



AQA GCSE Maths: Higher



Your notes

Forming & Solving Equations

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Your notes

Forming Equations from Words

Forming Linear Equations

How do I form expressions from words?

- You can turn common **phrases** into expressions

- Use **x** to represent an **unknown** value

2 less than "something"	$x - 2$
Double "something"	$2x$
5 lots of "something"	$5x$
3 more than "something"	$x + 3$
Half of "something"	$\frac{1}{2}x$ or $\frac{x}{2}$

- Common words indicating basic operations are:
 - Addition:** sum, total, more than, increase
 - Subtraction:** difference, less than, decrease
 - Multiplication:** product, lots of, times as many, double, triple
 - Division:** shared, split, grouped, halved, quartered
- Brackets** help keep the **order** correct
 - "something" add 1, then multiplied by 3
 - $(x + 1) \times 3$ which simplifies to $3(x + 1)$
 - Compare this to "something" multiplied by 3, then add 1
 - $x \times 3 + 1$ which simplifies to $3x + 1$
- You may have to **choose** which unknown to call **x**



Your notes

- If Adam is 10 years **younger** than Barry, then Barry is 10 years **older** than Adam
 - Either represent Adam's age as $x - 10$ and Barry's age as x
 - Or represent Adam's age as x and Barry's age as $x + 10$
- If Adam's age is **half** of Barry's age, then Barry's age is **double** Adam's age
 - So if Adam's age is x then Barry's age is $2x$
 - This makes the algebra easier (rather than using x for Barry's age and $\frac{1}{2}x$ for Adam's age)

How do I form equations?

- An **equation** is a statement with an **equals** sign that can be **solved**
- Try to put in the phrase "**is equal to**" to see where the equals goes
 - Lisa's age is double Aisha's age and the sum of their ages is ("is equal to") 27
 - Represent Aisha's age as x and Lisa's age is $2x$
 - The equation is $2x + x = 27$
 - When solving, always give the answer in context
 - $3x = 27$ so $x = 9$
 - In context: "Lisa is 18 years old and Aisha is 9 years old"
- Sometimes you might have **two** unknown values (x and y)
 - Use the information to form two **simultaneous** equations



Worked Example

A flowerbed has flowers of three different colours: red, yellow and purple.
The number of yellow flowers is three times the number of red flowers.
The number of purple flowers is 5 more than the number of yellow flowers.

If the difference between the number of purple flowers and red flowers is 29, find the number of yellow flowers.

Let the number of red flowers be x



Your notes

 x red flowers

Multiply this by 3 to get the number of yellow flowers

 $3x$ yellow flowers

Add 5 to the previous result to get the number of purple flowers

 $3x + 5$ purple flowers

Find the difference between the number of purple and red flowers (purple subtract red, as purple is larger)

$$3x + 5 - x$$

Set the difference equal to 29

$$3x + 5 - x = 29$$

Simplify the left-hand side ($3x - x = 2x$)

$$2x + 5 = 29$$

Solve the equation (subtract 5 then divide by 2)

$$2x = 29 - 5$$

$$2x = 24$$

$$x = \frac{24}{2}$$

$$x = 12$$

This is not the answer to the question asked

The number of yellow flowers is $3x$ so multiply this answer by 3**There are 36 yellow flowers**



Your notes

Forming Equations from Shapes

Forming Equations from Shapes

How do I form equations from shapes?

- You need to use all the information given on the diagram and any specific **properties** of that **shape**
- Common **2D shapes** that you should know properties for are
 - Triangles: **equilateral**, **isosceles**, **scalene**, **right-angled**
 - Quadrilaterals: **square**, **rectangle**, **kite**, **rhombus**, **parallelogram**, **trapezium**
- You may be asked about **perimeter**, **area** or **angles**
- You may be asked about **polygons**
 - **Regular** vs **irregular** polygons
 - **Interior** vs **exterior** angles
 - The **sum** of **interior** angles is $180(n-2)$ for an n -sided polygon
- You may be asked about **angles** in **parallel lines**
 - **Alternative**, **corresponding** and **co-interior**
- You may be asked about **3D shapes** involving **surface area** and **volume**
 - **Prisms** have constant cross sections
 - Volume is cross-section area multiplied by length

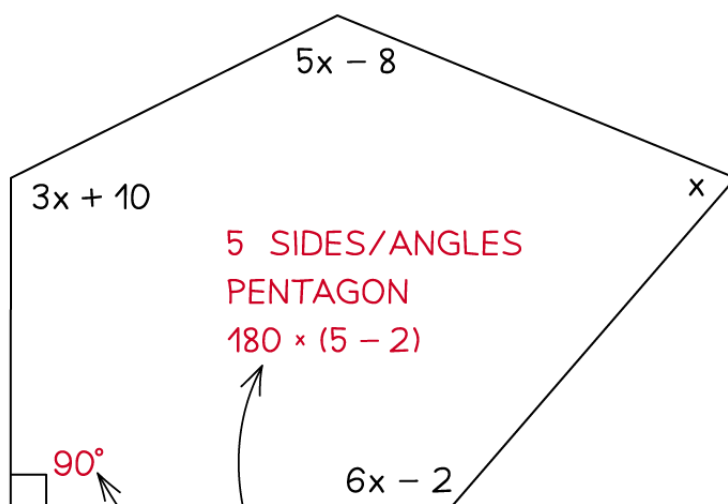
Is there anything else that can help?

- **Sketch** a diagram if none is given
- **Split** up uncommon shapes into the **sum** or **difference** of common shapes
- Look out for important extra information
 - For example, a trapezium "with a line of symmetry"
- With **irregular** shapes, **assume** all angles and lengths are different (unless told otherwise)
- Put **brackets** around **algebraic expressions** when substituting them into geometric properties



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e.g. THE DIAGRAM BELOW SHOWS AN IRREGULAR POLYGON.
ALL ANGLES ARE IN DEGREES.
FIND THE VALUE OF x .



ADDING TO A DIAGRAM
IS A GOOD PROBLEM
SOLVING TECHNIQUE

NOT TO SCALE - CAN'T MEASURE IT!
IRREGULAR POLYGON, BUT, 5 SIDES PENTAGON
INTERIOR ANGLES GIVEN
└ IS A RIGHT-ANGLE, 90°
SUM OF INTERIOR ANGLES IN ANY POLYGON

$$\begin{aligned}\text{SUM OF ANGLES} &= 180(n-2) \\ &= 180(5-2) \\ &= 180 \times 3 \\ &= 540\end{aligned}$$

$$3x + 10 + 5x - 8 + x + 6x - 2 + 90 = 540$$

$$15x + 90 = 540$$

$$15x = 450$$

$$x = 30^\circ$$

A GOOD ALTERNATIVE QUESTION HERE
WOULD BE TO FIND THE SMALLEST OR
LARGEST ANGLE (30° , 178° !)

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Your notes



Examiner Tips and Tricks

- Read the question carefully - does it want an angle? perimeter? total area? curved surface area? etc.
- For surface area and volume questions, check the list of formulas given in the exam.



Worked Example

A rectangle has a length of $3x + 1$ cm and a width of $2x - 5$ cm.

Its perimeter is equal to 22 cm.

(a) Use the above information to find the value of x .

The perimeter of a rectangle is $2 \times \text{length} + 2 \times \text{width}$

$$2(3x + 1) + 2(2x - 5)$$

Expand the brackets

$$6x + 2 + 4x - 10$$

Simplify by collecting like terms

$$10x - 8$$

This perimeter is 22, so set this expression equal to 22

$$10x - 8 = 22$$

Solve this equation by adding 8 then dividing by 10



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$$10x = 22 + 8$$

$$10x = 30$$

$$x = \frac{30}{10}$$

$$x = 3$$

$$\mathbf{x = 3}$$

(b) Find the area of the rectangle.

The area of a rectangle is its length multiplied by its width

Substitute the value of x from part (a) into the length and width given in the question

$$\text{length is } 3 \times 3 + 1 = 10$$

$$\text{width is } 2 \times 3 - 5 = 1$$

Find the area (multiply length by width)

$$10 \times 1$$

Include the correct units for area

$$\mathbf{\text{Area} = 10 \text{ cm}^2}$$



Your notes

Problem Solving with Equations

Equations & Problem Solving

What is problem solving?

- **Problem solving** means you are given a specific **situation** (real life or constructed) and you need to **form** and **solve equations** to find answers to the questions asked
- The equations can be any type that are in the course, including
 - **Linear**, e.g. $2(x + 4) = 3x$
 - **Quadratic**, e.g. $x^2 - 7x + 12 = 0$
- Answers must always be given **in context**
 - This means related to the situation using **words**, **phrases** and **units** from the question
 - For example, "The population density is 225 people per square km" (not just $x = 225$)

What type of algebra can come up in a problem solving question?

- Many questions will require you to **solve quadratic equations**
 - You need to be able to spot these
 - This may require bringing all the terms to one side to get " $= 0$ "
 - You are often free to **choose** which **quadratic method** to use to solve them
 - If you get two solutions, you may need to **justify** which solution is **correct**
- You may be asked to use **algebra** in other settings such as **geometry** or **numbers**
 - $P\%$ is $\frac{P}{100}$ as a decimal
 - If the ratio $x : (x + 2)$ is equivalent to $5 : 8$, then $\frac{x}{x + 2} = \frac{5}{8}$
- You may have **unfamiliar equations** to solve



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- such as multiplying both sides by x in $\frac{12}{x} = 7 - x$ to form a quadratic



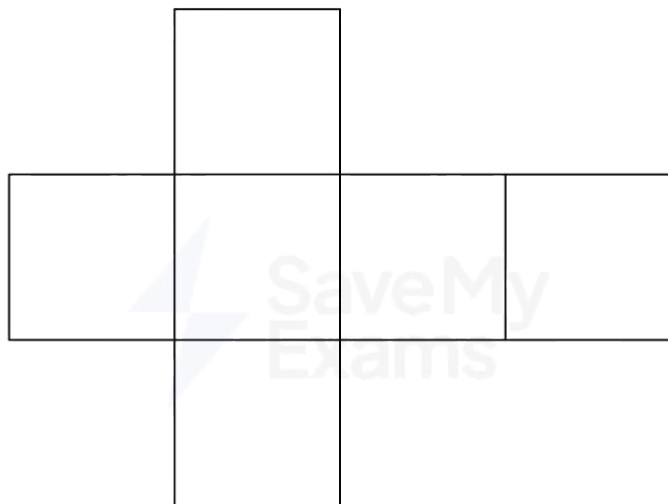
Examiner Tips and Tricks

- If part (a) asks you to prove an equation and part (b) uses that equation, you can still do part (b) without having done part (a)!
 - This means you won't lose all the marks if you can't do part (a)



Worked Example

The net of a cube is shown below.



NET OF A CUBE

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Let x cm be the side length of the cube.

Find an expression in terms of x for

(a) the perimeter of the net shown.



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Count the number of edges around the outside of the net

14 edges

Multiply this number by the side length, x

The perimeter of the net shown is $14x$ cm

(b) the area of the net shown.

Count the number of faces (squares) in the net

6 faces

Multiply this number by the area of one face, $x \times x = x^2$

The area of the net shown is $6x^2$ cm²

When the net is folded into a cube, the difference between the volume of the cube and the surface area of the net is eight times the perimeter of the net.

(c) Show that $x^3 - 6x^2 - 112 = 0$.

First find an expression for the volume of the cube

This will have dimensions $x \times x \times x$

The volume is x^3

Then find an expression for the difference between the volume and the surface area in part (b)

Subtract the surface area from the volume

$$x^3 - 6x^2$$

Set this difference equal to 8 times the perimeter of the net

Use the answer in part (a)

$$x^3 - 6x^2 = 8 \times 14x$$

Simplify 8×14 and bring all the terms to one side

$$x^3 - 6x^2 = 112x$$

$$x^3 - 6x^2 - 112x = 0$$

You are nearly at the correct equation given in the question



Your notes

Cancel both sides by X (as X cannot be zero)

To show this, you can factorise out an X first

$$X(x^2 - 6x - 112) = 0$$

$$x^2 - 6x - 112 = 0$$

(d) Hence, find the exact volume of the cube when the net is folded.

Hence means use the previous results

The volume of the cube is X^3 which involves an unknown, X

To find X , solve the equation in part (c), for example using the quadratic formula

$$\begin{aligned}x &= \frac{6 \pm \sqrt{(-6)^2 - 4 \times 1 \times (-112)}}{2} \\&= \frac{6 \pm \sqrt{484}}{2} \\&= \frac{6 \pm 22}{2}\end{aligned}$$

Find the two possible answers

14 or -8

The side length X cannot be a negative number

$$x = 14$$

Substitute this into X^3 to find the volume

$$14^3$$

The question asks for the answer to be exact, so do not round

The volume is 2744 cm^3