

$\sqrt{2}$

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$P = A (A^T A)^{-1} A^T$$

$$A^T A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$$

Найдем $(A^T A)^{-1}$

$$\left(\begin{array}{cc|cc} 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{array} \right) \rightarrow \left(\begin{array}{cc|cc} 1 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & 0 & 1 \end{array} \right)$$

$$A^T A^{-1} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix}$$

$$P = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \\ \frac{1}{2} & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$$

N2

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

Ортогонализирова

методом

Грамма-Шмидта

$$b_1 = a_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

Нормировка

$$b_1 = \begin{pmatrix} \frac{\sqrt{2}}{2} \\ 0 \\ \frac{\sqrt{2}}{2} \end{pmatrix}$$

$$b_2 = a_2 - b_1 \frac{\langle a_2, b_1 \rangle}{\langle b_1, b_1 \rangle} \oplus$$

$$\oplus \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$Q = \begin{pmatrix} \frac{\sqrt{2}}{2} & 0 \\ 0 & 1 \\ \frac{\sqrt{2}}{2} & 0 \end{pmatrix}$$

$$R = Q^T A = \begin{pmatrix} \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} \sqrt{2} & 0 \\ 0 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$P = B(B^T B)^{-1} B^T$$

$$B^T B = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 2 \\ 2 & 5 \end{pmatrix}$$

Найдем $(B^T B)^{-1}$

$$\left(\begin{array}{cc|cc} 2 & 2 & 1 & 0 \\ 2 & 5 & 0 & 1 \end{array} \right) \rightarrow \left(\begin{array}{cc|cc} 1 & 0 & \frac{5}{6} & -\frac{1}{3} \\ 0 & 1 & -\frac{1}{3} & \frac{1}{5} \end{array} \right)$$

$$P = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \frac{5}{6} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{1}{5} \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{4}{6} & \frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} \\ \frac{5}{6} & -\frac{1}{3} \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix} = \begin{pmatrix} \frac{5}{6} & \frac{1}{3} & \frac{1}{6} \\ \frac{1}{3} & \frac{1}{3} & -\frac{1}{3} \\ \frac{1}{6} & -\frac{1}{3} & \frac{5}{6} \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \quad a_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad a_2 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

Аналогично как с A

$$b_1 = a_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad b_{1\text{норм}} = \begin{pmatrix} \frac{\sqrt{2}}{2} \\ 0 \\ \frac{\sqrt{2}}{2} \end{pmatrix}$$

$$b_2 = a_2 - \frac{\langle a_2, b_1 \rangle}{\langle b_1, b_1 \rangle} b_1 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} - \frac{2}{12} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

$$b_{2\text{норм}} = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \end{pmatrix}$$

$$Q = \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{3}}{3} \\ 0 & \frac{\sqrt{3}}{3} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{3}}{3} \end{pmatrix}$$

$$R = Q^T B = \begin{pmatrix} \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{3} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{3} \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} \sqrt{2} & \sqrt{2} \\ 0 & \sqrt{3} \end{pmatrix}$$