AQA GCSE Maths: Higher

Your notes

Linear Graphs

Contents

Equations of Straight Lines (y = mx + c)

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Finding Equations of Straight Lines

What is the equation of a straight line?

- The general **equation** of a **straight line** is $y = mx + c$ where
  - m is the **gradient**
  - **c** is the ***y*-intercept**
    - The value where it **cuts** the ***y*-axis**
- $y = 5x + 2$ is a straight line with
  - gradient 5
  - *y*-intercept 2
- $y = 3 - 4x$ is a straight line with
  - gradient -4
  - *y*-intercept 3

How do I find the equation of a straight line from a graph?

- Find the **gradient** by drawing a triangle and using
  - $\text{gradient} = \dfrac{\text{rise}}{\text{run}}$
    - **Positive** for uphill lines, **negative** for downhill
- Read off the ***y*-intercept** from the graph
  - Where it cuts the *y*-axis
- **Substitute** these values into $y = mx + c$

What if no y-intercept is shown?

- If you **can't** read off the ***y*-intercept**
  - find any **point** on the line
  - **substitute** it into the **equation**

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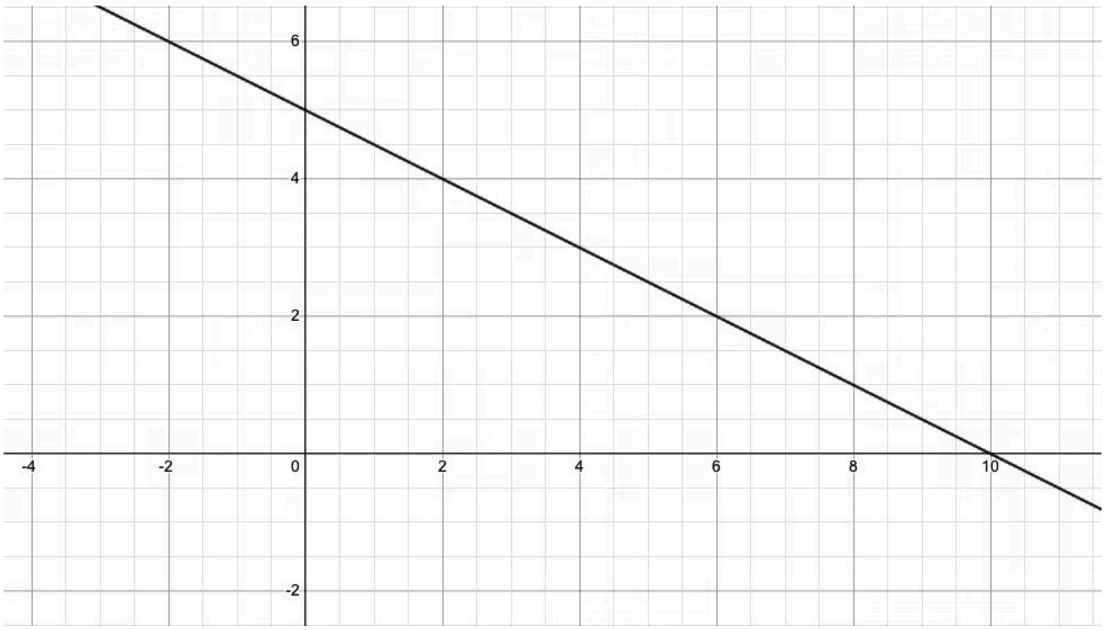
  - **solve** to **find c**
- For example, a line with gradient 6 passes through (2, 15)
  - The y-intercept is unknown
    - Write $y = 6x + c$
  - Substitute in $x = 2$ and $y = 15$
    - $15 = 6 \times 2 + c$
    - $15 = 12 + c$
  - Solve for c
    - $c = 3$
  - The equation is $y = 6x + 3$

What are the equations of horizontal and vertical lines?

- A **horizontal** line has the equation $y = c$
  - c is the y-intercept
- A **vertical** line has the equation $x = k$
  - k is the x-intercept
- For example
  - $y = 4$
  - $x = -2$

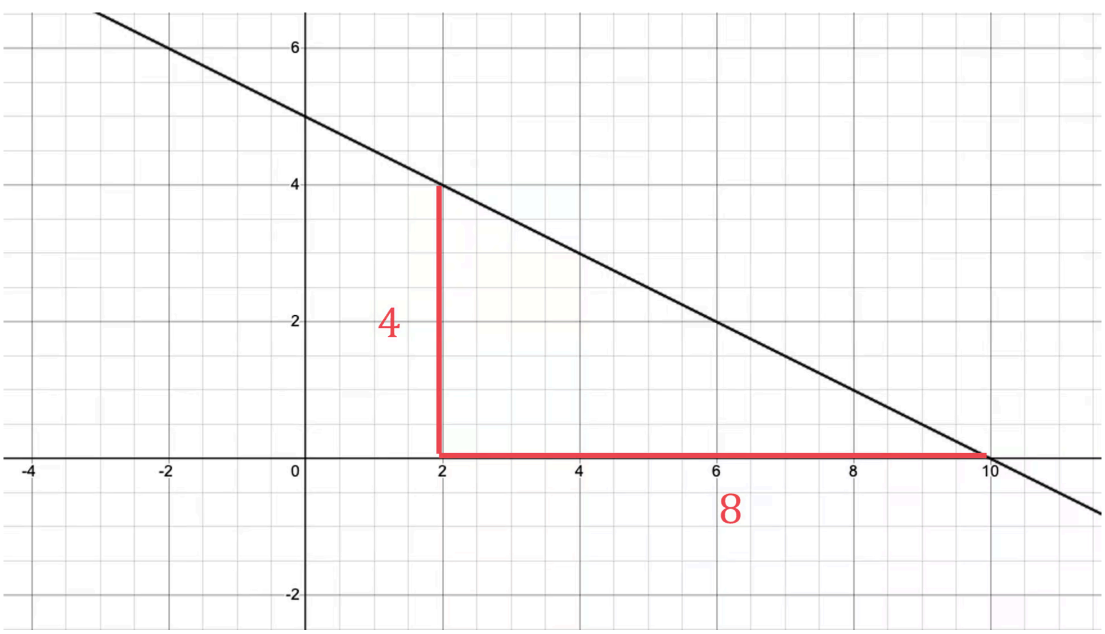
Worked Example

(a) Find the equation of the straight line shown in the diagram below.



Find m, the gradient
Identify any two points the line passes through and work out the rise and run

Line passes through (2, 4) and (10, 0)



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The rise is 4
The run is 8

Calculate the fraction $\frac{\text{rise}}{\text{run}}$

$$\frac{\text{rise}}{\text{run}} = \frac{4}{8} = \frac{1}{2}$$

The slope is downward (downhill), so it is a negative gradient

$$\text{gradient, } m = -\frac{1}{2}$$

Now find the y-intercept
The line cuts the y-axis at 5

$$y\text{-intercept, } c = 5$$

Substitute the gradient, m, and the y-intercept, c, into $y = mx + c$

$$y = -\frac{1}{2}x + 5$$

(b) Find the equation of the straight line with a gradient of 3 that passes through the point (5, 4).

Substitute $m = 3$ into $y = mx + c$
Leave c as an unknown letter

$$y = 3x + c$$

Substitute $x = 5$ and $y = 4$ into the equation
Solve the equation to find c

$$4 = 3 \times 5 + c$$
$$4 = 15 + c$$
$$-11 = c$$

You now know c
Replace c with -11 to complete the equation of the line

$$y = 3x - 11$$

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Drawing Straight Line Graphs

Drawing Linear Graphs

How do I draw a straight line from a table of values?

- You may be given a **table of values** with **no** equation
- Use the x and y values to form a point with **coordinates** (x, y)
  - Then **plot** these points
  - Use a ruler to draw a **straight line** through them
    - All points should lie on the line
- For example
  - The points below are (-3, 0), (-2, 2), ... etc

x	-3	-2	-1	0	1	2	3
y	0	2	4	6	8	10	12

How do I draw a straight line using y = mx + c?

- Use the **equation** to create your own **table of values**
  - Choose points that are **spread out** across the axes given
- For example, plot $y = 2x + 1$ on axes from $x = 0$ to $x = 10$
  - Substitute in $x = 0$, $x = 5$ and $x = 10$ to get y coordinates
    - Then plot those points

x	0	5	10
y	1	11	21

How do I draw a straight line without using a table of values?

- Assuming the equation is in the form $y = mx + c$

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- **Start** at the ***y*-intercept**, c
- Then, for every **1 unit** to the **right**, go up ***m* units**
  - *m* is the gradient
  - If *m* is **negative**, go **down**
  - If *m* is a **fraction**, remember that gradient is change in *y* divided by change in *x*
    - A gradient of $\frac{a}{b}$ would be a units up for every b units right
- This creates a sequence of **points** which you can then join up
  - Be **careful** of counting squares if axes have **different scales**
    - 1 unit might not be 1 square

What if the equation is not in the form y = mx + c?

- Equations will not always be presented in the form ***y* = *mx* + c**
- **Rearranging** to *y* = *mx* + c will make **plotting** these graphs **easier**
- Consider the equation $3x + 5y = 30$
  - Subtract 3*x* from both sides
    - $5y = -3x + 30$
  - Divide both sides by 5
    - $y = -\frac{3}{5}x + 6$
  - It can now be seen that the gradient is $-\frac{3}{5}$ and the *y*-intercept is 6
- Make sure you only have 1 *y* on one side, rather than say, 5*y*

How can I plot equations in the form ax + by = c?

- Instead of rearranging, equations in the form ***ax* + *by* = c**, like the example above, can also be plotted by **considering the *x* and *y* intercepts** instead
  - Substitute in ***x* = 0** to find the *y*-intercept
  - Substitute in ***y* = 0** to find the *x*-intercept

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- E.g. for $3x+5y=30$
  - When $x=0$, $5y=30$, so $y=6$
  - When $y=0$, $3x=30$, so $x=10$
- The points (0, 6) and (10, 0) can then be plotted and joined with a straight line

Examiner Tips and Tricks

- Always plot at least 3 points (just in case one of your end points is wrong!)

Worked Example

On the same set of axes, draw the graphs of $\frac{y+1}{3}=x$ and $y=-\frac{3}{5}x+3$.

Rearrange $\frac{y+1}{3}=x$ into the form $y=mx+c$ to make it easier to plot

$$\begin{aligned} \frac{y+1}{3} &= x \\ y+1 &= 3x \\ y &= 3x-1 \end{aligned}$$

For $y=3x-1$, create a table of values

x	0	1	2
y	-1	2	5

Plot the points (0, -1), (1, 2) and (2, 5)
Connect with a straight line

Alternatively, start at the *y*-intercept (0, -1) and mark the next points 3 units up for every 1 unit to the right

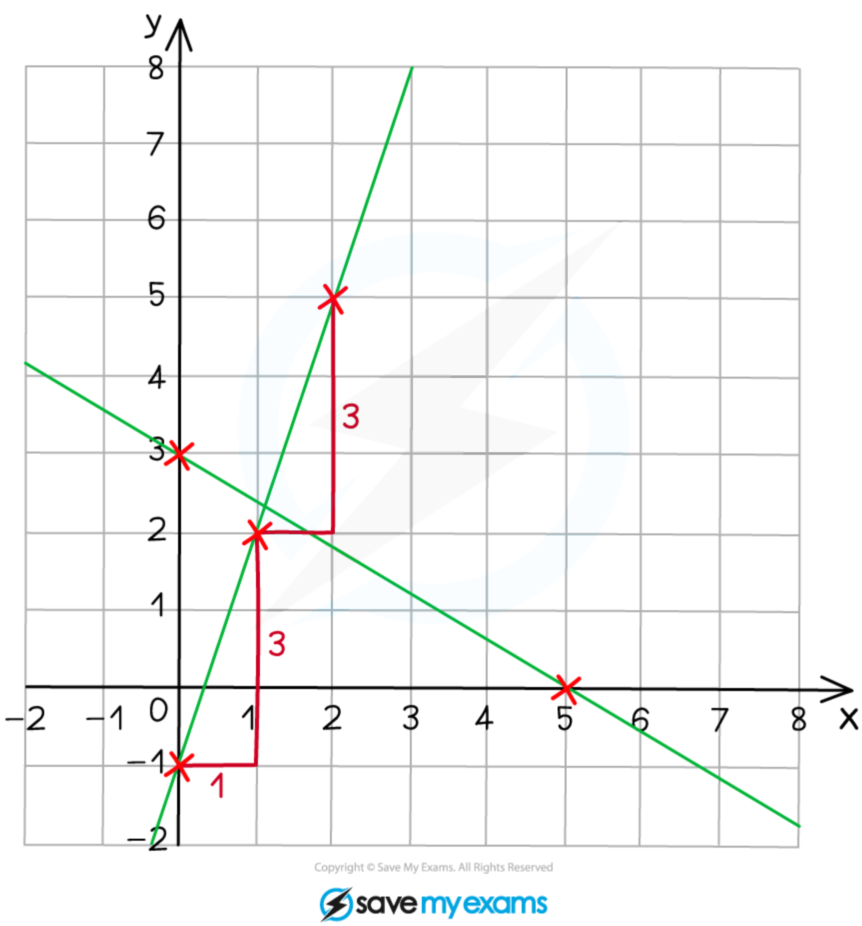
For $y = -\frac{3}{5}x + 3$, create a table of values

Because of the fraction, $x = 5$ is a good point to include

x	0	3	5
y	3	1.2	0

Plot the points (0, 3), (3, 1.2) and (5, 0)
Connect with a straight line

Alternatively, start at the y-intercept (0, 3) and mark the next points 3 units down for every 5 units to the right



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Parallel Lines

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Parallel Lines

What are parallel lines?

- **Parallel lines** are **straight lines** with the **same gradient**
  - Two parallel lines will **never meet**
    - They just stay side-by-side forever
- The **equation** of the line **parallel** to $y = mx + c$ is $\boldsymbol{y = mx + d}$
  - $y = 2x + 1$ and $y = 2x + 5$ are parallel
  - $y = 2x + 1$ and $y = 3x + 1$ are **not** parallel

How do I find the equation of a parallel line?

- For example, to find the **equation** of the line **parallel** to $y = 2x + 1$ which **passes through** the **point** (3, 14)
  - write the parallel line as $y = 2x + d$
    - using the same gradient of 2
  - **substitute** $x = 3$ and $y = 14$ into this equation
    - $14 = 2 \times 3 + d$
    - $14 = 6 + d$
  - **solve** to **find *d***
    - $d = 8$
  - The equation is $y = 2x + 8$

Worked Example

Find the equation of the line that passes through the point (2, 1), which is parallel to the straight line $y = 3x + 7$.

Parallel means the gradient will be the same
Use $y = mx + d$ where $m = 3$

$$y = 3x + d$$

Substitute in $x = 2$ and $y = 1$

$$1 = 3 \times 2 + d$$

Simplify the equation

$$1 = 6 + d$$

Solve the equation to find d (by subtracting 6 from both sides)

$$-5 = d$$

Write out the final answer in the form $y = mx + d$

$$\boldsymbol{y = 3x - 5}$$

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Perpendicular Lines

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Perpendicular Lines

What are perpendicular lines?

- **Perpendicular lines** are **straight lines** which meet at **right-angles** (90°)
- One line may be referred to as a **normal** to the other line

How are the gradients of perpendicular lines related?

- Gradients m_1 and m_2 are **perpendicular** if $m_1 \times m_2 = -1$
  - For example
    - 1 and -1
    - $\frac{1}{3}$ and -3
    - $-\frac{2}{3}$ and $\frac{3}{2}$
- The two gradients are **negative reciprocals** of one another
- We can use $m_2 = -\frac{1}{m_1}$ to find a perpendicular gradient

How can I tell if two lines are perpendicular?

- Given two lines in the form $y = mx + c$, simply check if their gradients (m) are **negative reciprocals** of one another
  - $y = \frac{1}{3}x + 10$ and $y = -3x - 18$ **are perpendicular**
  - $y = \frac{1}{7}x + 16$ and $y = 7x - 8$ **are not perpendicular**
- One or both of the equations may not be written in the form $y = mx + c$
  - In this case, you should **rearrange both equations into the form $y = mx + c$**

- Their **gradients** can then be easily **compared**

How do I find the equation of a line perpendicular to another?

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- You need to be able to find the **equation** of line that **passes through a particular point** and is **perpendicular** to another line
  - E.g. $5y = 4x + 30$ which **passes through** the **point** (8, 3)
- **Rearrange** the equation into the form $y = mx + c$ so that its gradient can be identified more easily
  - $y = \frac{4}{5}x + 6$
- Find the **gradient of the perpendicular** line
  - The gradient of the original line is $\frac{4}{5}$
  - Therefore the **gradient of the perpendicular** line is $-\frac{5}{4}$
  - The perpendicular line has an equation in the form $y = -\frac{5}{4}x + c$
- **Substitute the given point** into the equation for the perpendicular and **solve for** c
  - Substitute (8, 3), into $y = -\frac{5}{4}x + c$
  - $3 = -\frac{5}{4}(8) + c$
  - $c = 13$
- Substitute the value of c to find the **equation of the perpendicular**
  - The **equation** of the perpendicular line is $y = -\frac{5}{4}x + 13$
    - This could also be written as $4y = -5x + 52$ or equivalent

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Worked Example

The line *L* has equation $y - 2x + 2 = 0$.

Find the equation of the line perpendicular to *L* which passes through the point $(2,\ -3)$.

Leave your answer in the form $ax + by + c = 0$ where a, b and c are integers.

Rearrange *L* into the form $y = mx + c$ so we can identify the gradient

$$y - 2x + 2 = 0$$
$$y = 2x - 2$$

Gradient of *L* is 2

The gradient of the line perpendicular to *L* will be the negative reciprocal of 2

$$m = -\frac{1}{2}$$

Substitute the point $(2,\ -3)$ into the equation $y = -\frac{1}{2}x + c$

Solve for c

$$y = -\frac{1}{2}x + c$$
$$-3 = -\frac{1}{2}(2) + c$$
$$-3 = -1 + c$$
$$c = -2$$

Write the full equation of the line

$$y = -\frac{1}{2}x - 2$$

The question asks for the line to be written in the form $ax + by + c = 0$ where a, b and c are integers

Move all the terms to the left hand side

$$\frac{1}{2}x + y + 2 = 0$$

Then multiply every term by 2, to ensure they are all integers

$$x + 2y + 4 = 0$$

Your notes

How do I find the equation of a perpendicular bisector?

- A perpendicular bisector of a line segment cuts the line segment **in half** at a **right angle**
- Finding the **equation of the perpendicular bisector** of a line segment is very similar to finding the equation of a any perpendicular
  - Find the coordinates of the **midpoint** of the line segment
    - The perpendicular bisector will pass through this point
  - Find the **gradient of the line segment**
  - Then find the **negative reciprocal** of this gradient
    - This will be the **gradient of the perpendicular bisector,** m
  - Write the **equation** of the perpendicular bisector in the form $y = mx + c$
  - **Substitute** the **midpoint** of the line segment into the **equation of the perpendicular bisector**
    - **Solve** to find c
  - Write the full **equation** of the perpendicular bisector in the form $y = mx + c$
  - **Rearrange** the equation if the question requires a different form

Worked Example

Find the equation of the perpendicular bisector of the line segment joining the points (4, -6) and (8, 6).

Find the coordinates of the **midpoint** of the line segment
The perpendicular bisector will pass through this point

$$\left(\frac{4+8}{2}, \frac{-6+6}{2}\right) = (6, 0)$$

Find the gradient of the line segment

$$\frac{-6-6}{4-8}=\frac{-12}{-4}=3$$

Find the negative reciprocal of this

This will be the gradient of the perpendicular bisector, m

$$m=-\frac{1}{3}$$

Write the equation of the perpendicular bisector in the form $y = mx + c$

$$y=-\frac{1}{3}x+c$$

Substitute in the midpoint (6, 0) and solve to find c

$$0=-\frac{1}{3}(6)+c$$
$$0=-2+c$$
$$c=2$$

Write the full equation of the perpendicular bisector

$$\boldsymbol{y=-\frac{1}{3}x+2}$$

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