AQA GCSE Maths: Higher

Your notes

Linear Equations

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Solving Linear Equations

What are linear equations?

- A **linear equation** is one that can be written in the **form** $ax + b = c$
  - a, b, and c are **numbers** and x is the **variable**
    - $2x + 3 = 5$
    - $3x + 4 = 1$
    - $x - 5 = -3$
- The greatest power of x is 1
  - There are no terms like x^2

How do I solve linear equations?

- You need to use **operations** like adding, subtracting, multiplying and dividing to get ***x*** on its **own**
- Any operation you do to **one side** of the **equation** must also be done to the **other side**
- For example, to solve $2x + 1 = 9$ look at the +1 on the left
  - Undo this by subtracting 1 from **both sides** and **simplifying**

$$2x + 1 = 9$$

$$(-1) \qquad\qquad (-1)$$

$$2x = 8$$

- This equation is now **easier** to solve
- $2x$ is $2 \times x$ so undo this by dividing **both** sides by 2 and **simplifying**

$$2x = 8$$

$$(\div 2) \qquad\qquad (\div 2)$$

$$x = 4$$

- The **solution** to the equation is $x = 4$

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- Adding 1 was undone by subtracting 1
- Multiplying by 2 was undone by dividing by 2
  - **Addition** and **subtraction** are said to be **inverse** (opposite) operations
  - **Multiplication** and **division** are also **inverse** operations

Does the order of steps matter?

- As long as each step is applied correctly, the order in which inverse operations are applied **does not matter**
  - Applying the operations in one order may be easier than another
- Consider $4x + 8 = 12$
  - It is **easier** to **first subtract** 8 from both sides

$$4x = 4$$

- **Then divide** both sides by 4

$$x = 1$$

- If you want to first divide by 4, a common **mistake** is to write $x + 8 = 3$
  - This is **incorrect** as 8 has not been divided by 4
  - You must divide **every term** by 4

$$\frac{4x}{4} + \frac{8}{4} = \frac{12}{4}$$

$$x + 2 = 3$$

- Then subtract 2 from both sides

$$x = 1$$

How do I solve linear equations with negative numbers?

- For example, $2 - 3x = 10$
  - Subtract 2 from both sides and simplify

$$2 - 3x = 10$$

$$(-2) \qquad\qquad (-2)$$

$$-3x = 8$$

- Then divide both sides by -3 and simplify

$$-3x = 8$$

$$(\div -3) \qquad\qquad (\div -3)$$

$$x = -\frac{8}{3}$$

- Some people prefer to write $2 - 3x = 10$ as $-3x + 2 = 10$, which is equivalent
  - You then subtract 2 and divide by -3 as before

Examiner Tips and Tricks

Substitute your answer back into the original equation to check you got it right!

Worked Example

Solve the equation

$$9 - 7x = 5$$

Subtract 9 from both sides of the equation

$$-7x = -4$$

Divide both sides by -7
Remember that a negative divided by a negative will result in a positive number

$$x = \frac{-4}{-7}$$

$$\boldsymbol{x = \frac{4}{7}}$$

How do I solve linear equations with brackets?


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- If a linear equation involves brackets, **expand** the brackets first
- For example, solve $2(x-3)=10$
  - Expand the brackets

$$2x-6=10$$

- Then solve as shown previously
  - Add 6 then divide by 2

$$2x=16$$
$$x=8$$

- **Expanding** brackets **first** will always work, but you can also divide first
  - Dividing **both** sides of $2(x-3)=10$ by 2 gives $(x-3)=5$
    - which gives $x=8$
  - This method works but can lead to harder **fractions**

How do I solve linear equations with fractions?

- If a linear equation contains **fractions**, **multiply both sides** by the **lowest common denominator**
- For example, $\frac{x}{5}+4=\frac{9}{2}$
  - The lowest common denominator of 5 and 2 is 10
  - Multiply **all terms** on both sides by 10

$$\left(10\times\frac{x}{5}\right)+(10\times4)=10\times\frac{9}{2}$$
$$\frac{10x}{5}+40=\frac{90}{2}$$

- **Simplify** the fractions

$$2x+40=45$$

- Now solve as before, by subtracting 40, then dividing by 2

$$2x = 5$$
$$x = \frac{5}{2}$$


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- Unless the question specifies otherwise, you can leave the answer like this
  - A **decimal** or **mixed number** would also be **accepted**

What if the unknown is on the denominator?

- For example $\frac{4}{x-2} = 3$
  - Multiply both sides of the equation by the denominator

$$\frac{4}{x-2} \times (x-2) = 3(x-2)$$

- Simplify the fractions, and expand any brackets

$$4 = 3(x-2)$$
$$4 = 3x - 6$$

- Now solve as before, by adding 6 to both sides, then dividing by 3

$$10 = 3x$$
$$\frac{10}{3} = x$$

Worked Example

(a) Solve the equation

$$5(3-4x) + 1 = 26$$

Expand the bracket

$$15 - 20x + 1 = 26$$

Simplify

$$16 - 20x = 26$$

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It can be helpful to consider 16 - 20*x* as -20*x* + 16
Subtract 16 from both sides

$$-20x = 10$$

Divide both sides by -20 and simplify

$$x = \frac{10}{-20}$$

$$x = -\frac{1}{2}$$

$$\boldsymbol{x = -\frac{1}{2}}$$

-0.5 is also accepted

(b) Solve the equation

$$\frac{5x}{4} = \frac{1}{2}$$

The lowest common denominator of 4 and 2 is 4
Multiply both sides by 4

$$4 \times \frac{5x}{4} = 4 \times \frac{1}{2}$$

Simplify (cancel) the fractions

$$5x = 2$$

To solve this equation, divide both sides by 5

$$\boldsymbol{x = \frac{2}{5}}$$

0.4 is also accepted

How do I solve linear equations with x terms on both sides?

- **Collect the *x* terms** (or whichever variable is involved) **together on one side**
  - To do this, **remove all the *x* terms from one side**

- It is easiest to remove the **smallest *x* term** to avoid negatives

- For example, $4x - 7 = 11 + x$
- Remove the *x* term on the right-hand side, by subtracting *x* from both sides

$$4x - 7 = 11 + x$$

$$(-x) \qquad\qquad (-x)$$

$$3x - 7 = 11$$

- There are no longer any *x* terms on the right
- This now has the same form as previously seen
  - Solve by adding 7 then dividing by 3

$$3x - 7 = 11$$
$$3x = 18$$
$$x = 6$$

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Worked Example

Solve the equation

$$4 - 5x = 6x - 29$$

Remove the *x* terms from either side
We will remove them from the left as -5*x* is smaller than 6*x*

Add 5*x* to both sides

$$4 = 11x - 29$$

Get 11*x* on its own by adding 29 to both sides

$$33 = 11x$$

Divide both sides by 11 to find *x*

$$3 = x$$

$$\boldsymbol{x = 3}$$