AQA GCSE Maths: Higher

Your notes

3D Pythagoras & Trigonometry

Contents

Your notes

3D Pythagoras & Trigonometry

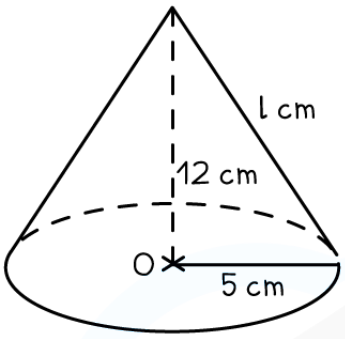
How do I use Pythagoras' theorem in a 3D shape?

- You can often find **right-angled triangles** within 3D shapes
  - If two sides of the triangle are known, you can use **Pythagoras' theorem**

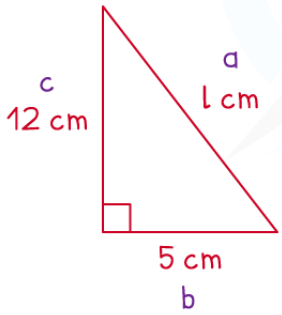
Your notes

PYTHAGORAS' THEOREM IN 3D

e.g. A CONE HAS BASE RADIUS 5 cm AND A PERPENDICULAR HEIGHT OF 12 cm.



FIND THE LENGTH, l cm, OF THE SLANTED HEIGHT OF THE CONE



DRAW THE 2D TRIANGLE OUT TO MAKE IT EASIER TO SEE.

"$a^2 = b^2 + c^2$"

$l^2 = 5^2 + 12^2$

$l^2 = 25 + 144$

$l^2 = 169$

$l = 13$ cm

USE PYTHAGORAS' THEOREM TO SOLVE THE PROBLEM.

Is there a 3D version of the Pythagoras' theorem formula?

- There is a **3D version** of Pythagoras' theorem: $d^2 = x^2 + y^2 + z^2$

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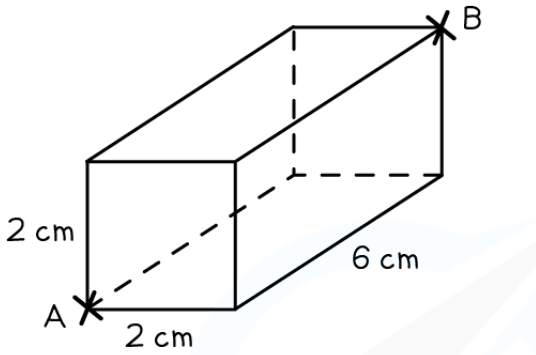
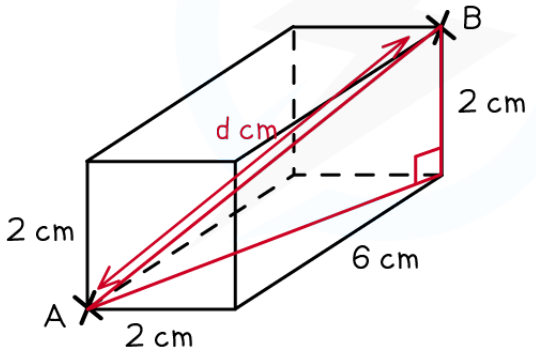
- d is the distance between two points
- x, y and z are the distances in the **three different perpendicular directions** between the two points

e.g. FIND THE DISTANCE BETWEEN A AND B.





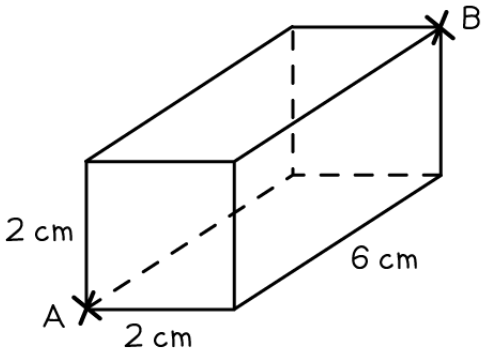
$d^2 = 2^2 + 6^2 + 2^2 = 44$

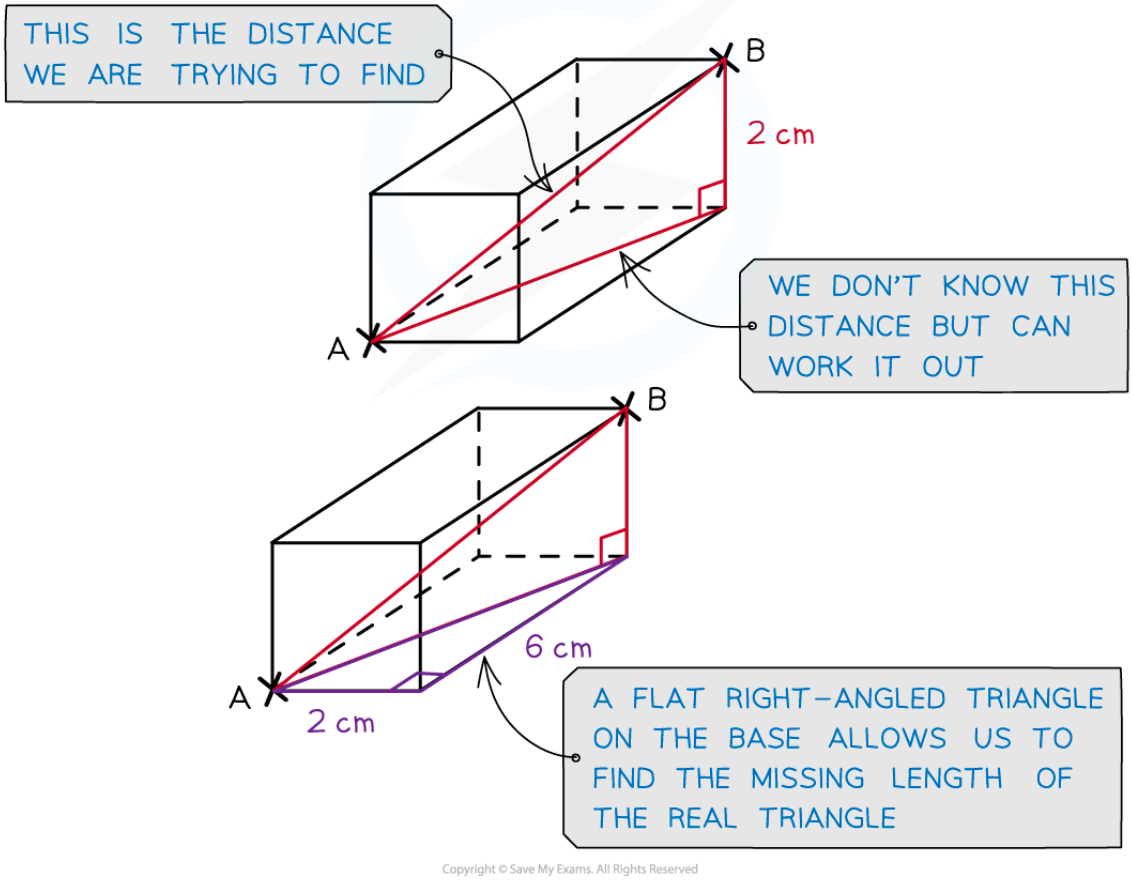
LENGTH OF AB IS $\sqrt{44} = 6.63$ (2 DP)

- However, all 3D situations can be broken into **two 2D problems**
  - Form two right-angle triangles

Your notes

e.g. FIND THE DISTANCE BETWEEN A AND B.





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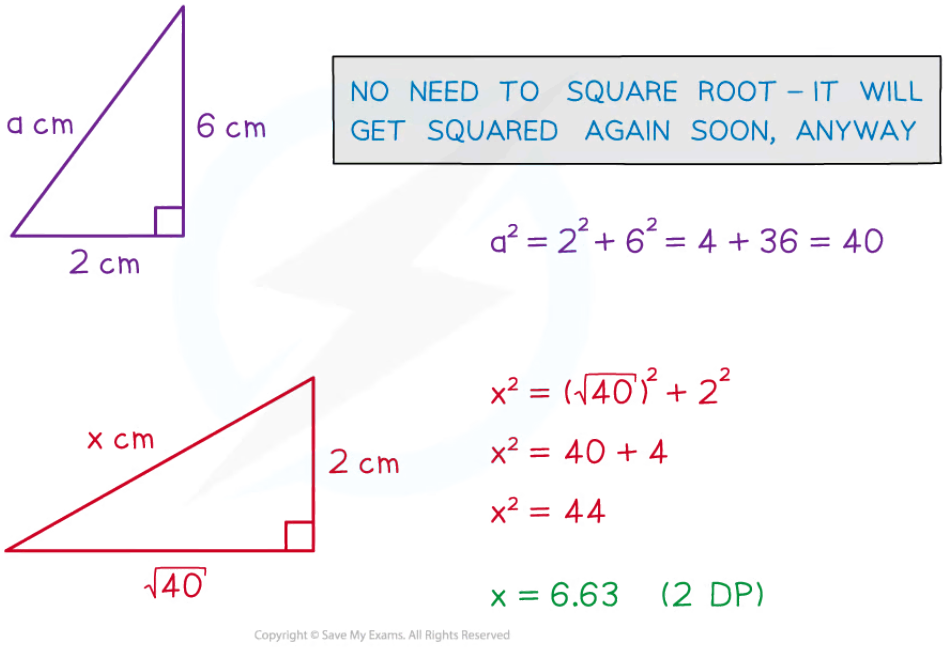


Examiner Tips and Tricks

- You are not given the 3D Pythagoras formula in the exam
  - You can always split 3D problems into two 2D problems (which don't need this formula)

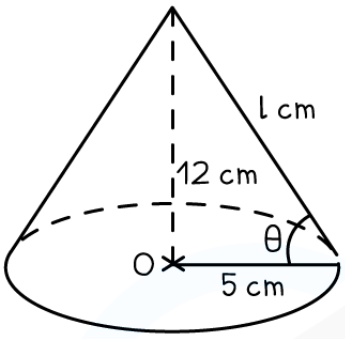
How do I use SOHCAHTOA in 3D?

- Again, look for **right-angled triangles** to use with **SOHCAHTOA**
  - You may need combinations of triangles that lead to the missing **side** or **angle**

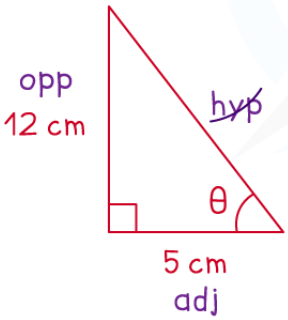
Your notes

SOHCAHTOA IN 3D

e.g. A CONE HAS BASE RADIUS 5 cm AND A PERPENDICULAR HEIGHT OF 12 cm.



FIND THE ANGLE θ, GIVING YOUR ANSWER TO ONE DECIMAL PLACE.



DRAW THE 2D TRIANGLE OUT TO MAKE IT EASIER TO SEE.

$$TOA - \tan\theta = \frac{opp}{adj}$$

$$\tan\theta = \frac{opp}{adj} = \frac{12}{5} = 2.4$$

$$\theta = \tan^{-1}(2.4)$$

REMEMBER TO USE SHIFT WHEN FINDING ANGLES

$$\theta = 67.4 \quad (1\ DP)$$

How do I find the angle between a line and a plane?

- To find the **angle** between a **line** and a **plane**:

- first **identity the plane** in the question
- then **draw** on the **line** in the question (if it is not already drawn on)
- form a **right-angled triangle** between the line and plane
  - The **height** of the triangle must be **perpendicular to the plane**
- then use **SOHCAHTOA**

- The angle between a line and a plane is sometimes **not obvious**
  - If unsure, put a different point on the same line and see if it helps
    - Try to create a right-angled triangle
  - Some people like to imagine the line as a fishing rod, from which they lower the hook (and fishing line) vertically down to the plane!
    - Others like to imagine they are sitting inside the 3D object, looking around it (like corners of a room)


Your notes

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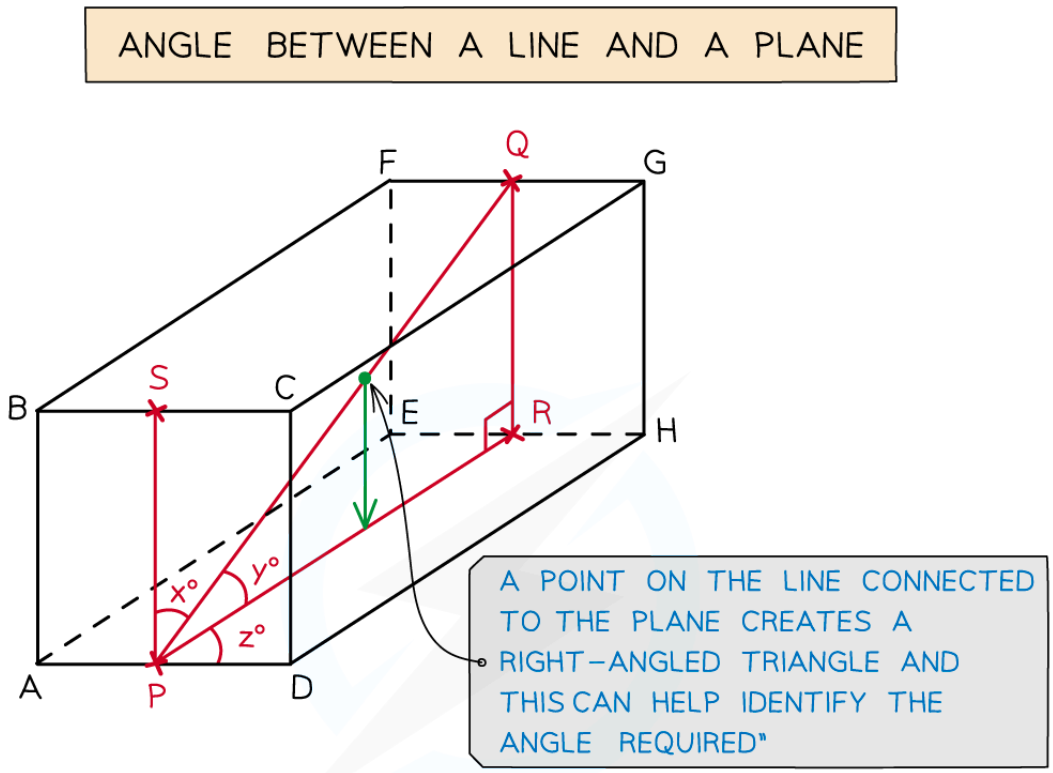


How do I apply 3D Pythagoras and trigonometry to more complicated problems?

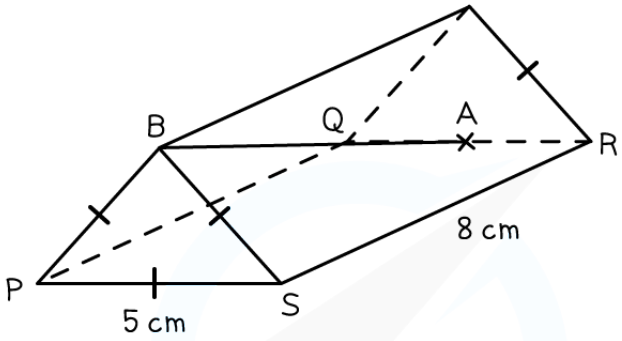
- Always split up a complicated problem into **2D right-angled triangles**
  - Some questions may requires more than one 2D right-angled triangle

- Some 2D triangles on the diagram are still drawn in 3D
  - It helps to **redraw** these 2D triangles **flat on the page** (not at angles)
  - You can then spot any uses of Pythagoras' theorem and SOHCAHTOA

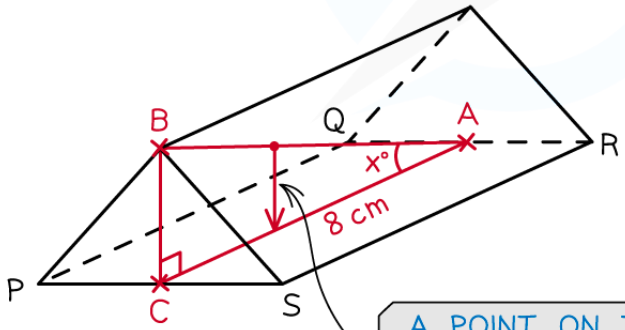
Your notes

PROBLEM SOLVING

e.g. IN THE TRIANGULAR PRISM BELOW, POINT A IS THE MID-POINT OF QR AND B IS A VERTEX OF THE FACE PBS WHICH IS AN EQUILATERAL TRIANGLE.



FIND THE ANGLE BETWEEN THE LINE AB AND THE PLANE PQRS.

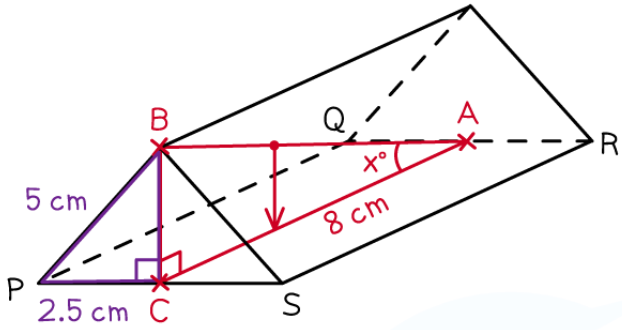


DRAW IN A RIGHT ANGLED TRIANGLE AND DETERMINE ANY KNOWN DISTANCES, AND THE ANGLE BEING ASKED FOR.

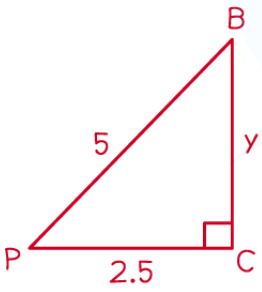
A POINT ON THE LINE AB CONNECTED TO THE PLANE PQRS CREATES A RIGHT ANGLED TRIANGLE

ONLY ONE SIDE OF TRIANGLE ABC IS KNOWN BUT BC CAN BE CALCULATED BY PYTHAGORAS' THEOREM.

Your notes



PB = 5 cm AS TRIANGLE PBS IS EQUILATERAL
PC = 2.5 cm AS A IS MIDPOINT OF QR,
SO C IS MIDPOINT OF PS.

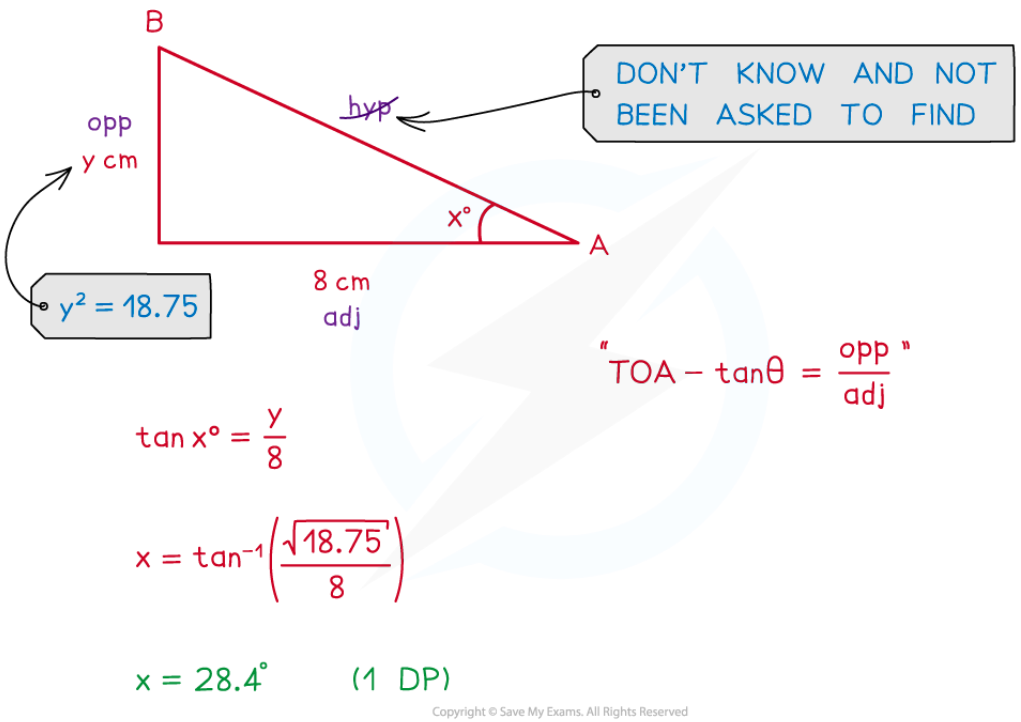


DRAW THE 2D TRIANGLES SEPARATELY TO HELP KEEP THINGS CLEAR

$y^2 = 5^2 - 2.5^2$

$y^2 = 18.75$

LEAVE AS y^2 FOR NOW TO AVOID ROUNDING



Your notes

Examiner Tips and Tricks

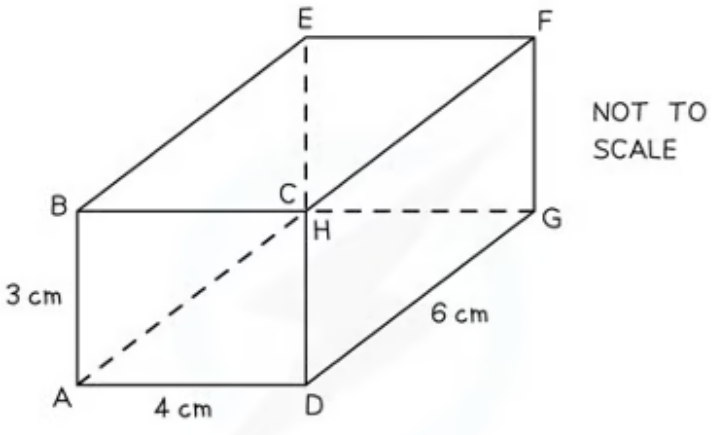
- If you are stuck in the exam with a complicated 3D diagram, it is always better to just start finding any lengths and angles in the shape, as:
  - these may end up being useful
  - you may score more marks than if you had left the question blank

Worked Example

A pencil is being put into a cuboid shaped box.

The box has dimensions 3 cm by 4 cm by 6 cm.

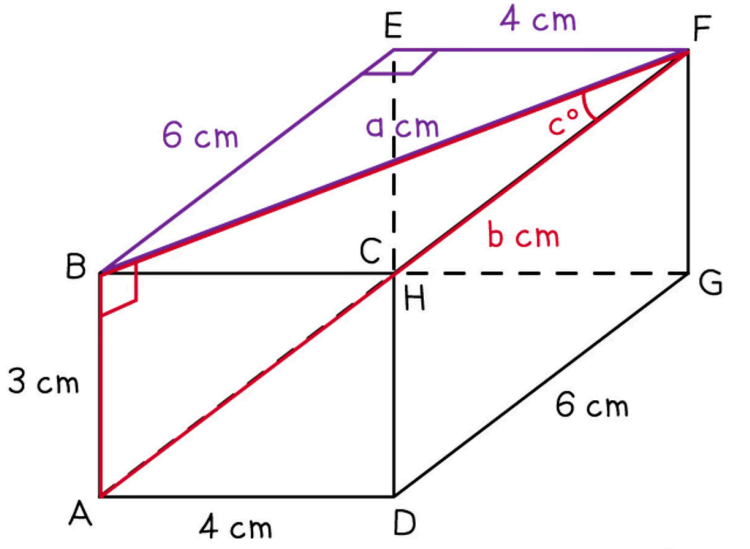
Your notes



(a) Find the length of the longest pencil that can fit inside the box.

The longest possible pencil will fit between diagonally opposite vertices, e.g. AF

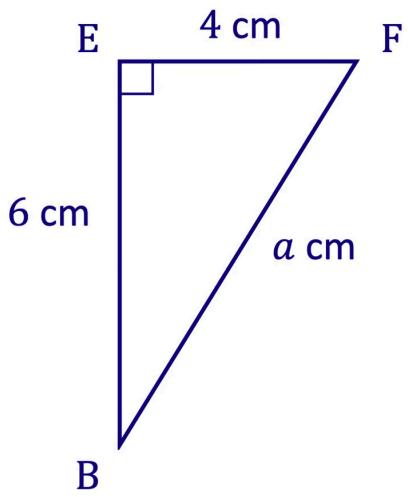
Form a 2D right-angled triangle, such as triangle ABF



Method 1

Your notes

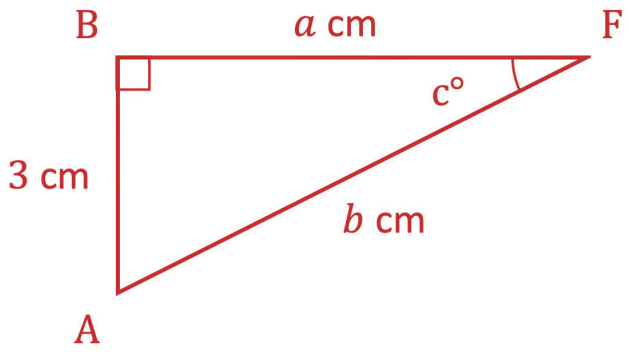
To find the length AF, there are a few different options
One option is to find length BF (from triangle BEF) then AF (from triangle ABF)
Draw triangle BEF flat and use Pythagoras' theorem to find BF



$$a^2 = 4^2 + 6^2$$
$$a^2 = 16 + 36$$
$$a^2 = 52$$

Draw triangle ABF flat and use Pythagoras' theorem to calculate AF

Your notes



$$b^2 = 3^2 + a^2$$
$$b^2 = 9 + 52$$
$$b^2 = 61$$
$$b = \sqrt{61} = 7.81024...$$

The longest pencil that can fit inside the box is 7.81 cm (to 3 s.f.)

Method 2

Apply the **3D version** of Pythagoras' theorem: $d^2 = x^2 + y^2 + z^2$

The distance in the x direction is 4 cm
The distance in the y direction is 6 cm
The distance in the z direction is 3 cm

$$d^2 = 4^2 + 6^2 + 3^2$$
$$d = \sqrt{4^2 + 6^2 + 3^2}$$
$$d = \sqrt{61} = 7.81024...$$

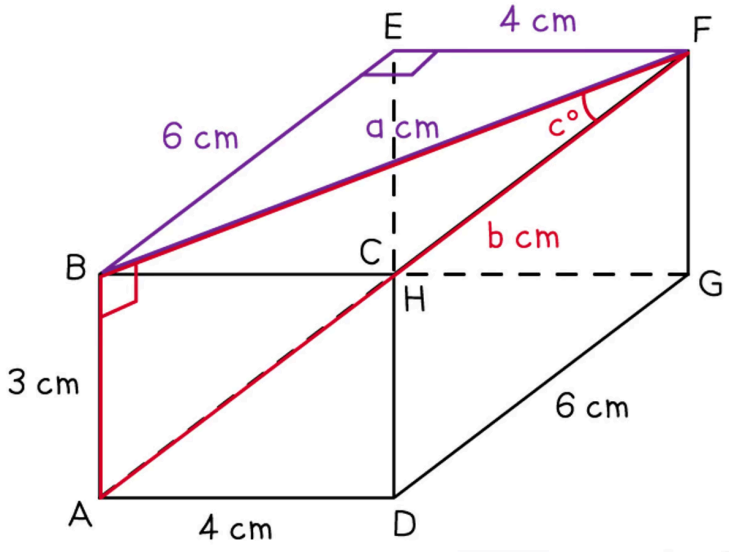
The longest pencil that can fit inside the box is 7.81 cm (to 3 s.f.)

(b) Find the angle that the pencil would make with the plane BEFC.

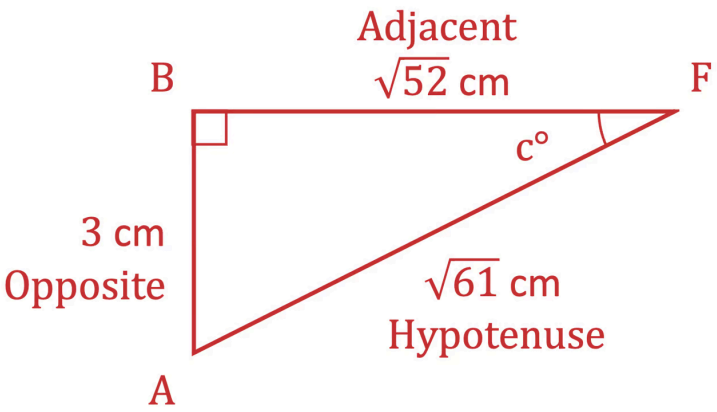
The plane BEFC is the horizontal top surface of the box

To see the angle between AF and the plane, form a triangle with side AB (which is a height perpendicular to the plane)

The angle needed is marked c



Triangle ABF is the 2D triangle needed, but it is currently drawn in 3D
Draw triangle ABF flat on the paper and write on the lengths from part (a)



Your notes

This is now ready for SOHCAHTOA

We know all three sides so could use any trig ratio, for example $\tan\theta = \frac{O}{A}$

Use this to find angle c

$$\tan c = \frac{3}{\sqrt{52}}$$

$$c = \tan^{-1}\left(\frac{3}{\sqrt{52}}\right)$$

$$c = 22.58853...$$

The angle between plane BEFC and the pencil is 22.6° (to 1 d.p.)

Your notes