Question 2 (6 marks for coding portion, 14 marks for non-coding portion)

Samarium-149 is a by-product of the fissioning of the U-235 nucleus. Due to the high neutron capture cross-section of samarium-149, its production impacts the operation of a nuclear reactor (i.e., this is an important fission product poison). The reaction scheme for ^{149}Sm is shown on the next page.

The following assumptions and information are available:

- The half-life of Nd-149 is sufficiently shorter than that of Pm-149 such that it can be assumed that Pm-149 is formed directly from the fissioning of U-235.
- Sm-149 is a stable isotope. During normal operation of a nuclear reactor, one of the ways it can be removed is by burnup, or the neutron absorption of Sm-149 to form Sm-150.
- Assume that fission does not produce Sm-149 directly.
- Assume a neutron flux of 1×10^{14} n/(cm^2 s).

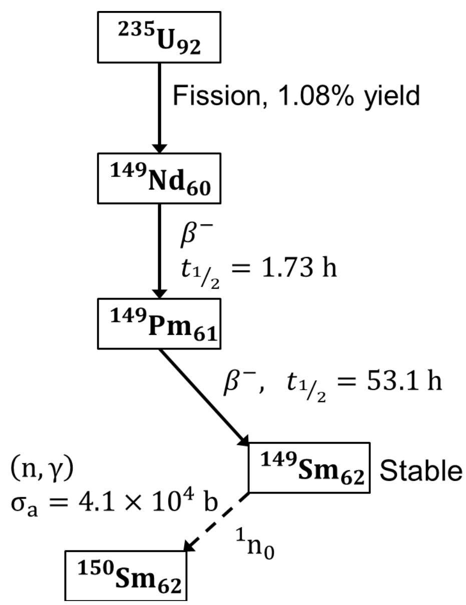

Figure 1. Reaction pathway for the production of Sm-149 from U-235 fission.

(a) Write out all the reactions in Figure 1, i.e., the splitting of U-235 to form Nd-149 (assume 2 neutrons and another fission product, X, which you need to identify, are formed), the decays of Nd-149 and Pm-149, and the neutron activation of Sm-149 to form Sm-150. **(2 marks)**

(b) Write the differential equations for Pm-149 and Sm-149, considering all relevant production and consumption terms from the provided reaction scheme and all assumptions. **(2 marks)**

(c) Derive an expression for the equilibrium concentration of Pm-149. What is the equilibrium concentration of Pm-149? Report your answer in atoms/cm^3. **(3 marks)**

(d) The burnup of Sm-149 to form Sm-150 generates a stable isotope, meaning Sm-150 will accumulate inside of a nuclear reactor. Is the burnup of Sm-149 an effective removal mechanism for fission product poisoning? Explain. **(3 marks)**

Coding Portion (4 marks)

(e) As done in Coding Exercise #3, set up a function that solves the system of ODEs from **(a)** using the solve_ivp function from the scipy package in Python. Plot the concentration (atoms/cm^3) of Sm-149 and Pm-149 over a period of at least 3 weeks (you may use any time scale you wish, but hours are suggested). **(4 marks)**

(f) What are the equilibrium concentrations of Pm-149 and Sm-149, based on your plot from **(e)**? When are these saturation concentrations reached? Does the equilibrium concentration of Pm-149 match your calculation in **(c)**? **(2 marks)**

(g)

Coding Portion (2 marks)

i. Re-run your code from **(e)** with a different neutron flux (e.g., choose a flux that is a factor of 2-10 times greater or lower than the 1×10^{14} n/(cm^2 s) flux provided initially). Compare your plot of Sm-149 concentration at this new flux relative to the one generated in **(e)**. **(2 marks)**

Note: Make sure the plot is formatted appropriately (axis titles, scaling of axes, etc.)
Save this plot as .png or .jpeg file and upload it to the D2L Assignment 3 Dropbox.

ii. Has the magnitude of the saturation concentrations of Sm-149 and Pm-149. or the time required to reach these saturation concentrations, changed significantly? **Explain this behaviour.** Hint: it may be useful to derive the expression for the equilibrium concentration of Sm-149 here. **(2 marks)**

BONUS (2 marks): Plot the Sm-149 concentration at three of four different neutron fluxes all on the same plot. You do not need to include the Pm-149 concentration.

Hint: one of the ways to accomplish this is to make the neutron flux an input argument in the function that defines the system of ODEs (similarly to how this was done for the decay constants in Coding Exercise #3). A list of neutron fluxes of choice can then be iterated through, calling the solve_ivp function and the plot at each iteration.

Include the saved .png or .jpeg figure of your plot output with your assignment submission (this would replace the upload of just the one new neutron flux plot from part (f)).