

$$\begin{aligned}
 S &= -\frac{1}{16} \sum_{x,\mu} \text{tr} \left\{ g_x^{-1} [g_{x+\hat{\mu}} - g_{x-\hat{\mu}}] \right. \\
 &\quad \left. - [g_{x+\hat{\mu}} - g_{x-\hat{\mu}}]^{-1} g_x \right\}^2 \\
 &= -\frac{1}{16} \sum_{x,\mu} \text{tr} \left\{ g_x^{-1} [g_{x+\hat{\mu}} - g_{x-\hat{\mu}}] \right. \\
 &\quad \times g_x^{-1} [g_{x+\hat{\mu}} - g_{x-\hat{\mu}}] \\
 &\quad - \cancel{g_x} \left[ \cancel{g_{x+\hat{\mu}}} - \cancel{g_{x-\hat{\mu}}} \right] \\
 &\quad \times \cancel{[g_{x+\hat{\mu}} - g_{x-\hat{\mu}}]^{-1} g_x} \\
 &\quad - \cancel{[g_{x+\hat{\mu}} - g_{x-\hat{\mu}}]^{-1} g_x} \\
 &\quad \times \cancel{g_x^{-1} [g_{x+\hat{\mu}} - g_{x-\hat{\mu}}]} \\
 &\quad + \cancel{[g_{x+\hat{\mu}} - g_{x-\hat{\mu}}]^{-1} g_x} \\
 &\quad \times \cancel{[g_{x+\hat{\mu}} - g_{x-\hat{\mu}}]^{-1} g_x} \Big\} \\
 &= -\frac{1}{16} \sum_{x,\mu} \text{tr} \left\{ -2 \right. \\
 &\quad + g_x^{-1} [g_{x+\hat{\mu}} - g_{x-\hat{\mu}}] \\
 &\quad \times g_x^{-1} [g_{x+\hat{\mu}} - g_{x-\hat{\mu}}] \\
 &\quad + [g_{x+\hat{\mu}} - g_{x-\hat{\mu}}]^{-1} g_x \\
 &\quad \times [g_{x+\hat{\mu}} - g_{x-\hat{\mu}}]^{-1} g_x \Big\}
 \end{aligned}$$

$$\partial_y^{\alpha} S = -\frac{1}{16} \sum_{x,\mu} \frac{d}{ds} \text{tr} \left\{ g_x^{-1} [g_{x+\hat{\mu}} - g_{x-\hat{\mu}}] \right. \\ \left. \times g_x^{-1} [g_{x+\hat{\mu}} - g_{x-\hat{\mu}}] \right\}$$

$$= -\frac{1}{16} \sum_{x,\mu} \text{tr} \left\{ S_x \frac{d}{ds} e^{-sT^a} \right. \\ \left. - H.c. \right\}_{g \rightarrow g e^{sT^a}} \\ \times g_x^{-1} [g_{x+\hat{\mu}} - g_{x-\hat{\mu}}] \\ \times g_x^{-1} [g_{x+\hat{\mu}} - g_{x-\hat{\mu}}]$$

$$+ g_x^{-1} \left[ \delta_{x+\hat{\mu},y} \frac{d}{ds} g_{x+\hat{\mu}} e^{sT^a} \right. \\ \left. - \delta_{x-\hat{\mu},y} \frac{d}{ds} g_{x-\hat{\mu}} e^{sT^a} \right] \\ \times g_x^{-1} [g_{x+\hat{\mu}} - g_{x-\hat{\mu}}]$$

$$+ \dots - H.c. \Big\}_{s=0}$$

$$= -\frac{1}{16} \sum_{\mu} \text{tr} \left\{ -T^a g_y^{-1} [g_{y+\hat{\mu}} - g_{y-\hat{\mu}}] \right. \\ \left. \times g_y^{-1} [g_{y+\hat{\mu}} - g_{y-\hat{\mu}}] \right\} \\ + g_{y-\hat{\mu}}^{-1} g_y T^a g_{y-\hat{\mu}}^{-1} [g_y - g_{y-2\hat{\mu}}] \\ - g_{y+\hat{\mu}}^{-1} g_y T^a g_{y+\hat{\mu}}^{-1} [g_{y+2\hat{\mu}} - g_y] \\ - g_y^{-1} [g_{y+\hat{\mu}} - g_{y-\hat{\mu}}] T^a g_y^{-1} [g_{y+\hat{\mu}} - g_{y-\hat{\mu}}] \\ + g_{y-\hat{\mu}}^{-1} [g_y - g_{y-2\hat{\mu}}] g_{y-\hat{\mu}}^{-1} g_y T^a \\ - g_{y+\hat{\mu}}^{-1} [g_{y+2\hat{\mu}} - g_y] g_{y+\hat{\mu}}^{-1} g_y T^a \Big\}_{H.c.}$$

$$Z = -g_x \partial_x^\alpha S T^\alpha g_x^{-1}$$

$$= \frac{1}{8} \sum_\mu g_x \text{tr} \left\{ -T_{ij}^\alpha \left[ g_x^{-1} [g_{x+\hat{\mu}} - g_{x-\hat{\mu}}] g_x^{-1} [g_{x+\hat{\mu}} - g_{x-\hat{\mu}}] \right]_{ji} \right.$$

$$+ T_{ij}^\alpha \left( g_{x-\hat{\mu}}^{-1} [g_x - g_{x-\hat{\mu}}] g_{x-\hat{\mu}}^{-1} g_x \right)_{ji}$$

$$- T_{ij}^\alpha \left( g_{x+\hat{\mu}}^{-1} [g_{x+\hat{\mu}} - g_x] g_{x+\hat{\mu}}^{-1} g_x \right)_{ji}$$

$$- H.c. \left\{ T_{k\ell}^\alpha g_x^{-1} \right\}$$

$$= -\frac{1}{16} \sum_\mu g_x \left[ -g_x^{-1} (g_{x+\hat{\mu}} - g_{x-\hat{\mu}}) g_x^{-1} (g_{x+\hat{\mu}} - g_{x-\hat{\mu}}) \right.$$

$$+ g_{x-\hat{\mu}}^{-1} (g_x - g_{x-\hat{\mu}}) g_{x-\hat{\mu}}^{-1} g_x$$

$$\left. - g_{x+\hat{\mu}}^{-1} (g_{x+\hat{\mu}} - g_x) g_{x+\hat{\mu}}^{-1} g_x - H.c. \right] g_x^{-1}$$

$$= \frac{1}{16} \sum_\mu \left[ g_{x+\hat{\mu}} g_x^{-1} (g_{x+\hat{\mu}} - g_{x-\hat{\mu}}) g_x^{-1} \right.$$

$$- g_{x-\hat{\mu}} g_x^{-1} (g_{x+\hat{\mu}} - g_{x-\hat{\mu}}) g_x^{-1}$$

$$- g_x g_{x-\hat{\mu}}^{-1} (g_x - g_{x-\hat{\mu}}) g_{x-\hat{\mu}}^{-1}$$

$$\left. + g_x g_{x+\hat{\mu}}^{-1} (g_{x+\hat{\mu}} - g_x) g_{x+\hat{\mu}}^{-1} - H.c. \right]$$