

MECE 6397

Spring 2026

Piero Risi Mortola

Assignment 1

Workflow:

1. Set up the context of the chat. I have found this is crucial since ChatGPT seems to love giving wordy/winding answers.

Your role is a dynamics systems modeling expert. Focus only on what I am asking you to do, be concise.

Understood. I'll answer strictly as a dynamics systems modeling expert, focused and concise.

🔗 ↻ ⚡ ⌂ ⌂ ...

2. Give chat a screenshot of the lumped parameter model we will be using. Asked chat to solve for the effective stiffness and mass to cross-reference the expected values.

What is K and M in terms of the beam's geometry and properties?

🔗 ⌂

Assuming **axial vibration** of a **uniform fixed-free beam** with tip displacement $x(t)$:

Effective stiffness

$$K = \frac{EA}{L}$$

Effective mass

$$M = \frac{1}{3} \rho A L$$

Justification:

- Axial strain is uniform → stiffness reduces exactly to EA/L .
- Axial mode shape for a fixed-free rod is linear → consistent-mass reduction gives $M = \int_0^L \rho A \phi^2(x) dx = \rho A L / 3$.

🔗 ↻ ⚡ ⌂ ⌂ ...

3. Asked the chat to solve for the damping coefficient, ζ , and the natural frequency, ω_n .
 Answers match expectations.

$$\ddot{x}(t) + 2\zeta\omega_n\dot{x}(t) + \omega_n^2x(t) = \frac{1}{M}F_{Input}$$

Using the general form of a second order differential equation, define zeta and omega_n in terms of M, K and B

Start with your model:

$$\ddot{x} + \frac{B}{M}\dot{x} + \frac{K}{M}x = \frac{1}{M}F(t)$$

Match to:

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2x = \frac{1}{M}F(t)$$

So,

$$\omega_n = \sqrt{\frac{K}{M}}, \quad \zeta = \frac{B}{2\sqrt{KM}} = \frac{B}{2M\omega_n}.$$

□ ⚡ ⚡ ⌂ ⌂ ...

4. When checking for any errors, chat keeps track of the assumptions that we have made. It also serves as another way to check whether chat is on the same page.

So far are there any errors

□ ⚡

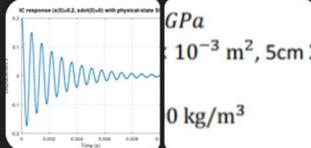
No errors so far **given the assumptions** (axial vibration, uniform bar, using a single-DOF reduction).

Two caveats to keep straight:

- Your written “fixed–free beam” picture is fine for **axial** motion, but “cantilever beam” often implies **bending**; for bending, K and M are different.
- B is a lumped viscous damper term; it’s not directly a basic material property unless you assume a damping model (e.g., Kelvin–Voigt, Rayleigh, or fit to data).

□ ⚡ ⚡ ⌂ ⌂ ...

5. I then asked it to estimate ζ and B from the plot, using the attached parameters for the beam. It first gave me a different answer than the slides, so I asked it to recheck the peak values. The new results, $\zeta \approx 0.046$ and $B \approx 5.3 \times 10^3$, are close enough to the actual values.



GPa
 $10^{-3} \text{ m}^2, 5\text{cm} \times$
 0 kg/m^3

From this experimental data, estimate zeta and B in units of Ns/m given the initial conditions $x(0)=0.2$, $xdot(0) = 0$.

Thought for 5m 17s >

Using the **log decrement** from successive positive peaks in the trace:

- Pick early peaks (to avoid noise floor). From the plot I read approx $x_1 \approx 0.130$, $x_2 \approx 0.0867$, $x_3 \approx 0.0548$, $x_4 \approx 0.0306$, $x_5 \approx 0.0126 \text{ m}$.
 Log decrements: $\delta_i = \ln(x_i/x_{i+1}) \rightarrow \text{average } \bar{\delta} \approx 0.583$.

$$\zeta = \frac{\bar{\delta}}{\sqrt{(2\pi)^2 + \bar{\delta}^2}} \approx 0.092$$

Now compute B using your lumped model ($2\zeta\omega_n = B/M$, $\omega_n = \sqrt{K/M}$):

Beam properties (axial SDOF reduction):

$$K = \frac{EA}{L} = \frac{(200 \times 10^9)(2.5 \times 10^{-3})}{1} = 5.0 \times 10^8 \text{ N/m}$$

$$M = \frac{1}{3}\rho AL = \frac{1}{3}(7900)(2.5 \times 10^{-3})(1) = 6.58 \text{ kg}$$

$$B = 2\zeta\sqrt{KM} \approx 2(0.092)\sqrt{(5.0 \times 10^8)(6.58)} \approx 1.06 \times 10^4 \text{ N}\cdot\text{m}$$

Estimates: $\zeta \approx 0.09$, $B \approx 1.1 \times 10^4 \text{ N}\cdot\text{m}$.

↶ ↷ ↹ ↻ ⋮



I am not getting that same zeta value, recheck your peaks

Thought for 4m 13s >

You're right — my earlier peaks were off.

Re-reading the **positive** peaks from the plot (image-digitized), I get approximately:

$$x_1 = 0.147, x_2 = 0.110, x_3 = 0.0820, x_4 = 0.0621, x_5 = 0.0456 \text{ m}$$

Log decrements:

$$\delta_i = \ln\left(\frac{x_i}{x_{i+1}}\right) \approx [0.289, 0.295, 0.278, 0.309]$$

$$\bar{\delta} \approx 0.293$$

Convert to damping ratio (no small- δ assumption):

$$\zeta = \frac{\bar{\delta}}{\sqrt{(2\pi)^2 + \bar{\delta}^2}} \approx \frac{0.293}{\sqrt{39.48 + 0.086}} \approx 0.046$$

Now compute B . Using your axial SDOF reductions:

$$K = \frac{EA}{L} = 5.0 \times 10^8 \text{ N/m}, \quad M = \frac{1}{3}\rho A L = 6.58 \text{ kg}$$

$$B = 2\zeta\sqrt{KM} \approx 2(0.046)\sqrt{(5.0 \times 10^8)(6.58)} \approx 5.3 \times 10^3 \text{ N}\cdot\text{cdotp}\text{s}/\text{m}$$

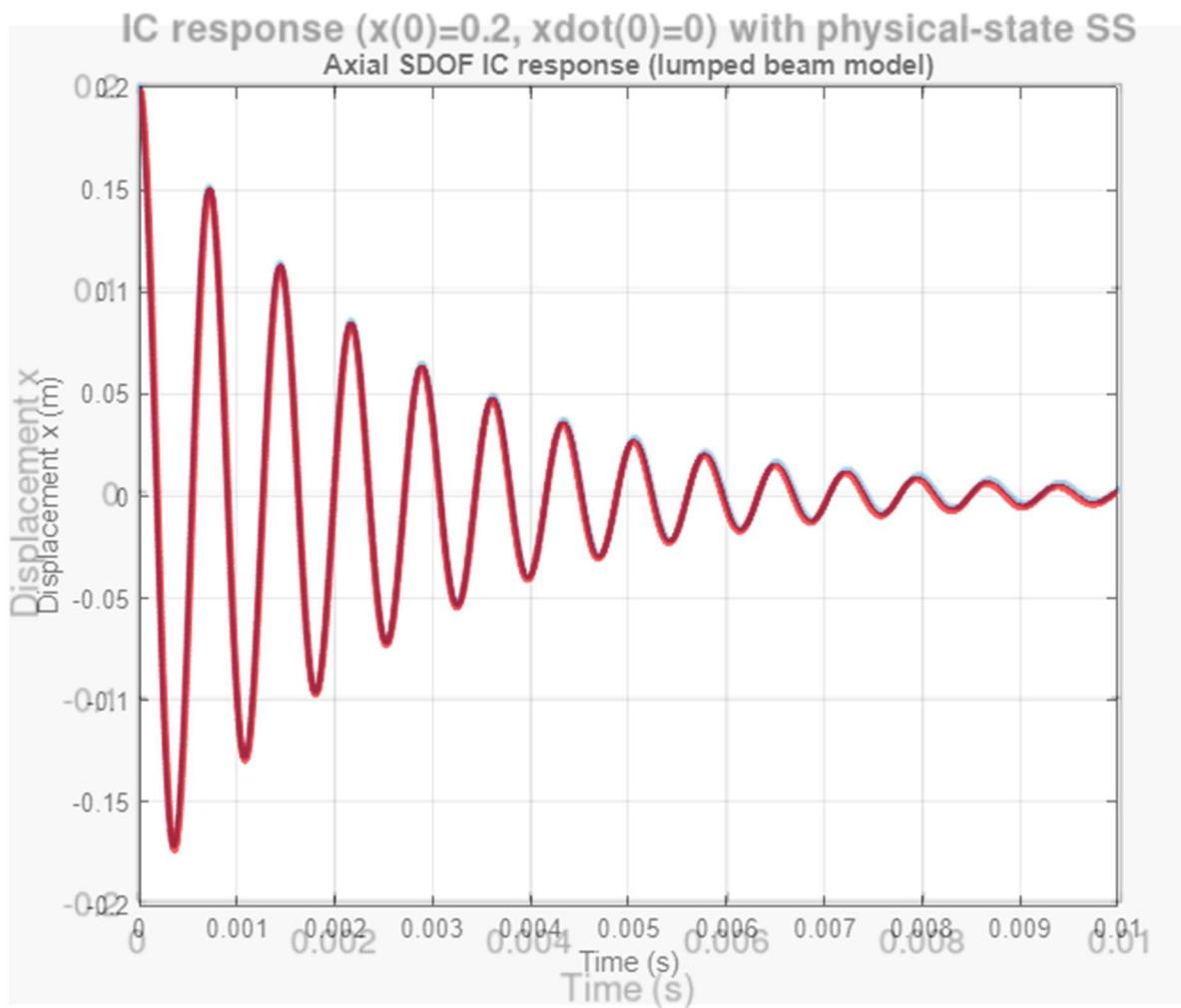
Result: $\zeta \approx 0.046$, $B \approx 5.3 \times 10^3 \text{ N}\cdot\text{cdotp}\text{s}/\text{m}$.

◻ ⚡ ⚡ ⌄ ⌄ ...

MATLAB Code:

```
1 %% Piero Risi Mortola
2 % Assignment 1 - MECE 6397
3 %
4 % Axial SDOF beam IC response (lumped model) + comparison plot
5 % Model: xddot + (B/M) xdot + (K/M) x = 0
6 % ICs: x(0)=0.2 m, xdot(0)=0 m/s
7
8 clear; clc; close all;
9
10 %% Given beam properties
11 E = 200e9; % Pa (N/m^2)
12 A = 2.5e-3; % m^2
13 L = 1.0; % m
14 rho = 7900; % kg/m^3
15
16 %% Lumped parameters for axial vibration (uniform bar, fixed-free)
17 K = E*A/L; % N/m
18 M = (1/3)*rho*A*L; % kg (effective axial modal mass)
19
20 %% Identified damping from data
21 zeta = 0.046; % (-) estimated damping ratio
22 wn = sqrt(K/M); % rad/s (undamped natural frequency)
23 B = 2*zeta*wn*M; % N*s/m (viscous damping coefficient)
24
25 %% Initial conditions
26 x0 = 0.2; % m
27 v0 = 0.0; % m/s
28 xIC = [x0; v0];
29
30 %% Simulation time
31 tEnd = 0.01; % s
32 tspan = [0 tEnd];
33
34 %% ODE definition (free response: F(t)=0)
35 ode = @(t, x) [ ...
36     x(2); ...
37     -(B/M)*x(2) - (K/M)*x(1) ...
38 ];
39
40 %% Simulate
41 opts = odeset('RelTol',1e-8,'AbsTol',1e-10);
42 [t, x] = ode45(ode, tspan, xIC, opts);
43
44 %% Plot displacement
45 figure;
46 plot(t, x(:,1), 'LineWidth', 3, 'Color','r');
47 grid on;
48 xlabel('Time (s)');
49 ylabel('Displacement x (m)');
50 title('Axial SDOF IC response (lumped beam model)');
51
52 %% Print key numbers (sanity check)
53 fprintf('K = %.3e N/m\n', K);
54 fprintf('M = %.3f kg\n', M);
55 fprintf('wn = %.2f rad/s (%.2f Hz)\n', wn, wn/(2*pi));
56 fprintf('zeta = %.4f (-)\n', zeta);
57 fprintf('B = %.3e N*s/m\n', B);
58
```

The MATLAB code generated the following plot in red. This is overlayed the plot from the slides and they do match up closely.



Experiences with AI:

- Used for guidance when learning new topics, creating outlines of concepts, etc.
- Have used it to double check work in various projects ranging from tensegrity systems to optimal controls. I have found it tends to go into irrelevant topics.
- When asking for information that I am not familiar enough to tell if what it is saying is correct, I will ask for sources or look up the topic myself. There has been a lot of times that it has been completely off, though it is often because my questions were not worded well enough.
- For this assignment, the generated code used `ode45()` to run the simulation rather than `lsim()` in the slides. Probably useful to have the state space representation for later use.