AQA GCSE Maths: Higher

Your notes

Graphing Inequalities

Contents

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Representing Inequalities as Regions

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Finding Regions using Inequalities

What are 2D inequalities?

- Recall that an inequality in one variable **(1D inequality)** represents a relationship that is **not equal**
  - An inequality of $x < 7$, represents **all values** smaller than 7
    - There are an infinite number of values than can satisfy this inequality
- A **2D inequality** represents a relationship between **two expressions** that is **not equal**
  - The inequality $y > x$ represents **all pairs of numbers *x* and *y*** where the y value is greater than the x value
    - There are an infinite number of pairs of values that would satisfy this inequality
    - These pairs of numbers can be thought of as **coordinates**
    - On a graph, all coordinates **above** the line $y = x$ would satisfy this inequality
- If a 2D inequality includes either the symbol ≤ or ≥, then coordinates **on the line itself** also satisfies the inequality
  - E.g. $y \leq 2x$ represents all of the pairs of numbers where the value of y is less than two lots of the value of x
    - This is the region **below** the line $y = 2x$, but also being on the line $y = 2x$ satisfies the inequality

How do we draw inequalities on a graph?

- A set of **2D inequalities** can be shown **graphically** using **straight lines** and **shaded regions**
- To draw the correct lines:
  - Replace the inequality sign with "=" and draw that line
    - Use a **solid** line for ≤ or ≥ (to indicate the line is included)
    - Use **dotted** line for < or > (to indicate the line is not included)
- To decide which side of the line is the **wanted side:**
  - if "$y \leq$ ..." or "$y <$ ..." then the wanted region is **below** the line
  - if "$y \geq$ ..." or "$y >$ ..." then the wanted region is **above** the line

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- If you are unsure
  - substitute the coordinates from a point on one side of the line into the inequality
  - determine whether or not the inequality holds true on that side
- For **vertical lines**:
  - the wanted region for $x < k$ is to the **left** of $x = k$
  - the wanted region for $x > k$ is to the **right** of $x = k$
- To do the shading:
  - **Shade the unwanted sides** of each line (unless the question says otherwise)
    - You are shading away any parts you don't want
    - This will leave behind a clear region that is the wanted region (rather than trying to look for the wanted region under multiple shades)
    - Label the wanted region *R* (unless the question says otherwise)
- (Be careful if using graphing software, as some shade the wanted sides)

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Worked Example

Show, graphically, the region that is satisfied by all three inequalities below:

$$3x + 2y \geq 12 \quad y < 2x \quad x < 3$$

Label this region *R*.

First draw the three straight lines: $3x + 2y = 12$, $y = 2x$ and $x = 3$

Use your knowledge of **Straight Line Graphs**, $y = mx + c$
You may wish to rearrange $3x + 2y = 12$ to the form $y = mx + c$ first

$$2y = -3x + 12$$

$$y = -\frac{3}{2}x + 6$$

The line $3x + 2y \geq 12$ is a solid line because of the "≥"

The lines $y < 2x$ and $x < 3$ are dotted lines because of the "<"

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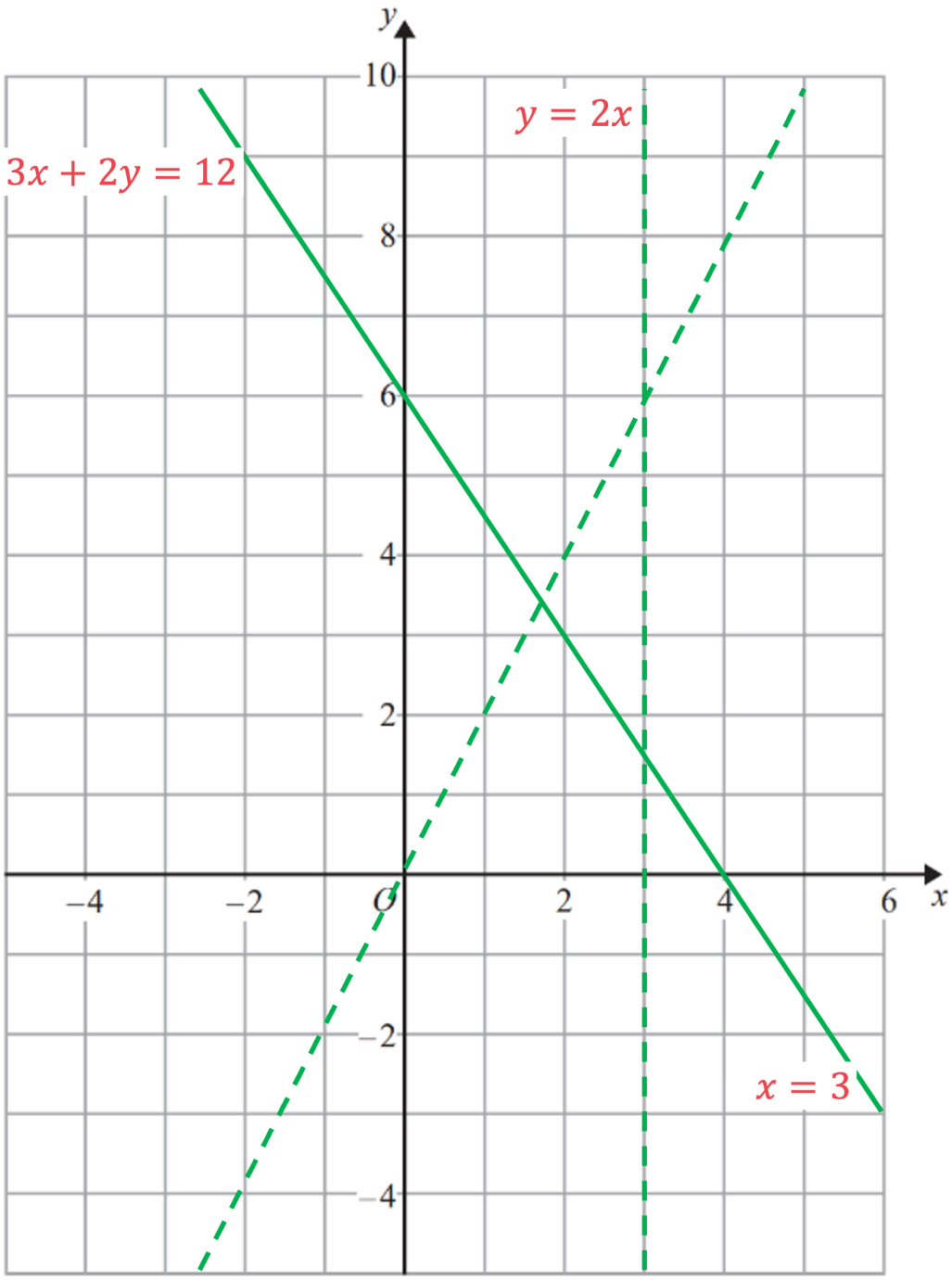

Now we need to shade the **unwanted** regions

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For $3x + 2y \geq 12$ (or $y \geq -\frac{3}{2}x + 6$), the unwanted region is below the line

We can check this with the point (0, 0)

"$3(0) + 2(0) \geq 12$" is false therefore (0, 0) does lie in the unwanted region for $3x + 2y \geq 12$

For $y < 2x$, the unwanted region is above the line
Check with another point, for example (1, 0)

"$0 < 2(1)$" is true, so (1, 0) lies in the wanted (i.e. unshaded) region for $y < 2x$

For $x < 3$, shade the unwanted region to the right of $x = 3$
If unsure, check with a point

Finally, don't forget to label the region *R*

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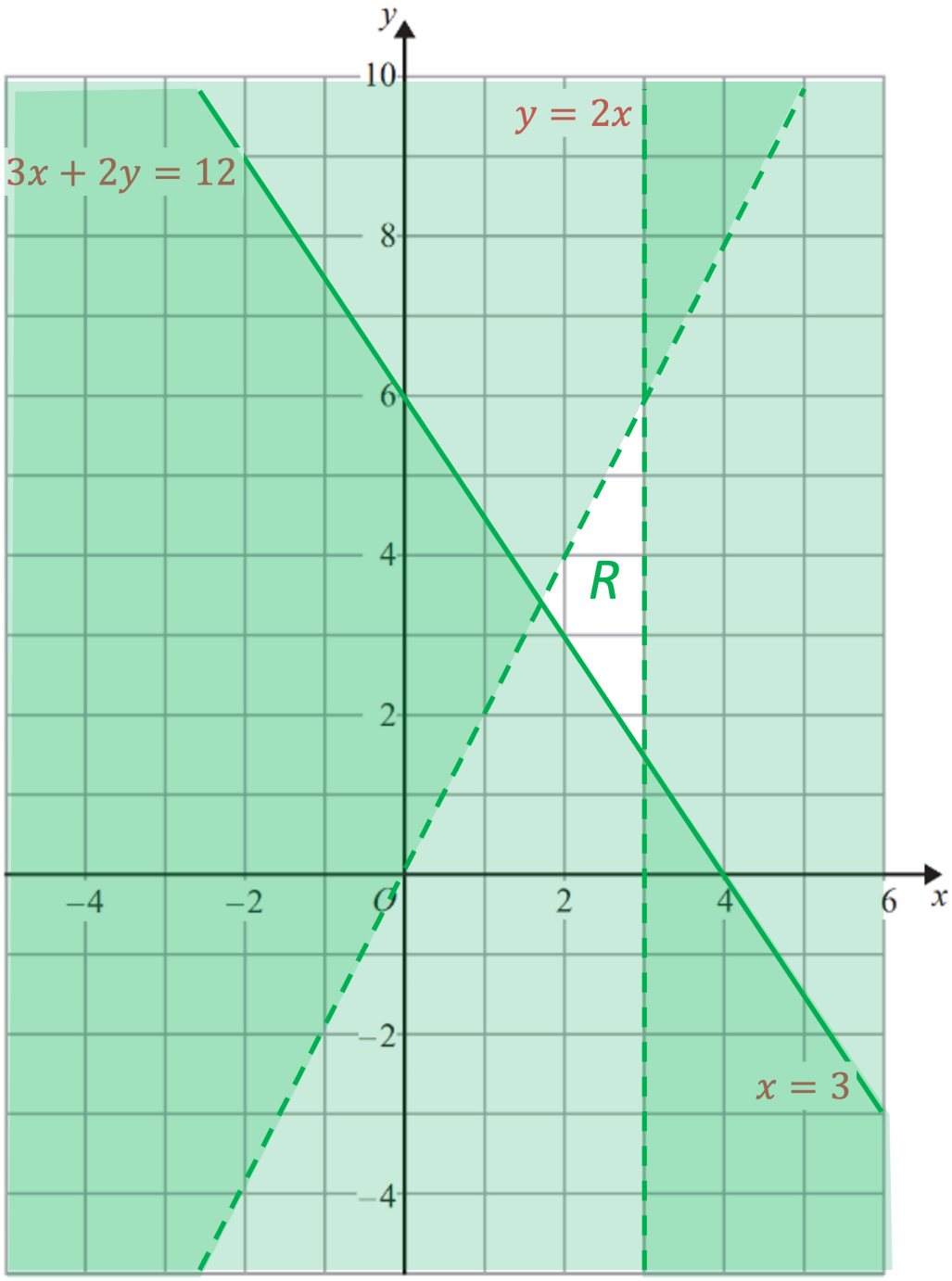

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Finding Inequalities from Regions

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Interpreting Graphical Inequalities

How do I know which inequalities are shown on a graph of shaded regions?

- To **identify the inequalities** represented by the shaded regions on a graph:
  - Find the **equation** of each line on the graph
    - You may have to calculate the **gradient** and find the **y-intercept** to use $y = mx + c$
    - **Vertical** lines have the form $x = k$
    - **Horizontal** lines have the form $y = k$
  - **Remember** that lines are drawn using:
    - a **solid** line for ≤ or ≥, indicating a line **included** in the region
    - a **dotted** line for < or >, indicating a line **not included** in the region
  - **Replace** the = sign with the relevant inequality
    - ≤ or < if region is **below** line
    - ≥ or > if region is **above** line
    - (Use a point to test if not sure)

Examiner Tips and Tricks

Always read the question carefully to see if the diagram shades the wanted region or the unwanted region.

Worked Example

Write down the three inequalities which define the shaded region shown on the axes below.

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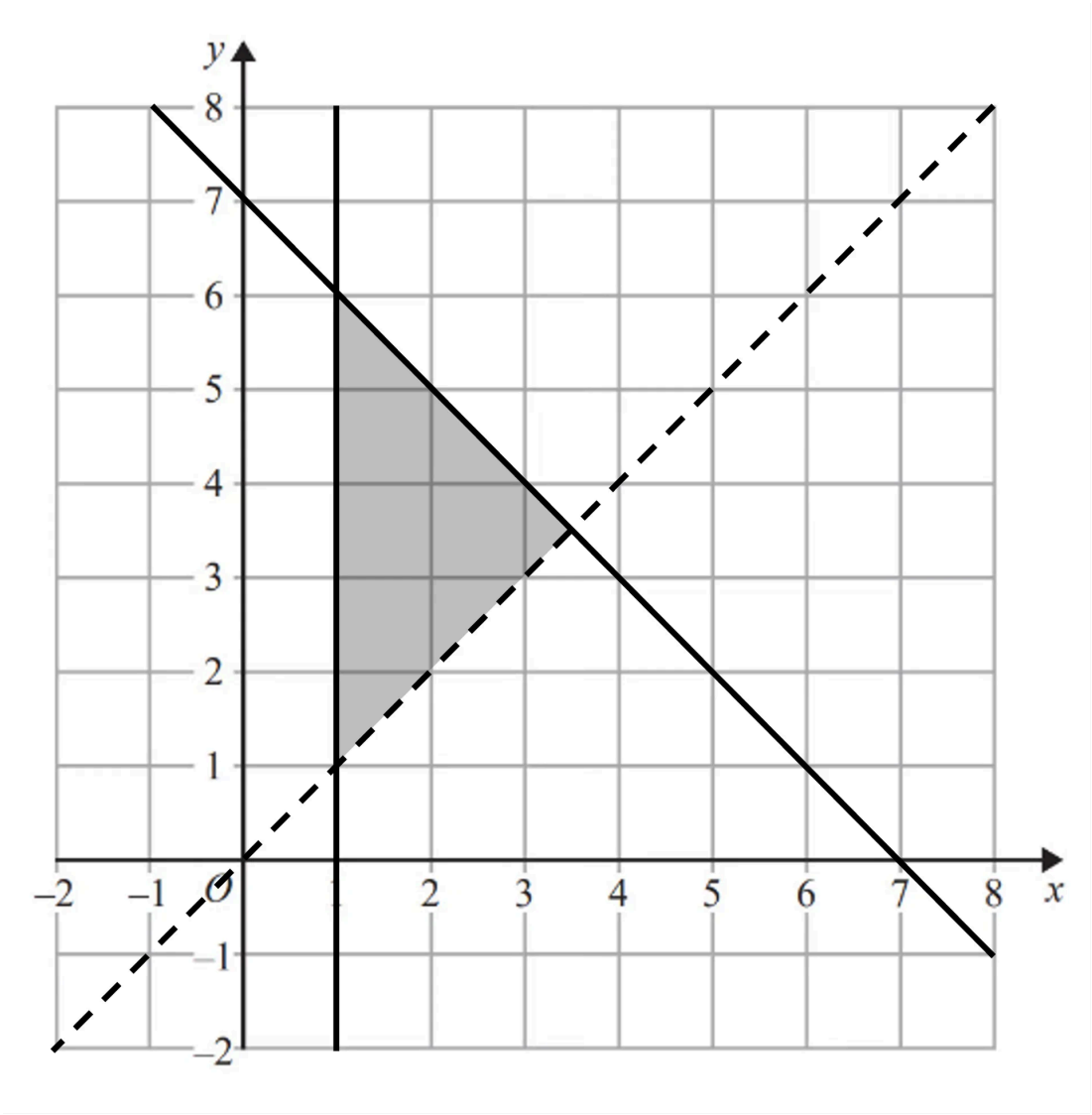

Find the equations of the three lines shown (ignoring inequality signs for now)

You may be able to see the lines $x = 1$ and $y = x$

The other line has the form $y = mx + c$ with y-intercept 7 and gradient -1

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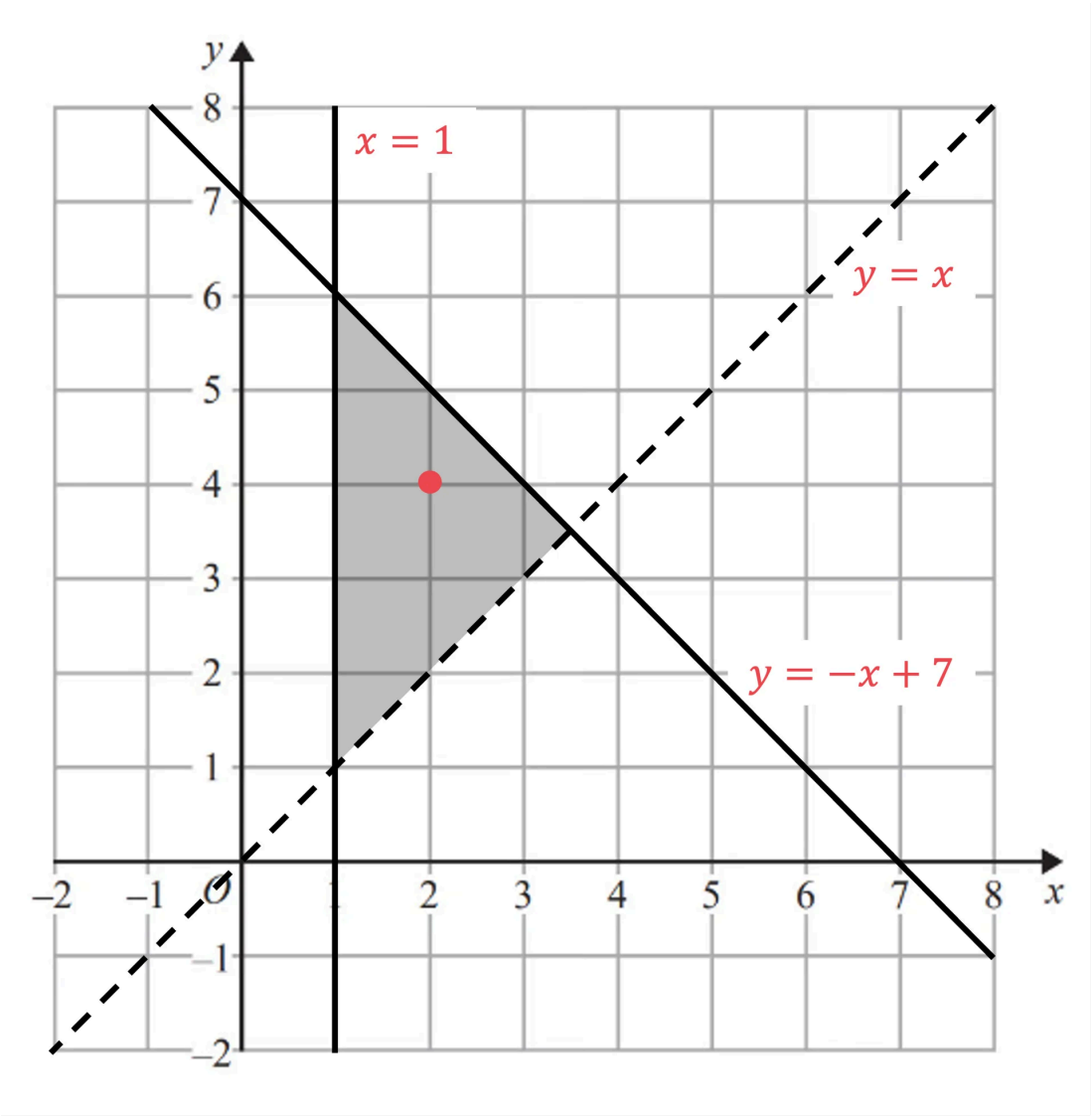

Now decide which inequality signs to use

For $y = x$, the shaded region is above the line, and the line is dotted, so the inequality is

$$y > x$$

If unsure, check by substituting in coordinates from the shaded region
For example, using (2, 4) as marked on the graph above

"$4 > 2$" is true, so the inequality $y > x$ is correct

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For $y = -x + 7$, the shaded region is below the line, and the line is solid, so the inequality is

$$y \leq -x + 7$$

Again, check by substituting (2, 4) into the inequality

"$4 \leq -2 + 7$" is true, so the inequality $y \leq -x + 7$ is correct

For $x = 1$, the shaded region is to the right of the solid line so the inequality is

$$x \geq 1$$

Write all three inequalities together as your final answer

$y > x$, $y \leq -x + 7$ and $x \geq 1$


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