AQA GCSE Maths: Higher

Your notes

Introduction to Algebra

Contents

Your notes

Algebraic Notation

What is algebra?

- **Algebra** is a topic in mathematics that uses **letters** to represent **general** (or **unknown**) **numbers**

  - x and y are two unknown numbers

    - More information is needed to find their values

- Letters are also called **variables**

How do I write calculations in algebra?

- Writing mathematical ideas in letters is called using **algebraic notation**

- You can use + and – to mean **add** and **subtract**

  - $a + b$
    $c + d - e$

- However **no** symbol is used for **multiplication**

  - ab (means $a \times b$)
    $3ab$ (means $3 \times a \times b$)

- **Fractions** are used for **division**

  - $\dfrac{a}{b}$ (means $a \div b$)

- You can combine the ideas above

  - $ab + \dfrac{c}{3}$ (means $a \times b + c \div 3$)

    - The **order of operations** for numbers still works

      - work out $a \times b$ and $c \div d$ before adding them together

- **Powers** (**indices**) and **roots** are the same as with numbers

  - a^2 means $a \times a$
    $4a^2$ means $4 \times a^2$

- With the order of operations, a^2 happens before multiplying by 4
  - $\sqrt{a}$ means the square root of a
- **Brackets** work in the same way as they do with numbers

  - $3(a + b)$ means $3 \times (a + b)$

    - Add the a and b first, then multiply the result by 3

Worked Example

Raheem is playing a game and starts with q points.
He then earns a further 6 points.

(a) Use algebra to write down the total number of points that Raheem has.

Raheem has q + 6 points

The game then decides to doubles his total number of points.

(b) Use algebra to write down the number of points that Raheem now has.

Raheem has 2(q + 6) points

Algebraic Vocabulary

Your notes

Algebraic Vocabulary

What is a term?

- A **term** is either:
  - a letter (**variable**) on its own, or a variable raised to a power
    - For example, x or x^2
  - a number on its own
    - For example, 20
    - These are also called **constants** as they can't change value
  - or a number **multiplied** by a letter
    - For example, $5x$
- The number in front of a letter is called a **coefficient**
  - The coefficient of x in $6x$ is 6
  - The coefficient of y in $-5y$ is -5
- Terms can include **powers** and more than one letter,
  - $6xy$
  - $4x^2$
  - ab^3c

What is a factor?

- A **factor** is any number or letter that **divides** a term **exactly**
  - There is no remainder
    - The factors of $3x$ are 1, 3, x and $3x$
    - The factors of $4xy$ are 1, 2, 4, x, $2x$, $4x$, y, $2y$, $4y$, xy, $2xy$ and $4xy$
- A term can be separated into **factors** that **multiply** together to give that term
  - Two factors of $5x$ are 5 and x

Your notes

- 5 × x = $5x$
- To **factorise** means to write something as a **multiplication** of **factors**
- When comparing two terms, a **common factor** is one that divides both
  - Compare $6xy$ with $4x$
    - Common factors are 1, 2, x and $2x$
    - The **highest** (or greatest) common factor is $2x$

What is an expression?

- An **expression** is an algebraic statement that does **not** have an **equals sign**
  - There is nothing to solve
- An expression is made by adding, subtracting, multiplying or dividing **terms**
  - $2x + 5y$
  - $b^2 - 2cd$
  - $$\frac{6y}{5t}$$
  - A single term is still an expression
- Expressions can be **simplified** (made easier)
  - $x + x + x$ simplifies to $3x$

What is an equation?

- An **equation** is an algebraic statement with an **equals sign** between a **left-hand side** and a **right-hand side**
  - Both sides are **equal** in value
  - For example, if $2x$ has the same value as 10, then $2x = 10$
- An equation can be **solved** by finding the values of the letters that make both sides equal
  - The equation $2x = 10$ is solved when x has the value of 5
    - $x = 5$ is called the **solution**

What is a formula?

- A **formula** is a rule, definition or relationship between different quantities, written in shorthand using **letters**

- For example, weight, w, is mass, m, multiplied by gravitational acceleration, g

  - The formula is $w = mg$
- It is common to **substitute** numbers into a formula

  - But a formula on its own cannot be solved
- To turn a **formula** into an **equation**, more information is needed

  - For example, if $w = 50$ and $m = 5$, the formula $w = mg$ becomes the equation $50 = 5g$

What is an identity?

- An **identity** is an algebraic statement with an identity sign, $\equiv$, between a left-hand side and a right-hand side that is **true** for **all values of x**

  - E.g. $x + x \equiv 2x$

  - This means $x + x$ is **identical** to $2x$, or that $x + x$ can also be written as $2x$

- An identity **cannot** be **solved**

- **All numbers** can be **substituted** into an identity and it will remain true

  - E.g. $x + x \equiv 2x$ is true for $x = 1$, $x = 2$, $x = 3$ ... (even $x = -0.01$, $x = \pi$ etc)

  - Unlike with equations, where only the solutions work

    - E.g. $2x = 10$ is not true for $x = 1$, $x = 2$, $x = 3$ ... only for $x = 5$

- Identities can be used to **write** algebraic expressions **in different forms**

  - E.g. find p and q if $3(x + y) + 2y \equiv px + qy$

    - $3(x + y)$ expands to $3x + 3y$

    - The coefficient of x on the left is 3 and on the right is p, so $p = 3$

    - The coefficient of y on the left is $3 + 2$ and on the right is q, so $q = 5$

    - Therefore $3(x + y) + 2y$ is identical to $3x + 5y$

    - This method is called **equating coefficients**

💡

Examiner Tips and Tricks

Knowing the differences between an **expression**, an **equation** and a **formula** will help you to understand the wording of exam questions.

Worked Example

(a) From the list below, write down

(i) an expression,

(ii) an equation.

$$2x + 5 = 4 \qquad 7x - 9 \qquad x = vt - w$$

(i) An expression does not have an equals sign

$7x - 9$ is the expression

(ii) An equation has an equals sign and can be solved

$2x + 5 = 4$ is the equation

(b) If $x = 10$, $v = 2$ and $w = 3$, use the formula shown to write an equation in t.

$x = vt - w$ is the formula shown (a group of different quantities forming a relationship)

Substitute $x = 10$, $v = 2$ and $w = 3$ into the formula

$10 = 2t - 3$

Substitution

What is substitution?

- **Substitution** means **replacing** a letter (variable) in a formula with a given **number**

How do I substitute numbers into a formula?

- Write down the **formula**

- **Replace** (substitute) the letters in the formula with the given numbers

    - If substituting in a **negative number**, it is important to put **brackets** around it

        - For example, (-3)

- **Simplify** any numerical calculations

- **Calculate** the final value

- Sometimes the result is an **equation** which you can then **solve**

💡

Examiner Tips and Tricks

- On your calculator, don't forget to type out **brackets** around any substituted **negative** numbers!

✏️

Worked Example

(a)Find the value of the expression $2x(x + 3y)$ when $x = 2$ and $y = -4$.

Substitute the numbers given
Use brackets () around negative numbers

$$2 \times 2 \times (2 + 3 \times (-4))$$

Complete the calculation
Show every step of working, following the order of operations correctly

$$= 2 \times 2 \times (2 - 12)$$
$$= 2 \times 2 \times (-10)$$
$$= 4 \times (-10)$$

-40

(b) The formula $P = 2l + 2w$ is used to find the perimeter, P, of a rectangle of length l and width w.

Given that the rectangle has a perimeter of 20 cm and a width of 4 cm, find its length.

Substitute the values you are given for P and w into the formula

$$20 = 2 \times l + 2 \times 4$$

Simplify

$$20 = 2l + 8$$

Solve the resulting equation to find the value of l
Start by subtracting 8 from both sides

$$12 = 2l$$

Divide both sides by 2

$$l = 6 \text{ cm}$$

Your notes

Collecting Like Terms

What happens if there is more than one term?

- **Terms** can be **added** and **subtracted**

  - The **numbers** in front of the letters are called **coefficients**

- Each term has a positive or negative **sign** in front

  - In $2x - 3y$ the sign of the x term is positive and the sign of the y term is negative

- **Subtractions** can be thought of as **adding** a **negative**

  - $2x - 3y$ is the same as $2x + (-3y)$

    - Just like $20 - 30$ is the same as $20 + (-30)$

- The **order** of two terms can be **swapped**, but the **signs** must **move** with their terms

  - $2x - 3y$ is the same as $-3y + 2x$

    - A **plus** is now needed in front of the $2x$

    - Just like $20 - 30$ is the same as $-30 + 20$

- If **no number** appears in front of a term, then its number is **1**

  - x is the same as $1x$

What is a like term?

- **Like terms** are terms with exactly the same **letters** and **powers**

  - The **numbers** in front can be **different**

    - For example:

    - $2x$ and $3x$

    - $4x^2$ and $6x^2$

    - $5xy$ and $-7xy$

  - These are **not** like terms:

    - $2x$ and $3y$ (different letters)

- 4x^2 and 6x^4 (different powers)

  - 5xy and 7xyz (different letters)
- Remember **multiplication** can be done in **any order**

  - xy and yx are **like** terms

  - So are 2xy and 3yx

How do I collect like terms?

- **Collecting like terms** means simplifying by **adding** or **subtracting** the **numbers** in front

  - 2x + 3x becomes 5x

  - 4y - 10y becomes -6y

    - A negative sign is needed here

- If there are **different types** of like terms, **collect** them **separately**

  - For 2x + 4y + 5x - 3y

    - Collecting the x's gives 2x + 5x = 7x

    - Collecting the y's gives 4y - 3y = y

    - The answer is 7x + y

💡

Examiner Tips and Tricks

- Don't leave terms like 1x in your final answer in an exam - always simplify them to just x.

✏️

Worked Example

Simplify

$$8a - 5b - 6a + 4b$$

Collect the a terms first

$$8a - 6a = 2a$$

Then collect the b terms
Don't forget the minus sign in front of the 5b

$$-5b + 4b = -b$$

Add together the two answers

$$2a + -b$$

Simplify the signs

$$\boldsymbol{2a - b}$$