

自然演繹 (NK) の推論規則:

$$(\wedge I) \quad \frac{\varphi \quad \psi}{\varphi \wedge \psi}$$

$$(\wedge E) \quad \frac{\varphi \wedge \psi}{\varphi} \quad \frac{\varphi \wedge \psi}{\psi}$$

$$(\rightarrow I) \quad \frac{\Gamma \quad \varphi}{\vdots} \Rightarrow \frac{\Gamma}{\varphi \rightarrow \psi}$$

$$(\rightarrow E) \quad \frac{\varphi \quad \varphi \rightarrow \psi}{\psi}$$

$$(\vee I) \quad \frac{\varphi}{\varphi \vee \psi} \quad \frac{\psi}{\varphi \vee \psi}$$

$$(\vee E) \quad \frac{\Gamma \quad \varphi}{\vdots} \text{ and } \frac{\Gamma \quad \psi}{\vdots} \Rightarrow \frac{\Gamma \quad \varphi \vee \psi}{\chi}$$

$$(\perp I) \quad \frac{\varphi \quad \neg \varphi}{\perp}$$

$$(\neg I) \quad \frac{\Gamma \quad \varphi}{\vdots} \Rightarrow \frac{\Gamma}{\neg \varphi}$$

$$(\neg E) \quad \frac{\neg \neg \varphi}{\varphi}$$

$$(\forall E) \quad \frac{\forall x(\varphi)}{\varphi[t/x]}$$

$$(\forall I) \quad \frac{\Gamma}{\vdots} \Rightarrow \frac{\Gamma}{\forall y(\varphi[y/x])} \quad \text{where } x, y \notin FV(\Gamma)$$

$$(\exists I) \quad \frac{\varphi[t/x]}{\exists x(\varphi)}$$

$$(\exists E) \quad \frac{\Gamma \quad \varphi[y/x]}{\vdots} \Rightarrow \frac{\Gamma \quad \exists x(\varphi)}{\psi} \quad \text{where } x, y \notin FV(\Gamma, \psi)$$

* 代入は可べ2代入条件を満たす可と可也.

V. 1, 3 のみで「原子論理記号」として

$$\varphi \wedge \psi \equiv \neg ((\neg \varphi) \vee (\neg \psi)),$$

$$\varphi \rightarrow \psi \equiv (\neg \varphi) \vee \psi.$$

$$^{\forall}x(\varphi) \equiv \neg(\exists x(\neg\varphi))$$

と区別する場合、 $(\wedge I), (\wedge E), (\rightarrow I), (\rightarrow E), (\vee E), (\vee I)$ は $(\vee I), (\vee E), (\perp I), (\perp E), (\exists I), (\exists E)$ から区別する:

$$\begin{array}{c}
 (\wedge I) \quad \frac{\varphi \quad [\neg \varphi]_1 (\perp I)}{\perp} \quad \frac{\psi \quad [\neg \psi]_1 (\perp I)}{\perp} \quad \frac{}{[(\neg \varphi) \vee (\neg \psi)]_2 (\vee I)} \\
 \hline
 \perp \\
 \hline
 \neg ((\neg \varphi) \vee (\neg \psi)) \quad 2 \quad (\neg I)
 \end{array}$$

$$\begin{array}{rcl}
 (\wedge E) & \frac{[\varphi]_1}{(\neg \varphi) \vee (\neg \psi)} & (vI) \\
 & \frac{(\neg \varphi) \vee (\neg \psi)}{\perp} & \neg((\neg \varphi) \vee (\neg \psi)) \quad (LI) \\
 & \frac{\perp}{\neg \neg \varphi} & 1 \quad (\neg I) \\
 & \frac{\neg \neg \varphi}{\varphi} & (\neg E)
 \end{array}$$

(もう1つも同様)

$$\begin{array}{c}
 (\rightarrow I) \quad \frac{\Gamma \quad [\varphi]_1}{\vdots} \\
 \frac{\varphi \quad [\neg \varphi]_2}{\perp} \quad (\perp I) \\
 \frac{\perp}{\neg \varphi} \quad (\neg I) \\
 \frac{\neg \varphi}{(\neg \varphi) \vee \varphi} \quad (\vee I) \\
 \frac{[\varphi]_2}{(\neg \varphi) \vee \varphi} \quad (\vee I) \\
 \frac{(\neg \varphi) \vee \varphi}{(\neg \varphi) \vee \varphi} \quad (\vee E)
 \end{array}$$

$$\frac{\frac{\frac{[\psi]_1}{\psi \vee (\neg \psi)} (vI)}{\frac{[\psi \vee (\neg \psi)]_2}{1} (vI)} \quad \frac{\frac{\neg \psi}{\psi \vee (\neg \psi)} (vI)}{\frac{[\psi \vee (\neg \psi)]_2}{1} (vI)} \quad \frac{\frac{[\psi \vee (\neg \psi)]_2}{1} (vI)}{2} (vI)$$

(\rightarrow E)

φ	$[\neg\varphi]_1$	$(\perp I)$	
\perp			$[\varphi]_1$
\vdash			φ
\vdash			$(\neg\varphi) \vee \varphi$
			1 ($\vee E$)

φ

$\frac{\perp \quad [\neg\varphi]_1}{\perp} 1$
 $\frac{\perp}{\neg\varphi} 1$
 $\frac{\neg\varphi}{\varphi} 1$

$$\begin{array}{l}
 (\forall E) \quad \frac{\frac{\frac{[\neg \varphi[t/x]]_1}{\neg(\exists x(\neg \varphi))} \quad \exists x(\neg \varphi)}{\perp} \quad (\exists I) \quad (\perp I)}{\neg \neg \varphi[t/x]} \quad 1 (\neg I) \\
 \hline
 \varphi[t/x] \quad (\neg E)
 \end{array}$$

$$\begin{array}{l}
 (\forall I) \quad \frac{\frac{\frac{\Gamma}{\vdots} \quad \neg \varphi[y/x][x/y]}{\varphi \quad [\neg \varphi]_1} \quad (\perp I)}{\perp} \quad \exists y(\neg \varphi[y/x]) \quad 1 (\exists E) \\
 \hline
 \neg \exists y(\neg \varphi[y/x]) \quad 2 (\neg I)
 \end{array}$$

$(\exists E)$ Σx と y の役割を交換して適用