AQA GCSE Maths: Higher

Your notes

Histograms

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Frequency Density

What is frequency density?

- **Frequency density** is given by the formula
  - $\text{frequency density} = \dfrac{\text{frequency}}{\text{class width}}$
- Frequency density is used with **grouped data** (i.e. data grouped by **class intervals**)
  - It is useful when the class intervals are of **unequal width**
  - It provides a measure of how **dense** data is within its **class interval**
    - relative to the width of the interval
  - For example,
    - 10 data values spread over a class interval of width 20 would have a frequency density of $\dfrac{10}{20} = \dfrac{1}{2}$
    - 20 data values spread over a class interval of width 100 would have a frequency density of $\dfrac{20}{100} = \dfrac{1}{5}$
    - As $\dfrac{1}{2} > \dfrac{1}{5}$

How do I calculate frequency density?

- In questions it is usual to be presented with grouped data in a **table**
- **Add two extra columns** to the table
  - one to work out and write down the **class width** of each interval
  - the second to then work out the **frequency density** for each group (row)

Worked Example

The table below shows information regarding the average speeds travelled by trains in a region of the UK.

The data is to be plotted on a histogram.

Work out the frequency density for each class interval.

Average speed s m/s	Frequency
$20 \leq s < 40$	5
$40 \leq s < 50$	15
$50 \leq s < 55$	28
$55 \leq s < 60$	38
$60 \leq s < 70$	14

Add two columns to the table

- one for class width
- one for frequency density

Writing the calculation in each box helps to keep accuracy

Average speed s m/s	Frequency	Class width	Frequency density
$20 \leq s < 40$	5	40 - 20 = 20	5 ÷ 20 = **0.25**
$40 \leq s < 50$	15	50 - 40 = 10	15 ÷ 10 = **1.5**
$50 \leq s < 55$	28	55 - 50 = 5	28 ÷ 5 = **5.6**
$55 \leq s < 60$	38	60 - 55 = 5	38 ÷ 5 = **7.6**
$60 \leq s < 70$	14	70 - 60 = 10	14 ÷ 10 = **1.4**

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Drawing Histograms

What is a histogram?

- A **histogram** looks similar to a bar chart, but there are important **differences**
- **Bar charts** are used for **discrete** (and sometimes **non-numerical**) data
  - In a bar chart, the **height** (or **length**) of a bar determines the frequency
  - There are usually **gaps** between the bars
- **Histograms** are used with **continuous data,** grouped into **class intervals** (usually of **unequal width**)
  - In a histogram, the **area of a bar** determines the frequency
    - This means it is difficult to tell anything simply from looking at a histogram
    - Some basic calculations will be needed for conclusions and comparisons to be made
  - There are **no gaps** between the bars

How do I draw a histogram?

- **Drawing** a histogram first requires the calculation of the **frequency densities** for each class interval (group)
  - Use $\text{frequency density} = \dfrac{\text{frequency}}{\text{class width}}$
  - Exam questions often ask you to **finish** an incomplete histogram, rather than start with a blank graph
- Once the frequency densities are known, the bars (rectangles) for each class interval can be **drawn**
  - with widths being measured on the horizontal (x) axis
  - and the height of each bar (the frequency density) being measured on the vertical (y) axis
  - As the data is continuous, the bars will be touching

Examiner Tips and Tricks

- Always work out and write down the frequency densities
  - It is easy to make errors and lose marks by going straight to the graph
  - Method marks may depend on showing you know to use frequency density rather than frequency

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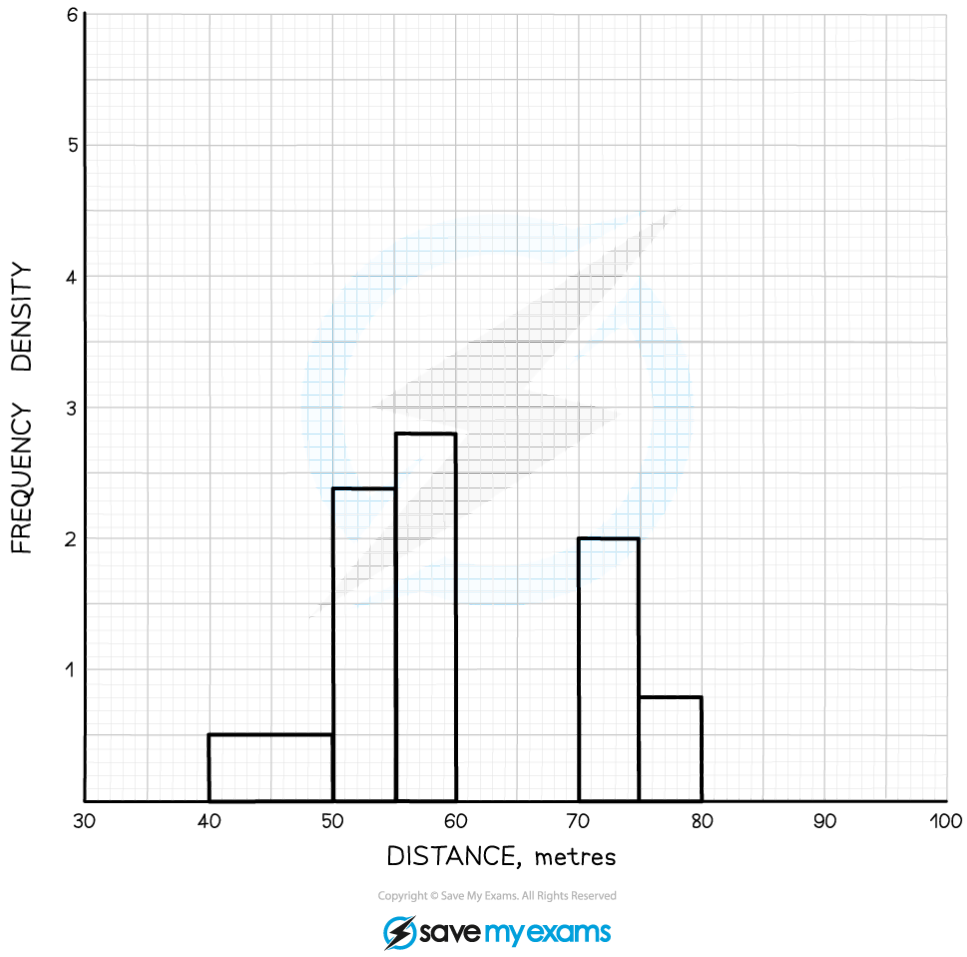
Worked Example

A histogram is shown below representing the distances achieved by some athletes throwing a javelin.



There are two classes missing from the histogram.

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These are:

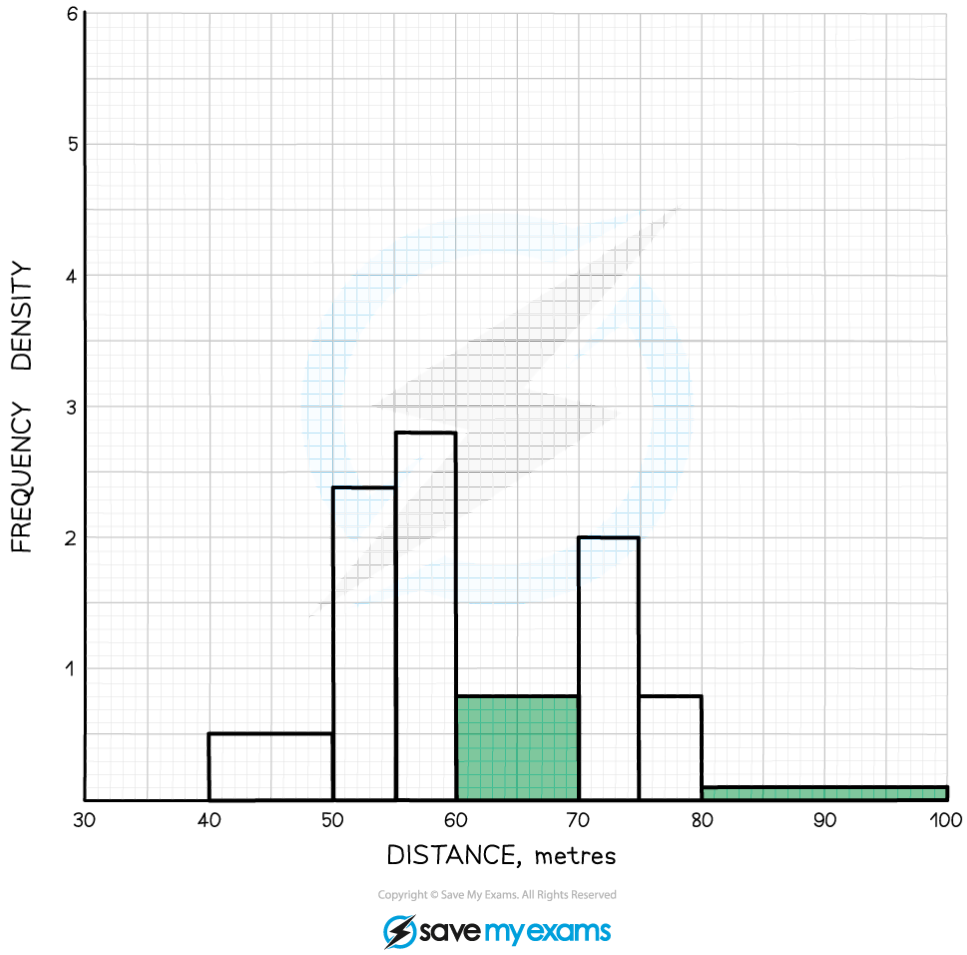
Distance, x m	Frequency
$60 \leq x < 70$	8
$80 \leq x < 100$	2

Add these to the histogram.

Before completing the histogram, remember to show clearly you've worked out the missing frequency densities

Distance, x m	Frequency	Class width	Frequency density
$60 \leq x < 70$	8	70 - 60 = 10	8 ÷ 10 = 0.8
$80 \leq x < 100$	2	100 - 80 = 20	2 ÷ 20 = 0.1

Now the bars can be drawn on the histogram
They should stretch along the *x*-axis from the start to the end of the class interval
The heights will be equal to the frequency densities



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Interpreting Histograms

How do I interpret a histogram?

- It is important to remember that the frequency density (y-) axis does not tell us frequency
  - The **area of the bar** is equal to the **frequency**
    - $\text{frequency} = \text{frequency density} \times \text{class width}$
- Note that a very **simple** histogram may have **equal class widths**
  - In this case the y-axis may be labelled 'frequency' instead of 'frequency density'
  - If 'frequency' is on the y-axis, then you **can** use the heights of the bars to directly determine the frequencies
- You may be asked to **estimate** the frequency of **part** of a bar/class interval within a histogram
  - Find the area of the bar for the part of the interval required
  - Once area is known, frequency can be found as above
- You can use histograms for two data sets to **compare** the data distributions
  - but only if they have the **same class intervals** and the **same frequency density scales**

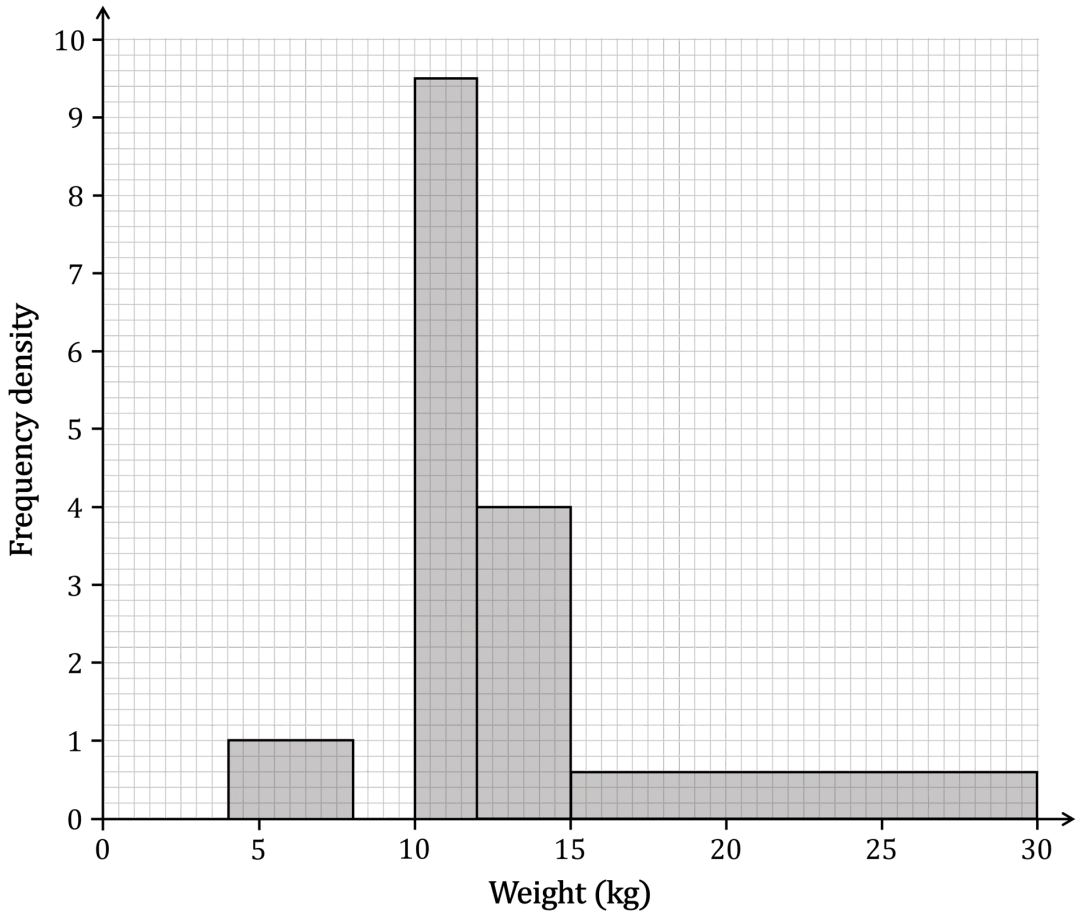
Examiner Tips and Tricks

- The **frequency density axis** will not always be labelled
  - Look carefully at the scale, it is unlikely to be 1 unit to 1 square

Worked Example

The table below and the corresponding histogram show the weight, in kg, of some newborn bottlenose dolphins.

Weight w kg	Frequency
$4 \leq w < 8$	4
$8 \leq w < 10$	16
$10 \leq w < 12$	19
$12 \leq w < 15$	12
$15 \leq w < 30$	



(a) Use the histogram to complete the table.

The frequency for the $15 \leq w < 30$ class interval is missing

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The bar for that class interval on the histogram has a

- **height (frequency density) of 0.6**
- **width of 30 – 15 = 15**

Rearrange $\text{frequency density} = \dfrac{\text{frequency}}{\text{class width}}$ **to get**

$$\text{frequency} = \text{frequency density} \times \text{class width}$$

$$\text{frequency} = 0.6 \times 15 = 9$$

Weight w kg	Frequency
$4 \leq w < 8$	4
$8 \leq w < 10$	16
$10 \leq w < 12$	19
$12 \leq w < 15$	12
$15 \leq w < 30$	**9**

(b) Use the table to complete the histogram.

The bar for the $4 \leq w < 8$ class interval is missing

That class interval has a

- **frequency of 16**
- **width of 10 – 8 = 2**

Use $\text{frequency density} = \dfrac{\text{frequency}}{\text{class width}}$ **to find the frequency density**

$$\text{frequency density} = \frac{16}{2} = 8$$

Draw a bar with that height on the histogram, between 8 and 10 on the horizontal axis

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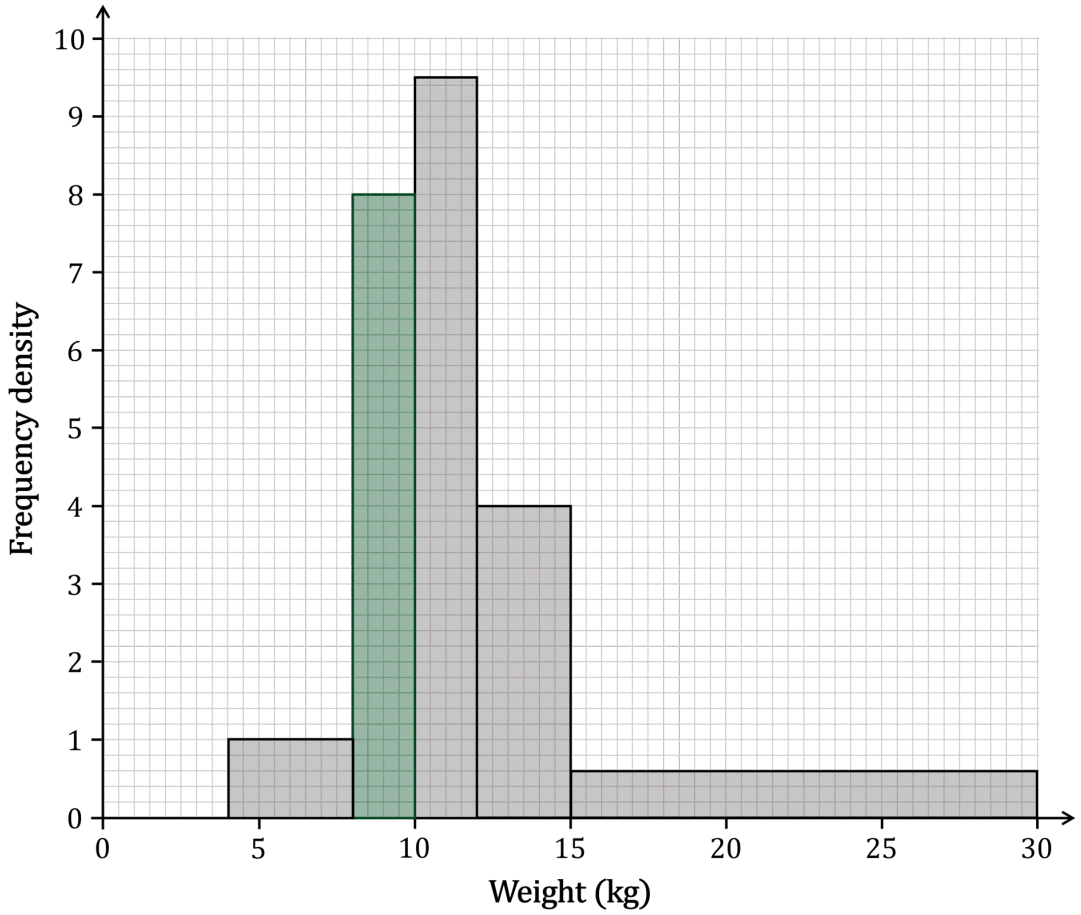
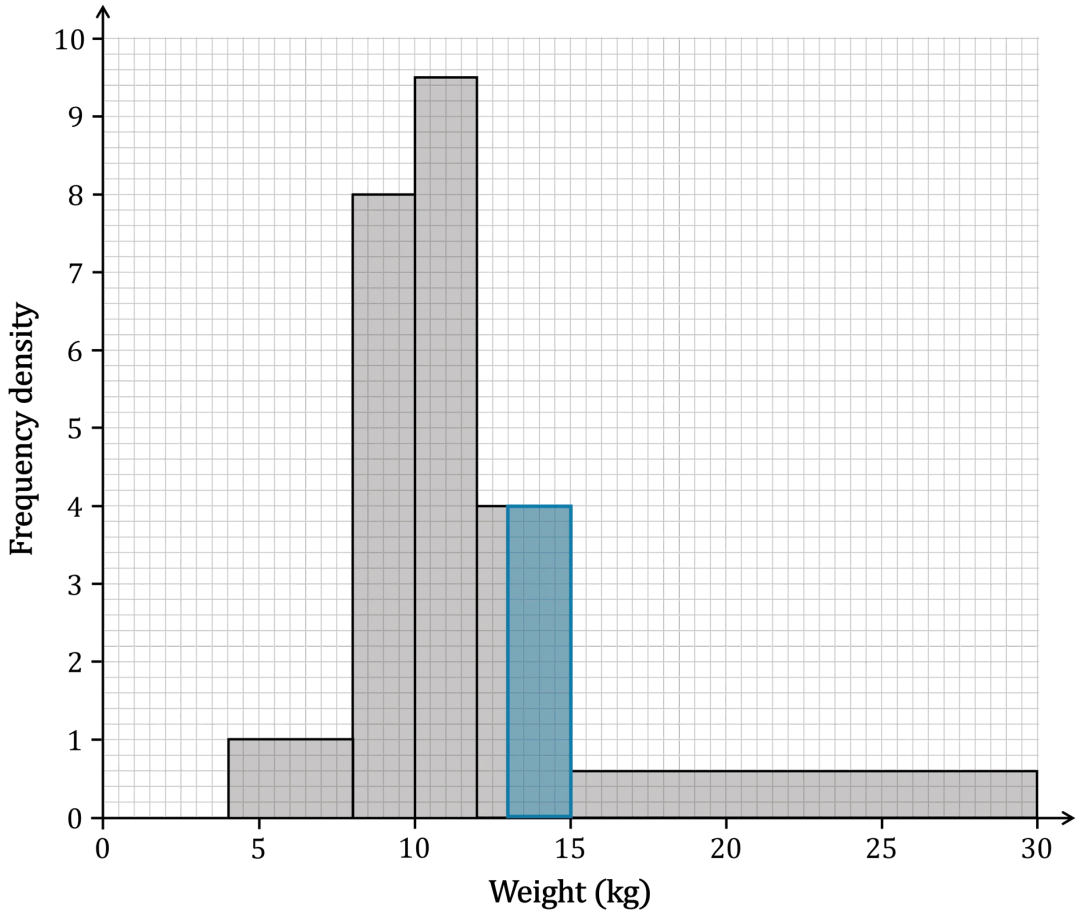


(c) Estimate the number of dolphins whose weight is greater than 13 kg.

We know from part a) that there are 9 dolphins in the $15 \leq w < 30$ class interval
So we need to estimate the number of dolphins that are in the interval $13 \leq w < 15$

For $13 \leq w < 15$, the histogram shows that

- **the frequency density is 4**
- **the width is 15–13 = 2**



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Now use $\text{frequency} = \text{frequency density} \times \text{class width}$ to estimate the number of dolphins in the $13 \leq w < 15$ interval
(Note that using the histogram in this way is actually a form of linear interpolation)

$$\text{frequency} = 4 \times 2 = 8$$

This is only an estimate because we don't actually know that dolphins are evenly distributed across the entire $12 \leq w < 15$ class interval

Now the total number of dolphins with a weight greater than 13 kg can be estimated

$$8 + 9 = 17$$

There are approximately 17 dolphins with a weight greater than 13 kg