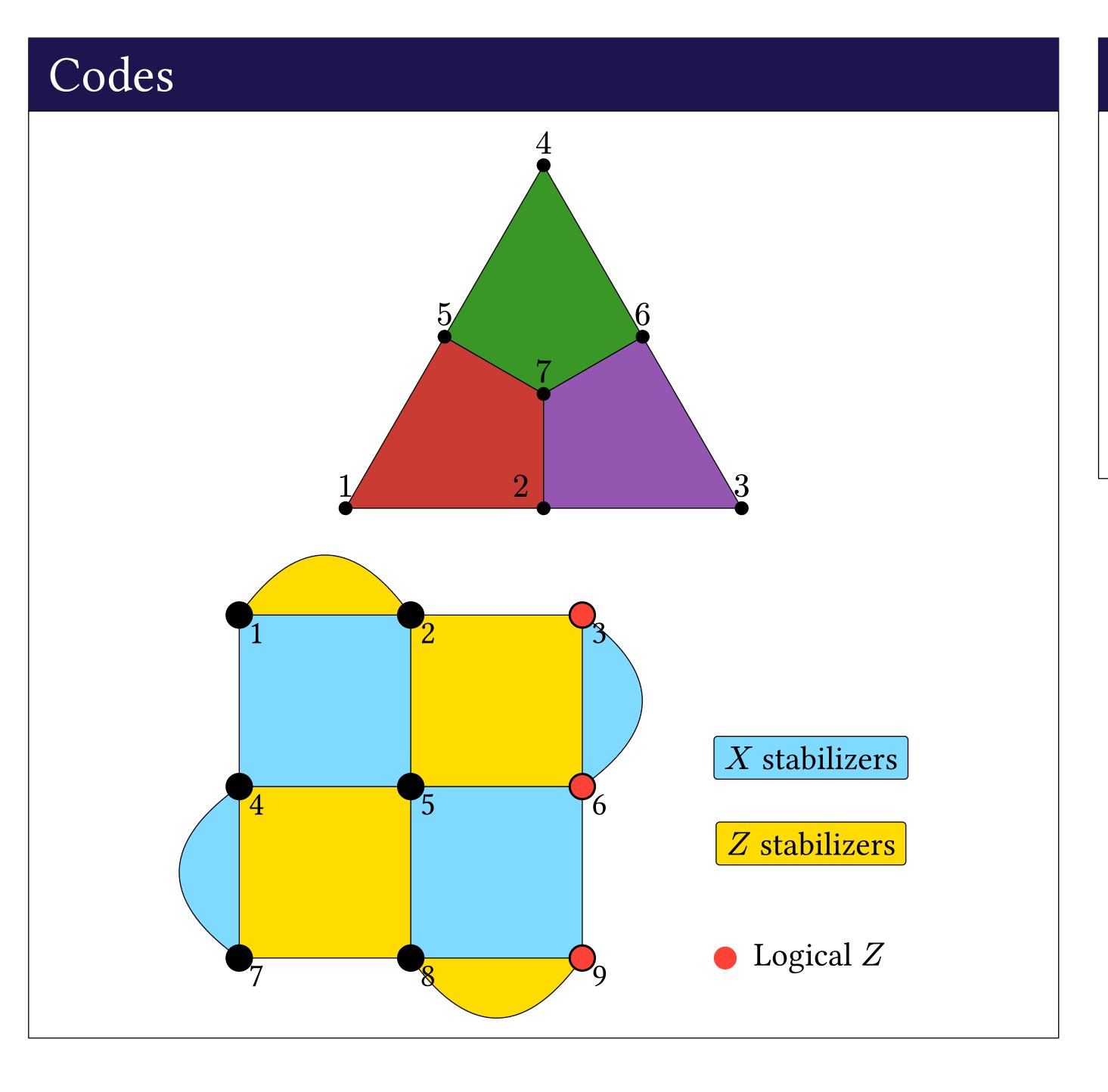
Zhongyi's Cheatsheet, 2025/03



Pauli

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$
 $XY = iZ, \qquad YZ = iX, \qquad ZX = iY$
 $YX = -iZ, \quad ZY = -iX, \quad XZ = -iY$
 $[X,Y] = 2iZ, \qquad [Y,Z] = 2iX, \qquad [Z,X] = 2iY$
 $[Y,X] = -2iZ, \qquad [Z,Y] = -2iX, \qquad [X,Z] = -2iY$

Quantum channels

A quantum channel: $\varphi \in T(X, Y)$

Kraus representation:

$$\varphi(\rho) = \sum_{i} E_{i} \rho E_{i}^{\dagger} \text{ (CP)}, \quad \sum_{i} E_{i}^{\dagger} E_{i} = I \text{ (TP)}$$

Choi isomorphism: $J:T(X,Y)\to L(Y\otimes X)$

$$J(\varphi) = \sum_{i,j} \varphi(|i\rangle\langle j|) \otimes |i\rangle\langle j|$$

Definition representation: $\varphi(\rho)=\mathrm{Tr}_Eig(U\rho U^\daggerig)$

Vectorization: $K(\varphi)\mathrm{vec}(\rho) = \mathrm{vec}(\varphi(\rho))$