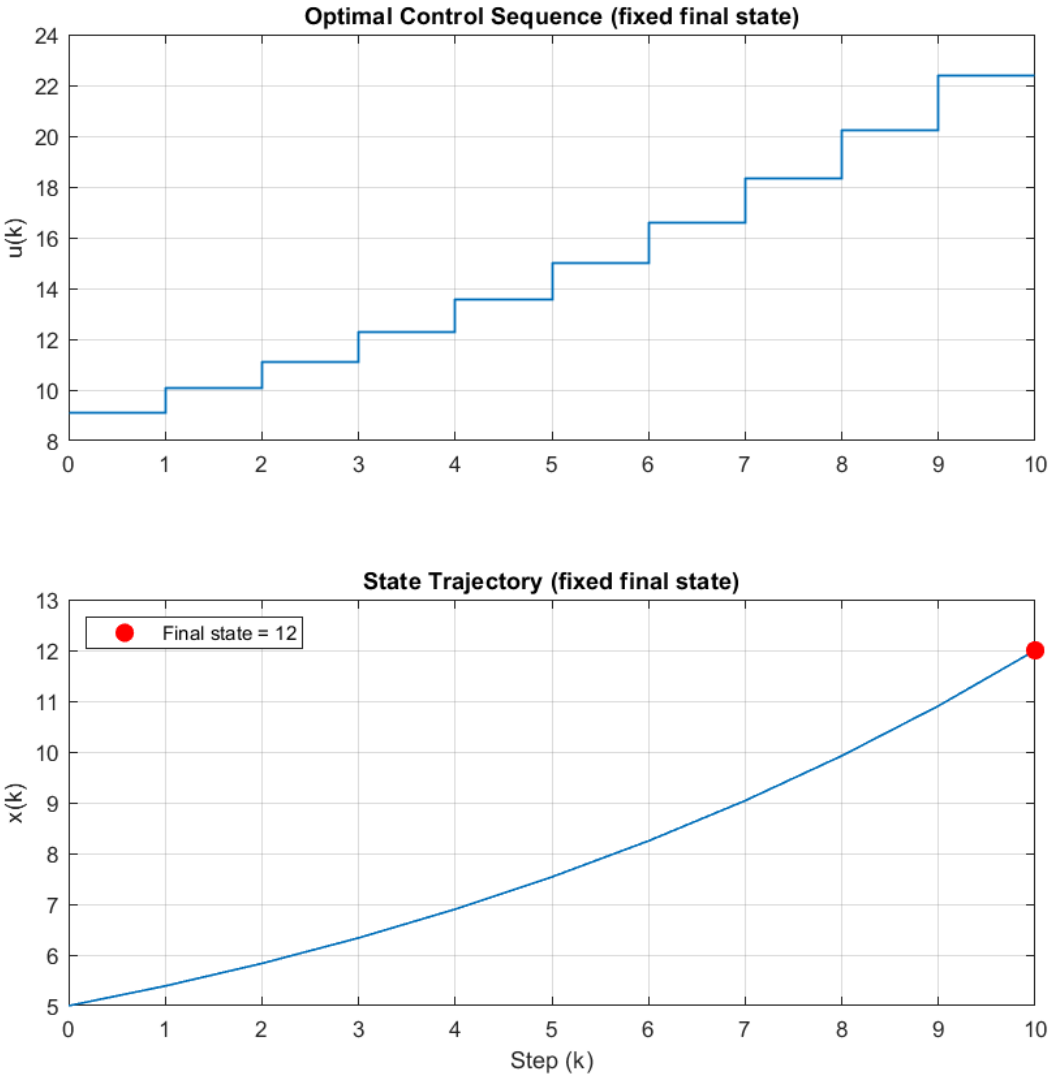
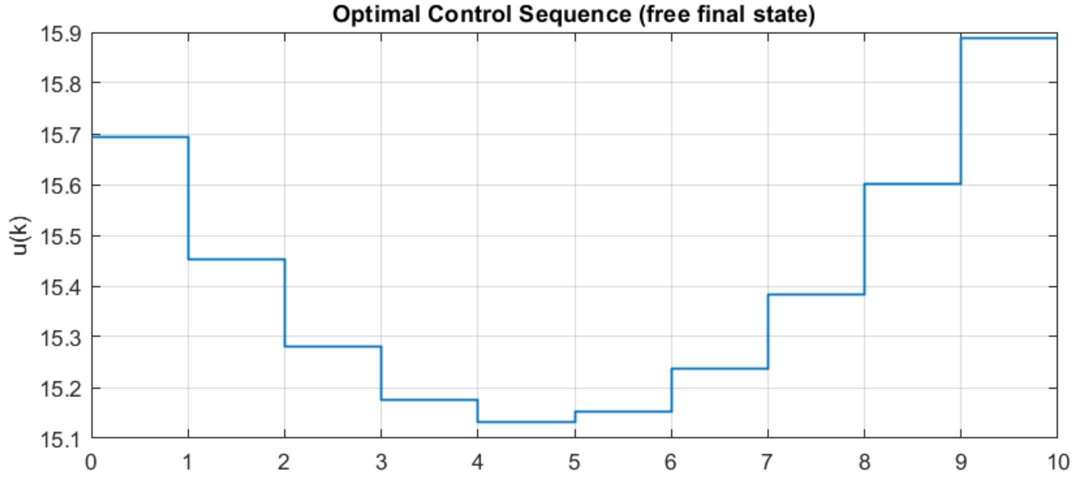
Piero Risi Mortola
Optimal Control Theory
Dr. Zheng Chen

Homework 3 - LQR controllers

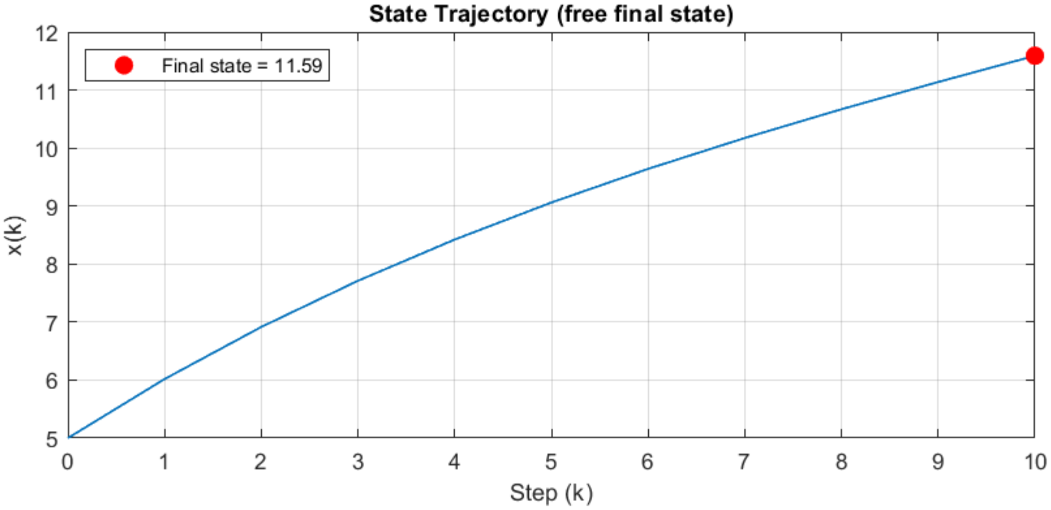
The circuit from example 2.3-1 from the textbook was modeled in MATLAB using two LQR controller: a fixed final state controller and a free final state controller. The state trajectories and the optimal control sequence are shown below.

The MATLAB code is pasted at the end.





Optimal Control Sequence (free final state)



State Trajectory (free final state)

```
clear all
clc
close all
% Parameters
a = 0.9048;
b = 0.0952;
r = 1;
q = 1;
sN = 100;
N = 10; % number of steps
S(1) = sN;
rN = 12; % target
% Free final state: adjust system to drive towards rN=12 not zero
x(1) = 5;
x_ref = rN; % xN = a*xN+b*u_ref
u_ref = (1-a)*x_ref/b;
% solving Riccati equation backwards in time
for k = 1:N
   S(k+1) = (a^2)*(r*S(k))/((b^2)*S(k)+r)+q;
   K(k) = a*b*S(k)/((b^2)*S(k)+r);
end
% applying the control to the system
for k=1:N
   u(k) = -K(end-k+1)*(x(k)-x_ref)+u_ref;
   x(k+1) = a*x(k)+b*u(k);
end
%% Fixed final state - becomes open loop control
xf(1) = 5;
for k = 0:N-1
   uf(k+1) = ((1 - a^2)/(b*(1 - a^(2*N))))*(rN - a^N * xf(1))*a^(N-1-k);
   xf(k+2) = a*xf(k+1)+b*uf(k+1);
end
%% Plotting the trajectory and control signal
% Free final state
figure
subplot(2,1,1)
stairs(0:N,[u u(end)],'LineWidth',1)
ylabel("u(k)")
title("Optimal Control Sequence (free final state)")
grid on
subplot(2,1,2)
plot(0:N,x,'LineWidth',1)
hold on
plot(N, x(end), 'ro', 'MarkerSize', 8, 'MarkerFaceColor','r');
```

```
xlabel("Step (k)")
ylabel("x(k)")
title("State Trajectory (free final state)")
grid on
legend('',sprintf('Final state = %.2f', x(end)),'Location','northwest')
% Fixed final state
figure
subplot(2,1,1)
stairs(0:N,[uf uf(end)],'LineWidth',1)
ylabel("u(k)")
title("Optimal Control Sequence (fixed final state)")
grid on
subplot(2,1,2)
plot(0:N,xf,'LineWidth',1)
hold on
plot(N, xf(end), 'ro', 'MarkerSize', 8, 'MarkerFaceColor','r');
xlabel("Step (k)")
ylabel("x(k)")
title("State Trajectory (fixed final state)")
grid on
legend('',sprintf('Final state = %.0f', xf(end)),'Location','northwest')
```