AQA GCSE Maths: Higher

Your notes

Simple & Compound Interest, Growth & Decay

Contents

Simple Interest

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Simple Interest

What is simple interest?

- **Interest** is money that is regularly added to an original amount of money
  - This could be added yearly, monthly, etc
  - When saving money, interest helps increase the amount saved
  - With debt, interest increases the amount owed
- **Simple interest** refers to interest which is **based only on the starting amount**
  - Each interest payment (or charge in the case of debt) will be the **same**

How do I calculate simple interest?

- To find the **total simple interest earned**
  - Find a **percentage** (the percentage rate) of the starting amount
    - Use a **multiplier** to do this (e.g. 0.05 to find 5%)
  - **Multiply** this by the **number of time periods** (e.g. years) it is being applied for
- To find the **total balance** after the simple interest has been earned
  - Use the same method as above, and **add this on** to the starting amount

Examiner Tips and Tricks

- Double check:
  - Does the question ask for the interest earned, or the total amount at the end?
  - Do you need to round your answer? (e.g. to the nearest hundred)

Worked Example

Your notes

A bank account offers **simple** interest of 4% per year. Nigel puts £ 250 into this bank account, and leaves it to earn interest for 6 years.

(a) Find the total amount of interest earned over the 6 year period.

Each year, 4% of the starting amount is added as interest
Find 4% of £ 250 using a multiplier

$$0.04 \times 250 = 10$$

This amount of interest is earned every year, for 6 years

$$10 \times 6 = 60$$

£ 60 of interest is earned

(b) Find the total amount in the bank account at the end of the 6 year period.

Add the amount of interest earned, found in part (a), to the starting amount

$$250 + 60 = 310$$

£ 310

Worked Example

Noah invests £ 9000 at a rate of $n\%$ **simple** interest per year, for 5 years. At the end of 5 years there is £ 11 700 in the account. Find the value of n.

Find the total amount of interest earned over the 5 years

$$11\,700 - 9\,000 = £\,2\,700 \text{ total interest}$$

As we are dealing with simple interest, the same amount of interest is earned each year
Find the interest earned each year

$$2\,700 \div 5 = £\,540 \text{ interest per year}$$

Find what percentage of the original amount this represents

$$\frac{540}{9000} = 0.06 = 6\%$$

£ 540 is 6% of the original £ 9000

$$n = 6$$

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Compound Interest

What is compound interest?

- **Compound interest** is where interest is calculated on the **running total**, not just the starting amount
  - This is different from **simple interest** where interest is only based on the starting amount
- E.g. **£ 100** earns **10% interest** each year, for 3 years
  - At the end of year 1, **10% of £ 100 is earned**
    - The total balance will now be 100+10 = **£ 110**
  - At the end of year 2, **10% of £ 110 is earned**
    - The balance will now be 110+11 = **£ 121**
  - At the end of year 3, **10% of £ 121 is earned**
    - The balance will now be 121+12.1 = **£ 133.10**

How do I calculate compound interest?

- Compound interest **increases an amount by a percentage** and then **increases the new amount by the same percentage**
  - This process repeats each time period (yearly or monthly etc)
- We can use a **multiplier** to carry out the percentage increase multiple times
  - To increase **£ 300** by **5% once**, we would find $\mathbf{300 \times 1.05}$
  - To increase **£ 300** by **5%**, each year for **2 years**, we would find $(300 \times 1.05) \times 1.05$
    - This could be rewritten as $\mathbf{300 \times 1.05^2}$
  - To increase **£ 300** by **5%**, each year for **3 years**, we would find $((300 \times 1.05) \times 1.05) \times 1.05$
    - This could be rewritten as $\mathbf{300 \times 1.05^3}$
- This can be extended to any number of periods that the interest is applied for
  - If **£ 2000** is subject to **4%** compound interest each year for **12 years**
  - Find $\mathbf{2000 \times 1.04^{12}}$, which is £ 3202.06
- Note that this method calculates the **total balance** at the end of the period, **not the interest earned**

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Compound interest formula

- An **alternative method** is to use the **following formula** to calculate the final balance
  - Final balance = $P\left(1+\frac{r}{100}\right)^n$ where
    - P is the original amount,
    - r is the % increase,
    - and n is the number of years
  - Note that $1+\frac{r}{100}$ is the same value as the multiplier
    - e.g. 1.15 for 15% interest
- This formula is **not given** in the exam

How do I solve reverse compound interest problems?

- You could be **told the final balance after** compound interest has been applied, and **need to find the original amount**
  - This could be referred to as a "**reverse compound interest**" problem
- For example if:
  - The final balance is £432
  - After 20% interest has been applied each year
  - For 3 years
- Using the same method as above, this can be written as an equation:
  - $432 = P \times 1.20^3$ where P is the original amount
  - Solve for P,
    - Divide both sides by 1.20^3
    - $432 \div 1.20^3 = P$
    - $P = £250$
- In general, to find the original amount:

- Divide the final amount by m^n where
  - m is the multiplier for the time period
  - and n is the number of time periods (usually years)

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Examiner Tips and Tricks

- Double check if the question uses **simple** interest or **compound** interest
- The formula for compound interest is **not** given in the exam

Worked Example

Jasmina invests £ 1200 in a savings account, which pays compound interest at the rate of 4% per year for 7 years.

To the nearest dollar, what is her investment worth at the end of the 7 years?

Method 1

We want an increase of 4% per year
This is equivalent to a multiplier of 1.04, or 104% of the original amount

This multiplier is applied 7 times
$\times 1.04 \times 1.04 \times 1.04 \times 1.04 \times 1.04 \times 1.04 \times 1.04 \ = \ 1.04^7$

Therefore the final value after 7 years will be

$$1200 \times 1.04^7 = \$ \, 1579.118135...$$

Round to the nearest dollar

$$\text{£ } 1579$$

Method 2

Using the formula for the final amount $P\left(1 + \frac{r}{100}\right)^n$

Substitute *P* is 1200, *r* = 4 and *n* = 7 into the formula

$$1200\left(1 + \frac{4}{100}\right)^7$$

£ 1579

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Depreciation

What does depreciation mean?

- Depreciation is where an item **loses value** over time
  - E.g. cars, mobile phones, etc
- Depreciation is usually calculated as a percentage decrease at the end of each year
  - This works the same as compound interest, but with a percentage **decrease**

How do I calculate depreciation?

- A similar method to **compound interest** can be used
- Change the **multiplier** to one which represents a **percentage decrease**
  - e.g. a **decrease of 15%** would be a **multiplier** of **0.85**
- If a car worth £ 16 000 depreciates by 15% each year for 6 years
  - Its value will be 16 000 × 0.85^6, which is £ 6034.39
- If you are asked to find the amount the value has depreciated by:
  - Find the difference between the starting value and the new value

Depreciation formula

- An **alternate method** is to use the **following formula** to calculate the final balance
  - Final balance = $P\left(1-\frac{r}{100}\right)^n$ where
    - P is the original amount,
    - r is the % increase
    - and n is the number of years
  - Note that all of $1-\frac{r}{100}$ is the **multiplier**
    - e.g. 0.75 for a 25% depreciation

- This formula is **not given** in the exam

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Worked Example

Mercy buys a car for £20 000. Each year its value depreciates by 15%.

Find the value of the car after 3 full years.

Identify the multiplier

$$100\% - 15\% = 85\%$$

$$m = 1 - 0.15 = 0.85$$

Raise to the power of number of years

$$0.85^3$$

Multiply by the starting value

$$£20\ 000 \times 0.85^3$$

£12 282.50

Alternative method

Use the formula for the final amount $P\left(1 - \frac{r}{100}\right)^n$

Substitute $P = 20\ 000$, $r = 15$ and $n = 3$ into the formula

$$20\ 000\left(1 - \frac{15}{100}\right)^3$$

£12 282.50

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Exponential Growth & Decay

The ideas of compound interest and depreciation can be applied to other (non-money) situations, such as increasing or decreasing populations.

What is exponential growth?

- When a quantity **grows exponentially** it is **increasing** from an original amount by a percentage each year for n years
  - Some questions use a different timescale, such as each day, or each minute
- Real-life examples of exponential growth include:
  - Population increases
  - Bacterial growth
  - The number of people infected by a virus

What is exponential decay?

- When a quantity **exponentially decays** it is **decreasing** from an original amount by a percentage each year for n years
  - Some questions use a different timescale, such as each day, or each minute
- Real-life examples of exponential decay include:
  - The temperature of hot water cooling down
  - The value of a car decreasing over time
  - Radioactive decay (the mass of a radioactive a substance over time)

How can I model a scenario as exponential growth or decay?

- Scenarios which exponentially grow or decay can be **modelled with an equation**
- A useful format for this equation is
  - $B = A \times k^n$ where:
    - A is the starting (initial) amount

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- B is the new amount
- k is the appropriate **multiplier or scale factor** for the growth or decay in the time period
  - E.g. $k = 0.8$ for a 20% decay, $k = 1.2$ for a 20% growth
- n is the number of time periods
- Note if $k > 1$ then it is exponential **growth**
  - If $0 < k < 1$ then it is exponential **decay**
  - k cannot be negative

How do I use the exponential growth & decay equation?

- You may need to **rearrange** the equation $B = A \times k^n$
  - To find A giving $A = \frac{B}{k^n}$
  - To find k giving $k^n = \frac{B}{A}$ so $k = \sqrt[n]{\frac{B}{A}}$
  - To find n, using **trial and improvement**
    - Test different whole-number values for n until both sides of the equation balance

How does exponential growth and decay relate to exponential graphs?

- Plotting the **exponential model** $B = A \times k^n$ on a graph where:
  - n is on the x-axis
  - and B is on the y-axis
  - gives the shape of an **exponential graph**
    - often written as $y = ak^x$

Examiner Tips and Tricks

- Look out for how the question wants you to give your final answer
  - It may want the final amount to the nearest thousand
  - Or the question may require you to round to the nearest integer for n

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Worked Example

An island has a population of 25 000 people.

The population increases exponentially by 4% every year.

Find the population after 13 years, giving your answer to the nearest hundred.

The question says "increases exponentially" so use $B = A \times k^n$ where $k > 1$

k comes from a percentage increase so add 0.04 to 1

$$k = 1 + 0.04$$

Substitute A = 25 000, k = 1.04 and n = 13 into the formula

$$25\ 000 \times 1.04^{13}$$

Work out the value on your calculator

41626.83...

Round to the nearest hundred

41 600 people

Worked Example

The temperature of a cup of coffee exponentially decays from 60°C by r% each hour. After 3 hours, the temperature is 18°C.

Find the value of r to 3 significant figures.

The question says "exponentially decays" so use $B = A \times k^n$ where $0 < k < 1$

Note that k is the multiplier (it is not equal to r in the question, but is related)

Your notes

Substitute A = 60 and n = 3 into the equation

$$60 \times k^3$$

The temperature after 3 hours is 18, so set the whole equation equal to 18

$$60 \times k^3 = 18$$

Solve this equation for k
Start by dividing both sides by 60

$$k^3 = 0.3$$

The left hand side is to the power of 3 (cubed)
So cube-root both sides and write out lots of decimal places

$$k = \sqrt[3]{0.3} = 0.669432950...$$

Find the percentage decrease represented by this number
It may help to think of an example, e.g. k = 0.6 represents a decrease of 40%

$$1 - 0.669432950... = 0.3305670499...$$

It represents a decrease by 33.05670...%
Round to 3 significant figures

r = 33.1