



# AQA GCSE Maths: Higher



Your notes

## Percentages

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Your notes

## Basic Percentages

# Basic Percentages

## What is a percentage?

- “Per-cent” simply means “out of 100” (or “ $\div 100$ ”)
- Rewriting fractions as percentages means they can be **compared** more easily
- You can do this by finding an equivalent fraction with a **denominator of 100**
  - $\frac{1}{2} = \frac{50}{100} = 50\%$
  - $\frac{2}{5} = \frac{40}{100} = 40\%$
  - $\frac{3}{4} = \frac{75}{100} = 75\%$
  - The three percentages are much easier to compare than the three fractions
- Percentages are also **equivalent to decimals**
  - 100% = 1
  - 10% = 0.1
  - 1% = 0.01
  - 0.1% = 0.001
  - 25% = 0.25
  - 2.5% = 0.025
  - 0.25% = 0.0025
- Notice that a **decimal** can be converted **to a percentage** by **multiplying by 100**
  - Therefore a **percentage** can be converted **to a decimal** by **dividing by 100**
- A fraction can be written as a percentage by finding the **decimal equivalent**
  - You could use your calculator to do this

- E.g.  $\frac{234}{650} = 234 \div 650 = 0.36 = 36\%$



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## How do I find a percentage of an amount without a calculator?

- There are some percentages of an amount that are easy to work out
  - To find **50%**, **halve** the amount
  - To find **25%**, **halve** the amount **twice** (finding a **quarter**)
  - To find **10%**, **divide** the amount **by 10**
  - To find **1%**, **divide** the amount **by 100**
- These percentages can then be used as **building blocks** to find other percentages, for example:
  - To find **20%**, find 10% and then double it
  - To find **5%**, find 10% and halve it
  - To find **0.1%**, find 1% and divide it by 10
  - To find **12%**, find 10% and 1%, then add together the 10% and two lots of the 1%
- To find a percentage **larger than 100%**, remember that **100% is the original amount**
  - To find **150%**, find 50% and add it on to the original amount

## How do I find a percentage of an amount with a calculator?

- Whilst the methods above can be used with a calculator it is more efficient to use **multipliers**
- A multiplier is the **decimal equivalent** of a percentage
- E.g. To find **12% of 650**
  - Write 12% as a decimal multiplier
    - 12% is equivalent to 0.12
  - Find the product of the amount and the multiplier, using your calculator
    - $0.12 \times 650 = 78$
  - So 12% of 650 is 78
- When finding a percentage **larger than 100%**, the multiplier will be **greater than 1**
  - The multiplier for finding 126% of an amount would be 1.26



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## How do I express one number as a percentage of another?

- Start by writing one number **as a fraction of the other**
- Find the **decimal equivalent** of this fraction using your calculator
  - or find an equivalent fraction with a **denominator of 100**
- Rewrite this as a percentage
- E.g. To find 7 as a percentage of 20

- Write as  $\frac{7}{20}$

- This is equivalent to 0.35 or  $\frac{35}{100}$

- So 7 is 35% of 20



### Worked Example

Shade 35% of the grid below.


Count the total number of squares in the grid

Total of 40 squares

Find 35% of 40

Start by finding 10%, to help find 5% and 30%



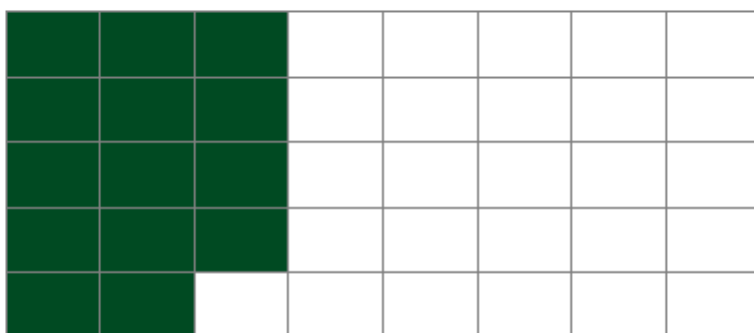
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$$\begin{aligned} 10\% \text{ of } 40 &= 4 \\ \text{so } 30\% \text{ of } 40 &= 4 \times 3 = 12 \\ \text{and } 5\% \text{ of } 40 &= 4 \div 2 = 2 \end{aligned}$$

$$35\% \text{ of } 40 = 12 + 2 = 14$$

Shade 14 squares

It doesn't matter which 14 you shade



### Worked Example

Amber owes \$ 1200 for a trip. She has to pay a deposit of \$ 150 to secure her place.

Express the deposit as a percentage of the price of the trip.

Write 150 as a fraction of 1200

$$\frac{150}{1200}$$

Find the value of this fraction as a decimal, using your calculator

$$\frac{150}{1200} = 0.125$$

Write this as the equivalent percentage (by multiplying by 100)

**12.5 %**



Your notes

## Percentage Increases & Decreases

# Percentage Increases & Decreases

## How do I increase by a percentage?

- A percentage **increase** makes an amount **bigger** by **adding** that percentage on to itself
- **Without a calculator**, use the **basic percentages** methods to find the percentage you are increasing by
  - Then **add** this on to the original amount
  - To increase 30 by 10%
    - 10% of 30 is 3
    - $30 + 3 = 33$
    - This is equivalent to finding 110% of 30
- **With a calculator** it is more efficient to use **multipliers**
  - A multiplier is the **decimal equivalent** of a percentage
    - A **percentage** can be converted to a decimal by **dividing by 100**
  - When **increasing** by a percentage, we are finding a percentage **greater than 100%**
  - To increase 80 by 15%
    - We are finding **115%** of 80, so the multiplier is **1.15**
    - $1.15 \times 80 = 92$

## How do I decrease by a percentage?

- A percentage **decrease** makes an amount **smaller** by **subtracting** that percentage from itself
- **Without a calculator**, use the methods outlined in **Basic Percentages** to find the percentage you are decreasing by
  - Then **subtract** this from the original amount
  - To decrease 30 by 10%
    - 10% of 30 is 3
    - $30 - 3 = 27$



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- This is equivalent to finding 90% of 30
  - Because  $100\% - 10\% = 90\%$
- **With a calculator** it is more efficient to use **multipliers**
  - When **decreasing** by a percentage, we are finding a percentage **smaller than 100%**
  - To decrease 80 by 15%
    - We are finding **85%** of 80, so the multiplier is **0.85**
      - Because  $100\% - 15\% = 85\%$
    - $0.85 \times 80 = 68$



### Worked Example

(a) Increase 200 kg by 21%.

*Method 1: Non-calculator*

By first finding 10% and 1%, find 21% of 200

$$\begin{aligned} 10\% \text{ of } 200 &= 20 \\ 1\% \text{ of } 200 &= 2 \\ 21\% \text{ of } 200 &= 20 + 20 + 2 = 42 \end{aligned}$$

Add this to the original amount

$$200 + 42$$

**242 kg**

*Method 2: Calculator*

An increase by 21% is equivalent to finding 121% of the original amount

So the multiplier is 1.21

$$1.21 \times 200$$

**242 kg**

(b) An item that costs £ 500 is discounted by 35%.

Find the new price of the item.

A discount of 35% means the price decreases by 35%

*Method 1: Non-calculator*



Your notes

By first finding 10% and 5%, find 35% of 500

$$10\% \text{ of } 500 = 50$$

$$5\% \text{ of } 500 = 25$$

$$35\% \text{ of } 500 = 50 + 50 + 50 + 25 = 175$$

Subtract this from the original amount

$$500 - 175$$

**£325**

*Method 2: Calculator*

A decrease of 35% is equivalent to finding 65% of the original amount ( $100 - 35 = 65$ )

So the multiplier is 0.65

$$500 \times 0.65$$

**£ 325**

## How do I deal with repeated percentage changes?

- In some problems there may be **several** changes by a percentage
- For example,
  - A shop **increases** the price of a product costing £80 **by 10%**,
    - equivalent to a multiplier of  $\times 1.10$
  - and then **discounts** the product **by 15%**,
    - equivalent to a multiplier of  $\times 0.85$
  - and then **discounts** the product by a **further 20%**
    - equivalent to a multiplier of  $\times 0.80$
- You can either:
  - **Multiply** the starting amount **by each multiplier in turn**
    - $((80 \times 1.10) \times 0.85) \times 0.80 = £59.84$
  - Or **combine** the multipliers first and then multiply by the "combined multiplier"
    - $1.10 \times 0.85 \times 0.80 = 0.748$ 
      - This shows it is equivalent to 74.8% of the original amount, or a discount of 25.2%
    - $80 \times 0.748 = £59.84$





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- In general, for  $n$  multipliers of values  $m_1, m_2, \dots, m_n$ 
  - The combined multiplier is  $m_1 \times m_2 \times \dots \times m_n$

## How do I find a percentage change?

- The **multiplier** that was used for a **percentage change** can be found using the formula:

$$m = \frac{\text{Amount after}}{\text{Amount before}}$$

- The value of  $m$  corresponds to the **multiplier** for the percentage change
  - A value **greater than 1** is a percentage **increase**
    - **1.05** corresponds to an **increase by 5%**
  - A value **less than 1** is a percentage **decrease**
    - **0.75** corresponds to a **decrease by 25%**

- **Alternatively** you can use the formula:

$$\text{Percentage Change} = \frac{\text{After} - \text{Before}}{\text{Before}} \times 100$$

- A **positive** value is a percentage **increase**
  - An answer of 12 means an increase of 12%
- A **negative** value is a percentage **decrease**
  - An answer of -28 means a decrease of 28%

## How do I find a percentage profit or loss?

- Similar strategies to the above can be used to find the **percentage profit or loss**
- Shops buy or produce items at a "cost price" and sell them at a "selling price"
- Using a **multiplier** method:

$$m = \frac{\text{Selling Price}}{\text{Cost Price}}$$

- A value **greater than 1** is a **profit**
  - **1.05** corresponds to a **5% profit**



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- A value **less than 1** is a **loss**
  - **0.75** corresponds to a **25% loss**
- **Alternatively** you can use the formula:
  - Percentage Profit =  $\frac{\text{Selling Price} - \text{Cost Price}}{\text{Cost Price}} \times 100$
  - A **positive** value is a **profit**
    - An answer of 12 means a 12% profit
  - A **negative** value is a **loss**
    - An answer of -28 means a 28% loss



### Examiner Tips and Tricks

- Use "common sense" to check your answer!
  - If an item is sold for more than it was bought for, you are expecting a profit, not a loss



### Worked Example

The number of students in a school changes from 250 to 310.

Describe the percentage change in number of students.

*Method 1*

Use the formula  $m = \frac{\text{Amount after}}{\text{Amount before}}$

$$\frac{310}{250} = 1.24$$

This multiplier is greater than 1, so corresponds to a percentage increase

**A percentage increase of 24%**

*Method 2*

Use the formula Percentage Change =  $\frac{\text{After} - \text{Before}}{\text{Before}} \times 100$



Your notes

$$\frac{310 - 250}{250} \times 100 = 24$$

The value is positive, so this is a percentage increase

**A percentage increase of 24%**



### Worked Example

Sophie purchases a car for £8000 and sells it several years later for £5600.

Describe the percentage profit or loss on the car.

*Method 1*

Use the formula  $m = \frac{\text{Selling Price}}{\text{Cost Price}}$

$$\frac{5600}{8000} = 0.7$$

The value is less than 1 so means it is a percentage loss

The selling price was 70% of the cost price, so a loss of 30%

**A loss of 30%**

*Method 2*

Use the formula Percentage Profit =  $\frac{\text{Selling Price} - \text{Cost Price}}{\text{Cost Price}} \times 100$

$$\frac{5600 - 8000}{8000} \times 100 = -30$$

The value is negative, so this is a percentage loss

**A loss of 30%**



Your notes

## Reverse Percentages

# Reverse Percentages

## What is a reverse percentage?

- A reverse percentage question is one where we are given the **value after a percentage increase or decrease** and asked to find the value **before** the change

## How do I solve reverse percentage questions?

- You should think about the **before quantity**
  - even though it is not given in the question
- Find the percentage change as a **multiplier,  $p$** 
  - This is the decimal equivalent of a percentage change
    - A percentage increase of 4% means  $p = 1 + 0.04 = 1.04$
    - A percentage decrease of 5% means  $p = 1 - 0.05 = 0.95$
- Use **before  $\times p = \text{after}$**  to write an **equation**
  - Get the **order** right: the percentage change happens to the "before", not to the "after"
- **Rearrange** the equation to make the "before" quantity the subject
  - **Divide** the "after" quantity by the multiplier,  $p$
  - $$\text{Before} = \frac{\text{After}}{\text{Multiplier}}$$

## What is a common mistake with reverse percentage questions?

- Here is an example: a price of a mobile increases by 10% to £220
  - To find the price before, you do **not** apply a 10% **decrease** to £220
    - That would give  $220 \times 0.9 = £198$  (incorrect)
  - Use **before  $\times p = \text{after}$**  instead
    - $\text{before} \times 1.1 = 220$



Your notes

- before =  $\frac{220}{1.1} = £200$  (correct)
- You cannot turn a percentage increase into a decrease with reverse percentage questions



### Examiner Tips and Tricks

- To spot a reverse percentage question, see if you are being asked to find a quantity in the past
  - Find the **old / original / before** amount ...



### Worked Example

Jennie has been working for a company for the last ten years.

She receives a pay rise of 5%.

Her new salary is £31 500 per year.

Find her salary before the pay rise.

Use "before"  $\times p$  = "after" to write an equation

The "before" amount is unknown and the "after" amount is 31 500

$$\text{"before"} \times 1.05 = 31\,500$$

Find the multiplier,  $p$  (by writing 5% as a decimal and adding it to 1)

$$p = 1 + 0.05 = 1.05$$

Find the value of "before" (by dividing both sides by 1.05)

$$\text{"before"} = \frac{31\,500}{1.05} = 30\,000$$

She was paid £30 000 before the pay rise

**Jennie was paid £30 000 per year before the pay rise**