Functions

In[1]:=
```
gdg[a_, nx_, ny_, nz_, μ_] := If[μ == 1,
  ConjugateTranspose[a[nx, ny, nz]].(a[nx + 1, ny, nz] - a[nx - 1, ny, nz]),
  If[μ == 2,
   ConjugateTranspose[a[nx, ny, nz]].(a[nx, ny + 1, nz] - a[nx, ny - 1, nz]),
   ConjugateTranspose[a[nx, ny, nz]].(a[nx, ny, nz + 1] - a[nx, ny, nz - 1])
  ]]
gdg[a_, nx_, ny_, nz_, μ_, sw_] := If[μ == 1,
  ConjugateTranspose[a[nx + sw, ny, nz]].
   (a[nx + 1 + sw, ny, nz] - a[nx - 1 + sw, ny, nz]),
  If[μ == 2,
   ConjugateTranspose[a[nx, ny + sw, nz]].
    (a[nx, ny + 1 + sw, nz] - a[nx, ny - 1 + sw, nz]),
   ConjugateTranspose[a[nx, ny, nz + sw]].
    (a[nx, ny, nz + 1 + sw] - a[nx, ny, nz - 1 + sw])
  ]]
actiondensity[a_, nx_, ny_, nz_] := Module[{d, l},
  Do[
   d[nx, ny, nz, μ] = gdg[a, nx, ny, nz, μ];
   l[nx, ny, nz, μ] = d[nx, ny, nz, μ] - ConjugateTranspose[d[nx, ny, nz, μ]];
   , {μ, 1, 3}
  ];
  Re[(-1/16) Sum[Tr[l[nx, ny, nz, μ].l[nx, ny, nz, μ]], {μ, 1, 3}]]
 ]
action[a_, L_] := Module[{s},
  s = Sum[actiondensity[a, nx, ny, nz]
    , {nx, 0, L - 1}, {ny, 0, L - 1}, {nz, 0, L - 1}];
  Re[s/L^3]
 ]
W3[a_, L_] := Module[{d, l},
  Do[
   d[nx, ny, nz, μ] = gdg[a, nx, ny, nz, μ];
   l[nx, ny, nz, μ] = d[nx, ny, nz, μ] - ConjugateTranspose[d[nx, ny, nz, μ]];
   , {nx, 0, L - 1}, {ny, 0, L - 1}, {nz, 0, L - 1}, {μ, 1, 3}
  ];
  Re[3/(4^3 * 24 Pi^2) * Sum[
     Tr[l[nx, ny, nz, 1].
       (l[nx, ny, nz, 2].l[nx, ny, nz, 3] - l[nx, ny, nz, 3].l[nx, ny, nz, 2])]
    , {nx, 0, L - 1}, {ny, 0, L - 1}, {nz, 0, L - 1}]]
 ]
setWing[a_, L_] := Module[{wmax = 2},
  Do[
   a[-wing, ny, nz] = a[L - wing, ny, nz];
   a[L - 1 + wing, ny, nz] = a[wing - 1, ny, nz];
```

```
    , {wing, 1, wmax}, {ny, 0, L - 1}, {nz, 0, L - 1}];
   Do[
    a[nx, -wing, nz] = a[nx, L - wing, nz];
    a[nx, L - 1 + wing, nz] = a[nx, wing - 1, nz];
    , {wing, 1, wmax}, {nx, -wmax, L - 1 + wmax}, {nz, 0, L - 1}];
   Do[
    a[nx, ny, -wing] = a[nx, ny, L - wing];
    a[nx, ny, L - 1 + wing] = a[nx, ny, wing - 1];
    , {wing, 1, wmax}, {nx, -wmax, L - 1 + wmax}, {ny, -wmax, L - 1 + wmax}];
  ]
```

In[7]:=
```
mapsuMatrix[m_, nx_, ny_, nz_, L_] := Module[
  {unity, sigmax, sigmay, sigmaz, thetat, thetax, thetay, thetaz, norm, x, y, z},
  unity = {{1, 0}, {0, 1}};
  sigmax = {{0, 1}, {1, 0}};
  sigmay = {{0, -I}, {I, 0}};
  sigmaz = {{1, 0}, {0, -1}};

  x = -Pi + 2 Pi nx / L;
  y = -Pi + 2 Pi ny / L;
  z = -Pi + 2 Pi nz / L;
  thetat = m + Cos[x] - 1 + Cos[y] - 1 + Cos[z] - 1;
  thetax = Sin[x]; thetay = Sin[y]; thetaz = Sin[z];
  norm = Sqrt[(thetat)^2 + (thetax)^2 + (thetay)^2 + (thetaz)^2];

  N[(thetat * unity + I * thetax * sigmax +
      I * thetay * sigmay + I * thetaz * sigmaz) / norm]
 ]
randomUnitaryMatrix[n_] := Module[{A, Q, R, diagR},
  A = RandomReal[{-1, 1}, {n, n}] + I RandomReal[{-1, 1}, {n, n}];
  {Q, R} = QRDecomposition[A];
  diagR = DiagonalMatrix[Exp[I Arg[Diagonal[R]]]];
  Q.diagR
 ]
```

In[9]:=
```
gfZ[a_, nx_, ny_, nz_] := Module[{d, dp, dm, l},
  d[nx, ny, nz, 1] = gdg[a, nx, ny, nz, 1];
  d[nx, ny, nz, 2] = gdg[a, nx, ny, nz, 2];
  d[nx, ny, nz, 3] = gdg[a, nx, ny, nz, 3];
  dp[nx, ny, nz, 1] = gdg[a, nx, ny, nz, 1, 1];
  dp[nx, ny, nz, 2] = gdg[a, nx, ny, nz, 2, 1];
  dp[nx, ny, nz, 3] = gdg[a, nx, ny, nz, 3, 1];
  dm[nx, ny, nz, 1] = gdg[a, nx, ny, nz, 1, -1];
  dm[nx, ny, nz, 2] = gdg[a, nx, ny, nz, 2, -1];
  dm[nx, ny, nz, 3] = gdg[a, nx, ny, nz, 3, -1];

  l[nx, ny, nz, 1] =
   a[nx + 1, ny, nz].d[nx, ny, nz, 1].ConjugateTranspose[a[nx, ny, nz]] -
    a[nx - 1, ny, nz].d[nx, ny, nz, 1].ConjugateTranspose[a[nx, ny, nz]] +
    a[nx, ny, nz].dp[nx, ny, nz, 1].ConjugateTranspose[a[nx + 1, ny, nz]] -
    a[nx, ny, nz].dm[nx, ny, nz, 1].ConjugateTranspose[a[nx - 1, ny, nz]] +
    a[nx + 2, ny, nz].ConjugateTranspose[a[nx, ny, nz]] +
    a[nx - 2, ny, nz].ConjugateTranspose[a[nx, ny, nz]];

  l[nx, ny, nz, 2] =
   a[nx, ny + 1, nz].d[nx, ny, nz, 2].ConjugateTranspose[a[nx, ny, nz]] -
    a[nx, ny - 1, nz].d[nx, ny, nz, 2].ConjugateTranspose[a[nx, ny, nz]] +
    a[nx, ny, nz].dp[nx, ny, nz, 2].ConjugateTranspose[a[nx, ny + 1, nz]] -
    a[nx, ny, nz].dm[nx, ny, nz, 2].ConjugateTranspose[a[nx, ny - 1, nz]] +
    a[nx, ny + 2, nz].ConjugateTranspose[a[nx, ny, nz]] +
    a[nx, ny - 2, nz].ConjugateTranspose[a[nx, ny, nz]];

  l[nx, ny, nz, 3] =
   a[nx, ny, nz + 1].d[nx, ny, nz, 3].ConjugateTranspose[a[nx, ny, nz]] -
    a[nx, ny, nz - 1].d[nx, ny, nz, 3].ConjugateTranspose[a[nx, ny, nz]] +
    a[nx, ny, nz].dp[nx, ny, nz, 3].ConjugateTranspose[a[nx, ny, nz + 1]] -
    a[nx, ny, nz].dm[nx, ny, nz, 3].ConjugateTranspose[a[nx, ny, nz - 1]] +
    a[nx, ny + 2, nz].ConjugateTranspose[a[nx, ny, nz]] +
    a[nx, ny - 2, nz].ConjugateTranspose[a[nx, ny, nz]];

  (1/16) Sum[l[nx, ny, nz, μ] - ConjugateTranspose[l[nx, ny, nz, μ]], {μ, 1, 3}]
 ]
gfZsu[a_, nx_, ny_, nz_] := Module[{nc, z},
  nc = Length[a[nx, ny, nz]];
  z[nx, ny, nz] = gfZ[a, nx, ny, nz];
  z[nx, ny, nz] - (1 / nc) Tr[z[nx, ny, nz]] × IdentityMatrix[nc]
 ]
```

In[11]:=
```
flowW3[a_, Δt_, tmax_, L_, Z_] :=
 Module[{tblW, tblS, gflow, z0, z1, z2, w0, w1, w2, j},
  tblW = Table[{t, 0}, {t, 0, tmax, Δt}];
  tblS = Table[{t, 0}, {t, 0, tmax, Δt}];
```

```
j = 1;
Do[
 gflow[nx, ny, nz] = a[nx, ny, nz], {nx, -2, L + 1}, {ny, -2, L + 1}, {nz, -2, L + 1}
];
tblW[[j, 2]] = W3[gflow, L];
tblS[[j, 2]] = action[gflow, L];
Monitor[Do[
  Do[
   w0[nx, ny, nz] = gflow[nx, ny, nz],
   {nx, -2, L + 1}, {ny, -2, L + 1}, {nz, -2, L + 1}
  ];
  Do[
   z0[nx, ny, nz] = Δt * Z[w0, nx, ny, nz],
   {nx, 0, L - 1}, {ny, 0, L - 1}, {nz, 0, L - 1}
  ];
  setWing[z0, L];
  Do[
   w1[nx, ny, nz] = MatrixExp[(1 / 4) z0[nx, ny, nz]].w0[nx, ny, nz]
   , {nx, -2, L + 1}, {ny, -2, L + 1}, {nz, -2, L + 1}
  ];
  Do[
   z1[nx, ny, nz] = Δt * Z[w1, nx, ny, nz],
   {nx, 0, L - 1}, {ny, 0, L - 1}, {nz, 0, L - 1}
  ];
  setWing[z1, L];
  Do[
   w2[nx, ny, nz] =
    MatrixExp[(8 / 9) z1[nx, ny, nz] - (17 / 36) z0[nx, ny, nz]].w1[nx, ny, nz]
   , {nx, -2, L + 1}, {ny, -2, L + 1}, {nz, -2, L + 1}
  ];
  Do[
   z2[nx, ny, nz] = Δt * Z[w2, nx, ny, nz],
   {nx, 0, L - 1}, {ny, 0, L - 1}, {nz, 0, L - 1}
  ];
  setWing[z2, L];
  j++;
  Do[
   gflow[nx, ny, nz] = MatrixExp[(3 / 4) z2[nx, ny, nz] -
        (8 / 9) z1[nx, ny, nz] + (17 / 36) z0[nx, ny, nz]].w2[nx, ny, nz]
   , {nx, -2, L + 1}, {ny, -2, L + 1}, {nz, -2, L + 1}
  ];
  tblW[[j, 2]] = W3[gflow, L];
  tblS[[j, 2]] = action[gflow, L];
  , {t, Δt, tmax, Δt}
 ], t];
{tblW, tblS}
]
```

T^3->SU(2)

In[12]:=
```
L = 20;
m = 3;
Do[
  g[nx, ny, nz] = mapsuMatrix[m, nx, ny, nz, L];
  , {nx, 0, L - 1}, {ny, 0, L - 1}, {nz, 0, L - 1}
 ];
setWing[g, L]
```

In[16]:=
```
W3[g, L]
action[g, L]
```

Out[16]= −1.86086

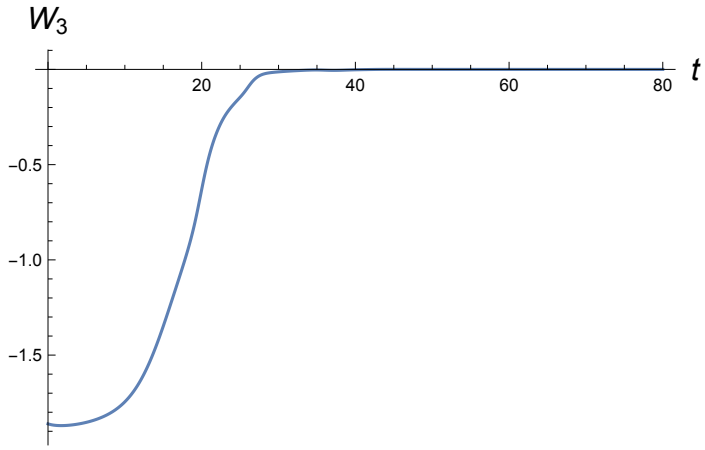
Out[17]= 0.221657

In[18]:=
```
(* tmax=40 for L=10; tmax=80 for L=20; tmax=160 for L=30 *)
```

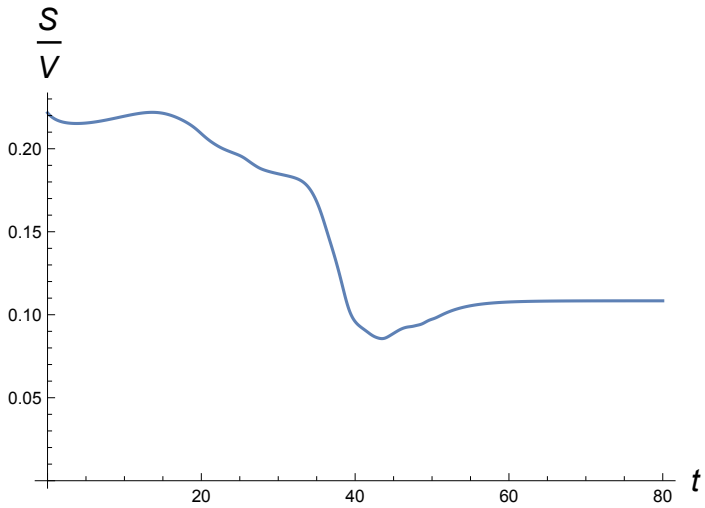
In[19]:=
```
δt = 0.1;
tm = 80;
tbl = flowW3[g, δt, tm, L, gfZsu];
```

In[22]:=
```
ListPlot[tbl[[1]], Joined → True, PlotRange → Full,
 AxesLabel → {Style[t, 16], Style[W3, 16]}]
ListPlot[tbl[[2]], Joined → True, PlotRange → Full,
 AxesLabel → {Style[t, 16], Style[S/V, 16]}]
```

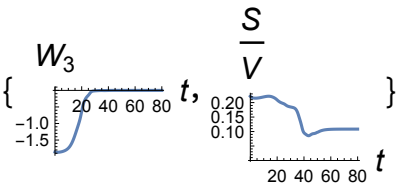
Out[22]=



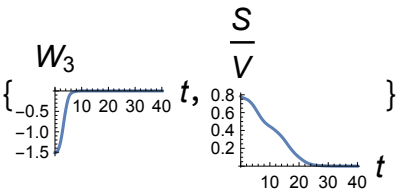
Out[23]=



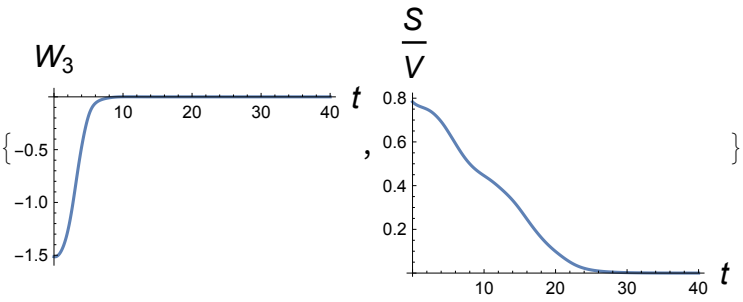

```
(* L=20; m=3; δt=0.1; tmax=80 *)
```



```
(* L=10; m=3; δt=0.1; tmax=40 *)
```



Out[24]=



Random

In[25]:=
```
L = 10;
Do[
  g[nx, ny, nz] = randomUnitaryMatrix[2];
  , {nx, 0, L - 1}, {ny, 0, L - 1}, {nz, 0, L - 1}
 ];
setWing[g, L]
W3[g, L]
action[g, L]
```

Out[28]= 0.0913653

Out[29]= 1.49561

In[30]:=
```
δt = 0.1;
tm = 40;
tbl = flowW3[g, δt, tm, L, gfZ];
```

13.2

In[•]:=
```
ListPlot[tbl[[1]], Joined → True, PlotRange → Full,
 AxesLabel → {Style[t, 16], Style[W3, 16]}]
ListPlot[tbl[[2]], Joined → True, PlotRange → Full,
 AxesLabel → {Style[t, 16], Style[S/V, 16]}]
```

Out[•]=