

Problem 1:

Path Traversed												Path Traversed												
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g=inf h=0	g=inf h=0	g=inf h=0	g=inf h=0	g=inf h=0	g=inf h=0	g=inf h=0	g=inf h=0	g=inf h=0	g=inf h=0	g=inf h=0	g=inf h=0	g=inf h=0	g=inf h=0	g=inf h=0	g=inf h=0	g=inf h=0	g=inf h=0	g=inf h=0	g=inf h=0	g=inf h=0	g=inf h=0	g=inf h=0		
g=inf h=0	g=inf h=0									g=inf h=0		g=inf h=0										g=inf h=0		
g=1 h=16	g=inf h=0	g=1 h=16										g=inf h=0												
g=2 h=15		g=inf h=0	g=inf h=0	g=2 h=15	g=3 h=14	g=4 h=13	g=5 h=12	g=6 h=11	g=7 h=10	g=8 h=9		g=inf h=0												
g=3 h=14	g=inf h=0	g=inf h=0	g=inf h=0	g=inf h=0	g=inf h=0	g=inf h=0	g=inf h=0	g=9 h=8		g=inf h=0														
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g=5 h=12	g=inf h=0	g=inf h=0	g=inf h=0	g=inf h=0	g=inf h=0	g=inf h=0	g=11 h=6		g=inf h=0															
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g=8 h=9	g=9 h=8	g=10 h=7	g=11 h=6	g=12 h=5	g=13 h=4	g=14 h=3	g=15 h=2	g=16 h=1	g=17 h=0					g=22 h=9	g=23 h=8	g=24 h=7	g=25 h=6	g=26 h=5	g=27 h=4	g=28 h=3	g=29 h=2	g=30 h=1	g=31 h=0	

The figure on the left shows the path traversed by the A* algorithm, which balances the $g(n)$ and $h(n)$ components of $f(n)$. This balance allows A* to find an optimal path since it accounts for both the cost incurred and the estimated cost to reach the goal. The figure on the right shows the path traversed by the Greedy Best-First algorithm, where $f(n) = h(n)$. This algorithm always pursues the locally best option because it only considers the estimated future cost to the goal. This is why the path immediately starts to move towards the bottom right corner before hitting walls and realizing that it must go around to reach the goal. A*, in contrast, picks the optimal path because of its ability to always think about the cost that has already been incurred. This contrast between A* and Greedy Best-First algorithms explains why A* is optimal with an admissible heuristic, while the Greedy Best-First is not.

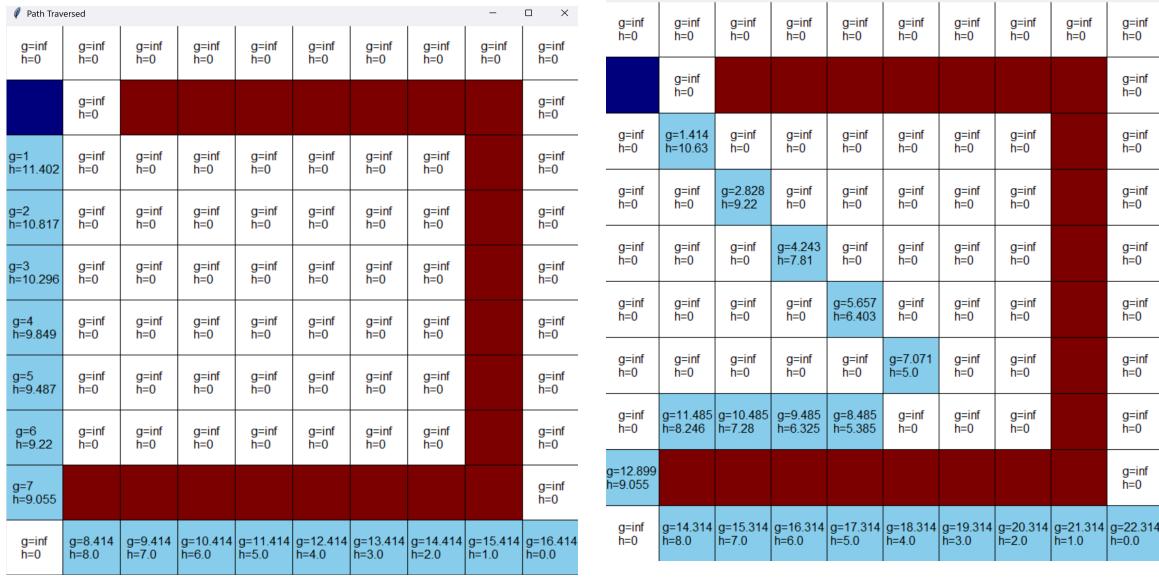
```
def __init__(self, root, maze):
```

```
    ### Set the search algorithm here: "A_star" or "Greedy"
    self.algorithm = "Greedy" # Change as needed
```

```
93     ##### Manhattan distance
94     #####
95     #####
96     def heuristic(self, pos):
97         return (abs(pos[0] - self.goal_pos[0]) + abs(pos[1] - self.goal_pos[1]))
```

```
# Calculate the new heuristic and evaluation function values
new_h = self.heuristic(new_pos)
if self.algorithm == "A_star":
    new_f = new_g + new_h # if A* search -- f(n) = g(n) + h(n) -- since h(n) is fixed for each cell, this is the same as comparing just g(n) values
else:
    new_f = new_h # if Greedy Best-First search -- f(n) = h(n); no g(n) component
```

Problem 2:



In this example, I replaced the Manhattan Distance heuristic with Euclidean Distance and allowed diagonal moves in addition to the standard four directions (N, S, E, W). Since each diagonal step has a cost of $\sqrt{2}$, the heuristic remains admissible because it never overestimates the true cost of reaching the goal. With these changes, the Greedy Best-First search produces a more diagonal-looking path. This is because the Euclidean Distance heuristic naturally pulls the agent toward the straight-line direction of the goal. A* still balances the path cost already incurred with the heuristic estimate, which means it continues to guarantee the optimal path while making use of diagonals when efficient.

```

41 class MazeGame:
42     def __init__(self, root, maze):
43
44         """ Set the search algorithm here: "A_star" or "Greedy"
45         self.algorithm = "Greedy" # Change as needed
46
47
48     def heuristic(self, pos):
49         return ((pos[0] - self.goal_pos[0]) ** 2 + (pos[1] - self.goal_pos[1]) ** 2) ** 0.5
50
51
52     def move(self, current_pos, direction):
53         new_pos = (current_pos[0] + direction[0], current_pos[1] + direction[1])
54
55         if 0 <= new_pos[0] < self.rows and 0 <= new_pos[1] < self.cols and not self.cells[new_pos[0]][new_pos[1]].is_wall:
56
57             # Ensuring that the costs of moving to a new position are set correctly, so that the heuristic remains admissible.
58             if direction == (0, 0):
59                 new_g = current_cell.g + 1 # The cost of moving to a new position is 1 unit if moving N, S, E, or W
60             else:
61                 new_g = current_cell.g + (2**0.5) # The cost of moving diagonal (NE, NW, SE, SW) is sqrt(2) units
62
63
64             # Calculate the new heuristic and evaluation function values
65             new_h = self.heuristic(new_pos)
66             if self.algorithm == "A_star":
67                 new_f = new_g + new_h # if A* search -- f(n) = g(n) + h(n) -- since h(n) is fixed for each cell, this is the same as comparing just g(n) values
68             else:
69                 new_f = new_h # if Greedy Best-First search -- f(n) = h(n); no g(n) component
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71
72             # Redraw cell with updated g() and h() values
73             self.canvas.create_rectangle(y * self.cell_size, x * self.cell_size, (y + 1) * self.cell_size, (x + 1) * self.cell_size, fill='skyblue')
74             # Round the values of g() and h() to 3 decimal places for better readability
75             text = f'g={round(self.cells[x][y].g, 3)}\nh={round(self.cells[x][y].h, 3)}'
76             self.canvas.create_text((y + 0.5) * self.cell_size, (x + 0.5) * self.cell_size, font=("Purisa", 12), text=text)
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93     #####
94     ##### Euclidean distance
95     #####
96     def heuristic(self, pos):
97         return ((pos[0] - self.goal_pos[0]) ** 2 + (pos[1] - self.goal_pos[1]) ** 2) ** 0.5
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Problem 3a:

- Used Manhattan Distance; No diagonal moves allowed; All moves are a cost of 1; $f(n) = \text{Alpha}^*g(n) + \text{Beta}^*(h(n))$

Figure	Alpha	Beta	Observed Behaviors
1	1.0	1.0	This is the A* search, as Alpha and Beta are perfectly balanced. The traversed path is the same as the A* path from Problem 1. 61 nodes expanded.
2	1.0	0.5	The traversed path mirrors that of the A* search. 76 nodes expanded. This algorithm has less of a heuristic component, so it explores more. This path is still optimal.
3	1.0	0.0	The traversed path also mirrors that of the A* search. 78 nodes expanded. Since this does include the heuristic, the search space is not limited at all. This makes it the same as Dijkstra's algorithm/UCS search, which explains why it expands the largest number of nodes. This path is still optimal.
4	0.5	1.0	This is a greedier search algorithm due to Beta being twice as large as Alpha. It makes sense that 56 nodes expanded because it is more greedy than a balanced approach. This would not be an optimal path.
5	0.0	1.0	This is the Greedy Best-First search, as the past costs (g-values) are not considered. The path pursues the next node by looking for the minimal h-value, which is why this path "hugs" the wall throughout its path. Only 50 nodes were expanded because of how greedy the search algorithm is. This would not be an optimal path.

Figures 1, 2, and 3:

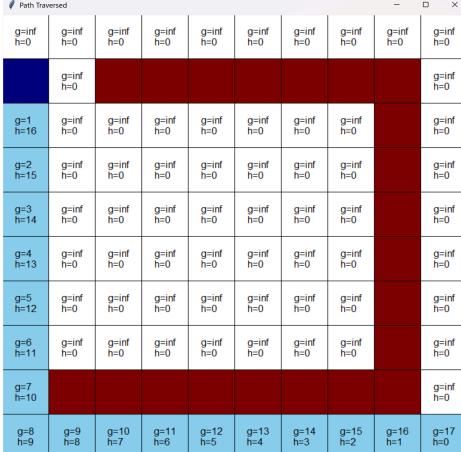


Figure 4:

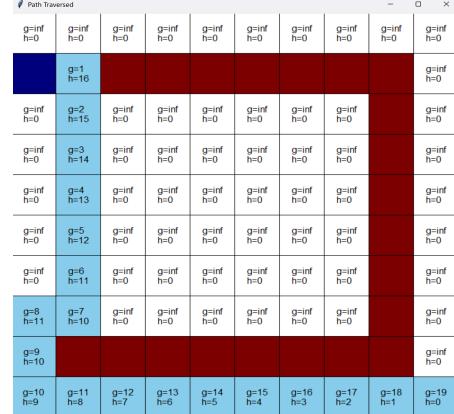
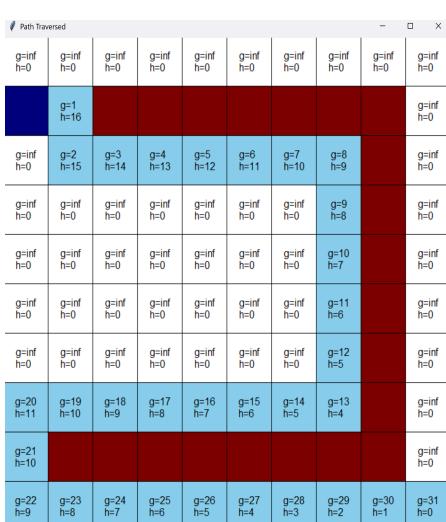


Figure 5:



When alpha becomes larger than Beta (heavy weight on $g(n)$), the search behaves more like Dijkstra's algorithm. It stays optimal, but explores more nodes in the process. The extreme in this case is when Beta=0, which is a purely Dijkstra's algorithm. When Beta becomes larger than Alpha, the search gets more goal-directed. This means it expands fewer nodes but risks taking sub-optimal paths. The extreme in this case is when Alpha=0, which is a purely Greedy Best-First algorithm. When Alpha and Beta are balanced, this balance gives an optimal path with fewer expansions than Dijkstra's.

Problem 3b:

- Changing values of Beta

Figure	Alpha	Beta	Observed Behaviors
1	1.0	0.0	The traversed path mirrors that of the A* search. 78 nodes expanded. Since this does include the heuristic, the search space is not limited at all. This makes it the same as Dijkstra's algorithm/UCS search, which explains why it expands the largest number of nodes. This path is still optimal.
2	1.0	0.5	The traversed path mirrors that of the A* search. 76 nodes expanded. This algorithm has less of a heuristic component, so it explores more. This path is still optimal.
3	1.0	1.0	This is the A* search, as Alpha and Beta are perfectly balanced. The traversed path is the same as the A* path from Problem 1. 61 nodes expanded.
4	1.0	2.0	This is a greedier search due to Beta being larger than Alpha. 56 nodes expanded, since the algorithm leans more on the heuristic. This would not be an optimal path.
5	1.0	3.0	The greedy influence increases further. 58 nodes expanded, as the search continues to prioritize heuristic guidance. This would not be an optimal path.
6	1.0	7.0	The search behaves very similarly to the Greedy Best-First search, strongly favoring heuristic guidance. 51 nodes expanded. This would not be an optimal path.
7	1.0	15.0	This search is nearly identical to Greedy Best_First search, as the heuristic weight dominates. 51 nodes expanded. This would not be an optimal path.

Figure 1, 2, and 3:

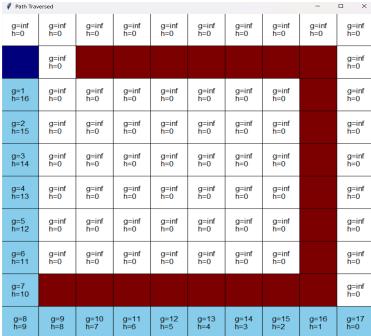


Figure 4:

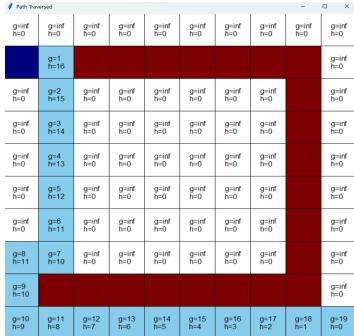


Figure 5:

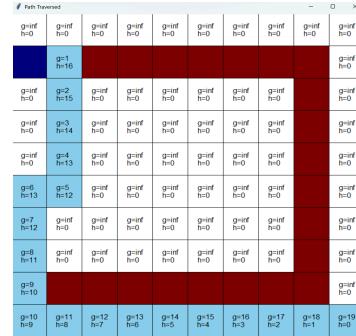


Figure 6:

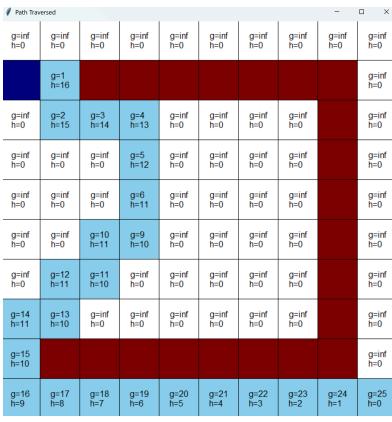
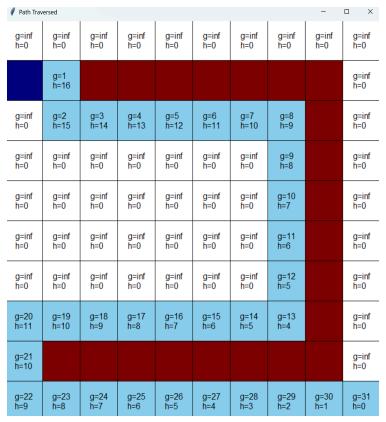


Figure 7:



As Beta increases relative to Alpha, the algorithm becomes more greedy. This causes it to expand fewer nodes but also makes it lose optimality. When Alpha and Beta are balanced (A*), the search is efficient and optimal. At the extremes, the search either mirrors Dijkstra's (when Beta = 0) or Greedy Best-First search (when Beta is **much** greater than Alpha). This shows the trade-off between completeness, optimality, and efficiency.

Code Snippets from Problem 3:

```
35 #####  
36 # A maze is a grid of size rows X cols  
37 #####  
38 class MazeGame:  
39     def __init__(self, root, maze):  
40         # Set the weights for g() and h() in the evaluation function f(n) = alpha * g(n) + beta * h(n)  
41         self.alpha = 1.0 # Change as needed  
42         self.beta = 15.0 # Change as needed  
43  
44         ##### Start state's initial values for f(n) = g(n) + h(n)  
45         # --- We use alpha and beta to weight g() and h() respectively  
46         self.cells[self.agent_pos[0]][self.agent_pos[1]].g = 0  
47         self.cells[self.agent_pos[0]][self.agent_pos[1]].h = self.heuristic(self.agent_pos)  
48         self.cells[self.agent_pos[0]][self.agent_pos[1]].f = self.alpha * 0 + self.beta * self.heuristic(self.agent_pos)  
49  
50 #####  
51 ##### Manhattan distance  
52 #####  
53 def heuristic(self, pos):  
54     return (abs(pos[0] - self.goal_pos[0]) + abs(pos[1] - self.goal_pos[1]))  
55  
56 ##### Stop if goal is reached  
57 if current_pos == self.goal_pos:  
58     self.reconstruct_path()  
59     # Printing information about the search for better analysis  
60     print(f"Alpha: {self.alpha}, Beta: {self.beta}")  
61     print(f"Number of expanded nodes: {expanded}")  
62     break  
63  
64 #####  
65 ##### Agent goes E, W, N, and S, whenever possible  
66 for dx, dy in [(0, 1), (0, -1), (1, 0), (-1, 0)]:  
67     new_pos = (current_pos[0] + dx, current_pos[1] + dy)  
68  
69     if 0 <= new_pos[0] < self.rows and 0 <= new_pos[1] < self.cols and not self.cells[new_pos[0]][new_pos[1]].is_wall:  
70  
71         ##### The cost of moving to a new position is 1 unit  
72         new_g = current_cell.g + 1  
73  
74         # Calculate the new heuristic and evaluation function values  
75         new_h = self.heuristic(new_pos)  
76  
77         # Use alpha and beta to weight g() and h() respectively  
78         new_f = self.alpha * new_g + self.beta * new_h
```