\boldsymbol{\sigma},\mathbf{c},N)&=\frac{\mu_s^*\hat{\theta}_s}{1+\hat{\theta}_s-\hat{\alpha}_sL_s^*}-\frac{\mu_w^*\hat{\theta}_w}{1+\hat{\theta}_w-\hat{\alpha}_wL_w^*}\\&=\frac{N[2+(N-2)\sigma_s][1+(N-1)\sigma_s]c_s}{(\sigma_s-1)^2\{N-1+[1+N^2(N-2)]\sigma_s\}}-\frac{N[2+(N-2)\sigma_w][1+(N-1)\sigma_w]c_w}{(\sigma_w-1)^2\{N-1+[1+N^2(N-2)]\sigma_w\}}.\end{aligned}$$

Note that in the case of CES demand, (σ_s,σ_w) is the only pair of demand parameters that determines which market is strong. Specifically, as in Fig. 2 above, the degree of substitution in the weak market, σ_w, must be sufficiently high as compared to σ_s for $p_s^*\geq p_w^*$ to hold. Interestingly, Fig. 3 shows that if differential costs are not allowed (i.e., the left panel), the region of $H(\sigma_s,\sigma_w;\mathbf{c},N)<0$ does not appear. Moreover, if $N=4$, neither the region of $H<0$ nor $G\geq 0$ appears, indicating that welfare assessment is not possible. However, once differential costs are allowed (i.e., the right panel), the region for $H<0$ appears: if (σ_s,σ_w) belongs to this region, we can *definitely* conclude that price discrimination increases social welfare. By comparing Fig. 3 with Fig. 2, we can also find that (δ_s,δ_w) in the linear demand and (σ_s,σ_w) in the CES demand play a similar role.

4.3. Multinomial logit demand with outside option

Lastly, we consider the following share/demand function that each firm i faces in market $m=s,w$:

$$x_{im}(p_{im},\mathbf{p}_{-i,m};\omega_m,\beta_m)=\frac{\exp(\omega_m-\beta_m p_{jm})}{1+\sum_{j=1}^{N}\exp(\omega_m-\beta_m p_{jm})}\in(0,1),$$

where $\omega_m>0$ is now the product-specific utility, and $\beta_m>0$ is the *price responsiveness* of the representative consumer in market m.[31] Then, under symmetric pricing, each firm's market share is given by

$$q_m(p;N)=\frac{\exp(\omega_m-\beta_m p)}{1+N\cdot\exp(\omega_m-\beta_m p)},$$

where the dependence on N is made explicit for clarity. For any $N\geq 2$, the own and cross price elasticities are $\epsilon_m^{own}(p)=\beta_m p\cdot[1-q_m(p;N)]$ and $\epsilon_m^{cross}(p)=\beta_m p q_m(p;N)$, respectively. Hence, the conduct parameter is given by

$$\theta_m(p)=\frac{1-2q_m(p;N)}{1-q_m(p;N)}.$$

Similarly, the two curvatures are also obtained as shown in Table 1. the symmetric discriminatory equilibrium price $p_m^*=p_m^*(c_m,\omega_m,\beta_m,N)$ satisfies:

$$\underbrace{p_m^*-c_m}_{=\mu_m^*}-\frac{1}{\beta_m(1-q_m^*)}=0$$

and

$$q_m^*\equiv q_m(p_m^*;N)=\frac{\exp(\omega_m-\beta_m p_m^*)}{1+N\cdot\exp(\omega_m-\beta_m p_m^*)}.$$

[31] Anderson et al. (1987) argue that the indirect utility of the representative consumer in market m is given by

$$V_m(\mathbf{p}_m)=\frac{\ln\left[\sum_{i=1}^{N}\exp(\omega_m-\beta_m p_{im})\right]}{\beta_m}.$$

This demand form can also be microfounded by the random utility model (see, e.g., Anderson et al. 1992, Ch. 2).

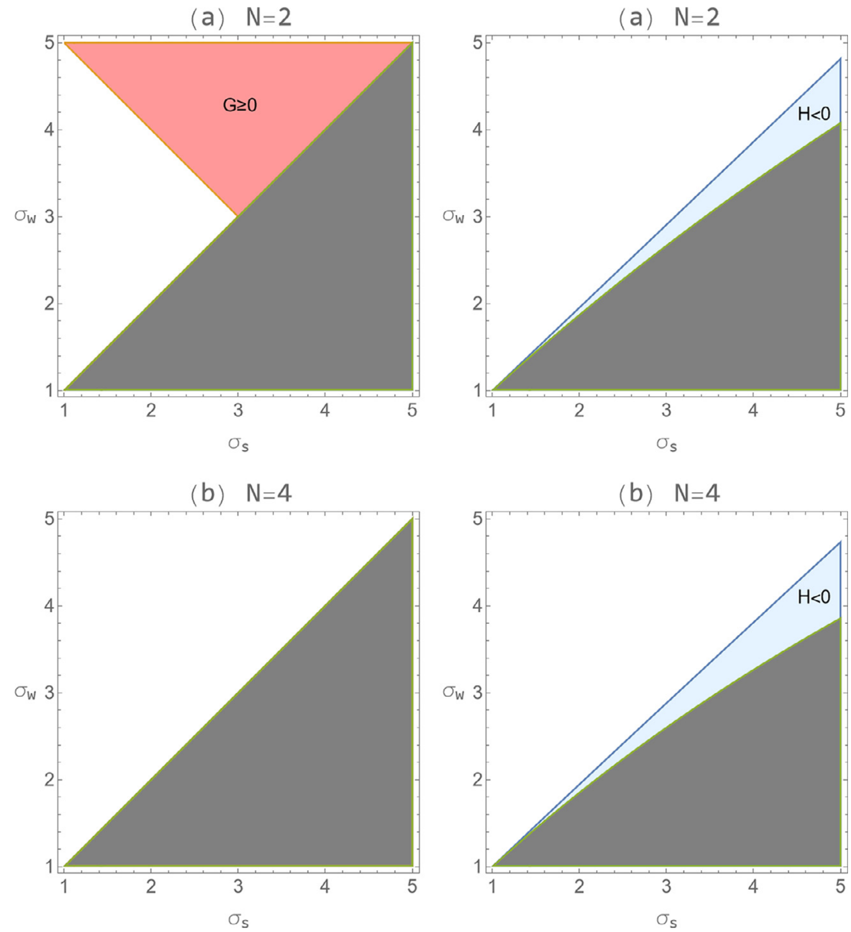

Fig. 3. CES demand when $N = 2$ (top) and when $N = 4$ (bottom). As in Fig. 2, the dark-shaded region of (σ_s, σ_w) is excluded for p_s^* to be actually higher than p_w^*. The regions for $H < 0$ and for $G \geq 0$ are colored when the two marginal costs are common (left) or different (right). Note also that $H < 0$ is *only a part* of the region where price discrimination improves social welfare and $G \geq 0$ is also *only a part* of the region where it reduces social welfare.

Both p_m^* and q_m^* should be jointly solved numerically. Similarly, the equilibrium uniform price $\overline{p} = \overline{p}(\mathbf{c}, \omega, \beta, N)$ satisfies

$$\sum_{m=s,w} q_m(\overline{p}; N)\{1 - \beta_m(\overline{p} - c_m)[1 - q_m(\overline{p}; N)]\} = 0,$$

which should also be numerically solved.

It is thus shown that

$$G(\boldsymbol{\beta}, \boldsymbol{\omega}, \mathbf{c}, N) = \frac{(\overline{p} - c_s)(1 - 2\overline{q}_s)}{2 - 3\overline{q}_s - \beta_s(\overline{p} - c_s)(1 - 2\overline{q}_s)} - \frac{(\overline{p} - c_w)(1 - 2\overline{q}_w)}{2 - 3\overline{q}_w - \beta_w(\overline{p} - c_w)(1 - 2\overline{q}_w)}$$

and

$$H(\boldsymbol{\beta}, \boldsymbol{\omega}, \mathbf{c}, N) = \frac{(p_s^* - c_s)(1 - 2q_s^*)}{2 - 3q_s^* - \beta_s(p_s^* - c_s)(1 - 2q_s^*)} - \frac{(p_w^* - c_w)(1 - 2q_w^*)}{2 - 3q_w^* - \beta_w(p_s^* - c_w)(1 - 2q_w^*)}.$$

Fig. 4 shows that the same type of argument carries over from the cases of linear demand. These examples exhibit similarity between linear and multinomial demands in terms of predictions based on our analysis using sufficient statistics. Recall that the cost structure is set common for both demands. It is observed that if linear or multinomial logit demand is assumed, intense competition in the weak market due to a lesser degree of product differentiation in that market is likely to justify a ban against price discrimination because this is the case where $G \geq 0$ is likely to hold.

5. Concluding remarks

This paper presents the theoretical implications of oligopolistic third-degree price discrimination with general non-linear demand, allowing cost differentials to exist across separate markets. In this sense, this paper, with the help of the method-

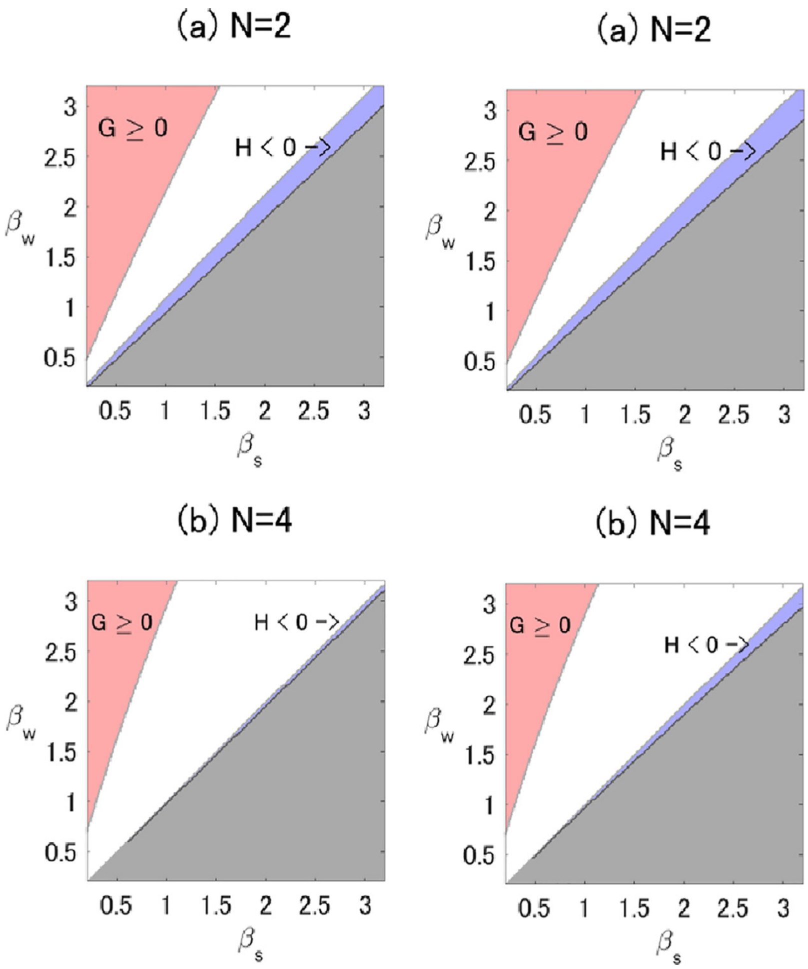

Fig. 4. Multinomial logit demand with $\omega_s = 1.5$ and $\omega_w = 1.0$ when $N = 2$ (top) and when $N = 4$ (bottom). As in Figs. 2, and 3, the region of (β_s, β_w) where the price coefficient for the strong market β_s relatively large as compared to β_w is excluded (the dark-shaded area). The regions for $H < 0$ and for $G \geq 0$ are colored when the two marginal costs are common (left) or different (right). Note also that $H < 0$ is *only a part* of the region where price discrimination improves social welfare and $G \geq 0$ is also *only a part* of the region where it reduces social welfare.

ology proposed by Weyl and Fabinger (2013), generalizes ACV's (2010) analysis of monopolistic third-degree price discrimination to complement Chen, Li, and Schwartz' (2021) analysis of oligopolistic differential pricing.

Our theoretical analysis, which accommodates firm heterogeneity, can also be utilized to empirically assess the welfare effects of third-degree price discrimination under oligopoly. In particular, in line with the "sufficient statistics" approach (Chetty, 2009; Kleven, 2021; Barnichon and Mesters, 2022), our predictions regarding the welfare effects do *not* rely on functional specifications, and are thus considered to be fairly robust, although these sufficient statistics can take different values, depending on functional specifications. However, once the numerical values of sufficient statistics are obtained, there should be no disagreement regarding welfare assessment.

As a promising direction, it would be interesting to apply our methodology to the analysis of the welfare effects of *wholesale/input* third-degree price discrimination (Katz, 1987; DeGraba, 1990; Yoshida, 2000; Inderst and Valletti, 2009; Villas-Boas, 2009; Arya and Mittenforf, 2010; Li, 2014; O'Brien, 2014; Miklós-Thal and Shaffer, 2021b; Miklós-Thal and Shaffer, 2021c and Gaudin and Lestage, 2023).[32] To do so, one would need to properly define the sufficient statistics at each stage of a vertical relationship. Another important issue to consider is the case of *multi-product* oligopolistic firms (Armstrong and Vickers, 2018, and Nocke and Schutz, 2018). What happens if price discrimination is allowed for some products, whereas uniform pricing is enforced for others? These and other important issues related to third-degree price discrimination await further research.

Data availability

No data was used for the research described in the article.

[32] See Jaffe and Weyl (2013) for such an attempt.

Appendix A. Proof of Proposition 1

First, for $t = t^*$, it is observed that

$$z_m(p_m^*) = \left(\frac{-q_m(p_m^*)}{\dfrac{\partial x_{Am}}{\partial p_A}(p_m^*, p_m^*)} \right) \theta_m(p_m^*)\rho_m(p_m^*)$$
$$= \mu_m(p_m^*)\theta_m(p_m^*)\rho_m(p_m^*)$$

and thus

$$\frac{W'(t^*)}{2} = \left(-\frac{\pi_s''\pi_w''}{\pi_s'' + \pi_w''} \right)(\mu_w^*\theta_w^*\rho_w^* - \mu_s^*\theta_s^*\rho_s^*) > 0$$

if $\mu_w^*\theta_w^*\rho_w^* > \mu_s^*\theta_s^*\rho_s^*$ holds.[33] Given the IRCW, this means that $W(t)$ is strictly increasing in $[0, t^*]$. This completes the proof for the first part.

Next, for $t < t^*$, note that

$$z_m(p_m) = \theta_m\left(\frac{-q_m}{\partial x_{Am}/\partial p_A} \right)\left(\frac{\partial x_{Am}/\partial p_A}{\pi_m''} \right)$$
$$= \mu_m\theta_m \underbrace{\left(\frac{\partial x_{Am}/\partial p_A}{\pi_s'' + \pi_w''} \right)}_{=\rho_m} \left(\frac{\pi_s'' + \pi_w''}{\pi_m''} \right).$$

Thus, it is verified that

$$\frac{W'(t)}{2} = \left(-\frac{\pi_s''\pi_w''}{\pi_s'' + \pi_w''} \right)\left(\mu_w\theta_w\rho_w\frac{\pi_s'' + \pi_w''}{\pi_w''} - \mu_s\theta_s\rho_s\frac{\pi_s'' + \pi_w''}{\pi_s''} \right)$$
$$= \underbrace{(-\pi_s''\pi_w'')}_{<0}\left(\frac{\mu_w(t)\theta_w(t)\rho_w(t)}{\pi_w''} - \frac{\mu_s(t)\theta_s(t)\rho_s(t)}{\pi_s''} \right),$$

which implies that given the IRCW, $W(t)$ is strictly decreasing in $[0, t^*]$ if $W'(0) \leq 0$. This completes the proof for the second part.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.ijindorg.2022.102893.

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[33] This derivation is partly due to Michal Fabinger.

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