Binary Classification

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Vector and Matrix Derivatives

向量和矩阵导数

Derivative of Scalar w.r.t. Scalar

Examples:

- $y = x^2; \quad \frac{dy}{dx} = 2x.$
- $y = e^x; \quad \frac{dy}{dx} = e^x.$

Derivative of Vector w.r.t. Scalar

- The derivative of a vector $\mathbf{y} \in \mathbb{R}^n$ w.r.t. a scalar $x \in \mathbb{R}$:

$$\frac{\partial \mathbf{y}}{\partial x} = \begin{bmatrix} \frac{\partial y_1}{\partial x} \\ \frac{\partial y_2}{\partial x} \\ \vdots \\ \frac{\partial y_n}{\partial x} \end{bmatrix}$$

- Example:

$$\mathbf{y} = \begin{bmatrix} 3x^2 \\ x+1 \\ \log x \\ e^x \end{bmatrix}, \qquad \frac{\partial \mathbf{y}}{\partial x} = \begin{bmatrix} 6x \\ 1 \\ 1/x \\ e^x \end{bmatrix}$$

Derivative of Scalar w.r.t. Vector

- The derivative of a scalar $y \in \mathbb{R}$ w.r.t. a vector $\mathbf{x} \in \mathbb{R}^m$:

$$\frac{\partial\, y}{\partial\, \mathbf{x}} = \begin{bmatrix} \frac{\partial y}{\partial x_1} \\ \frac{\partial y}{\partial x_2} \\ \vdots \\ \frac{\partial y}{\partial x_m} \end{bmatrix}$$

- Example 1:

$$y = \|\mathbf{x}\|_2^2 = \sum_{i=1}^{m} x_i^2, \qquad \frac{\partial\, y}{\partial\, \mathbf{x}} = 2\mathbf{x}.$$

Derivative of Scalar w.r.t. Vector

- The derivative of a scalar $y \in \mathbb{R}$ w.r.t. a vector $\mathbf{x} \in \mathbb{R}^m$:

$$\frac{\partial\, y}{\partial\, \mathbf{x}} = \begin{bmatrix} \frac{\partial y}{\partial x_1} \\ \frac{\partial y}{\partial x_2} \\ \vdots \\ \frac{\partial y}{\partial x_m} \end{bmatrix}$$

- Example 2:

$$y = \mathbf{x}^T \mathbf{z} = \sum_{i=1}^{m} x_i z_i, \qquad \frac{\partial\, y}{\partial\, \mathbf{x}} = \mathbf{z}.$$

Derivative of Scalar w.r.t. Vector

- The derivative of a scalar $y \in \mathbb{R}$ w.r.t. a vector $\mathbf{x} \in \mathbb{R}^m$:

$$\frac{\partial\, y}{\partial\, \mathbf{x}} = \begin{bmatrix} \frac{\partial y}{\partial x_1} \\ \frac{\partial y}{\partial x_2} \\ \vdots \\ \frac{\partial y}{\partial x_m} \end{bmatrix}$$

- Example 3:

$$y = \sum_{i=1}^{m} \log(1 + e^{-x_i}), \qquad \frac{\partial\, y}{\partial\, \mathbf{x}} = \begin{bmatrix} \frac{\partial \log(1+e^{-x_1})}{\partial x_1} \\ \vdots \\ \frac{\partial \log(1+e^{-x_m})}{\partial x_m} \end{bmatrix} = \begin{bmatrix} -\frac{1}{1+e^{x_1}} \\ \vdots \\ -\frac{1}{1+e^{x_m}} \end{bmatrix}$$

Derivative of Vector w.r.t. Vector

- The derivative of a vector $\mathbf{y} \in \mathbb{R}^n$ w.r.t. a vector $\mathbf{x} \in \mathbb{R}^m$:

$$\frac{\partial \mathbf{y}}{\partial \mathbf{x}} = \begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_2}{\partial x_1} & \cdots & \frac{\partial y_n}{\partial x_1} \\ \frac{\partial y_1}{\partial x_2} & \frac{\partial y_2}{\partial x_2} & \cdots & \frac{\partial y_n}{\partial x_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial y_1}{\partial x_m} & \frac{\partial y_2}{\partial x_m} & \cdots & \frac{\partial y_n}{\partial x_m} \end{bmatrix}$$

$m \times n$ matrix

The (i, j)-th entry is $\frac{\partial y_j}{\partial x_i}$

- Example 1:

$$\frac{\partial \mathbf{x}}{\partial \mathbf{x}} = \underbrace{\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}}_{m \times m}$$

Derivative of Vector w.r.t. Vector

- The derivative of a vector $\mathbf{y} \in \mathbb{R}^n$ w.r.t. a vector $\mathbf{x} \in \mathbb{R}^m$:

$$\frac{\partial \mathbf{y}}{\partial \mathbf{x}} = \begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_2}{\partial x_1} & \cdots & \frac{\partial y_n}{\partial x_1} \\ \frac{\partial y_1}{\partial x_2} & \frac{\partial y_2}{\partial x_2} & \cdots & \frac{\partial y_n}{\partial x_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial y_1}{\partial x_m} & \frac{\partial y_2}{\partial x_m} & \cdots & \frac{\partial y_n}{\partial x_m} \end{bmatrix}$$

$m \times n$ matrix

- Example 2:

$$\mathbf{y} = \begin{bmatrix} a_1 x_1^2 \\ a_2 x_2^2 \\ \vdots \\ a_m x_m^2 \end{bmatrix} \in \mathbb{R}^m, \qquad \frac{\partial \mathbf{y}}{\partial \mathbf{x}} = \underbrace{\begin{bmatrix} 2a_1x_1 & 0 & \cdots & 0 \\ 0 & 2a_2x_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 2a_mx_m \end{bmatrix}}_{m \times m}$$

Derivative of Vector w.r.t. Vector

- The derivative of a vector $\mathbf{y} \in \mathbb{R}^n$ w.r.t. a vector $\mathbf{x} \in \mathbb{R}^m$:

$$\frac{\partial \mathbf{y}}{\partial \mathbf{x}} = \begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_2}{\partial x_1} & \cdots & \frac{\partial y_n}{\partial x_1} \\ \frac{\partial y_1}{\partial x_2} & \frac{\partial y_2}{\partial x_2} & \cdots & \frac{\partial y_n}{\partial x_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial y_1}{\partial x_m} & \frac{\partial y_2}{\partial x_m} & \cdots & \frac{\partial y_n}{\partial x_m} \end{bmatrix}$$

$m \times n$ matrix

- Example 3:

$$\mathbf{A} \in \mathbb{R}^{n\times m}, \qquad \mathbf{y} = \mathbf{A}\mathbf{x} \in \mathbb{R}^n, \qquad \frac{\partial \mathbf{y}}{\partial \mathbf{x}} = \mathbf{A}^T \in \mathbb{R}^{m\times n}$$

Chain Rule

- Let $\mathbf{z} \in \mathbb{R}^{n_z}$ be a function of $\mathbf{y} \in \mathbb{R}^{n_y}$ and $\mathbf{y}$ be a function of $\mathbf{x} \in \mathbb{R}^{n_x}$.

$$\begin{tikzcd} \underset{\mathbf{x} \in \mathbb{R}^{n_x}}{\bullet} \arrow[r] & \underset{\mathbf{y} \in \mathbb{R}^{n_y}}{\bullet} \arrow[r] & \underset{\mathbf{z} \in \mathbb{R}^{n_z}}{\bullet} \end{tikzcd}$$

$$\underbrace{\frac{d\mathbf{z}}{d\mathbf{x}}}_{n_x \times n_z} = \underbrace{\frac{d\mathbf{y}}{d\mathbf{x}}}_{n_x \times n_y} \underbrace{\frac{d\mathbf{z}}{d\mathbf{y}}}_{n_y \times n_z}$$

Derivative of Scalar w.r.t. Matrix

- The derivative of a scalar $y \in \mathbb{R}$ w.r.t. a matrix $\mathbf{Z} \in \mathbb{R}^{p \times q}$:
  1. Vectorization: $\mathbf{x} = \text{vec}(\mathbf{Z}) \in \mathbb{R}^{pq \times 1}$.
  2. Compute $\frac{\partial\, y}{\partial\, \mathbf{x}} \in \mathbb{R}^{pq \times 1}$.
  3. Reshape the resulting $pq \times 1$ vector to $p \times q$ matrix.

Derivative of Vector w.r.t. Matrix

- The derivative of a vector $\mathbf{y} \in \mathbb{R}^{n}$ w.r.t. a matrix $\mathbf{Z} \in \mathbb{R}^{p\times q}$:
  1. Vectorization: $\mathbf{x} = \mathrm{vec}(\mathbf{Z}) \in \mathbb{R}^{pq\times 1}$.
  2. Compute $\frac{\partial\, \mathbf{y}}{\partial\, \mathbf{x}} \in \mathbb{R}^{pq\times n}$.
  3. Reshape the resulting $pq\times n$ matrix to $p\times q\times n$ tensor.

Binary Classification

Tasks

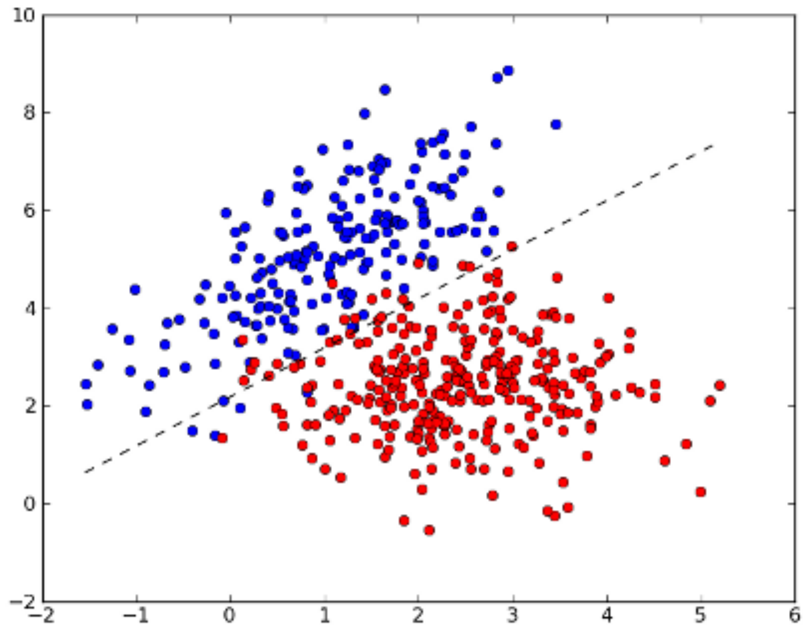
Methods

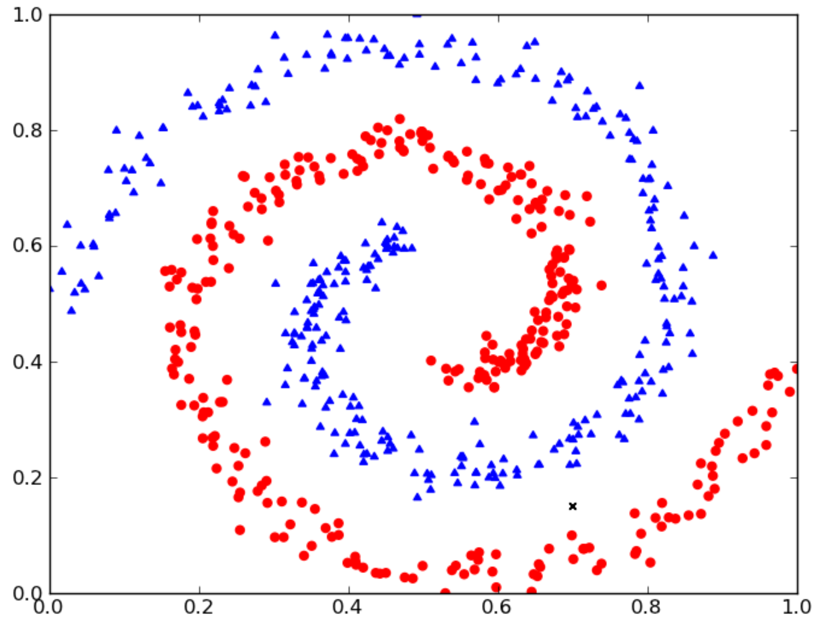
Algorithms

Binary Classification

Input: feature vectors $\mathbf{x}_1, \cdots, \mathbf{x}_n \in \mathbb{R}^d$ and labels $y_1, \cdots, y_n \in \{-1, +1\}$.

Output: a function $f: \mathbb{R}^d \mapsto \{-1, +1\}$.





Linear Classification

Nonlinear Classification

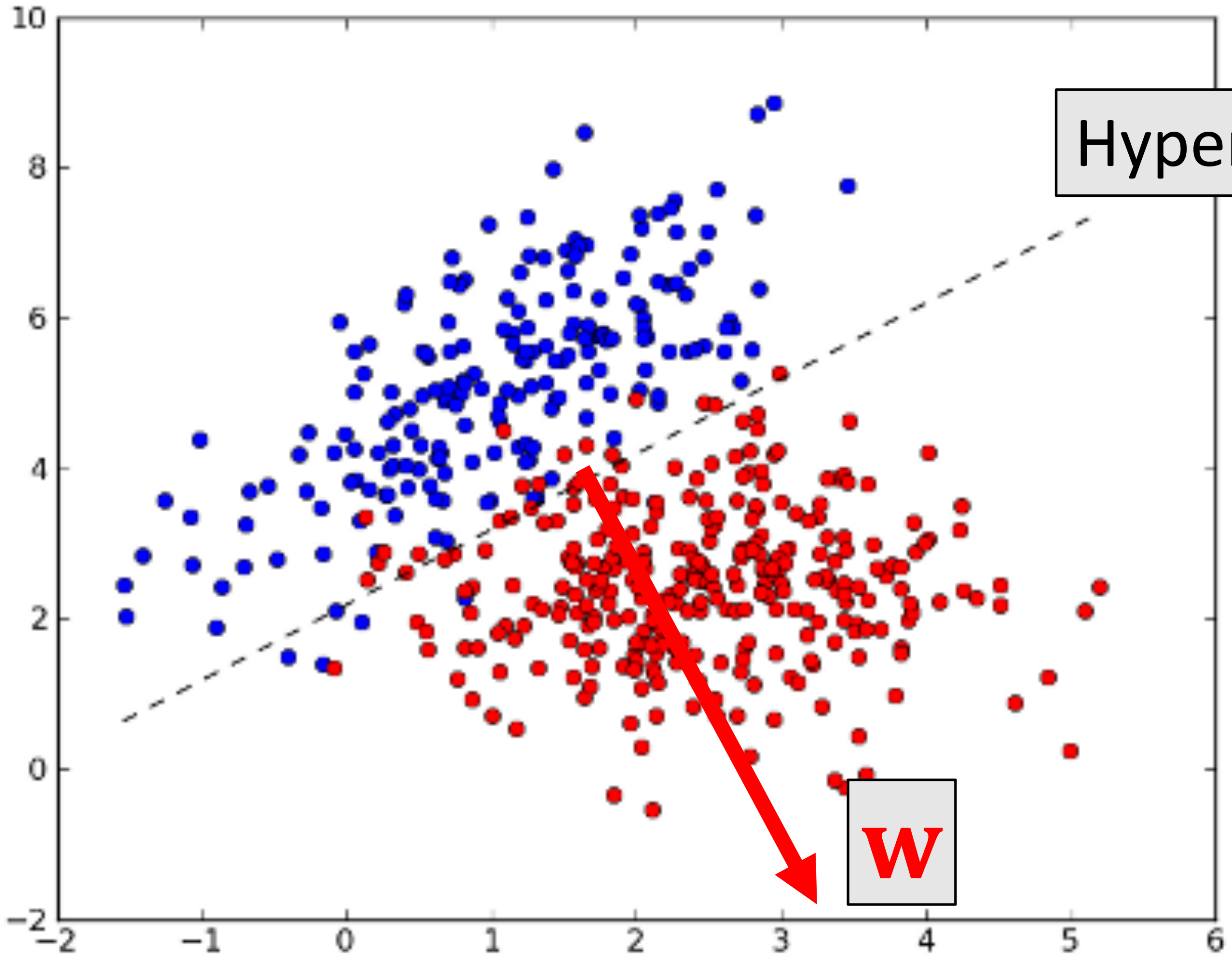
Logistic Regression (Linear Classifier)

Tasks

Methods

Algorithms

Linear Classifier



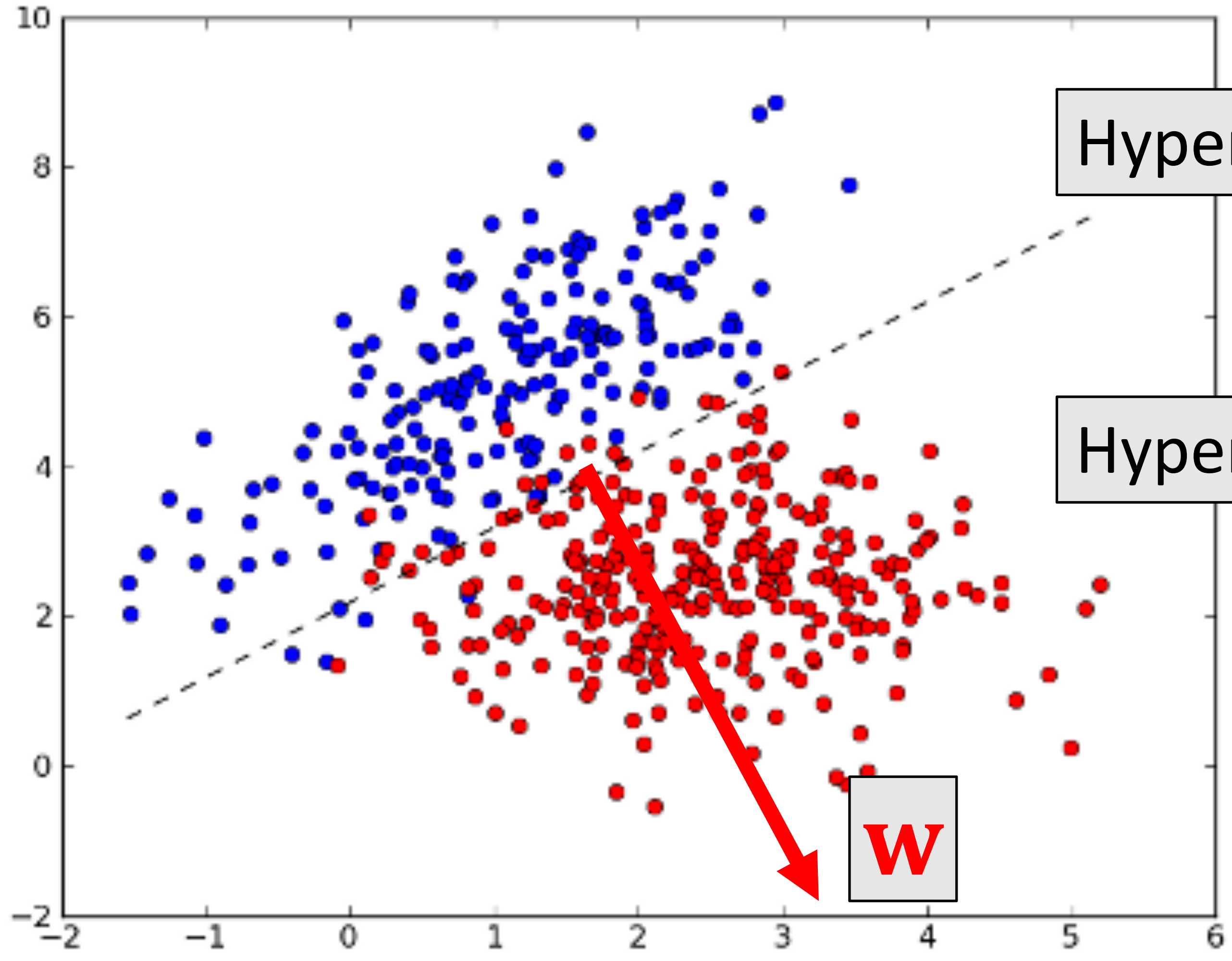
Hyperplane: $\mathbf{w}^T\mathbf{x} + b = 0$

Define $\bar{\mathbf{x}}_j = [\mathbf{x}_j; 1] \in \mathbb{R}^{d+1}$

Define $\bar{\mathbf{w}} = [\mathbf{w};\ b] \in \mathbb{R}^{d+1}$

➔ $\mathbf{x}_j^T\mathbf{w} + b = \bar{\mathbf{x}}_j^T\bar{\mathbf{w}}$

Linear Classifier

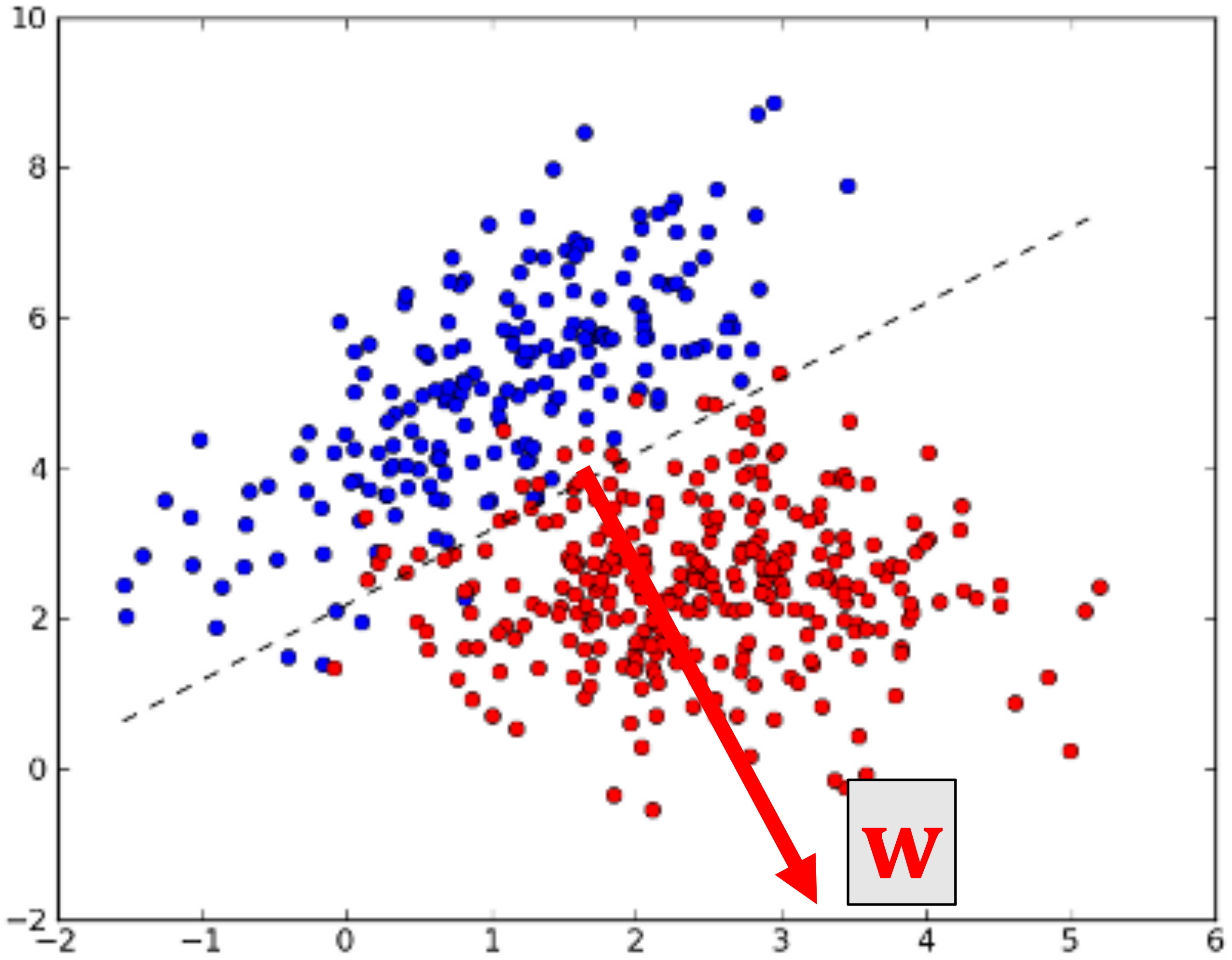


Hyperplane: $\mathbf{w}^T\mathbf{x} + b = 0$

Hyperplane: $\bar{\mathbf{w}}^T\bar{\mathbf{x}} = 0$

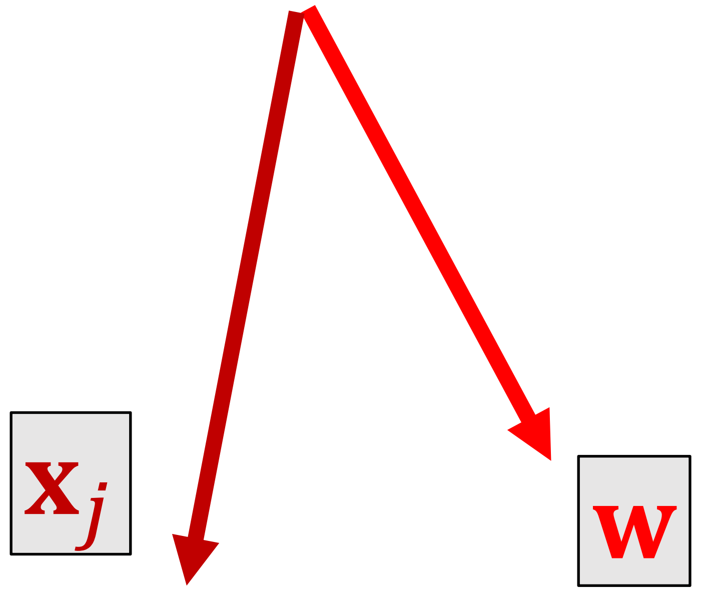
(The intercept is removed.)

Linear Classifier

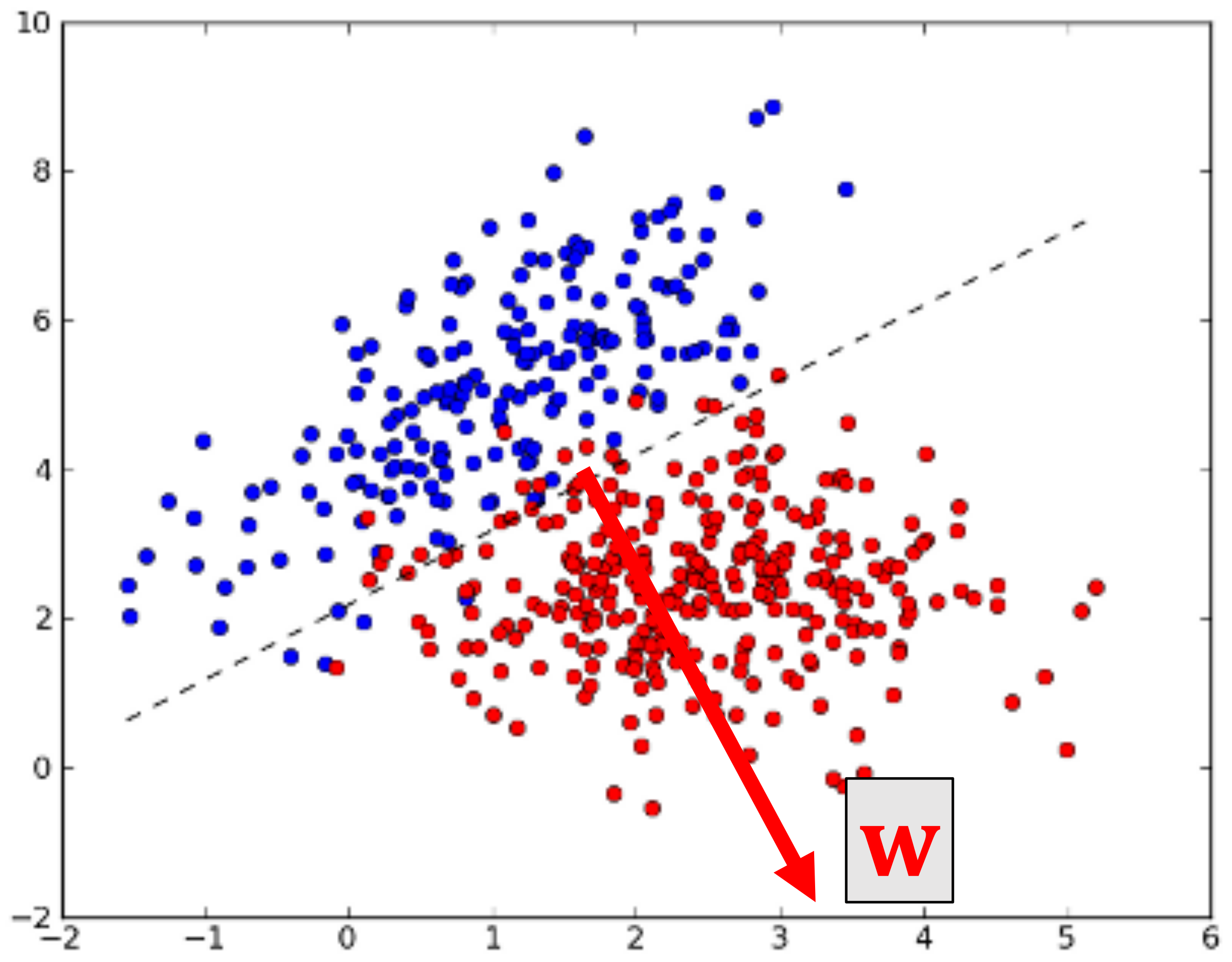


Learn a vector $\mathbf{w}$ such that

- If $y_j = +1$, then $\mathbf{w}^T\mathbf{x}_j > 0$.

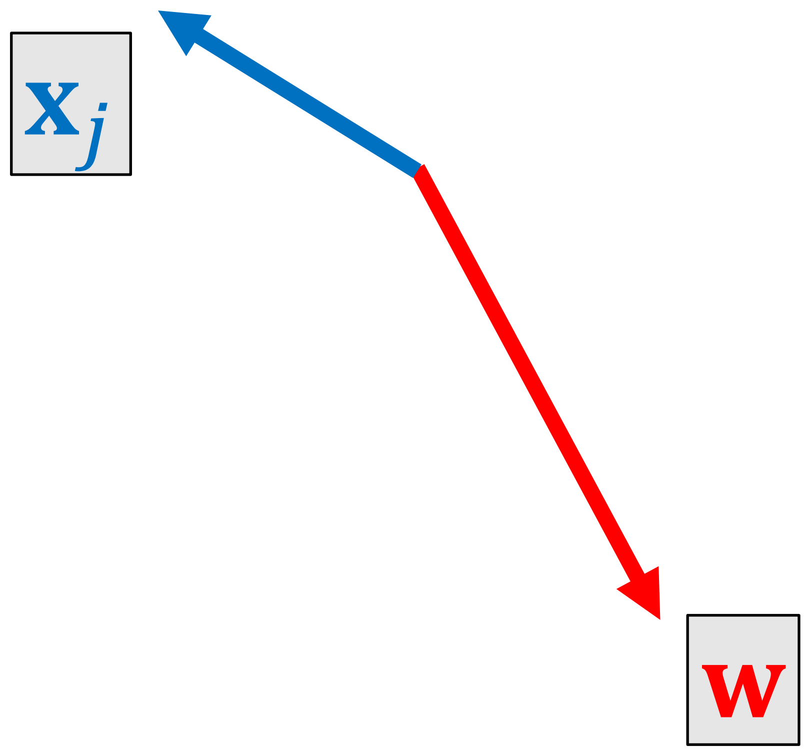


Linear Classifier

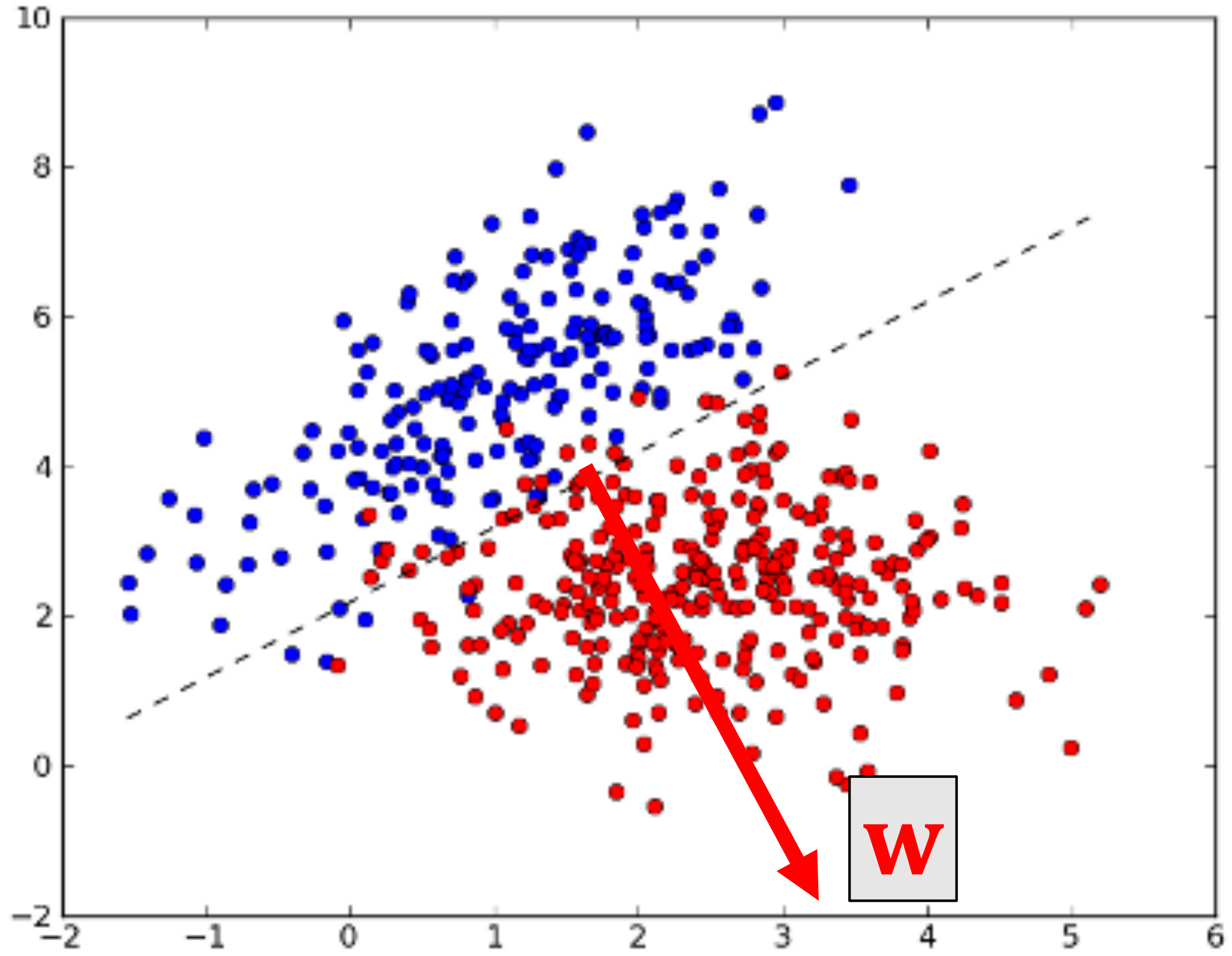


Learn a vector $\mathbf{w}$ such that

- If $y_j = +1$, then $\mathbf{w}^T \mathbf{x}_j > 0$.
- If $y_j = -1$, then $\mathbf{w}^T \mathbf{x}_j < 0$.



Linear Classifier



Learn a vector $\mathbf{w}$ such that

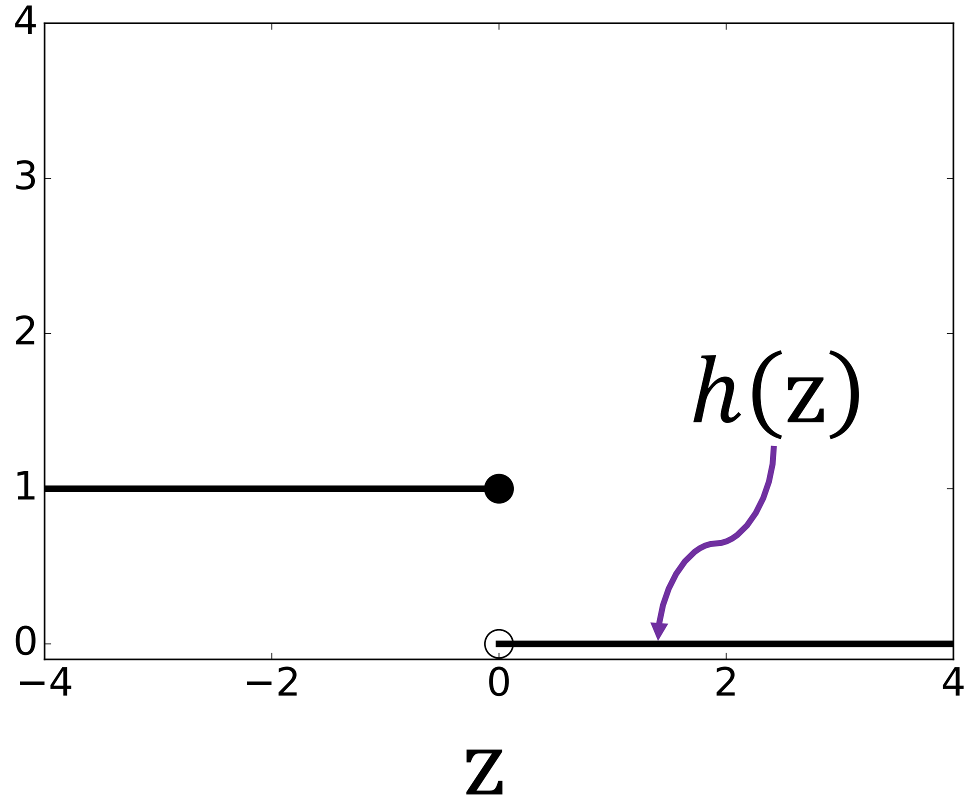
- If $y_j = +1$, then $\mathbf{w}^T \mathbf{x}_j > 0.$
- If $y_j = -1$, then $\mathbf{w}^T \mathbf{x}_j < 0.$

Key Idea:

Encourage $y_j \mathbf{w}^T \mathbf{x}_j$ to be positive

Directly Minimize the Classification Error?

Minimize $\sum_j h(y_j \mathbf{w}^T \mathbf{x}_j)$, where $h(z) = \begin{cases} 1, & \text{if } z < 0; \\ 0, & \text{if } z \geq 0. \end{cases}$



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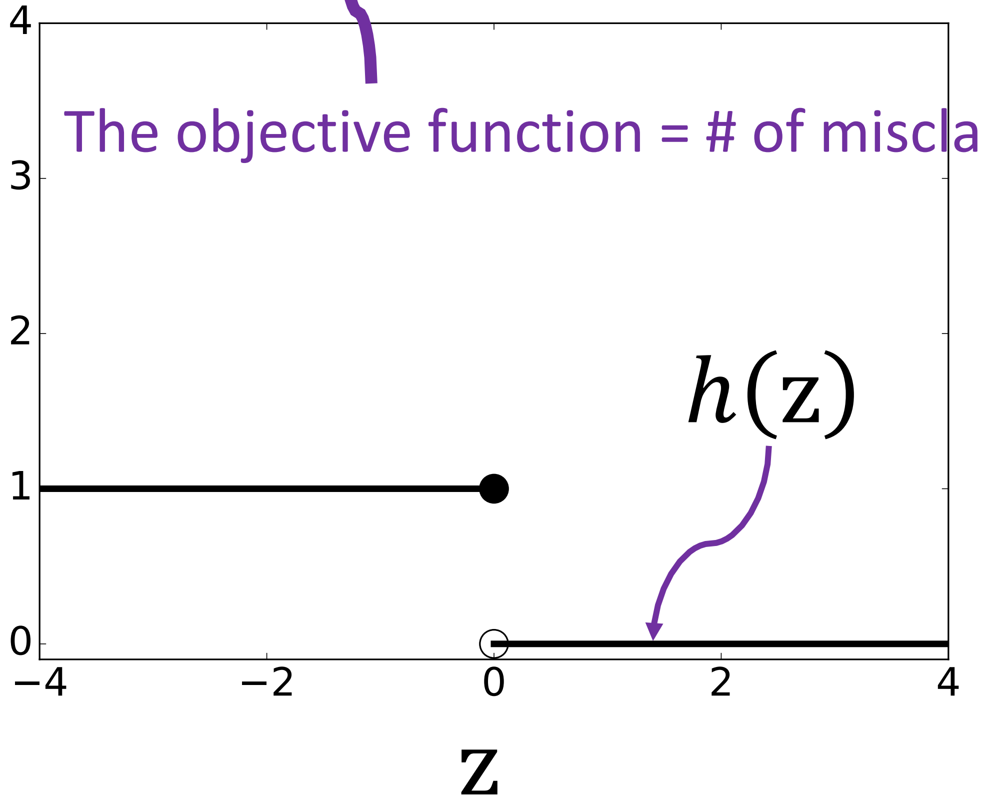
The objective function = # of misclassified training samples



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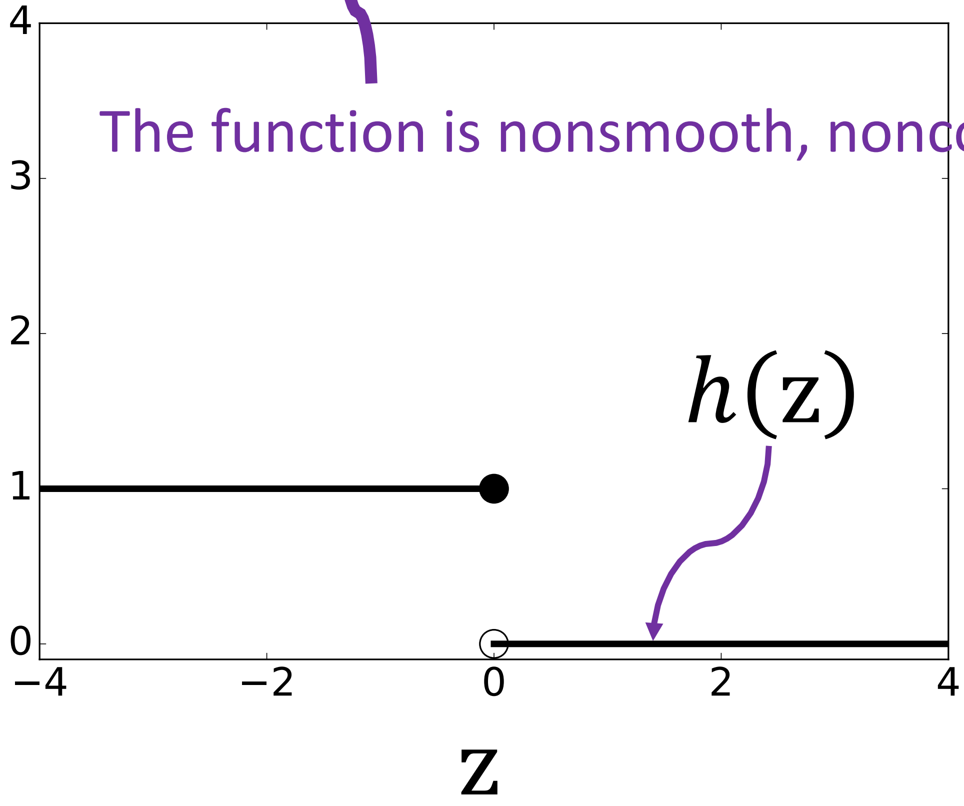
Directly Minimize the Classification Error?

Minimize $\sum_j h\left(y_j \mathbf{w}^T \mathbf{x}_j\right)$, where $h(z) = \begin{cases} 1, & \text{if } z < 0; \\ 0, & \text{if } z \geq 0. \end{cases}$

The function is nonsmooth, nonconvex, and hard to optimize.

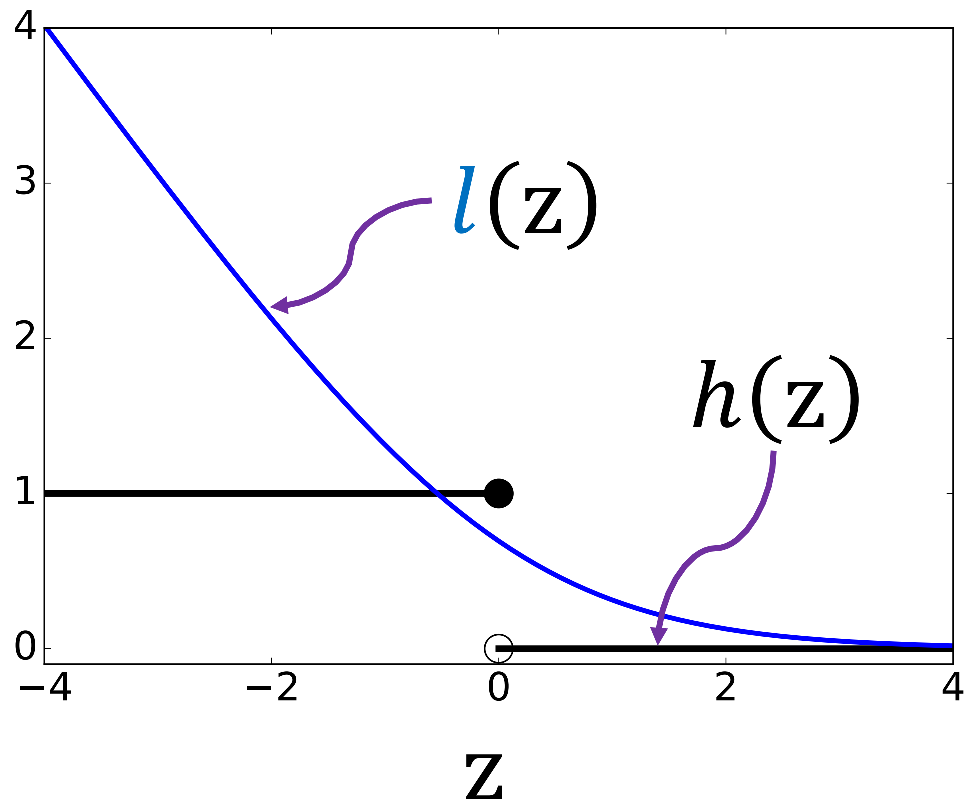


Key Idea:

Encourage $y_j \mathbf{w}^T \mathbf{x}_j$ to be positive

Logistic Regression

$$\boxed{\text{Minimize } \sum_j l(y_j \mathbf{w}^T \mathbf{x}_j), \text{ where } l(z) = \log(1 + e^{-z}).}$$



Key Idea:

Encourage $y_j \mathbf{w}^T \mathbf{x}_j$ to be positive

Logistic Regression

Tasks

Methods

Algorithms

Logistic Regression

Logistic regression: $\min_{\mathbf{w}} \frac{1}{n}\sum_{j=1}^{n} l\left(y_j \mathbf{w}^T \mathbf{x}_j\right)$, where $l(z) = \log(1 + e^{-z})$.

Tasks	Methods	Algorithms
Binary Classification	Logistic Regression	Gradient Descent (GD)
Multi-Class Classification	SVM	Accelerated GD
	Neural Networks	Stochastic GD

Gradient

Logistic regression: $\min_{\mathbf{w}} \frac{1}{n}\sum_{j=1}^{n} l\left(y_j \mathbf{w}^T \mathbf{x}_j\right)$, where $l(z) = \log(1 + e^{-z})$.

$$\begin{tikzcd}
\mathbf{w} \arrow[r] & z = y\,\mathbf{w}^T\mathbf{x} \arrow[r] & l = \log(1 + e^{-z})
\end{tikzcd}$$

Gradient

Logistic regression: $\min_{\mathbf{w}} \frac{1}{n}\sum_{j=1}^{n} l\left(y_j \mathbf{w}^T \mathbf{x}_j\right)$, where $l(z) = \log(1 + e^{-z})$.

$$\begin{tikzcd}
\mathbf{w} \arrow[r] & z = y\,\mathbf{w}^T\mathbf{x} \arrow[r] & l = \log(1+e^{-z})
\end{tikzcd}$$

- $\frac{\partial\, z}{\partial\, \mathbf{w}} = y\mathbf{x}$, $\quad \frac{\partial\, l(z)}{\partial\, z} = \frac{-e^{-z}}{1+e^{-z}} = -\frac{1}{1+e^{z}}$.
- $\boxed{\text{Chain rule: } \frac{\partial\, l}{\partial\, \mathbf{w}} = \frac{\partial\, z}{\partial\, \mathbf{w}} \cdot \frac{\partial\, l}{\partial\, z}} = (y\mathbf{x})\left(-\frac{1}{1+e^{z}}\right) = -\frac{y\mathbf{x}}{1+\exp(y\mathbf{w}^T\mathbf{x})}$.

算出梯度 ：损失函数 对 w的偏导数

Gradient

Logistic regression: $\min_{\mathbf{w}} \frac{1}{n}\sum_{j=1}^{n} l\left(y_j \mathbf{w}^T \mathbf{x}_j\right)$, where $l(z) = \log(1+e^{-z})$.

重要 Gradient at $\mathbf{w}_t$: $\mathbf{g}_t = \frac{1}{n}\sum_{j=1}^{n} \frac{-y_j\, \mathbf{x}_j}{1+\exp(y_j \mathbf{w}_t^T \mathbf{x}_j)}$.

- We have shown: $\frac{\partial\, l(y\mathbf{w}^T\mathbf{x})}{\partial\, \mathbf{w}} = \frac{-y\mathbf{x}}{1+\exp(y\mathbf{w}^T\mathbf{x})}$.
- Objective function: $f(\mathbf{w}) = \frac{1}{n}\sum_j l\left(y_j \mathbf{w}^T \mathbf{x}_j\right)$.
- $\frac{\partial f(\mathbf{w})}{\partial\, \mathbf{w}} = \frac{1}{n}\sum_j \frac{\partial\, l(y_j\mathbf{w}^T\mathbf{x}_j)}{\partial\, \mathbf{w}} = \frac{1}{n}\sum_j \frac{-y_j\, \mathbf{x}_j}{1+\exp(y_j\mathbf{w}^T\mathbf{x}_j)}$.

w_t^T ：在初始时刻t的权重

通项公式

Gradient Descent (GD) Algorithm

Logistic regression: $\min_{\mathbf{w}} \frac{1}{n}\sum_{j=1}^{n} l\left(y_j \mathbf{w}^T \mathbf{x}_j\right)$, where $l(z) = \log(1 + e^{-z})$.

Gradient at $\mathbf{w}_t$: $\mathbf{g}_t = \frac{1}{n}\sum_{j=1}^{n} \frac{-y_j\,\mathbf{x}_j}{1+\exp(y_j \mathbf{w}_t^T \mathbf{x}_j)}$.

GD repeat:

1. Compute gradient: $\mathbf{g}_t$ 计算梯度
2. Update: $\mathbf{w}_{t+1} = \mathbf{w}_t - \alpha\, \mathbf{g}_t$ 更新权值

Tune the step size (learning rate) α

Algorithms

- Gradient Descent (GD)
- Accelerated GD
- Stochastic GD

AGD Algorithm

Logistic regression: $\min_{\mathbf{w}} \frac{1}{n}\sum_{j=1}^{n} l\left(y_j \mathbf{w}^T \mathbf{x}_j\right)$, where $l(z) = \log(1 + e^{-z})$.

Gradient at $\mathbf{w}_t$: $\mathbf{g}_t = \frac{1}{n}\sum_{j=1}^{n} \frac{-y_j \mathbf{x}_j}{1+\exp(y_j \mathbf{w}_t^T \mathbf{x}_j)}$.

AGD repeat:

1. Compute gradient: $\mathbf{g}_t$
2. Update momentum: $\mathbf{v}_{t+1} = \beta \mathbf{v}_t + \mathbf{g}_t$
3. Update: $\mathbf{w}_{t+1} = \mathbf{w}_t - \alpha\, \mathbf{v}_{t+1}$

Tune α and β $(0 \leq \beta < 1)$

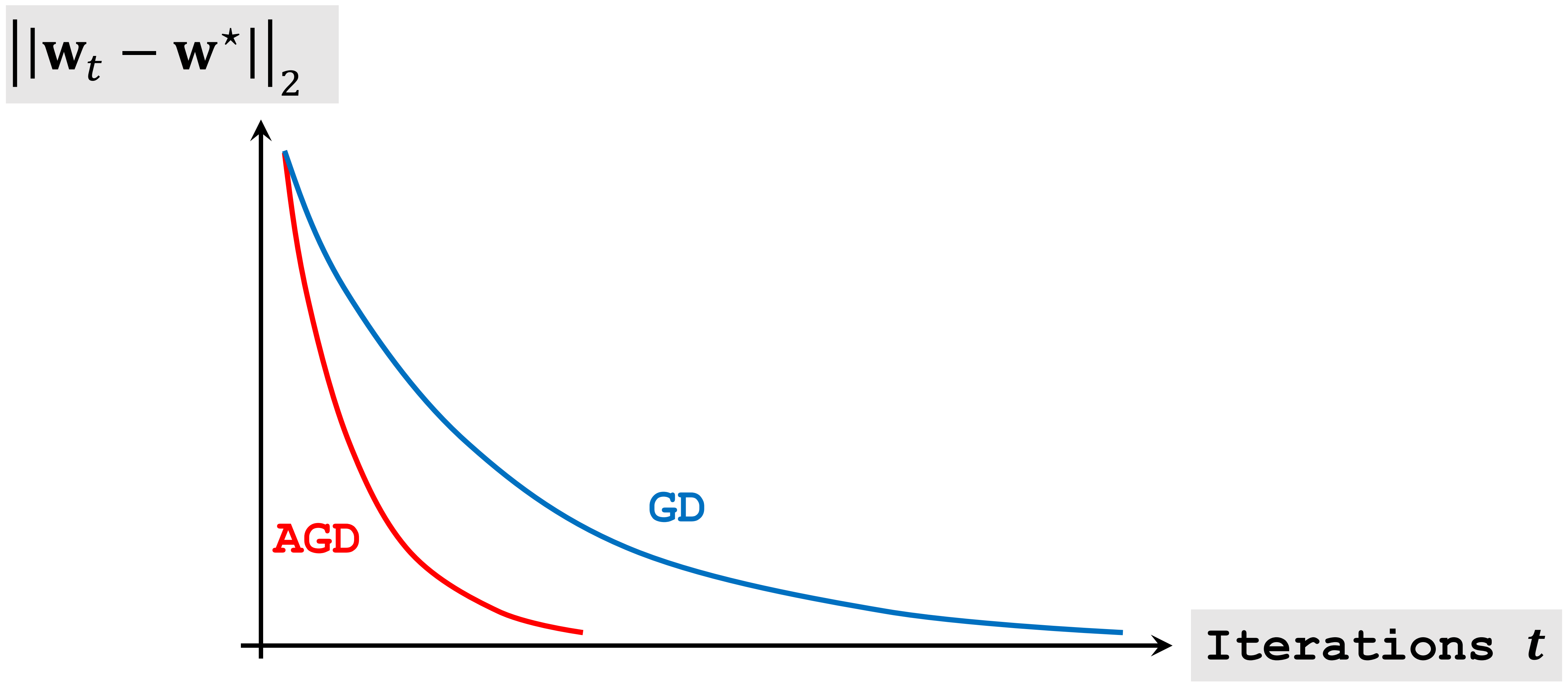
Algorithms

Gradient Descent (GD)

Accelerated GD

Stochastic GD

GD versus AGD



Time Complexity

Gradient at $\mathbf{w}_t$: $\mathbf{g}_t = \frac{1}{n}\sum_{j=1}^{n} \tilde{\mathbf{g}}_{t,j}$, where $\tilde{\mathbf{g}}_{t,j} = \frac{-y_j \mathbf{x}_j}{1+\exp(y_j \mathbf{w}_t^T \mathbf{x}_j)}$.

每次迭代的时间复杂度为

Per-iteration time complexity is $O(nd)$.

- $O(d)$ time for computing $\mathbf{w}_t^T \mathbf{x}_j$.
- $O(d)$ time for computing $\tilde{\mathbf{g}}_{t,j}$.
- $O(nd)$ time for computing all the $\tilde{\mathbf{g}}_{t,j}$.

Algorithms

Gradient Descent (GD)

Accelerated GD

Stochastic GD

SGD Algorithm

Gradient at $\mathbf{w}_t$: $\mathbf{g}_t = \frac{1}{n}\sum_{j=1}^{n} \tilde{\mathbf{g}}_{t,j}$, where $\tilde{\mathbf{g}}_{t,j} = \frac{-y_j \mathbf{x}_j}{1+\exp(y_j \mathbf{w}_t^T \mathbf{x}_j)}$.

The stochastic gradient is close to the full gradient:

$$\mathbf{g}_t = \mathbb{E}_j\big[\tilde{\mathbf{g}}_{t,j}\big],$$

where j is randomly sampled from $\{1, \cdots, n\}$.

SGD Algorithm

Gradient at $\mathbf{w}_t$: $\mathbf{g}_t = \frac{1}{n}\sum_{j=1}^{n} \tilde{\mathbf{g}}_{t,j}$, where $\tilde{\mathbf{g}}_{t,j} = \frac{-y_j \mathbf{x}_j}{1+\exp(y_j \mathbf{w}_t^T \mathbf{x}_j)}$.

SGD repeats

1. Randomly draw j from $\{1, 2, \cdots, n\}$.
2. Compute the stochastic gradient $\tilde{\mathbf{g}}_{t,j}$.
3. Update: $\mathbf{w}_{t+1} = \mathbf{w}_t - \alpha\, \tilde{\mathbf{g}}_{t,j}$.

Per-iteration time complexity is $O(d)$.

Algorithms

Gradient Descent (GD)

Accelerated GD

Stochastic GD

Accelerated SGD Algorithm

Gradient at $\mathbf{w}_t$: $\mathbf{g}_t = \frac{1}{n}\sum_{j=1}^{n} \tilde{\mathbf{g}}_{t,j}$, where $\tilde{\mathbf{g}}_{t,j} = \frac{-y_j \mathbf{x}_j}{1+\exp(y_j \mathbf{w}_t^T \mathbf{x}_j)}$.

Accelerated SGD repeats

1. Randomly draw j from $\{1, 2, \cdots, n\}$.
2. Compute the stochastic gradient $\tilde{\mathbf{g}}_{t,j}$.
3. Update momentum: $\mathbf{v}_{t+1} = \beta \mathbf{v}_t + \tilde{\mathbf{g}}_{t,j}$.
4. Update: $\mathbf{w}_{t+1} = \mathbf{w}_t - \alpha \mathbf{v}_{t+1}$.

SGD Algorithm

Gradient at $\mathbf{w}_t$: $\mathbf{g}_t = \frac{1}{n}\sum_{j=1}^{n} \tilde{\mathbf{g}}_{t,j}$, where $\tilde{\mathbf{g}}_{t,j} = \frac{-y_j \mathbf{x}_j}{1+\exp(y_j \mathbf{w}_t^T \mathbf{x}_j)}$.

Output of SGD:
- Option 1: output the last iteration $\mathbf{w}_{T+1}$
- Option 2: output the average of $\mathbf{w}$ produced by the last tens of iteration.

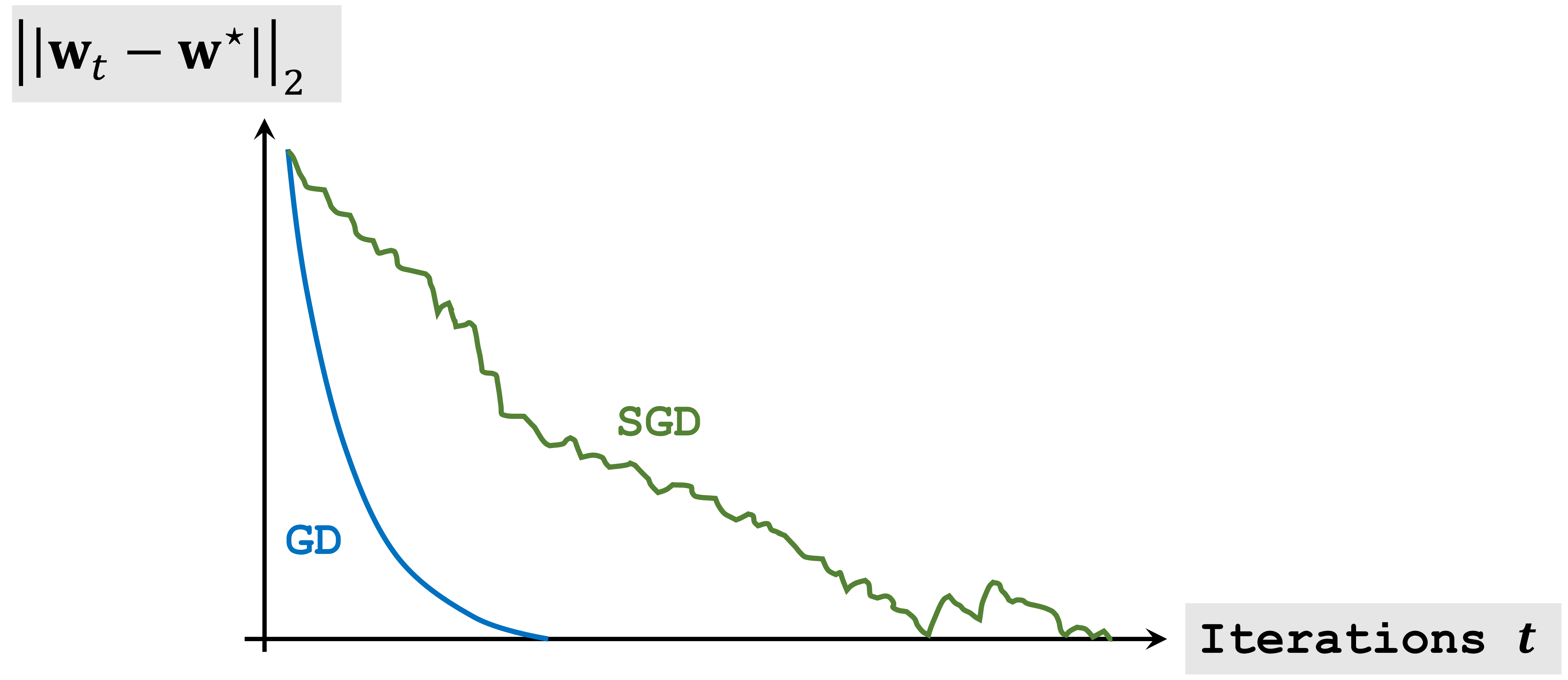
Algorithms

Gradient Descent (GD)

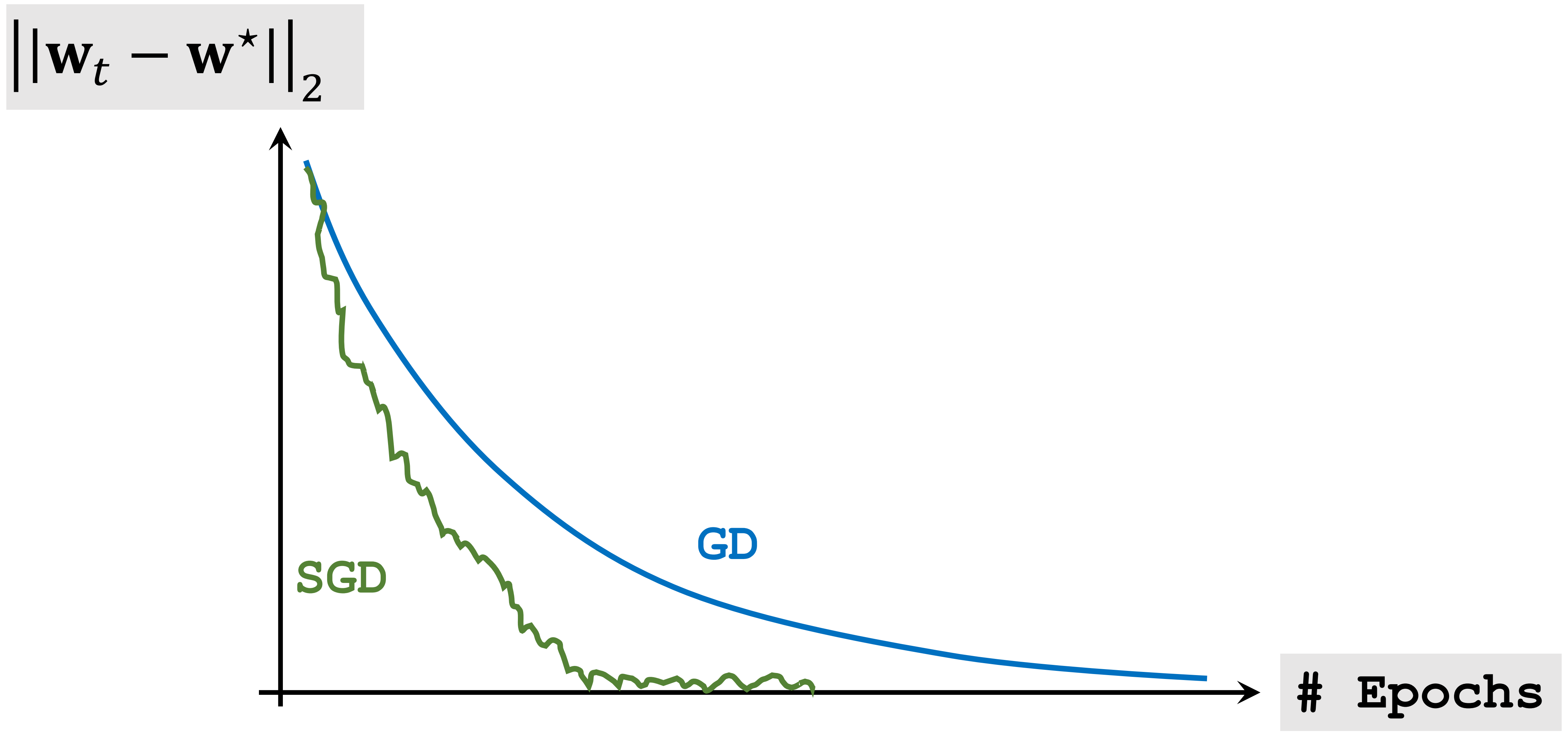
Accelerated GD

Stochastic GD

GD versus SGD



GD versus SGD



Training and Prediction

- Training: 训练 ： 在有限的 epoch 中，找到 最优的W*

$$\mathbf{w}^{\star} = \underset{\mathbf{w}}{\operatorname{argmin}} \sum_j l\left(y_j \mathbf{w}^T \mathbf{x}_j\right), \text{ where } l(\mathrm{z}) = \log(1 + e^{-z}).$$

- For a test feature vector $\mathbf{x}' \in \mathbb{R}^d$, make prediction by

$$\operatorname{sign}\left(\mathbf{x}'^T \mathbf{w}^{\star}\right).$$

Summary

- Logistic regression model for *linear binary* classification.

$$\mathbf{w}^\star = \underset{\mathbf{w}}{\operatorname{argmin}} \textstyle\sum_j l\left(y_j \mathbf{w}^T \mathbf{x}_j\right), \text{ where } l(\mathrm{z}) = \log(1 + e^{-z}).$$

- Compute the gradient using vector derivatives and the chain rule.
- Gradient-based algorithms: GD, AGD, SGD, etc.
- Make prediction using $\operatorname{sign}\left(\mathbf{x}'^T \mathbf{w}^\star\right)$.

Evaluate Binary Classification

Evaluate Binary Classification

- Error Rate $= \dfrac{\text{\# Classification Errors}}{\text{\# Samples}}$
- Accuracy = 1 - Error Rate

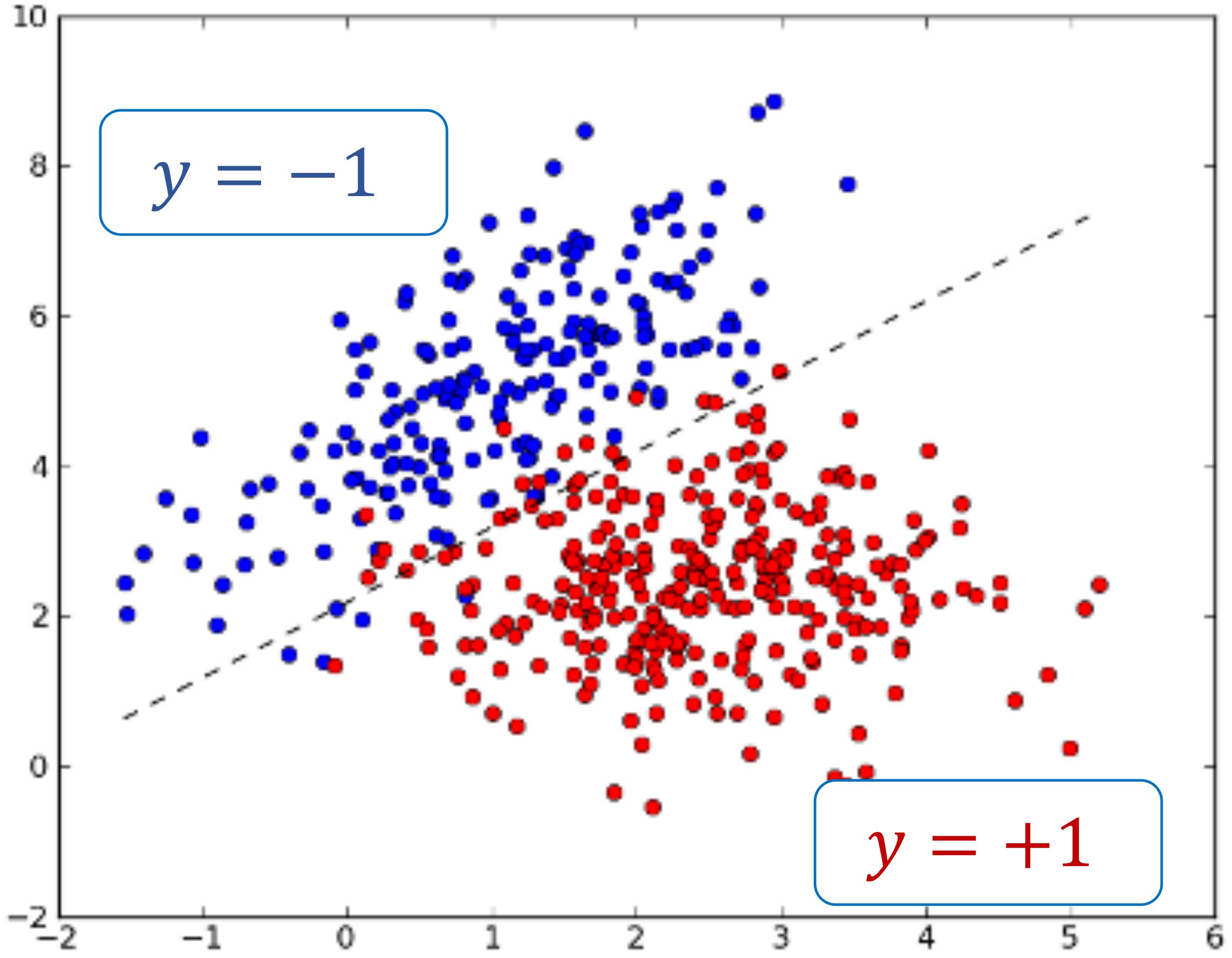
Evaluate Binary Classification

- Error Rate $= \dfrac{\text{\# Classification Errors}}{\text{\# Samples}}$
- Accuracy $= 1 -$ Error Rate

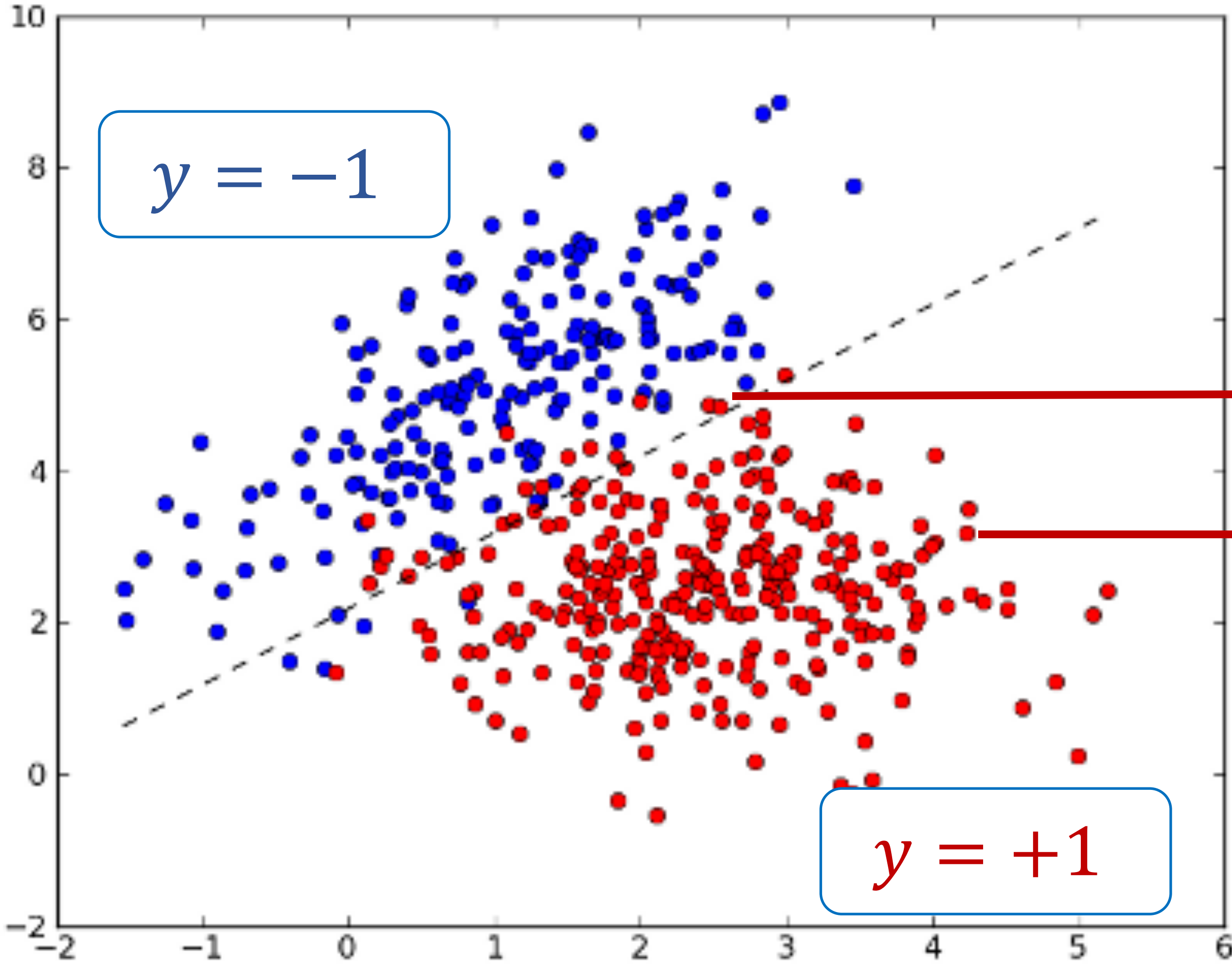
Error rate and **Accuracy** are not meaningful in class-imbalanced problems.

错误率和准确性在 类别不平衡问题 中 没有意义

Evaluate Binary Classification



Evaluate Binary Classification



True Positive Rate (TPR)

$$= \frac{\#\text{TP}}{\#\text{TP} + \#\text{FN}}$$

False Negative (FN)

True Positive (TP)

Evaluate Binary Classification

False Positive Rate (FPR)

$$= \frac{\#\mathrm{FP}}{\#\mathrm{FP} + \#\mathrm{TN}}$$

True Positive Rate (TPR)

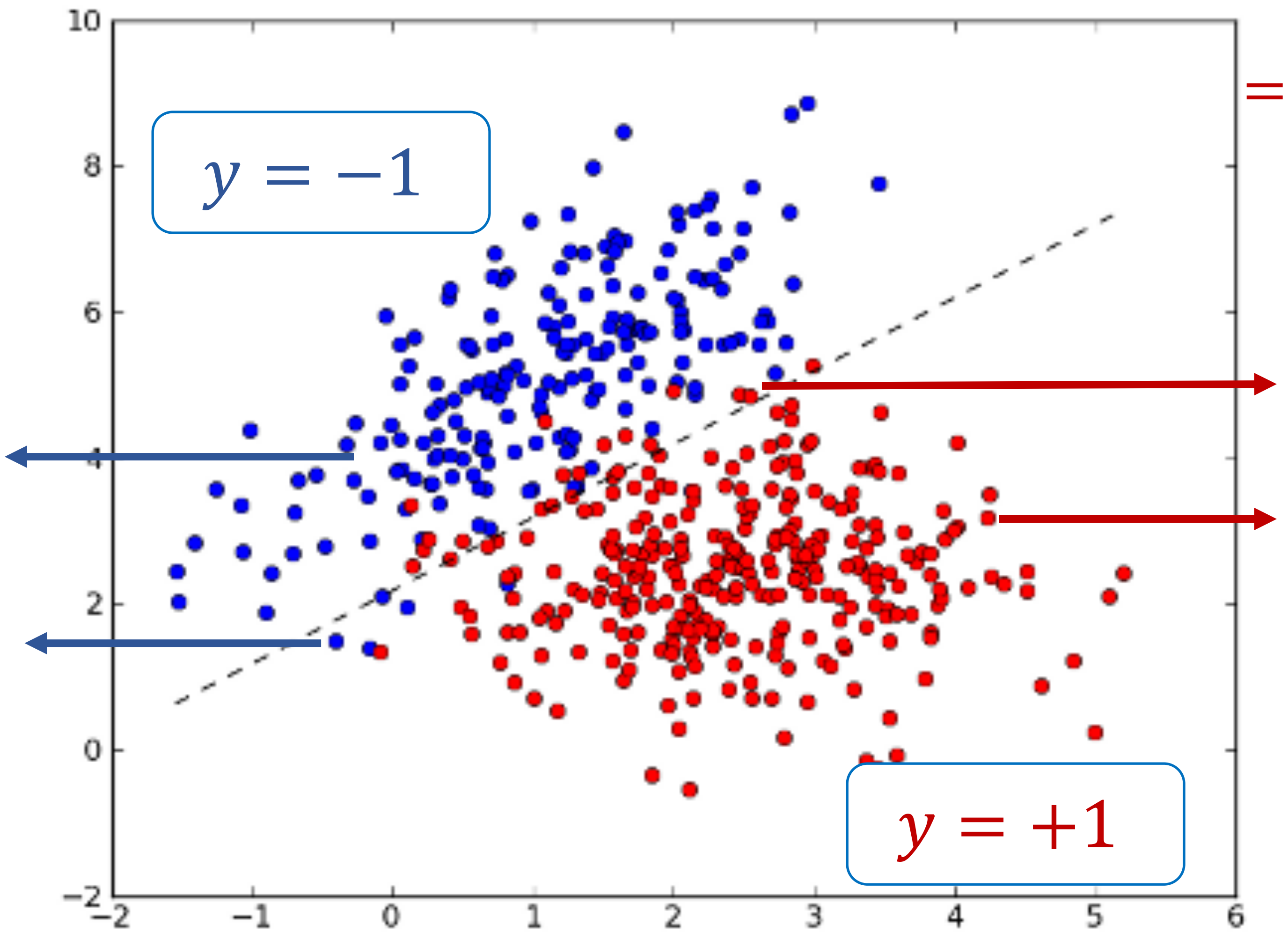
$$= \frac{\#\mathrm{TP}}{\#\mathrm{TP} + \#\mathrm{FN}}$$

True Negative (TN)

False Positive (FP)

False Negative (FN)

True Positive (TP)



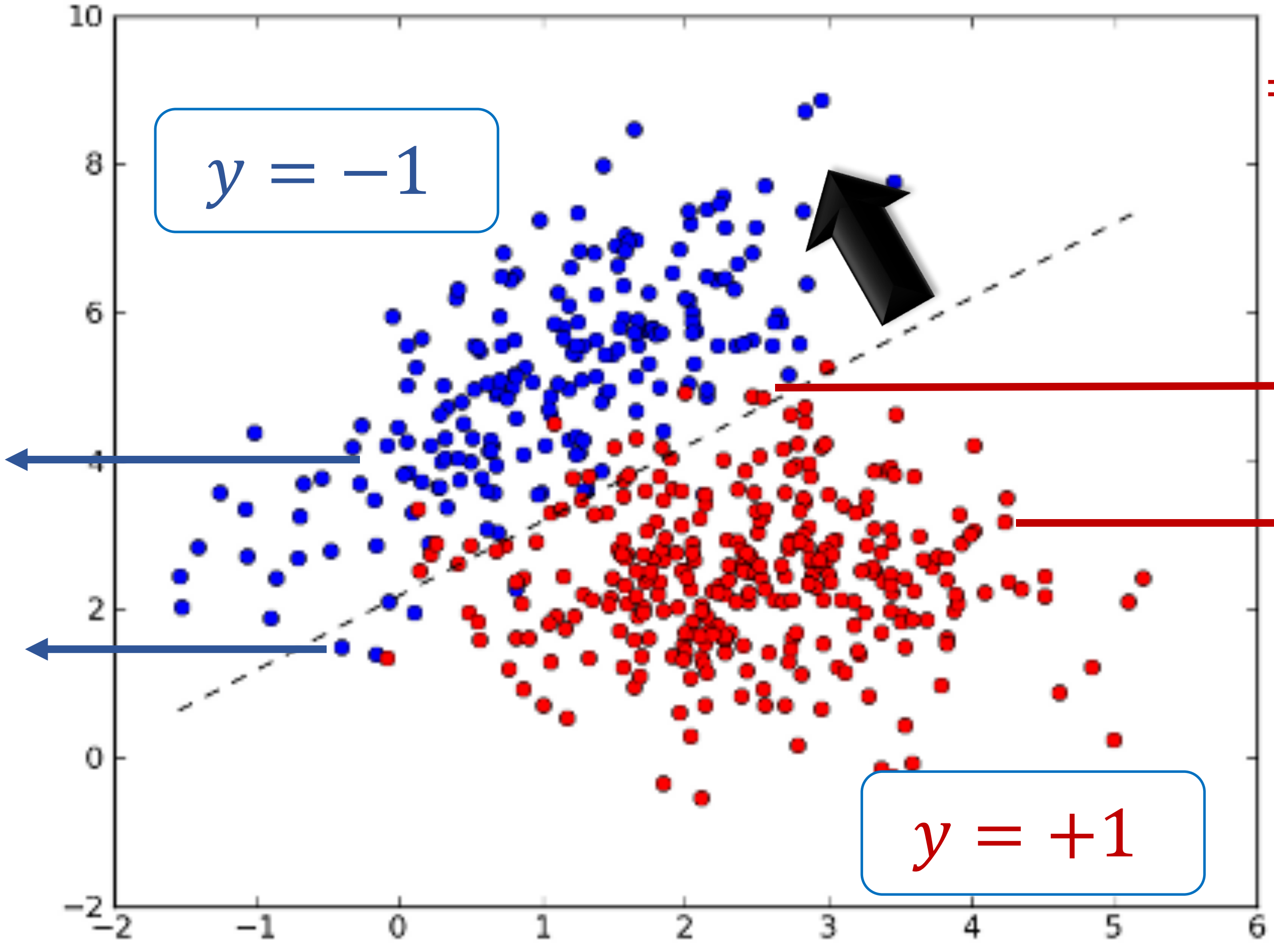
Evaluate Binary Classification

False Positive Rate (FPR)

$$= \frac{\#\text{FP}}{\#\text{FP} + \#\text{TN}} \nearrow$$

True Positive Rate (TPR)

$$= \frac{\#\text{TP}}{\#\text{TP} + \#\text{FN}} \nearrow$$



True Negative (TN)

False Positive (FP)

False Negative (FN)

True Positive (TP)

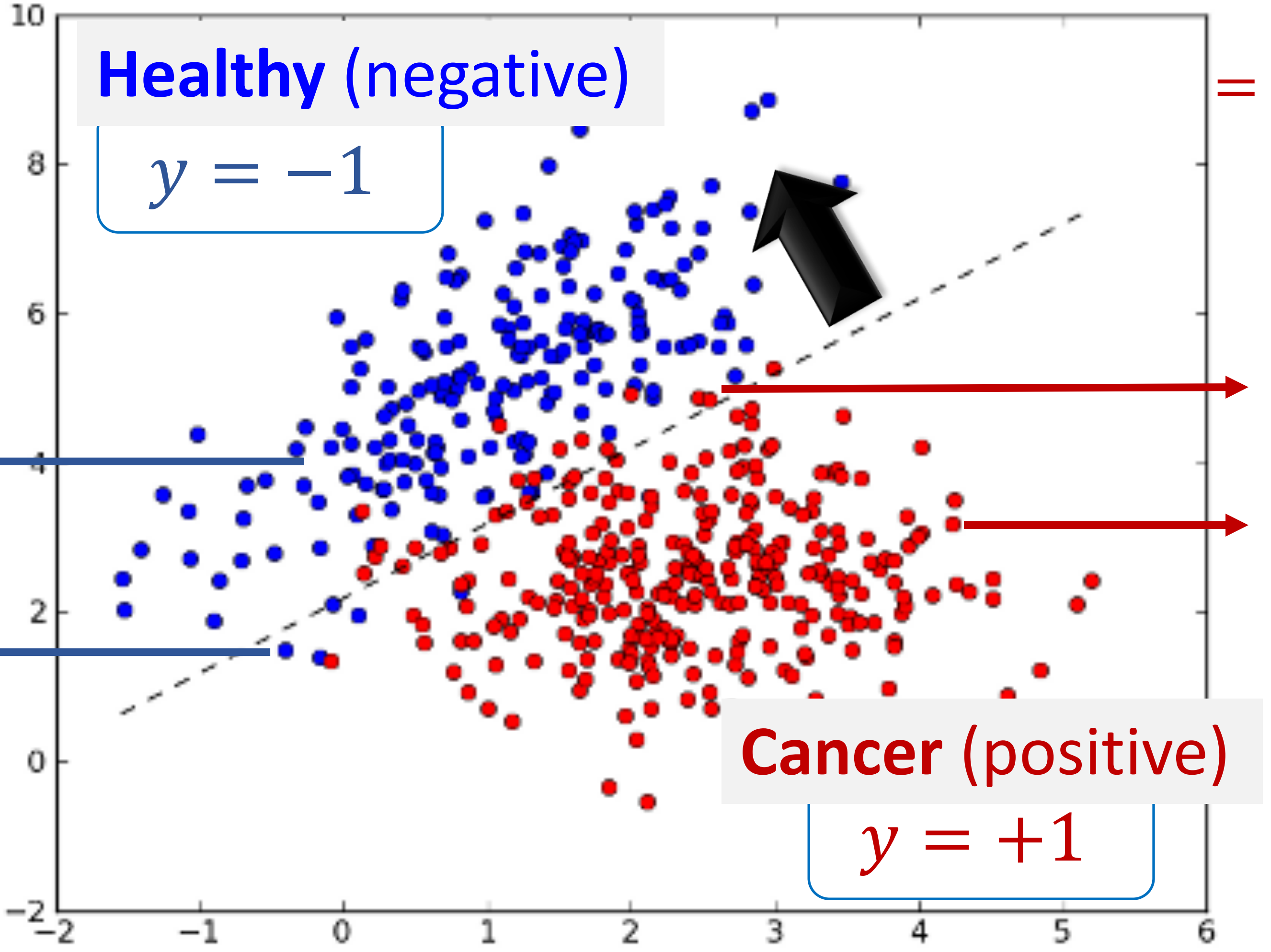
Evaluate Binary Classification

False Positive Rate (FPR)

$$= \frac{\#\text{FP}}{\#\text{FP} + \#\text{TN}} \nearrow$$

True Positive Rate (TPR)

$$= \frac{\#\text{TP}}{\#\text{TP} + \#\text{FN}} \nearrow$$



True Negative (TN)

False Positive (FP)

False Negative (FN)

True Positive (TP)

Evaluate Binary Classification

False Positive Rate (FPR)

$$= \frac{\#\text{FP}}{\#\text{FP} + \#\text{TN}}$$

True Positive Rate (TPR)

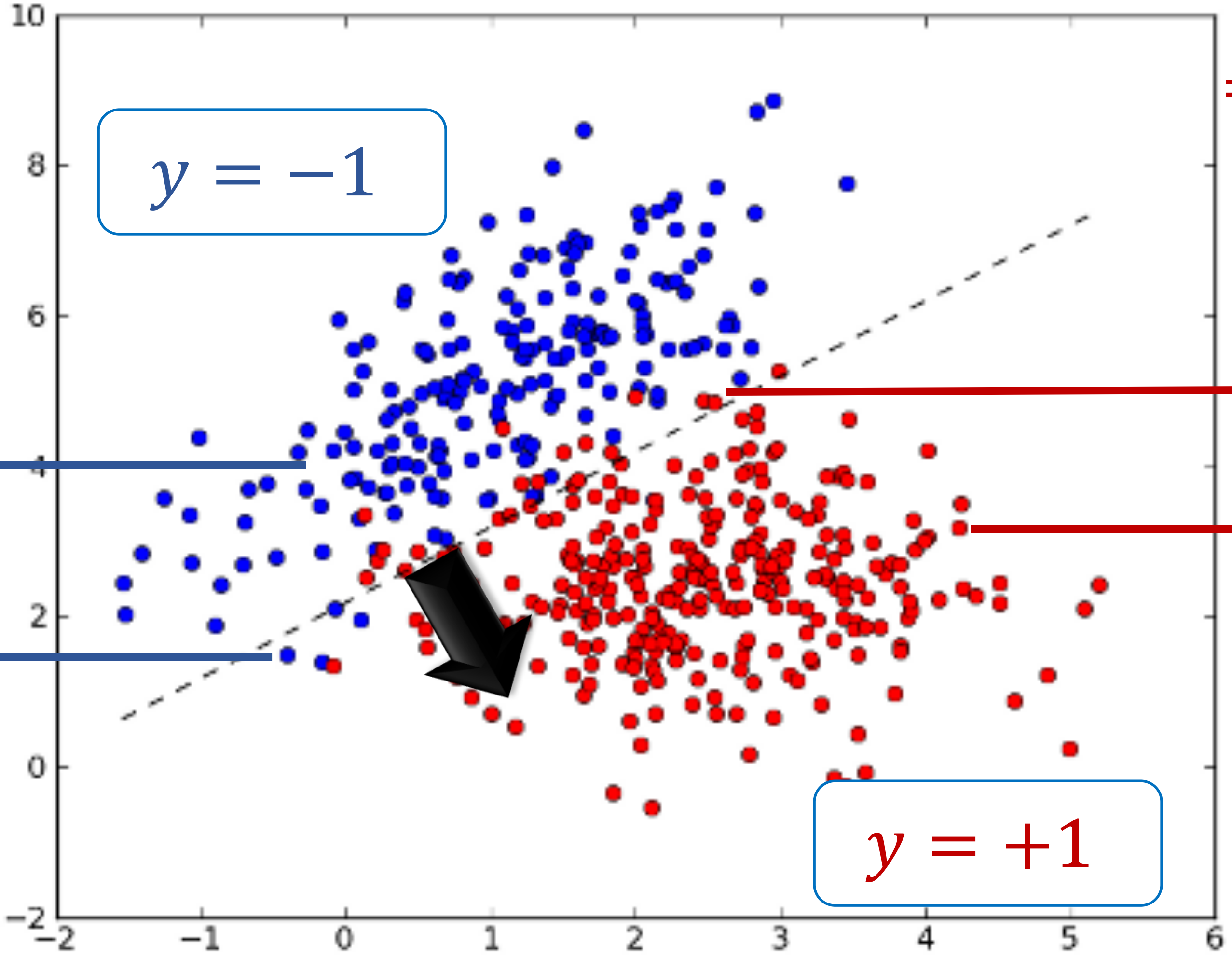
$$= \frac{\#\text{TP}}{\#\text{TP} + \#\text{FN}}$$

True Negative (TN)

False Positive (FP)

False Negative (FN)

True Positive (TP)



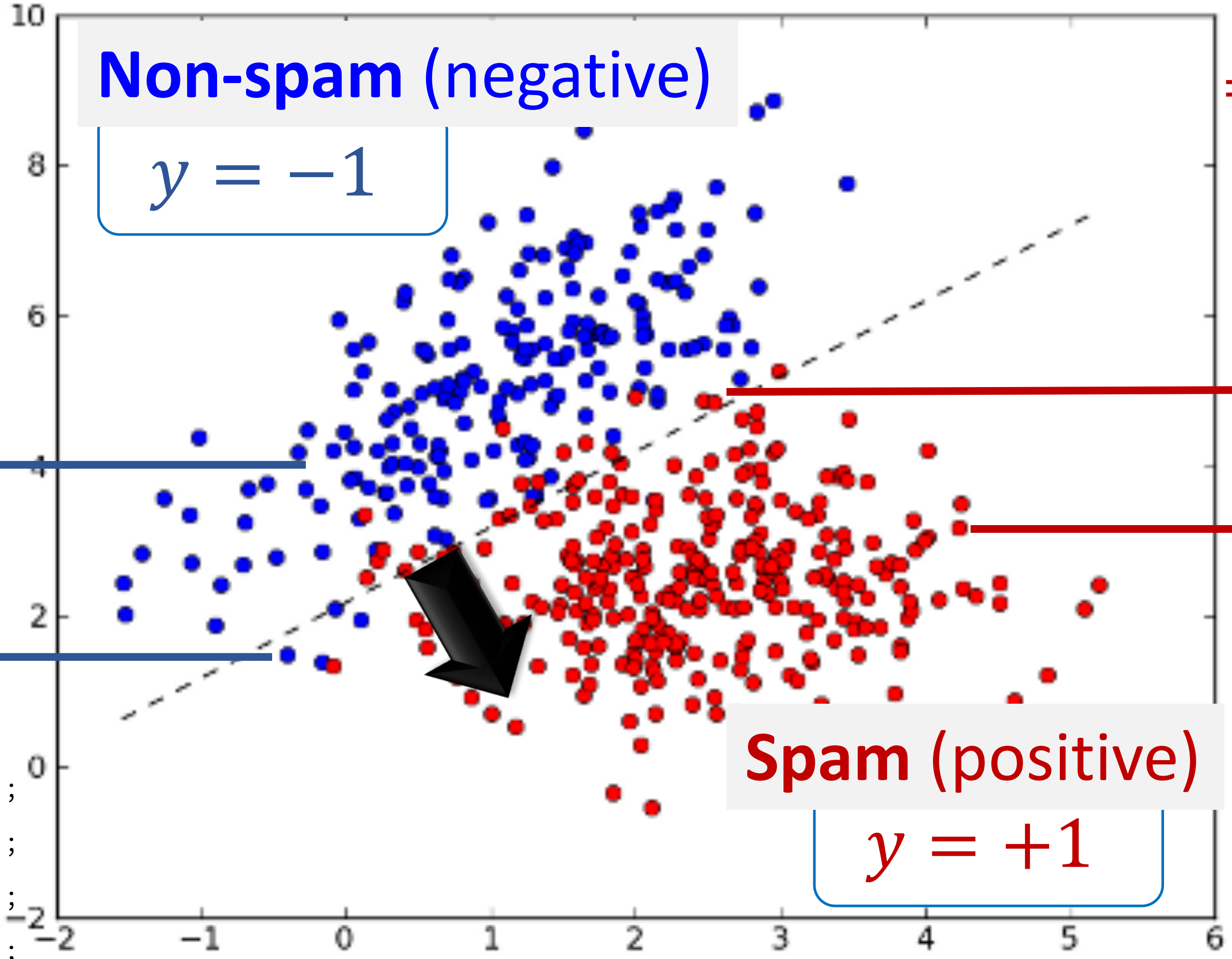
Evaluate Binary Classification

False Positive Rate (FPR)

$$= \frac{\#\mathrm{FP}}{\#\mathrm{FP} + \#\mathrm{TN}}$$

True Positive Rate (TPR)

$$= \frac{\#\mathrm{TP}}{\#\mathrm{TP} + \#\mathrm{FN}}$$



False Negative (FN)

True Positive (TP)

True Negative (TN)

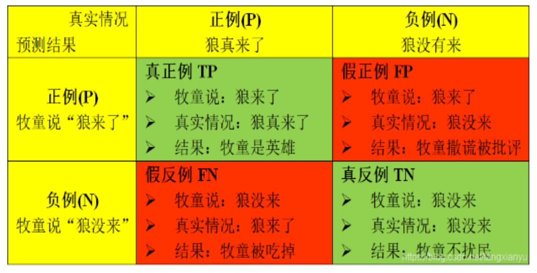
False Positive (FP)

TP：true positive， 被判定为正样本，事实上也是正样本；

TN：true negative， 被判定为负样本，事实上也是负样本；

FP：false positive，被判定为正样本，但事实上是负样本；

FN：false negative，被判定为负样本，但事实上是正样本；

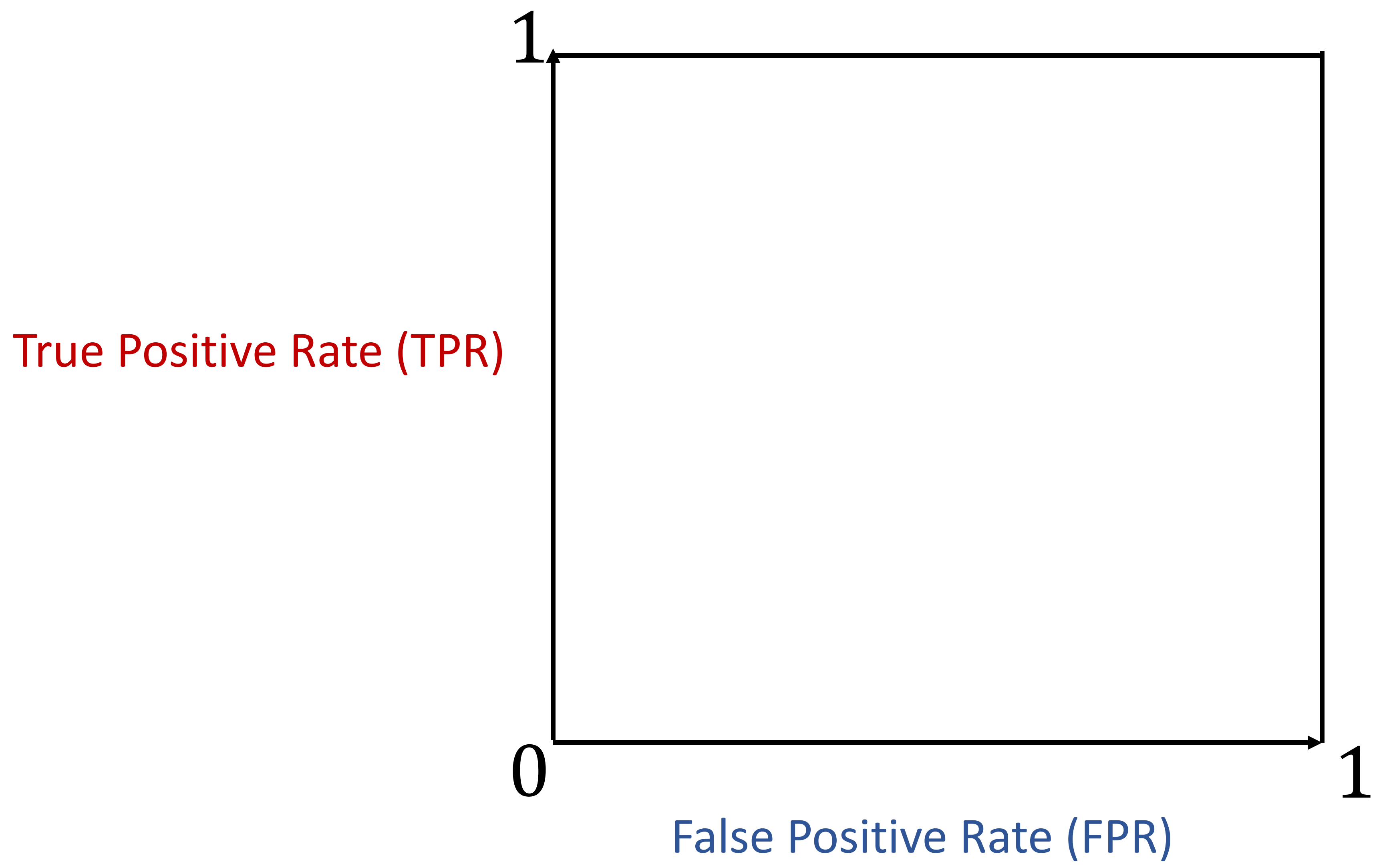


真实情况 预测结果	正例(P) 狼真来了	负例(N) 狼没有来
正例(P) 牧童说"狼来了"	真正例 TP ➢ 牧童说：狼来了 ➢ 真实情况：狼真来了 ➢ 结果：牧童是英雄	假正例 FP ➢ 牧童说：狼来了 ➢ 真实情况：狼没来 ➢ 结果：牧童撒谎被批评
负例(N) 牧童说"狼没来"	假反例 FN ➢ 牧童说：狼没来 ➢ 真实情况：狼来了 ➢ 结果：牧童被吃掉	真反例 TN ➢ 牧童说：狼没来 ➢ 真实情况：狼没来 ➢ 结果：牧童不扰民

https://blog.csdn.net/tangxianyu/article/details/102454611

Receiver Operating Characteristic (ROC) Curve

接收机工作特性（ROC）曲线

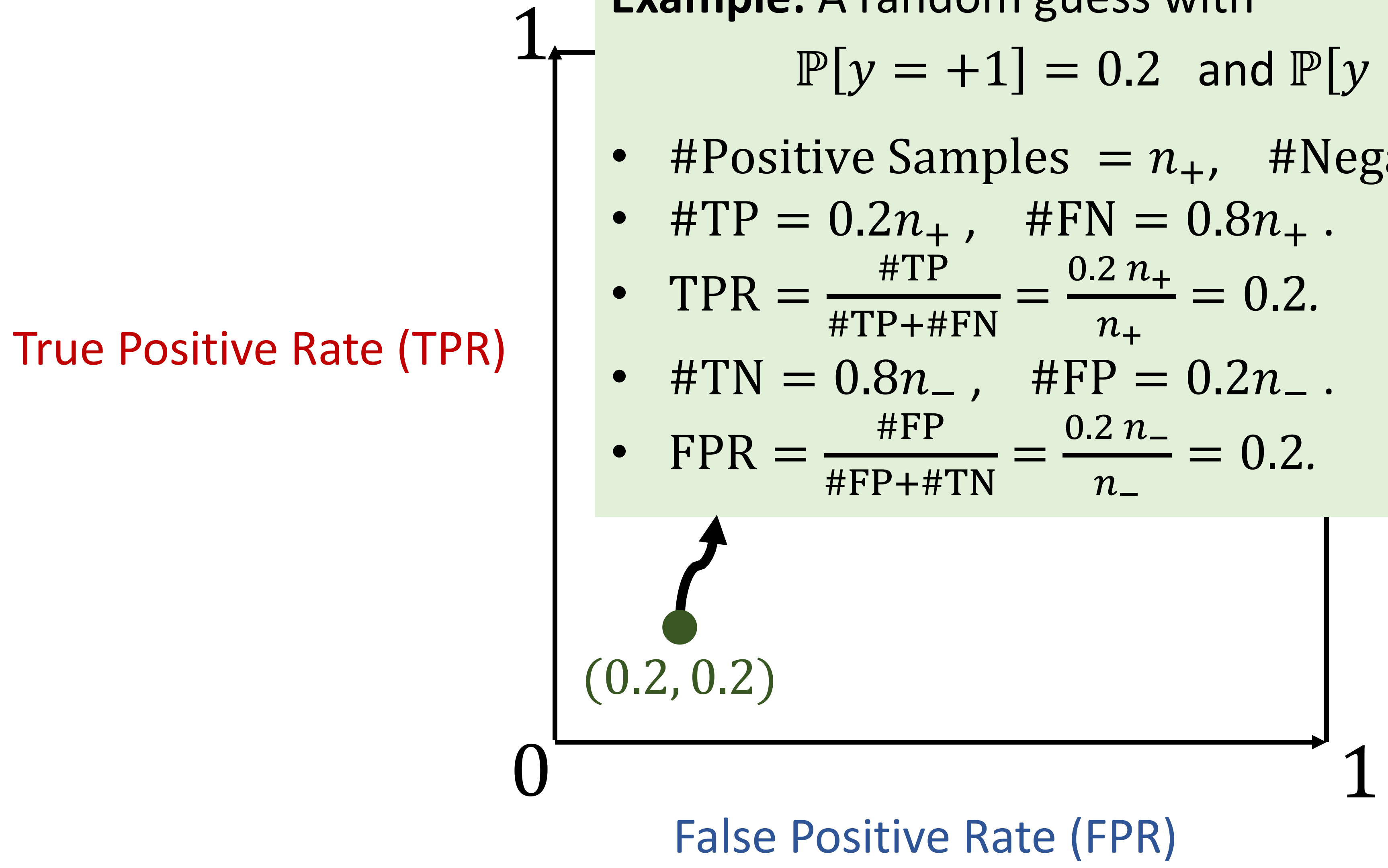


Receiver Operating Characteristic (ROC) Curve

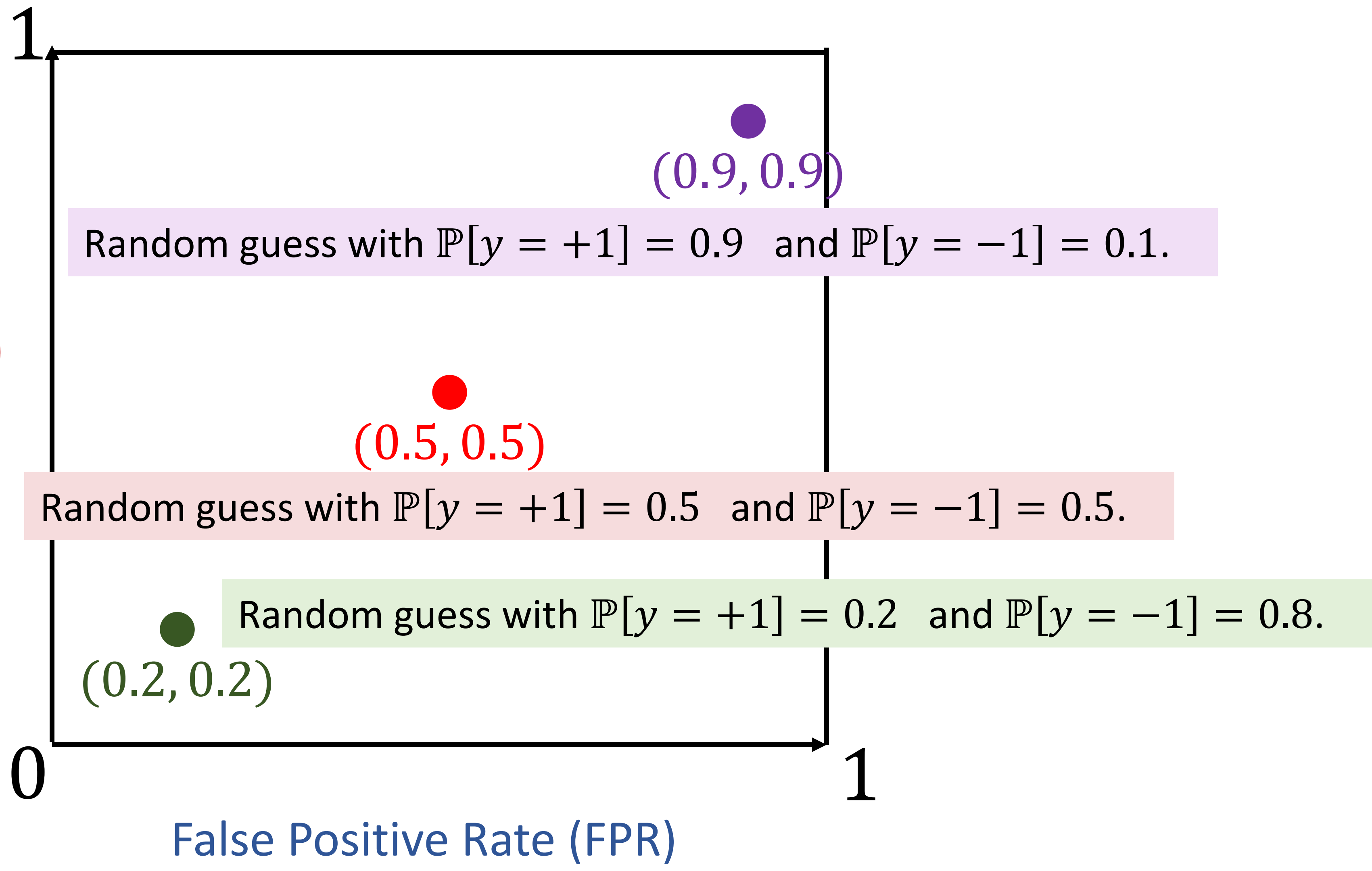
Example: A random guess with

$$\mathbb{P}[y = +1] = 0.2 \quad \text{and } \mathbb{P}[y = -1] = 0.8.$$

- #Positive Samples $= n_+$, #Negative $Samples$ $= n_-$.
- $\#\text{TP} = 0.2n_+$, $\#\text{FN} = 0.8n_+$.
- $\text{TPR} = \frac{\#\text{TP}}{\#\text{TP}+\#\text{FN}} = \frac{0.2\, n_+}{n_+} = 0.2.$
- $\#\text{TN} = 0.8n_-$, $\#\text{FP} = 0.2n_-$.
- $\text{FPR} = \frac{\#\text{FP}}{\#\text{FP}+\#\text{TN}} = \frac{0.2\, n_-}{n_-} = 0.2.$



Receiver Operating Characteristic (ROC) Curve

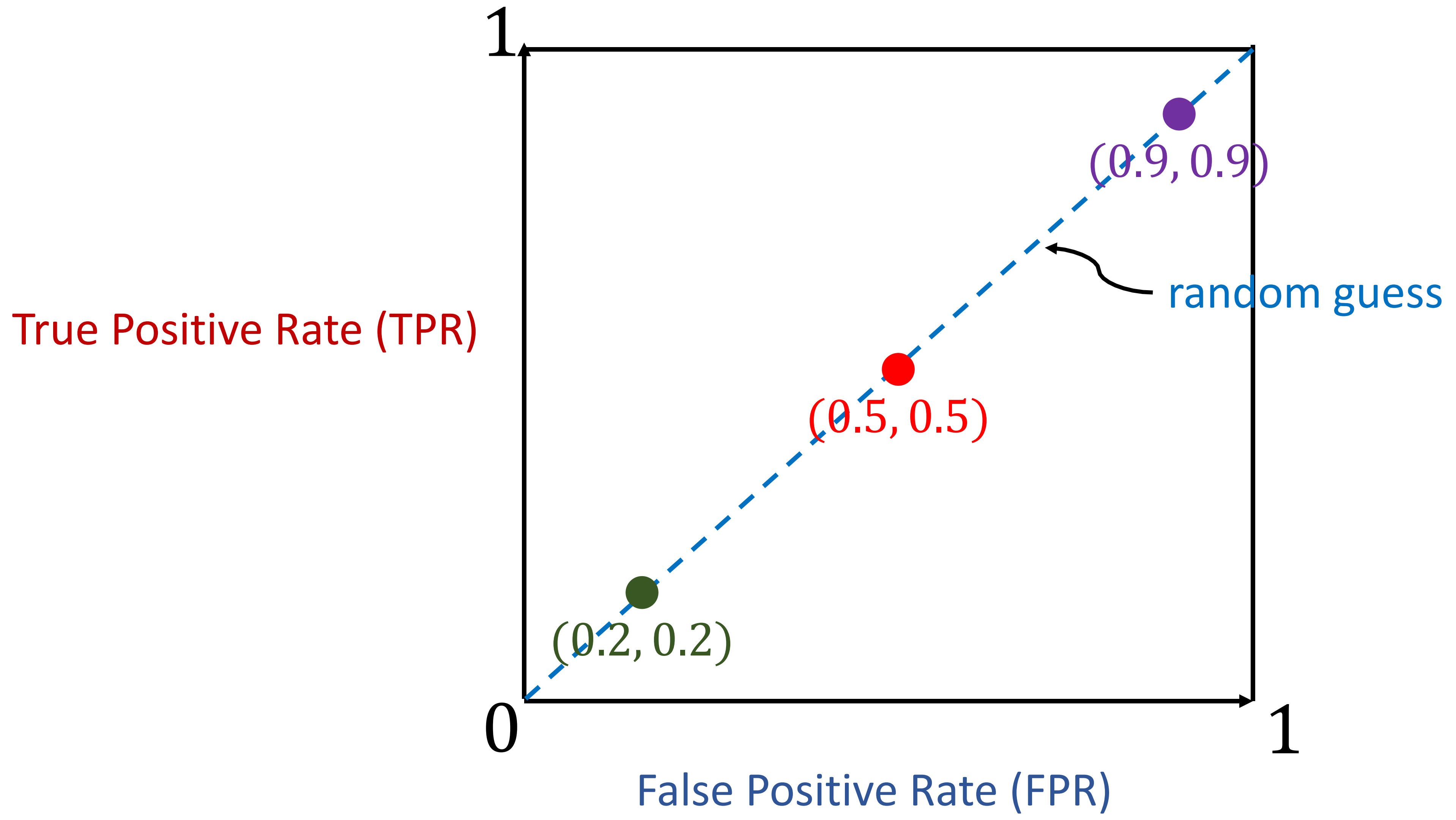


Random guess with $\mathbb{P}[y = +1] = 0.9$ and $\mathbb{P}[y = -1] = 0.1$.

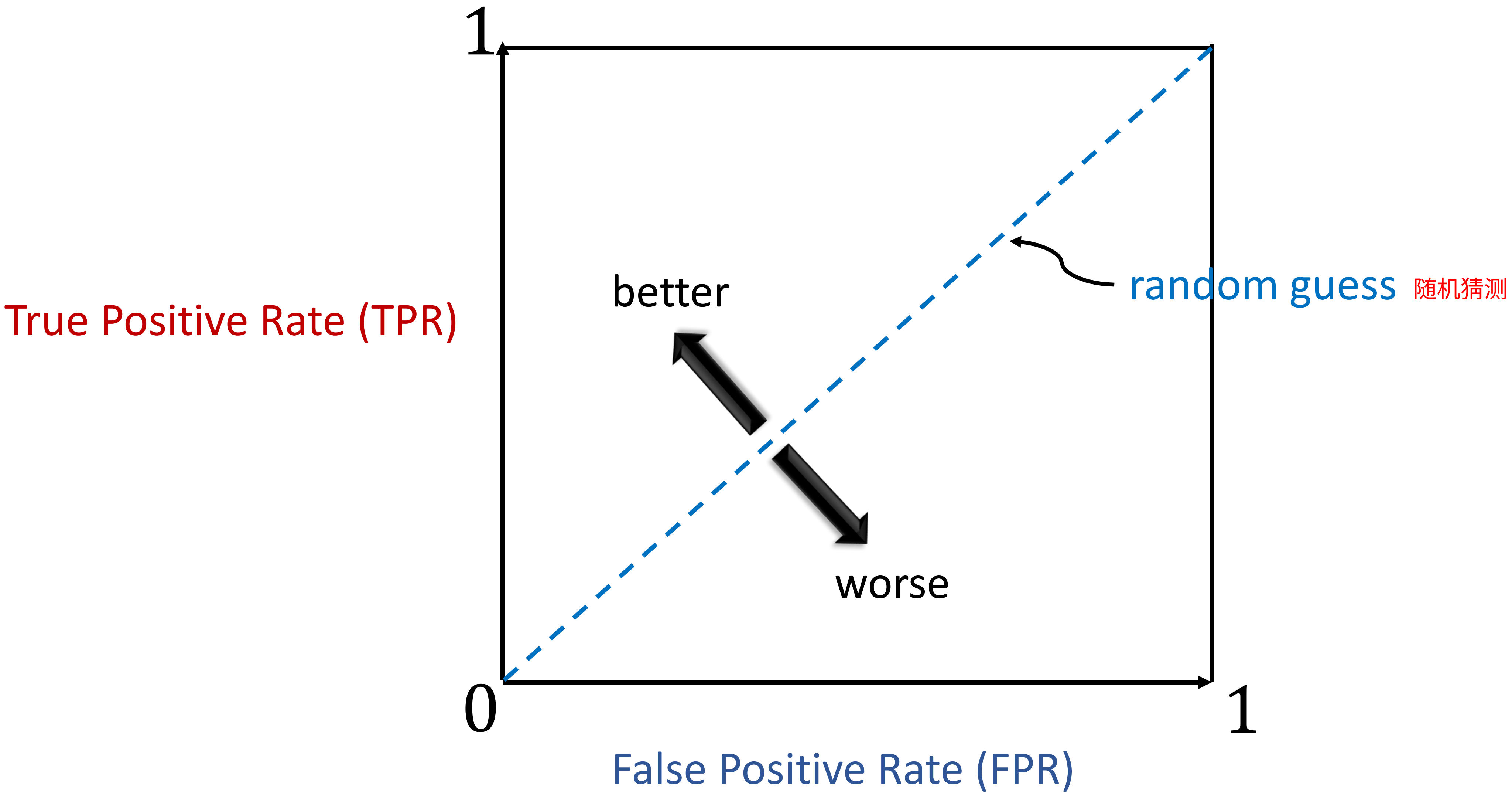
Random guess with $\mathbb{P}[y = +1] = 0.5$ and $\mathbb{P}[y = -1] = 0.5$.

Random guess with $\mathbb{P}[y = +1] = 0.2$ and $\mathbb{P}[y = -1] = 0.8$.

Receiver Operating Characteristic (ROC) Curve

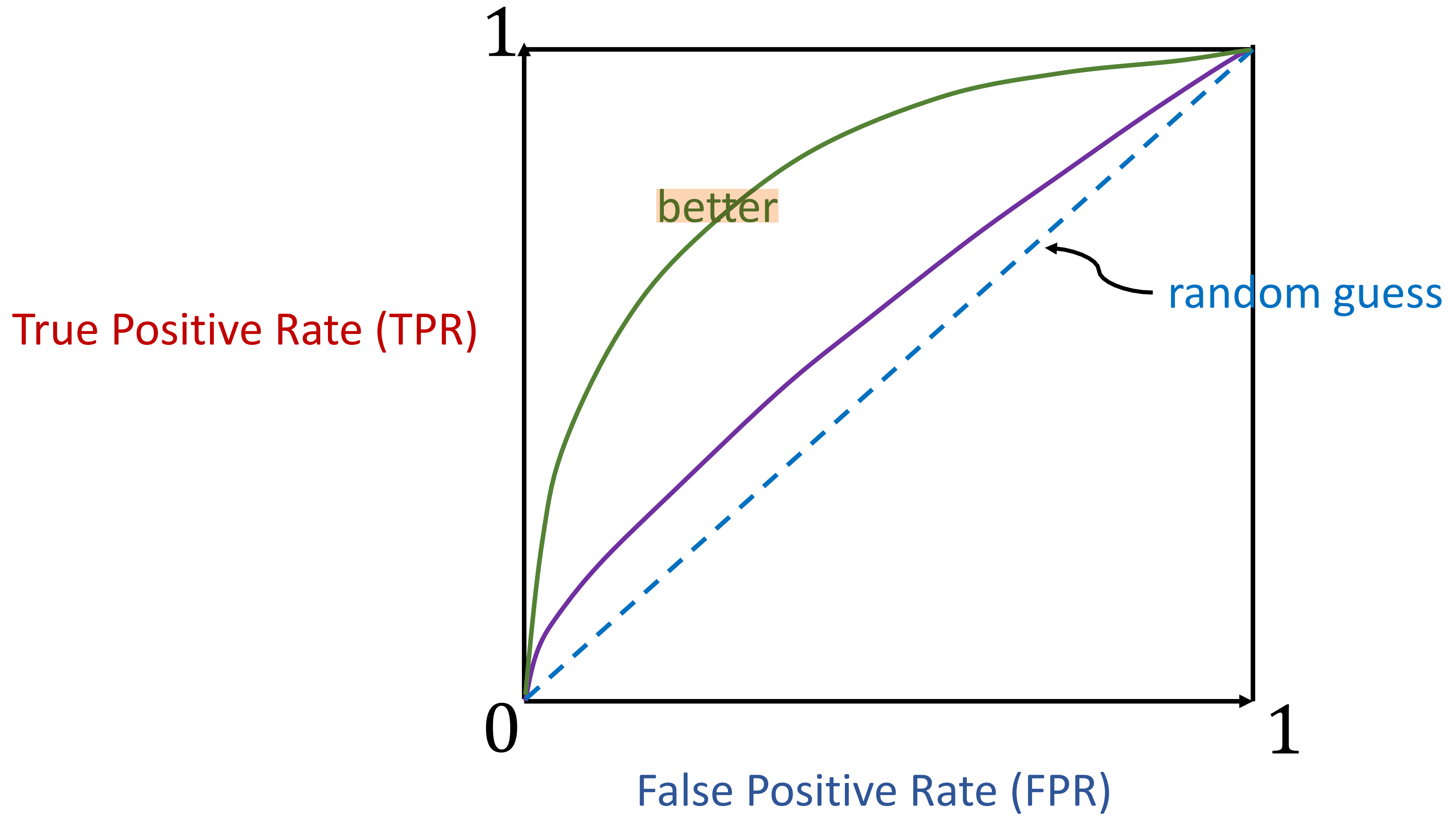


Receiver Operating Characteristic (ROC) Curve



Receiver Operating Characteristic (ROC) Curve

接收机工作特性（ROC）曲线



Thank you!