



AQA GCSE Maths: Higher



Your notes

Transformations of Graphs

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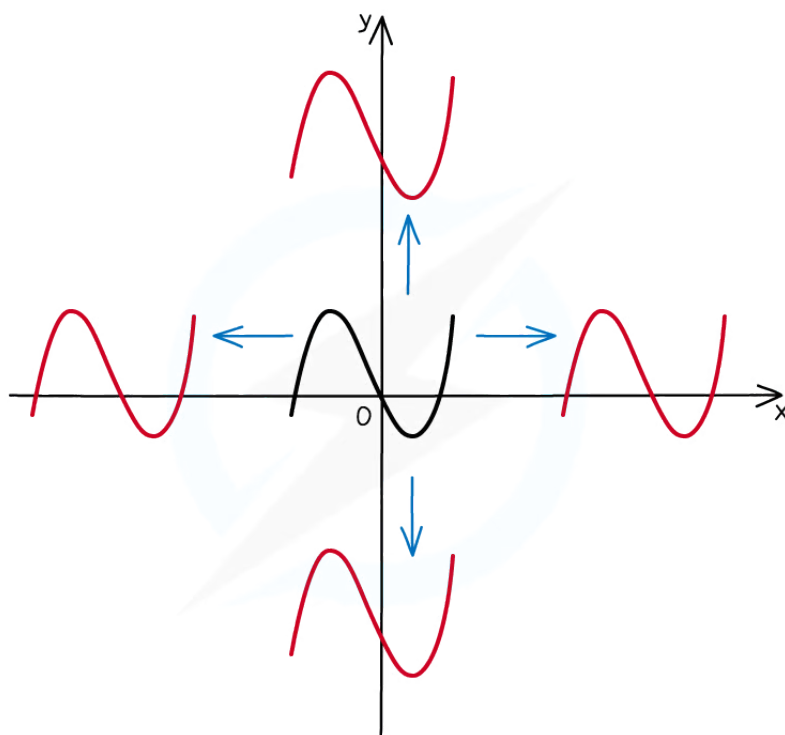
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Translations of Graphs

Translations of Graphs

What are translations of graphs?

- The **equation** of a graph can be changed in certain ways
 - This has an effect on the **graph**
 - How a graph changes is called a **graph transformation**
- A **translation** is a type of graph transformation that **shifts (moves)** a graph (up or down, left or right) in the xy plane
 - The shape, size, and orientation of the graph remain unchanged



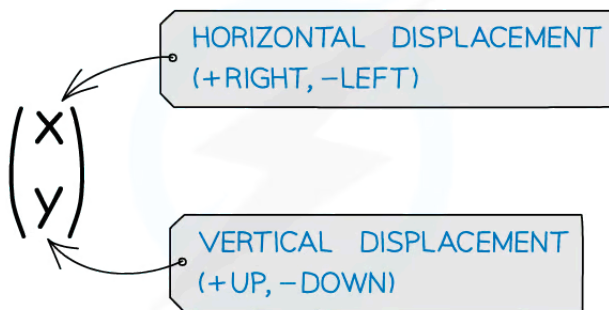
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- A particular translation is specified by a **translation vector**



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How do I translate graphs?

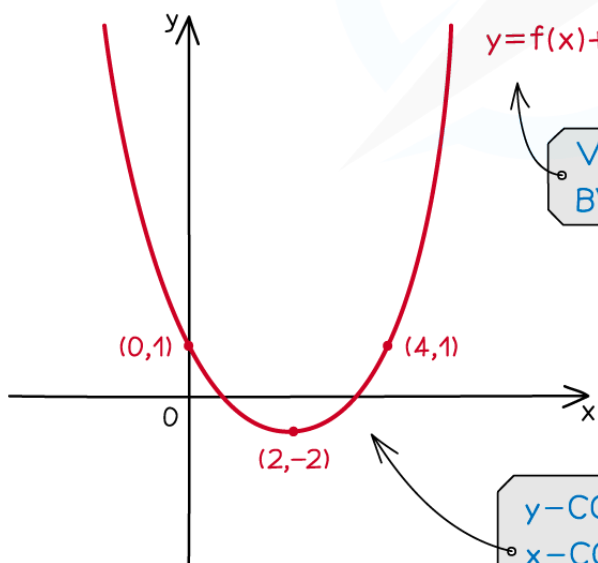
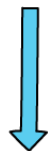
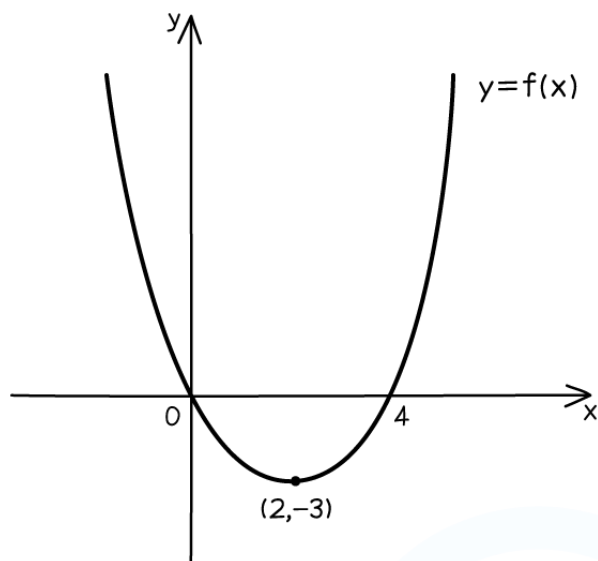
- Let $y = f(x)$ be the **equation** of the **original graph**

Vertical translations: $y = f(x) + a$

- $y = f(x) + a$ is a **vertical translation** by the vector $\begin{pmatrix} 0 \\ a \end{pmatrix}$
 - The graph moves **up for positive** values of a
 - The graph moves **down for negative** values of a
 - The **x-coordinates** stay the **same**



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VERTICAL TRANSLATION
BY THE VECTOR $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

y -COORDINATES CHANGE,
 x -COORDINATES STAY
THE SAME

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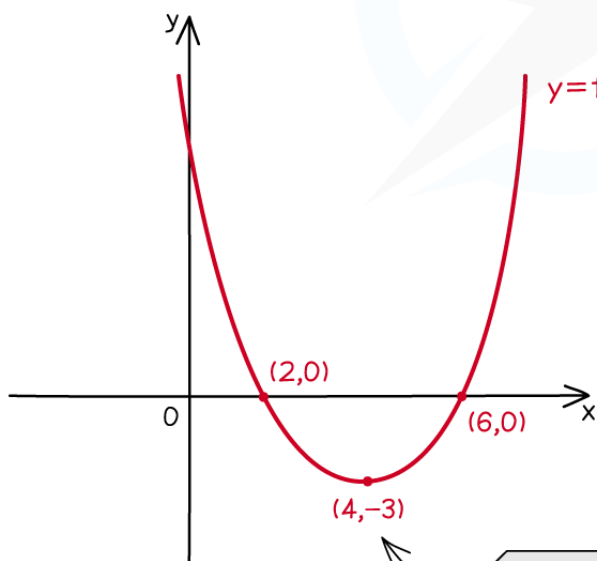
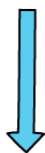
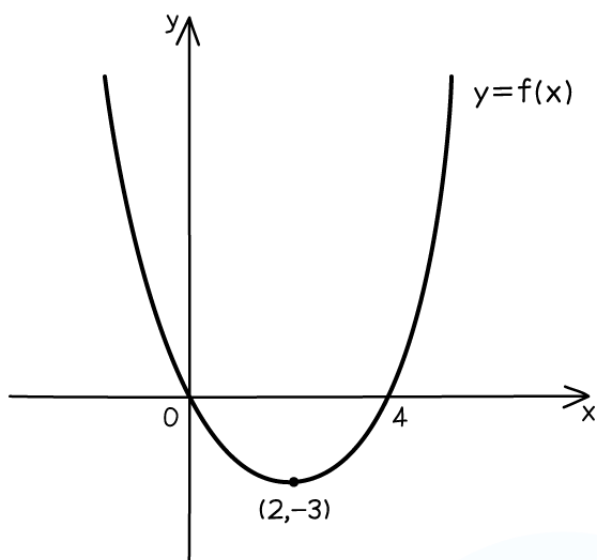
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Horizontal translations: $y=f(x+a)$

- $y=f(x+a)$ is a **horizontal translation** by the vector $\begin{pmatrix} -a \\ 0 \end{pmatrix}$
 - The graph moves **left for positive** values of a
 - This is often the opposite direction to which people expect
 - The graph moves **right for negative** values of a
 - The **y-coordinates** stay the **same**



Your notes



HORIZONTAL
TRANSLATION
BY THE VECTOR $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

x-COORDINATES CHANGE,
y-COORDINATES STAY
THE SAME

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What happens to asymptotes when a graph is translated?

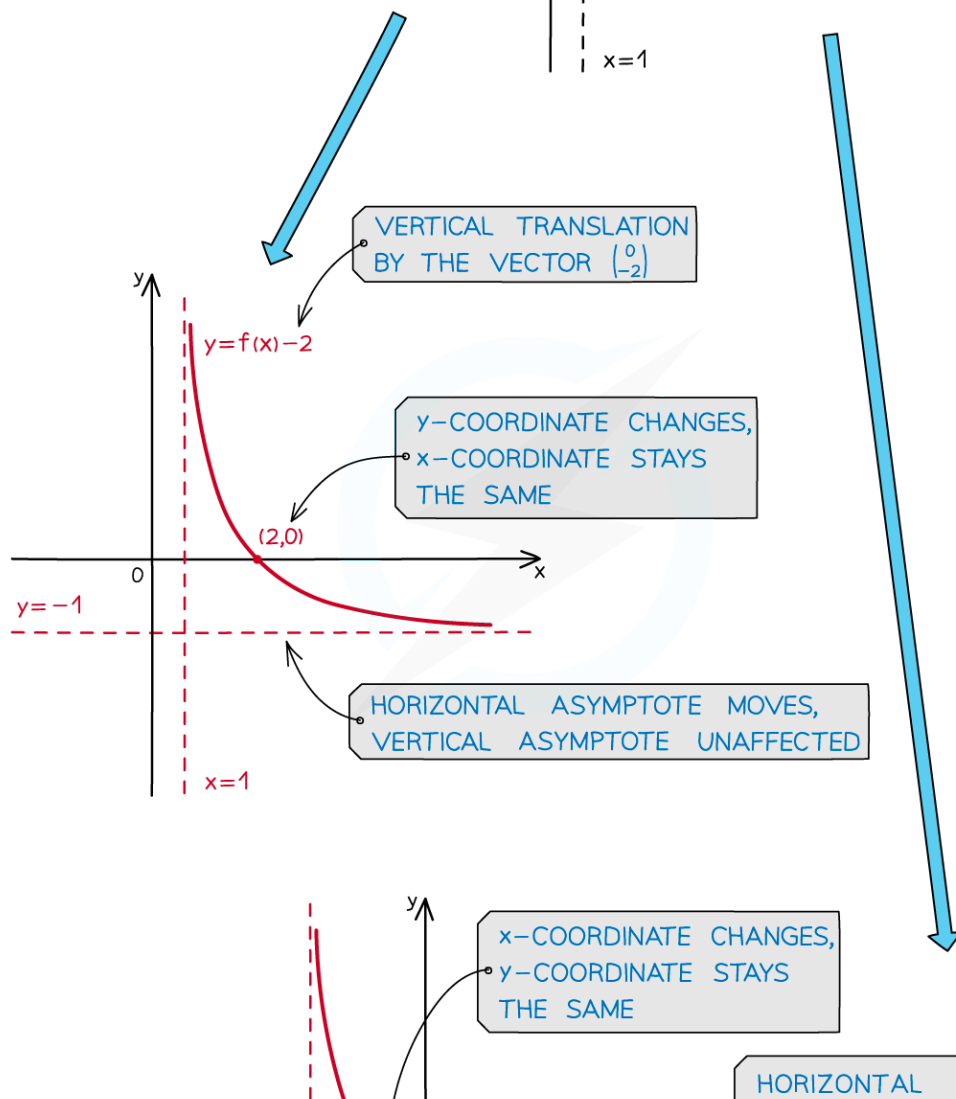
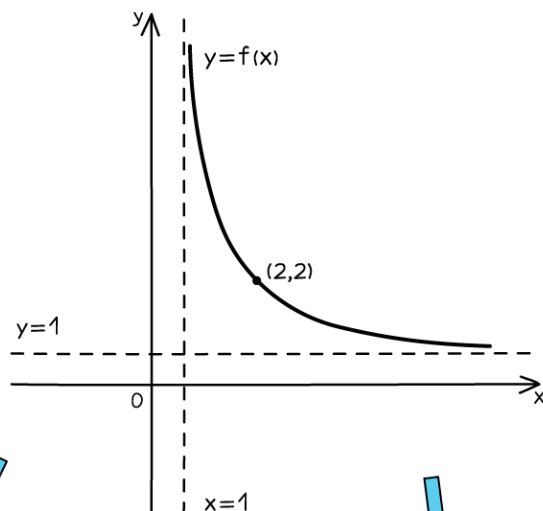
- Any **asymptotes** of $f(x)$ are also translated
 - An asymptote **parallel** to the **direction of translation** will **not** be affected

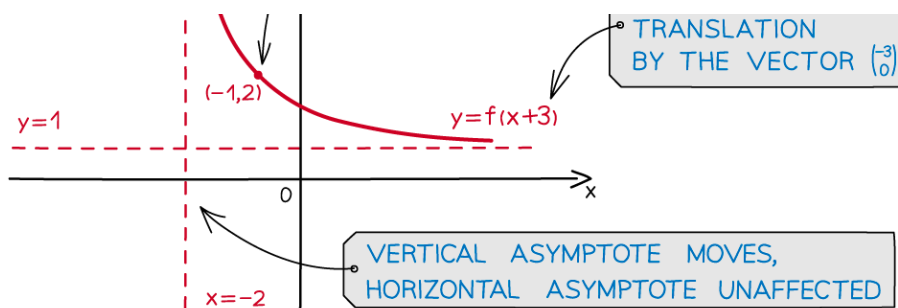


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How does a translation affect the equation of the graph?

- For a **horizontal translation** $y = f(x - a)$ of the graph $y = f(x)$
 - **a is subtracted from x** throughout the equation
 - Every instance of x in the equation is replaced with $(x - a)$
- E.g. the graph $y = x^2 - 3x + 7$ undergoes a **translation of 6 units to the right**
 - $y = f(x)$ becomes $y = f(x - 6)$
 - x is replaced throughout the equation by $(x - 6)$
 - $y = (x - 6)^2 - 3(x - 6) + 7$ is the **new** equation
 - The equation can be left in this form or expanded and simplified
 - $y = x^2 - 12x + 36 - 3x + 18 + 7$
 - $y = x^2 - 15x + 61$
- For a **vertical translation** $y = f(x) + a$ of the graph $y = f(x)$
 - **a is added to the equation as a whole**
- E.g. the graph $y = 4x^2 + 2x + 1$ undergoes a **translation of 5 units down**
 - $y = f(x)$ becomes $y = f(x) - 5$
 - 5 is subtracted from the equation as a whole
 - $y = 4x^2 + 2x + 1 - 5$
 - The equation can be left in this form or simplified

- $y = 4x^2 + 2x - 4$

How do I apply a combined translation?

- For a **horizontal translation** of p units and **vertical translation** of q units **combined**

- $y = f(x)$ becomes $y = f(x - p) + q$

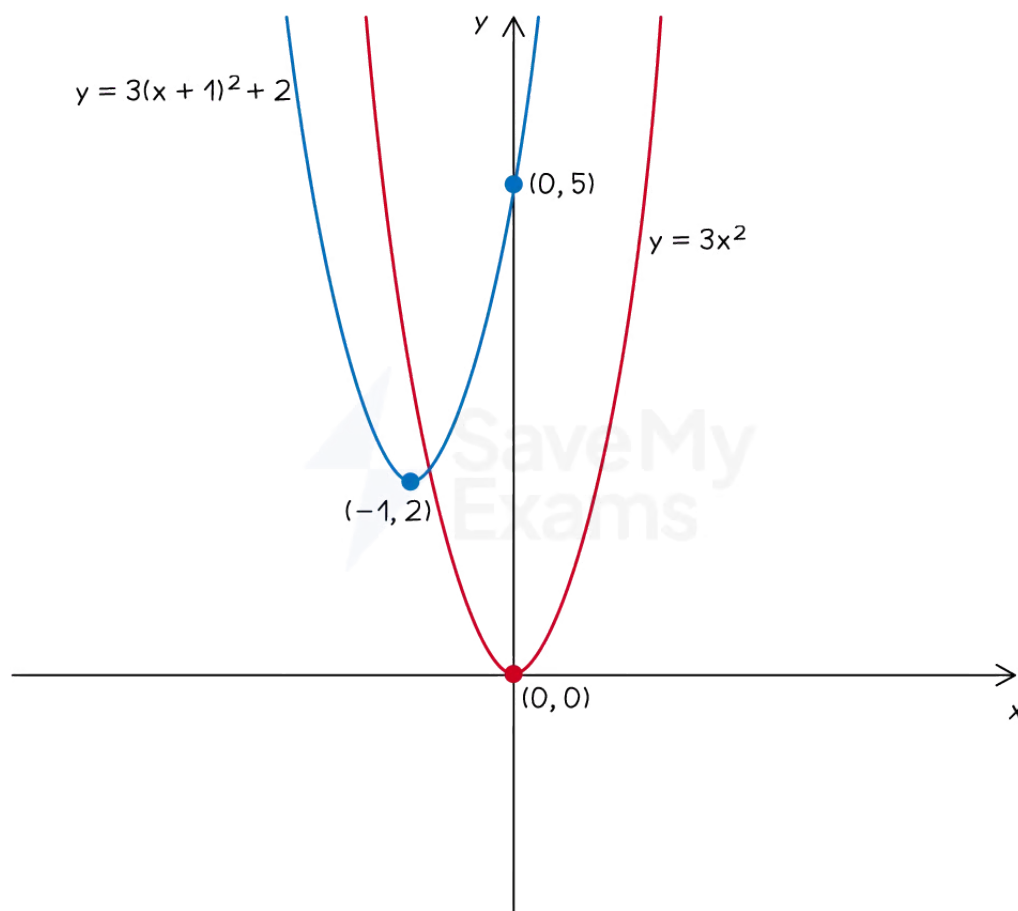
- E.g. the graph $y = 3x^2$ undergoes a **translation** of **2 units up** and **1 unit to the left**

- $y = f(x)$ will become $y = f(x + 1) + 2$

- x is replaced throughout the equation by $(x + 1)$

- 2 is added to the equation as a whole

- $y = 3(x + 1)^2 + 2$



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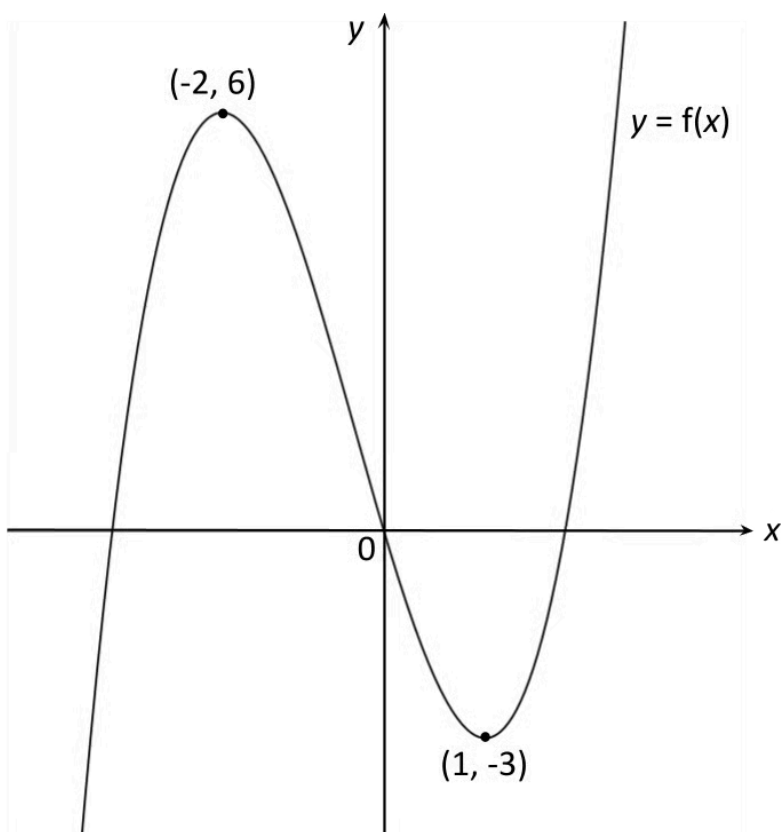
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- Note that when the equation is in the form $y = a(x - p)^2 + q$
 - the **vertex** is (p, q)
 - the value of a does not affect the vertex coordinates



Worked Example

The diagram below shows the graph of $y = f(x)$.



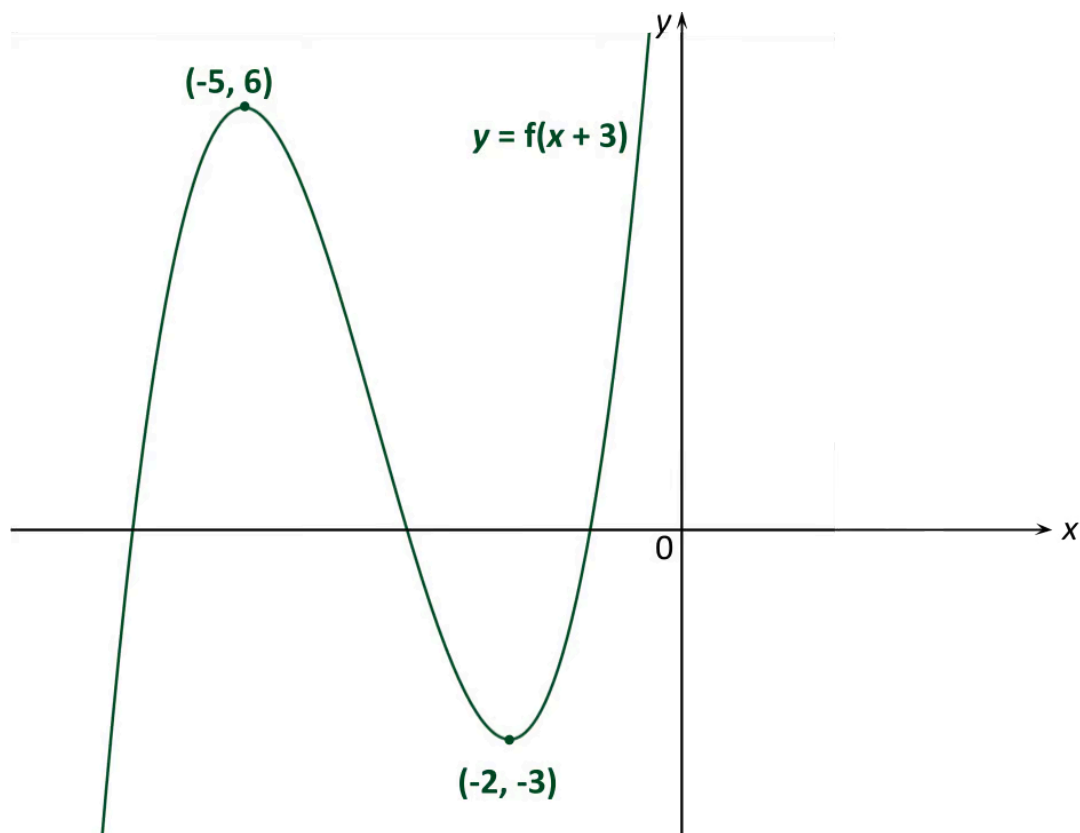
Sketch the graph of $y = f(x + 3)$.

The transformation of the graph is a horizontal translation with vector $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$ (3 units to the left)



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The x-coordinates of the points change (subtract 3 from each)
The y-coordinates of the points stay the same





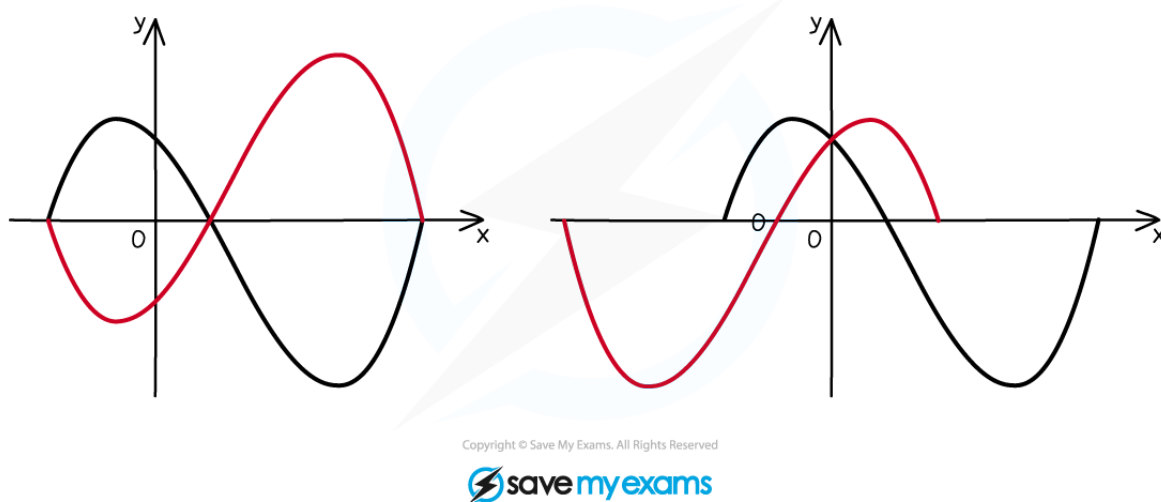
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Reflections of Graphs

Reflections of Graphs

What are reflections of graphs?

- **Reflections** of graphs are a type of **transformation** where the **curve** is **reflected** about one of the **axes**



A curve reflected in the x -axis (left) and y -axis (right)

How do I reflect graphs?

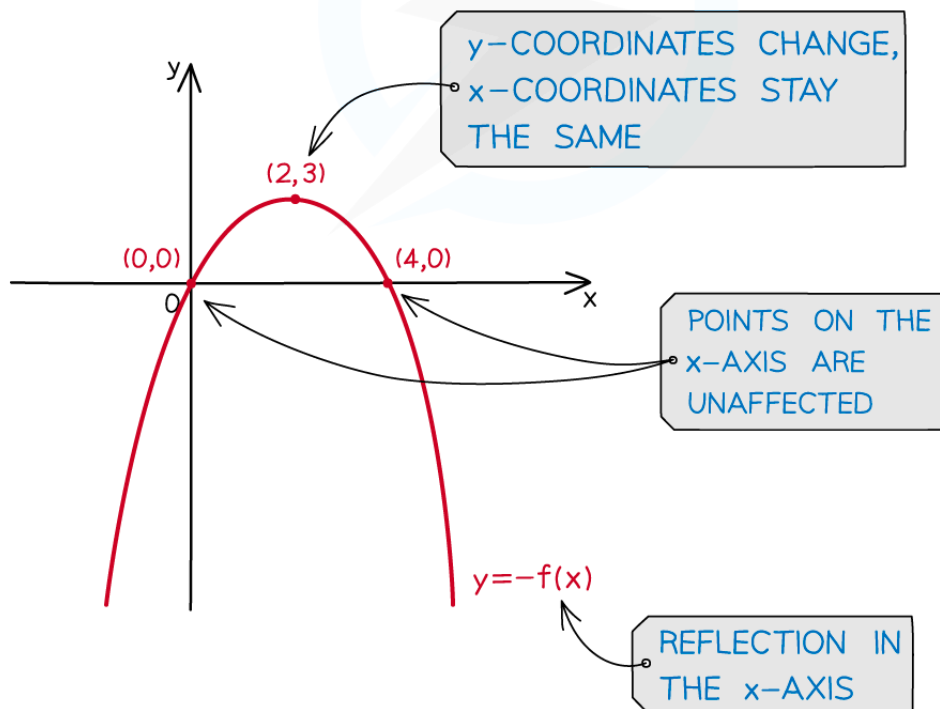
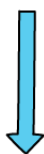
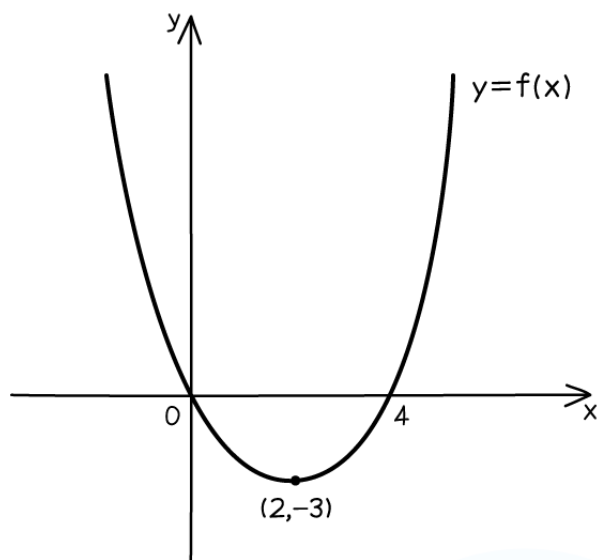
- Let $y = f(x)$ be the **equation** of the **original graph**

Vertical reflections: $y = -f(x)$

- $y = -f(x)$ is a reflection in the X -axis
 - The y coordinates change sign
 - The X coordinates are unaffected



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Horizontal reflections: $y=f(-x)$

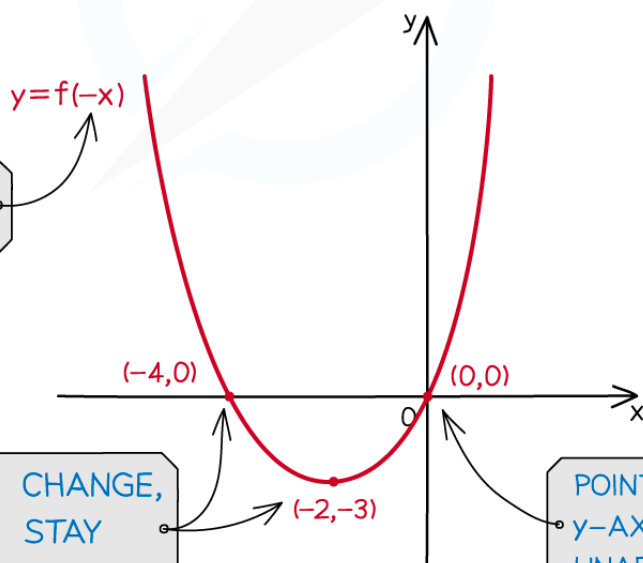
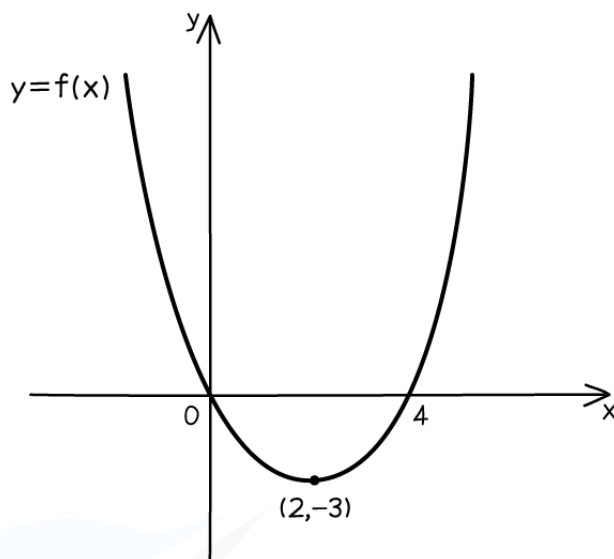
- $y=f(-x)$ is a reflection in the y -axis
 - The x coordinates change sign
 - The y coordinates are unaffected



Your notes



Your notes



REFLECTION IN
THE y -AXIS

x -COORDINATES CHANGE,
 y -COORDINATES STAY
THE SAME

POINTS ON THE
 y -AXIS ARE
UNAFFECTED

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What happens to asymptotes when a graph is reflected?

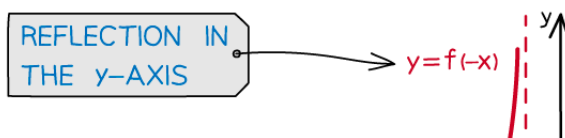
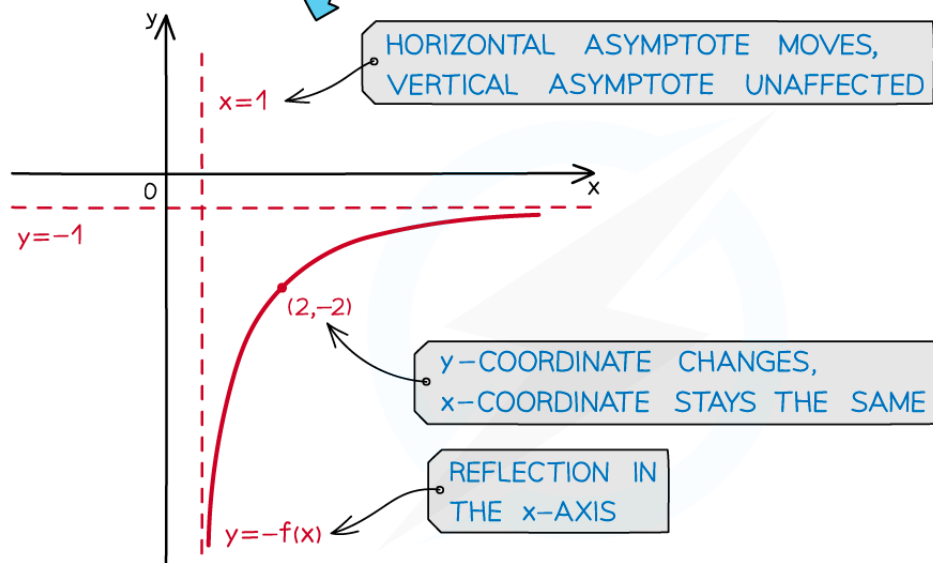
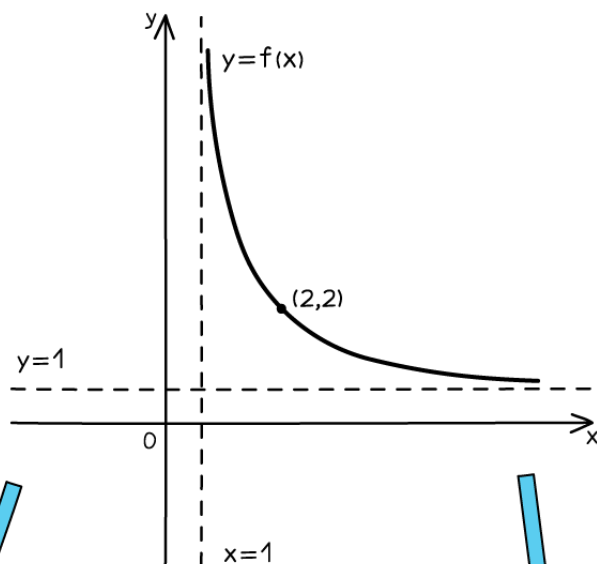
- Any **asymptotes** of $f(x)$ are also reflected

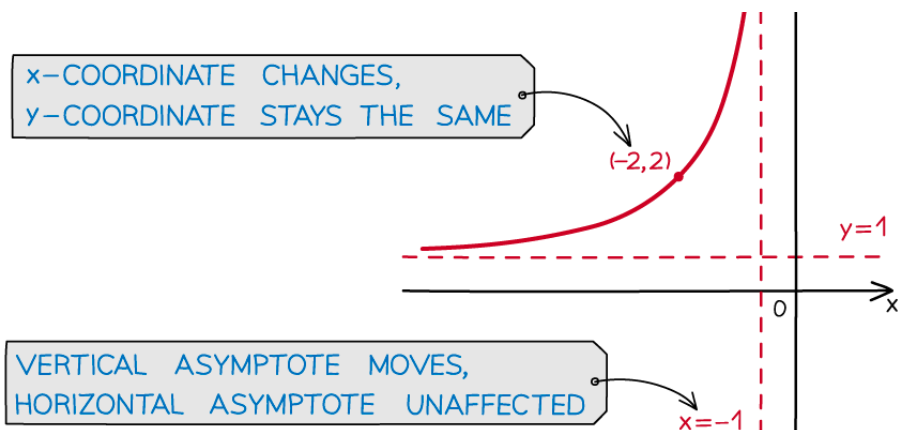


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Examiner Tips and Tricks

When reflecting graphs in the exam, reflect any key points on the graph first, then join them up with a smooth curve.

How does a reflection affect the equation of the graph?

- When a graph is reflected, you can **change its equation algebraically**
 - There is no need to sketch the graph
- Reflecting in the **X**-axis puts a **-** in front of the whole **equation**
 - For example, $y = x^2 + 2x$ becomes $y = -(x^2 + 2x)$
 - This simplifies to $y = -x^2 - 2x$
- Reflecting in the **y**-axis **replaces any X with (-X)** in the equation
 - For example, $y = x^2 + 2x$ becomes $y = (-x)^2 + 2(-x)$
 - This simplifies to $y = x^2 - 2x$

How do I apply a combined reflection?

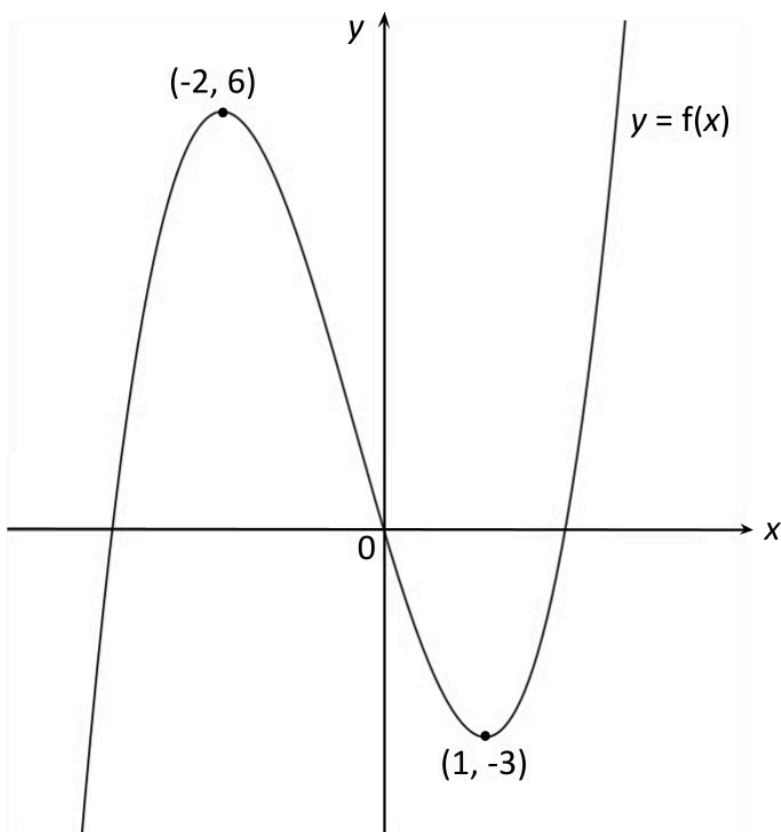
- The graph of $y = -f(-x)$ is a **combined reflection** in both the **X** and **y** axes
 - It **does not matter which order** you apply these in

- For example, reflect about the y -axis then about the x -axis



Worked Example

The diagram below shows the graph of $y = f(x)$.



Sketch the graph of $y = f(-x)$.

The transformation $y = f(-x)$ is a reflection in the y -axis

Reflect the points $(-2, 6)$ and $(1, -3)$ in the y -axis to get $(2, 6)$ and $(-1, -3)$

Sketch these points and join with a smooth curve through the origin



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