AQA GCSE Maths: Higher

Your notes

Combined & Conditional Probability

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Combined Probabilities

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Combined Probability

How do I calculate combined probabilities?

- You can calculate probabilities of **one event after another** without needing tree diagrams
  - These are called **combined** (or successive) probabilities
- There are two **rules** to learn
  - **And** means **multiply** and **or** means **add**
  - P(A **and** B) = P(A) x P(B)
  - P(AA **or** BB) = P(AA) + P(BB)
- Try to **rephrase** each question using and / or
  - For example, when flipping a coin twice:
    - P(two heads) = P(head **and** head)
    - P(both the same) = P(head **and** head **or** tail **and** tail) = P(HH) + P(TT)
- Remember that P(**not** A) = 1 - P(A)

Worked Example

A box contains 3 blue counters and 8 red counters.
A counter is taken at random and its colour is noted.
The counter is put back into the box.
A second counter is then taken at random, and its colour is noted.

Work out the probability that

(a) both counters are red,

P(both red) = P(red and red)
This is P(red) × P(red) using the 'and rule'

$$\frac{8}{11} \times \frac{8}{11}$$

$$\frac{64}{121}$$

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(b) the two counters are of different colours.

P(different colours) = P(blue and red or red and blue)
This is P(B and R) + P(R and B) using the 'or rule'
This is P(B) × P(R) + P(R) × P(B) using the 'and rule' twice

$$\frac{8}{11} \times \frac{3}{11} + \frac{3}{11} \times \frac{8}{11}$$
$$= \frac{24}{121} + \frac{24}{121}$$

$$\frac{48}{121}$$

Conditional Probability

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Conditional Probability

What is a conditional probability?

- A **conditional probability** is the probability of something happening (*A*) **given that** something else has **already happened** (*B*)
  - It is written $\mathrm{P}(A \mid B)$
    - Pronounced "the probability of ***A* given *B***"

How do I calculate conditional probabilities?

- Conditional probabilities must be out of a **smaller restricted set** of outcomes (not out of all possible events)
  - For example, if a computer randomly selects a digit from 1, 2, 3, 4, 5, 6, 7, 8, 9
    - P(it select a multiple of three) = $\frac{3}{9}$
    - This is **not** a conditional probability
    - There are 3 possibilities (3, 6, 9) out of **all** 9 possibilities
  - However, if you program the computer to only select from even numbers, then
    - P(it selects a multiple of three **given that** it selects an even number) = $\frac{1}{4}$
    - 1 possibility (6) out of **only** 4 possibilities (2, 4, 6, 8)
    - This **is** a conditional probability, written P(multiple of 3 | even) = $\frac{1}{4}$

Examiner Tips and Tricks

- Look out for conditional probability questions within larger questions on two-way tables, Venn diagrams or tree diagrams

- They often use the phrase **given that**

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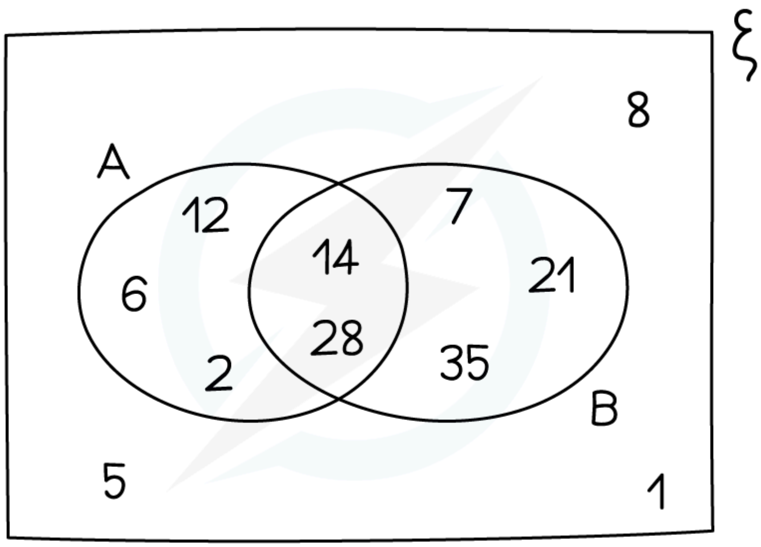
Worked Example

A Venn diagram is shown below.

Find $P(A|B)$, simplifying your answer.



$P(A|B)$ means the probability that *A* happens, given that *B* has already happened

Given that B has already happened, your probability will be out of the total number in *B*

Find the total number in *B*

$$14 + 28 + 7 + 35 + 21 = 105$$

Out of this total number in *B*, find how many are in *A*

$$14 + 28 = 42$$

Form a probability by dividing 42 by 105

$$P(A|B) = \frac{42}{105}$$

Simplify your answer

$$P(A|B) = \frac{2}{5}$$

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Combined Conditional Probabilities

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Combined Conditional Probabilities

What is a combined conditional probability?

- This is when you have two (or more) **successive events**, one after the other, and the **second** event **depends on** (is conditional on) the **first**

How do I calculate combined conditional probabilities?

- You need to **adjust** the number of outcomes as you go along
  - For example, selecting two cards from a pack of 52 playing cards **without replacing** the first card:
    - P(red 1st card) is 26 reds out of 52 cards
    - If the 1st card is not replaced, there are only 25 reds left out the remaining 51 cards
    - P(red 2nd card) is 25 reds out of 51 cards
    - P(red then red) = $\frac{26}{52} \times \frac{25}{51}$

Examiner Tips and Tricks

If a question says "two cards are drawn" then you may **assume** that they draw 1 card followed by another card **without replacement** (the maths is the same).

Can I a tree diagram for combined conditional probabilities?

- Yes, a **tree diagram** is a useful way to show combined conditional probabilities
  - For example, two counters are drawn at random from a bag of 3 blue and 8 red counters without replacement
    - The probabilities are shown below

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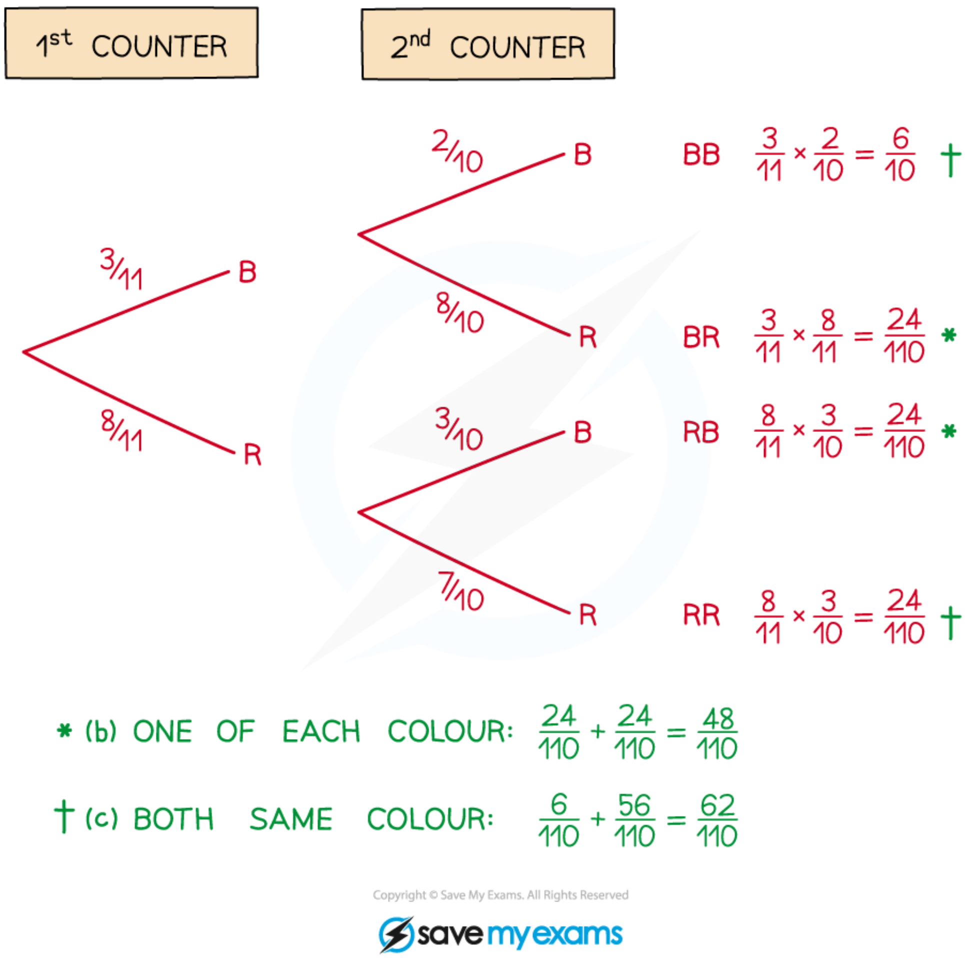

What if there are multiple possibilities within one question?

- You may need a **listing strategy** (e.g. *AAB*, *ABA*, *BAA*)
- You will need the **or** rule for multiple possibilities
  - P(*AB* **or** *BA* **or** *AA* **or**...) = P(*AB*) + P(*BA*) + P(*AA*) + ...
    - **Add** the cases together
- Remember that *AB* and *BA* are **not the same**
  - *AB* means *A* happened first, then *B*
  - *BA* means *B* happened first, then *A*

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Examiner Tips and Tricks

Try not to simplify your probabilities too early as it is easier to add probabilities together when they all have the same denominator!

Worked Example

A bag contains 10 yellow beads, 6 blue beads and 4 green beads.

A bead is taken at random from the bag and not replaced.

A second bead is then taken at random from the bag.

(a) Find the probability that both beads are different colours.

Let Y, B and G represent choosing a yellow, blue and green bead

Method 1

The probability of the beads being different colours is equal to 1 subtract the probability that the beads are the same colour

Find the probability of both beads being the same colour

$$\text{P(same colours)} = \text{P(YY)} + \text{P(BB)} + \text{P(GG)}$$

Calculate each conditional probability separately, remembering the number of beads changes after one is drawn and not replaced

For example, P(YY) = $\frac{10}{20} \times \frac{9}{19}$

$$\frac{10}{20} \times \frac{9}{19} + \frac{6}{20} \times \frac{5}{19} + \frac{4}{20} \times \frac{3}{19}$$

Multiply the pairs of fractions together and add their results

$$\frac{232}{380}$$

Subtract this from 1

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$$1-\frac{132}{380}=\frac{248}{380}$$

Simplify the answer

$$\frac{62}{95}$$

Method 2

List all the possibilities of different colours

Remember that YB (yellow first, then blue) is different to BY (blue first, then yellow)

YB, BY, YG, GY, BG, GB

Use the "or" rule to add the cases together

P(different colours) = P(YB) + P(BY) + P(YG) + P(GY) + P(BG) + P(GB)

Calculate each conditional probability separately, remembering the number of beads changes after one is drawn and not replaced

For example, P(YB) = $\frac{10}{20}\times\frac{6}{19}$

$$\frac{10}{20}\times\frac{6}{19}+\frac{6}{20}\times\frac{10}{19}+\frac{10}{20}\times\frac{4}{19}+\frac{4}{20}\times\frac{10}{19}+\frac{6}{20}\times\frac{4}{19}+\frac{4}{20}\times\frac{6}{19}$$

Multiply the pairs of fractions together and add their results

$$\frac{248}{380}$$

Simplify the answer

$$\frac{62}{95}$$

The second bead is not replaced and a third bead is taken at random from the bag.

(b) Find the probability that all three beads are the same colour.

List the possibilities

YYY, BBB, GGG

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Use the "or" rule to add between cases

P(all the same colour) = P(YYY) + P(BBB) + P(GGG)

Use conditional probabilities in each separate case, remembering the number of beads changes after each one is drawn and not replaced

$$\frac{10}{20} \times \frac{9}{19} \times \frac{8}{18} + \frac{6}{20} \times \frac{5}{19} \times \frac{4}{18} + \frac{4}{20} \times \frac{3}{19} \times \frac{2}{18}$$

Multiply the triplets of fractions together then add their results

$$\frac{720}{6840} + \frac{120}{6840} + \frac{24}{6840} = \frac{864}{6840}$$

Simplify the answer

$$\frac{12}{95}$$