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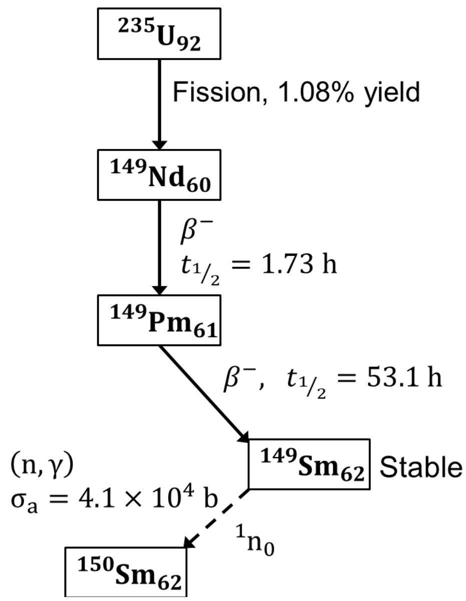
Assignment #3

Question 2 (6 marks for Coding Portion, 11 marks for Non-coding Portion)

Samarium-149 is a by-product of U-235 fission. Due to the high neutron capture cross-section of samarium-149, its production impacts the operation of a nuclear reactor (i.e., this is an important fission product poison). The reaction scheme for ^{149}Sm is shown:



The following assumptions and information are available:

- The half-life of Nd-149 is sufficiently shorter than that of Pm-149 such that it can be assumed that Pm-149 is formed directly from the fissioning of U-235.
- Sm-149 is a stable isotope. During normal operation of a nuclear reactor, one of the ways it can be removed is by burnup, or the neutron absorption of Sm-149 to form Sm-150.
- Assume that fission does not produce Sm-149 directly.
- Assume a neutron flux of 2×10^{14} n/(cm^2 s).

(a) Write the differential equations for Pm-149 and Sm-149, considering all relevant production and consumption terms from the provided reaction scheme and assumptions. **(2 marks)**

1 Promethium - 149 (Pm - 149) balance:

$^{235}U \rightarrow {}^{149}Nd \rightarrow {}^{149}Pm$

$t_{1/2}$ $^{149}Nd \downarrow$, assume: $^{235}U \longrightarrow {}^{149}Pd$

Accumulation = Production − Decay

$$\frac{dN_{Pm-149}}{dt} = \text{Production from U-235 fission} - \text{Decay of Pm-149 to Sm-149}$$

0.5 marks * Note: Don't need to add a burnup term for Pm-149 but σ_a is quite small here (assuming thermal neutrons)

fission yield

$$P = \gamma_{Pm-149} \Sigma_{f,U235}\,\phi = \underbrace{0.0072 N_{U-235}\,\sigma_{f\,U-235}\,\phi \times \gamma_{Pm-149}}$$

can sub in this or leave as $\gamma\Sigma\phi$ term for now

$$\text{Decay} = \lambda_{Pm-149} N_{Pm-149}$$

$$\boxed{\frac{dN_{Pm-149}}{dt} = \Sigma_{f\,U-235}\,\phi - \lambda_{Pm-149} N_{Pm-149}}$$

0.5 marks

② Samarium-149 (Sm-149) balance:

$$\frac{dN_{Sm-149}}{dt} = \underbrace{\text{Decay of Pm-149}} - \text{Burnup of Sm-149}$$

no decay term as Sm-149 is stable

$$P = \lambda_{Pm-149} N_{Pm-149}$$

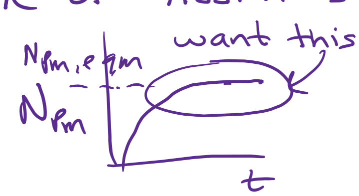
$$\text{Burnup} = \Sigma_{a\,Sm-149}\,\phi = N_{Sm-149} \times \sigma_{a\,Sm-149} \times \phi$$

$$\boxed{\frac{dN_{Sm-149}}{dt} = \lambda_{Pm-149} N_{Pm-149} - \sigma_{a,Sm-149}\,\phi\, N_{Sm-149}}$$

1 mark

(b) What is the equilibrium concentration of Pm-149? Report your answer in atoms/cm^3. **(2 marks)**

Assuming this is during normal operation, rate of fission is constant, so const. production of Pm-149 ↳



From (a):

$$\frac{dN_{Pm-149}}{dt} = \Sigma_{f\,U-235}\,\phi - \lambda_{Pm-149} N_{Pm-149}$$

At equilibrium: $dN_{Pm-149}/dt = 0$, 0.5 marks

0.5 marks

$$N_{Pm-149} = \frac{\gamma_{Pm-149} \Sigma_{f\,U235}\,\phi}{\lambda}$$

$$N_{U-235} = 0.0072(0.0483 \times 10^{24}\ a/cm^3)$$

$$\sigma_{f\,U-235} = 587 \times 10^{-24}\ cm^2$$

Pm-149

$\Sigma_{f\,U\text{-}235} = 0.0072 \times 587 \times 0.0483 = 0.204\ cm^{-1}$

0.5 marks

$\lambda_{Pm\text{-}149} = \frac{0.693}{(53.1\,h) \times \left(\frac{3600\,s}{h}\right)} = 3.62 \times 10^{-6}\ s^{-1}$

$N_{Pm\text{-}149,\,eq'm} = \frac{(0.0108)(0.204\,cm^{-1})(2 \times 10^{14}\,n/cm^2 \cancel{s})}{(3.62 \times 10^{-6}\ \cancel{s^{-1}})}$

$\gamma_{Nd} = 1.08\%$
$= 0.0108$
assuming U-235 → Pm-149, use this as the yield for Pm-149

$$\boxed{N_{Pm\text{-}149,\,eq'm} \simeq 1.22 \times 10^{17}\ atoms/cm^3}$$ 0.5 marks.

(c) The burnup of Sm-149 to form Sm-150 generates a stable isotope, meaning Sm-150 will accumulate inside of a nuclear reactor. Is the burnup of Sm-149 an effective removal mechanism for fission product poisoning? Explain. **(3 marks)**

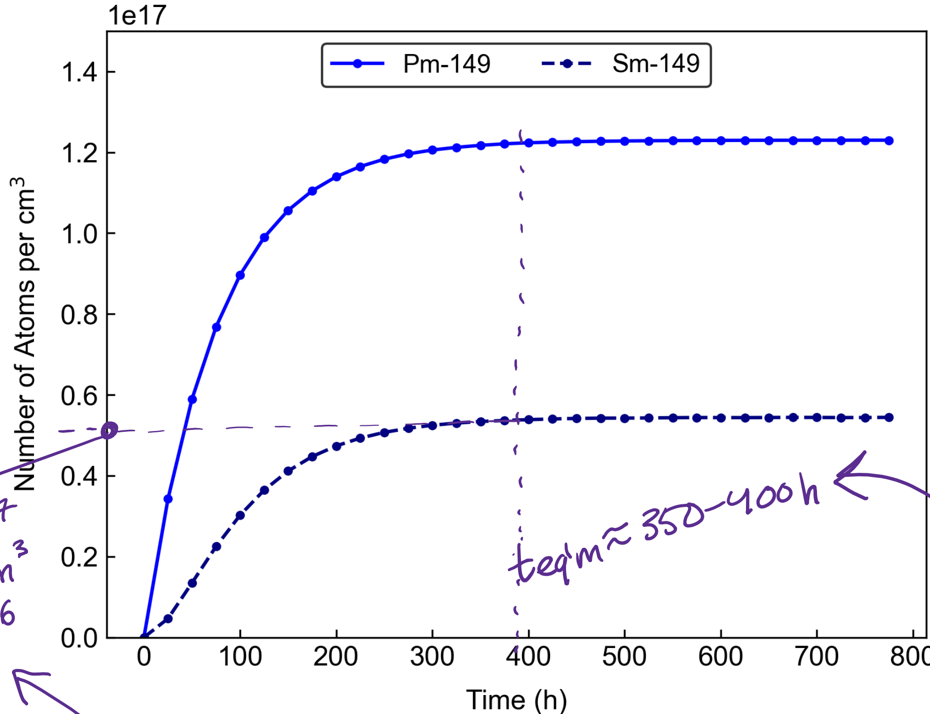
2 marks

Yes, Sm-150 stays in reactor until refuelling, but it has negligible neutron absorption, therefore, it is effective at removing some of the Sm-149.

1 mark

(d) As done in Coding Exercise #3, set up a function that solves the system of ODEs from part (a) using the solve_ivp function from the scipy package in Python. Plot the concentration (atoms/cm^3) of Sm-149 and Pm-149 over a period of at least 3 weeks (you may use any time scale you wish, but hours are suggested). **(4 marks for Coding Portion)**

Note: Make sure the plot is formatted appropriately (axis titles, reasonable scaling of axes, etc.) Save this plot as .png or .jpeg file and upload it to the D2L Assignment 3 Dropbox.



N_{Sm-149} eq'm $\approx 0.55 \times 10^{17}$ atom/cm^3 or 5.5×10^{16} a/cm^3

1 mark

$t_{eq'm} \approx 350$–400 h

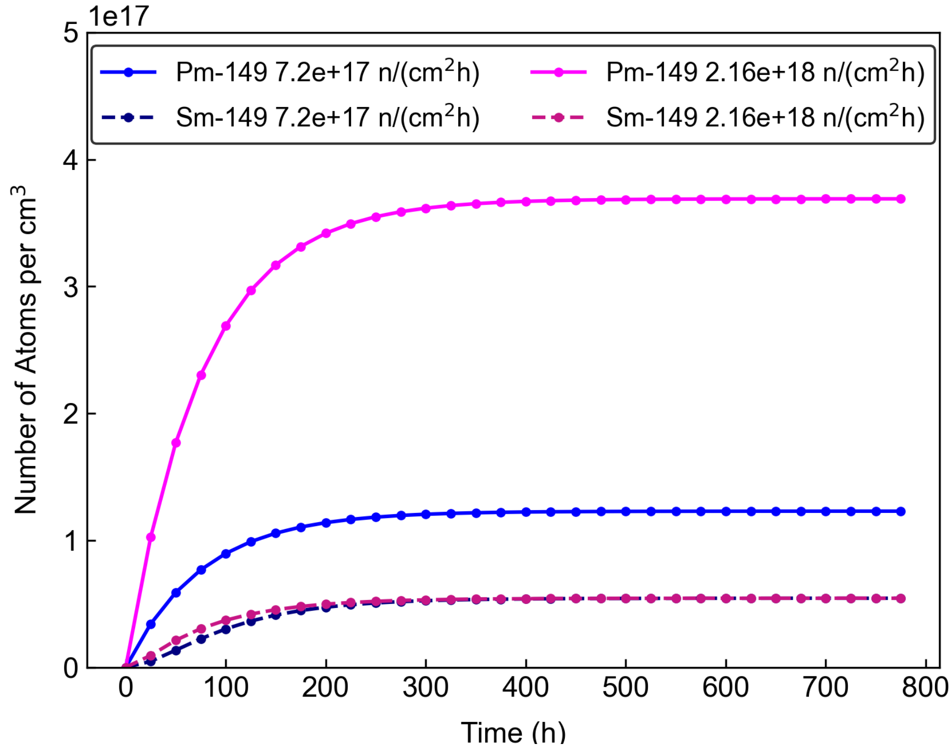
1 mark

(e) What is the equilibrium concentration of Sm-149 based on your plot from part (c)? When is this saturation concentration reached? Does the equilibrium concentration of Pm-149 match your calculation in part (b)? **(2 marks)**

(f)

i. Re-run your code from part (c) with a different neutron flux (e.g., choose a flux that is a factor of 2-10 times greater or lower than the 2×10^{14} n/(cm^2 s) flux provided initially). Compare your plot of Sm-149 concentration at this new flux relative to the one used in part (c). **(2 marks for Coding Portion)**

<u>Note:</u> Make sure the plot is formatted appropriately (axis titles, scaling of axes, etc.)
Save this plot as .png or .jpeg file and upload it to the D2L Assignment 3 Dropbox.



ii. Has the magnitude of the saturation concentration or the time required to reach the saturation concentration changed significantly? **Explain this behaviour. (2 marks)**

From the balance on Sm-149 in (a), at equilibrium,

$\frac{dN_{Sm-149}}{dt} = 0$, therefore:

$$0 = \lambda_{Pm-149} N_{Pm-149} - \sigma_{a,Sm-149} \phi N_{Sm-149}$$

$$N_{Pm\text{-}149,\,eq'm} = \frac{\gamma_{Pm\text{-}149}\,\Sigma_{f\,U\text{-}235}\,\phi}{\lambda_{Pm\text{-}149}}$$ (from part (b))

$$0 = \cancel{\lambda_{Pm\text{-}149}}\left(\frac{\gamma_{Pm\text{-}149}\,\Sigma_{f\,U\text{-}235}\,\phi}{\cancel{\lambda_{Pm\text{-}149}}}\right) - \sigma_{a,\,Sm\text{-}149}\,\phi\,N_{Sm\text{-}149,\,eqm}$$

$$\gamma_{Pm\text{-}149}\,\Sigma_{f\,U235}\,\cancel{\underline{\phi}} = \sigma_{a\,Sm\text{-}149}\,N_{Sm\text{-}149,\,eqm}\,\cancel{\underline{\phi}}$$

cancels out!

Therefore:

1.5 marks $$N_{Sm\text{-}149,\,eqm} = \frac{\gamma_{Pm\text{-}149}\,\Sigma_{f\,U235}}{\sigma_{a\,Sm\text{-}149}}$$

independent of flux!

0.5 marks So, <u>no</u> [Sm-149] does not change at a different flux, or reactor power.

if eq'n not derived, flux term cancelling out needs to be explained verbally.

BONUS (2 marks): Plot the Sm-149 concentration at three of four different neutron fluxes all on the same plot. You do not need to include the Pm-149 concentration.

<u>Hint</u>: one of the ways to accomplish this is to make the neutron flux an input argument in the function that defines the system of ODEs (similarly to how this was done for the decay constants in Coding Exercise #3). A list of neutron fluxes of choice can then be iterated through, calling the solve_ivp function and the plot at each iteration.

Include the saved .png or .jpeg figure of your plot output with your assignment submission (this would replace the upload of just the one new neutron flux plot from part (f)).