



AQA GCSE Maths: Higher



Your notes

Bearings, Scale Drawing, Constructions & Loci

Contents

- * Bearings
- * Scale
- * Constructing Triangles
- * Constructions
- * Loci

Bearings

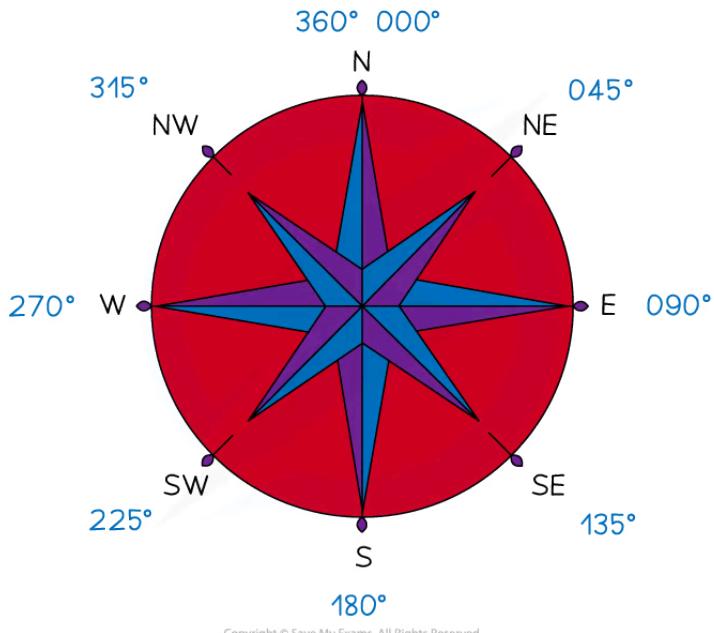


Your notes

Bearings

What are bearings?

- **Bearings** are a way of describing an **angle**
 - They are commonly used in **navigation**
- There are **three rules** which must be followed when using a bearing:
 - They are measured **from North**
 - North is usually straight up on a scale drawing or map, and should be labelled on the diagram
 - They are measured **clockwise**
 - The angle should always be written with **3 digits**
 - 059° instead of just 59°
- Knowing the **compass directions** and their respective bearings can be helpful





Your notes

How do I find a bearing between two points?

- Identify where you need to **start**
 - "The bearing **of A from B**" means **start at B** and find the bearing to A
 - "The bearing **of B from A**" means **start at A** and find the bearing to B
- Draw a **North line** at the **starting point**
- Draw a **line between the two points**
- **Measure** the angle between the **North line** and the **line joining the points**
 - Measure **clockwise** from North
 - Write the angle using **3 figures**

How do I draw a point on a bearing?

- You might be asked to plot a point that is a **given distance** from another point and on a **given bearing**
- **STEP 1**
Draw a **North line** at the point you wish to measure the bearing **from**
 - If you are given the bearing **from A to B** draw the North line at A
- **STEP 2**
Measure the **angle** of the bearing given **from the North line** in the **clockwise direction**
- **STEP 3**
Draw a line and add the **point B** at the **given distance**

How do I find the bearing of B from A if I know the bearing of A from B?

- If the **bearing of A from B** is **less than 180°**
 - **Add 180°** to it to find the **bearing of B from A**
- If the **bearing of A from B** is **more than 180°**
 - **Subtract 180°** from it to find the **bearing of B from A**

How do I answer trickier questions involving bearings?

- Bearings questions may involve the use of **Pythagoras** or **trigonometry** to find missing distances (lengths) and directions (angles)
 - You should always **draw a diagram** if there isn't one given



Examiner Tips and Tricks

- Make sure you have all the **equipment** you need for your maths exams
 - A rubber and pencil sharpener can be essential as these questions are all about accuracy
 - Make sure you can see and read the markings on your ruler and protractor
- **Always** draw a big, clear diagram and annotate it, be especially careful to label the angles in the correct places!



Your notes



Worked Example

A ship sets sail from the point P, as shown on the map below.

It sails on a bearing of 105° until it reaches the point Q, 70 km away. The ship then changes path and sails on a bearing of 065° for a further 35 km, where its journey finishes.

Show on the map below the point Q and the final position of the ship.



Your notes

SCALE: 1cm = 10 km



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Draw in a north line at the point P

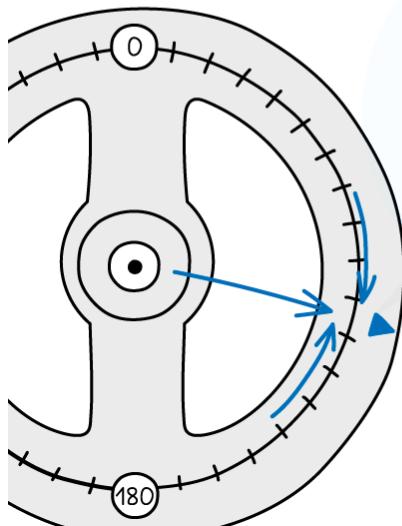
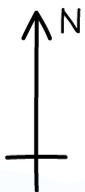
Measure an angle of 105° clockwise from the north line

Making sure you are accurate, carefully make a small but visible mark on the map



Your notes

SCALE: 1cm = 10 km



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Draw a line from P through the mark you have made. Make this line long so that you can easily measure along it accurately



Your notes

SCALE: 1cm = 10 km



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Use the scale given on the map (1 cm = 10 km) to work out the number of cm that would represent 70 km

$$70 \text{ km} = 70 \div 10 = 7 \text{ cm}$$

Accurately measure 7 cm from the point P along the line and make a clear mark on the line
Label this point Q



Your notes

SCALE: 1cm = 10 km



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A bearing of 065 means 65° clockwise from the North

First, draw a North line at the point Q, then carefully measure an angle of 65° clockwise from this line.
Make a mark and then draw a line from Q through this mark

Using the scale, find the distance in cm along the line you will need to measure.

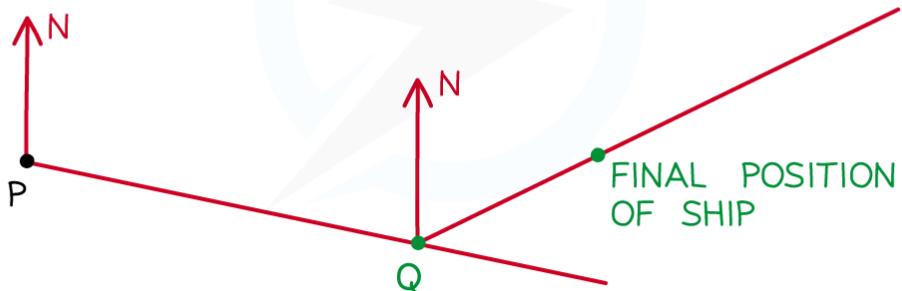
$$35 \text{ km} = 35 \div 10 = 3.5 \text{ cm}$$

Accurately measure 3.5 cm from the point Q along this new line and make a clear mark on the line
This is the final position of the ship.



Your notes

SCALE: 1cm = 10 km



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Scale



Your notes

Scale

What is a scale?

- For **accurate drawings** and **constructions** scale refers to a **ratio**
 - This ratio describes the relationship between the **drawn size** and the **real-life size**
- **Maps** are usually drawn to a scale
- The ratio will work for **any unit of length** applied to both sides
 - For example, the scale 1: 50 000 could mean 1 cm = 50 000 cm, 1 km = 50 000 km or even 1 yard = 50 000 yards
 - If you're measuring the length from a map it will be easiest to measure in cm



Examiner Tips and Tricks

- When working with lots of different units and converting between them, make sure to use "common sense" checks
 - e.g. When converting 500 km into metres, am I expecting a bigger or smaller number?

Maps

How can I use a scale to find the actual lengths from a map?

- A map can be used to calculate the real-life distances between points
- **STEP 1**
Use a ruler to **measure** the **distance accurately** on the **map**
 - For example, measuring a length from A to B as 5.8 cm
- **STEP 2**
Use the **scale** to find the **actual distance** in the **same units**
 - For example, if the scale is 1: 150 000 the actual distance = $5.8 \text{ cm} \times 150\,000 = 870\,000 \text{ cm}$
- **STEP 3**
Convert the actual distance to a **more suitable unit**

- For example, $870\ 000\ \text{cm} = 8\ 700\ \text{m} = 8.7\ \text{km}$



Worked Example

A map is drawn where a length of 5 cm is equal to an actual distance of 0.6 km.

(a) Write the scale that is used for the map.

Convert both parts of the scale to the same units

The answer needs to be in the form $1:n$ so convert 0.6 km into cm using $1\ \text{m} = 100\ \text{cm}$ and $1\ \text{km} = 1000\ \text{m}$

$$\begin{aligned}0.6\ \text{km} &= 0.6 \times 1000\ \text{m} = 600\ \text{m} \\600\ \text{m} &= 600 \times 100\ \text{cm} = 60\ 000\ \text{cm}\end{aligned}$$

Now the ratio has the same units 5 cm : 60 000 cm, you can remove the units

$$5 : 60\ 000$$

Write in the form $1:n$ by dividing both sides by 5

$$\mathbf{1:12\ 000}$$

(b) The width of a park on the map is 17 mm.

Find the actual width of the park, giving your answer in metres.

Convert 17 mm into cm

$$17\ \text{mm} = 1.7\ \text{cm}$$

Use the scale to find 1.7 cm on the map in real life

$$1.7\ \text{cm} \times 12\ 000 = 20\ 400\ \text{cm}$$

Convert to metres

$$20\ 400\ \text{cm} \div 100$$

$$\mathbf{204\ m}$$

(c) The distance from the mouth of the ocean to the first bridge over a river is 125 metres.

Find this distance on the map.

Convert 125 metres to cm

$$125\ \text{m} = (125 \times 100\ \text{cm}) = 12\ 500\ \text{cm}$$

Use the scale to find 12500 cm in real life on the map



Your notes

$12\ 500\ \text{cm} \div 12\ 000 = 1.0416\dots\ \text{cm}$

1.04 cm



Your notes

Scale Drawings

How can I use a scale to find lengths for an accurate drawing?

- A scale can be used to produce an accurate drawing or model of an object

- **STEP 1**

Convert the scale into a **ratio** where one side is **1 cm** and the other side uses the **units the real distance** is measured in

- For example, if the real distance is in km and the scale is 1:500 000,
 - $1:500\ 000 = 1\text{cm} : 500\ 000\ \text{cm} = 1\text{cm} : 5\ 000\ \text{m} = 1\text{cm} : 5\ \text{km}$
 - So 1cm on the map, represents 5 km in real life

- **STEP 2**

Use this ratio to **convert** the **actual distance** to the **scale distance**

- For example, if the actual distance = 20 km, the scale distance will be $20 \div 5 = 4\ \text{cm}$



Your notes

Constructing Triangles

Constructing Triangles

What are triangle constructions?

- A **triangle construction** is an accurate drawing that usually uses:
 - a sharp **pencil**
 - a **ruler**
 - a **protractor**
 - and/or a pair of **compasses**
- You will be given information about the size of some of the **angles** and/or **sides** of a triangle
- Depending on the type of triangle, you will need to follow a specific method and you may need different equipment
- The types of triangle you may be asked to construct include:
 - **SSS** - you are given the lengths of **all three sides**
 - **SAS** - you are given the lengths of **two sides** and the **angle in between them**
 - **ASA** - you are given the size of **two angles** and the length of the **side in between them**

How do I construct an SSS triangle?

▪ STEP 1

Use a **ruler** to draw the **longest** side as the **horizontal** base near the bottom of the space you have been given

- This needs to be **accurate**
- Write its length (with units) just underneath

▪ STEP 2

Open your **compasses** so that the **length** from the **compass point to the tip of your pencil** is exactly the **length** of one of the **remaining sides**

- Being extra careful not to change the length, put the compass point on one end of the horizontal line you have drawn and **draw an arc** above the horizontal line
- You can draw a full circle rather than an arc if you prefer



Your notes

■ STEP 3

Open your **compasses** so that the length from the **compass point to the tip of your pencil** is **exactly the length** of the **third side**

- Being extra careful not to change the length, put the compass point on the other end of the horizontal line and **draw another arc**, making sure that it **crosses the first arc**

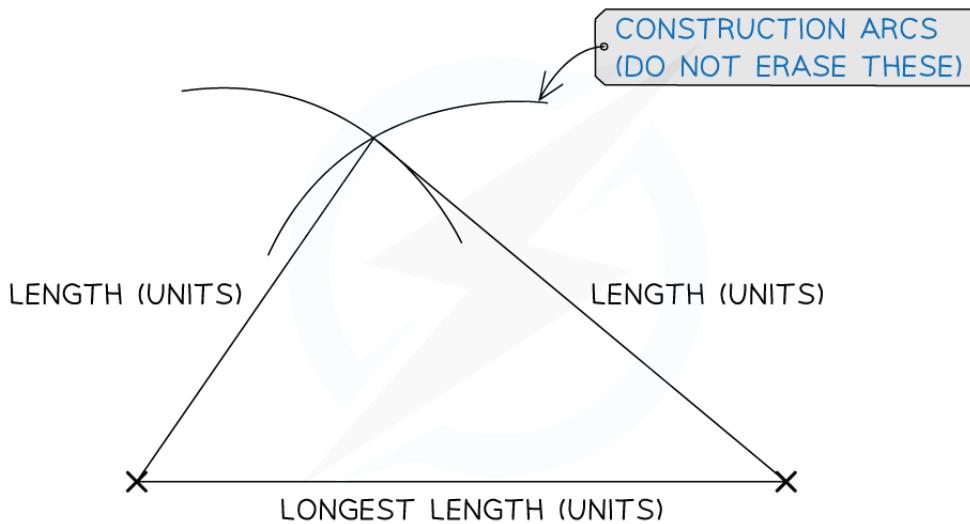
■ STEP 4

Use your **ruler** to draw **straight lines** from each end of the horizontal line to the point **where the arcs cross**

■ STEP 5

Use your **ruler** to **check** that the two new lines are **exactly equal** to the lengths given in the question

- When you are confident that they are accurate, label the lines with their lengths and units
- **Do not rub out your arcs** as the examiner will use these to check your work
- Sometimes the instructions will include a **triangle name** such as triangle ABC
 - Make sure you label each vertex with the correct letters



Worked Example

Using a ruler and pair of compasses only, construct a triangle with sides 6 cm, 7 cm and 10 cm. Leave in your construction arcs.



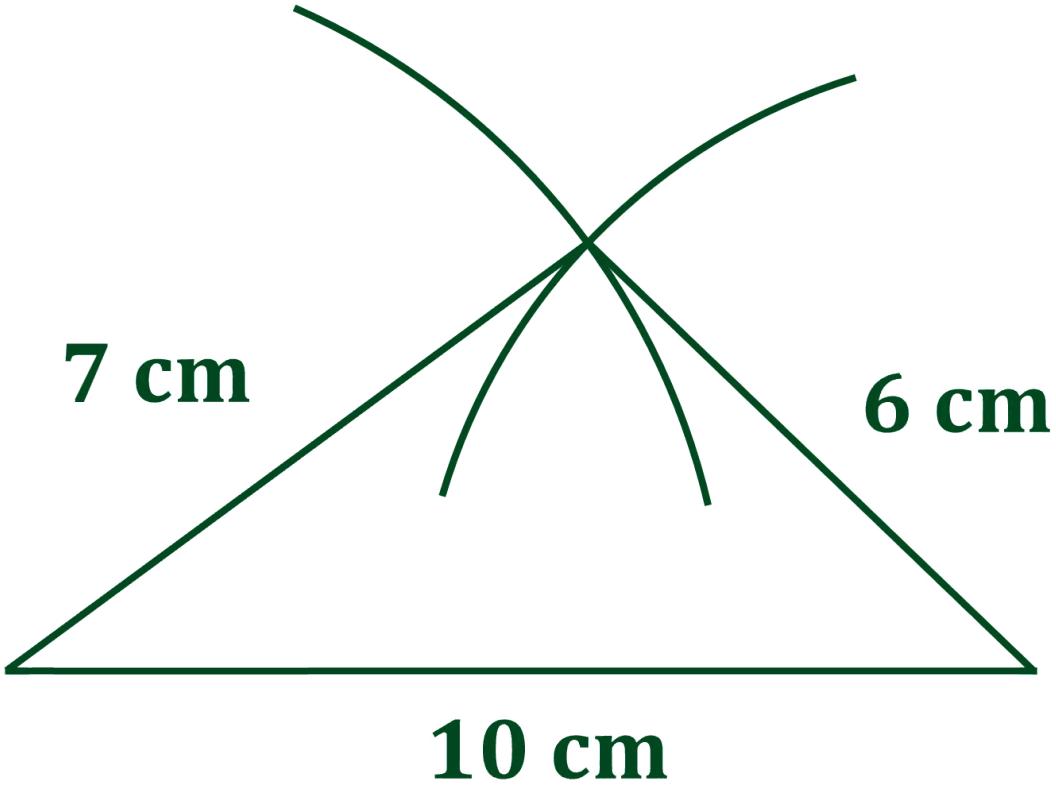
Your notes

Draw the 10 cm line as the horizontal base

Place the point of the compasses at each end and draw an arc with radius 6 cm from one end and another with radius 7 cm from the other end

The third vertex of the triangle is the point at which they intersect

Use your ruler to measure each side and check for accuracy



How do I construct an SAS triangle?

- **STEP 1**

Use a **ruler** to draw the longest given side as the horizontal base

- This needs to be accurate, measure it carefully with your **ruler**
- Write its length (with units) just underneath

- **STEP 2**

Measure the **angle** from the end of the line

- Place the centre point of the **protractor** on one end of the side that you have just drawn, measure the given angle, and make a mark to indicate where it is



Your notes

- Use a ruler to draw a straight line from where you placed the protractor, through the mark, and extend it further

STEP 3

Draw the second **straight line**

- Measure along the line you have just drawn in STEP 2 from the end where it connects to the first horizontal line
- Make a mark on the line when you have measured the length of the second given side

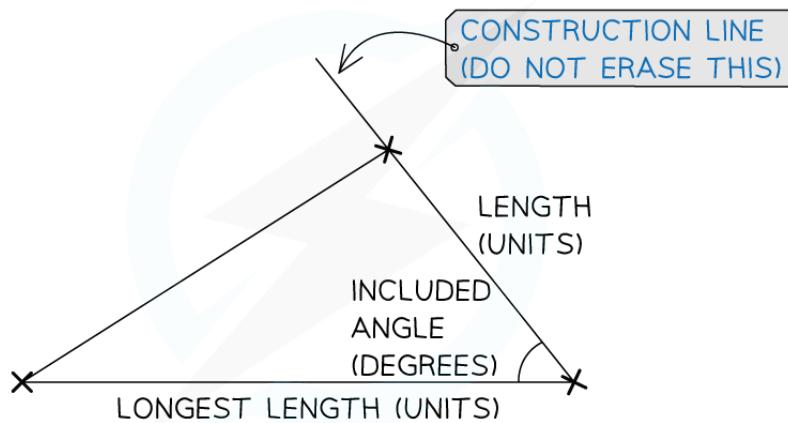
STEP 4

Use your **ruler** to draw a straight line from other end of the first horizontal line to the mark you have just made on the second line

STEP 5

Use your **protractor** and **ruler** to **check** that the measured angle and sides are exactly equal to the sizes given in the question

- When you are confident that they are accurate, label the sides and the angle
- It is important that you **do not rub out your construction lines** as the examiner will use these to check your work

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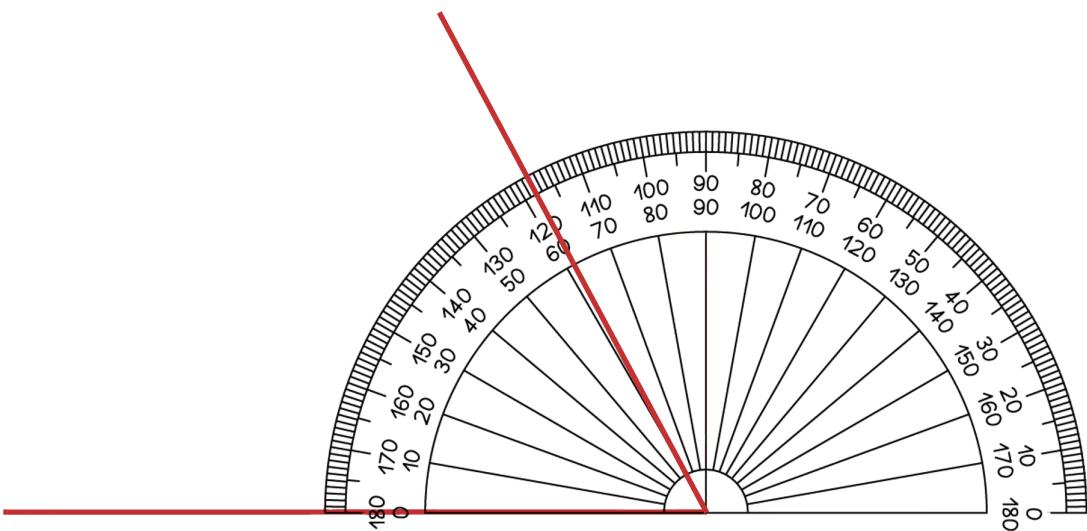
Worked Example

Using a **ruler** and a **protractor** only, construct a triangle with sides 9 cm and 6 cm and an included angle of 62° .

Draw the 9 cm line as the horizontal base



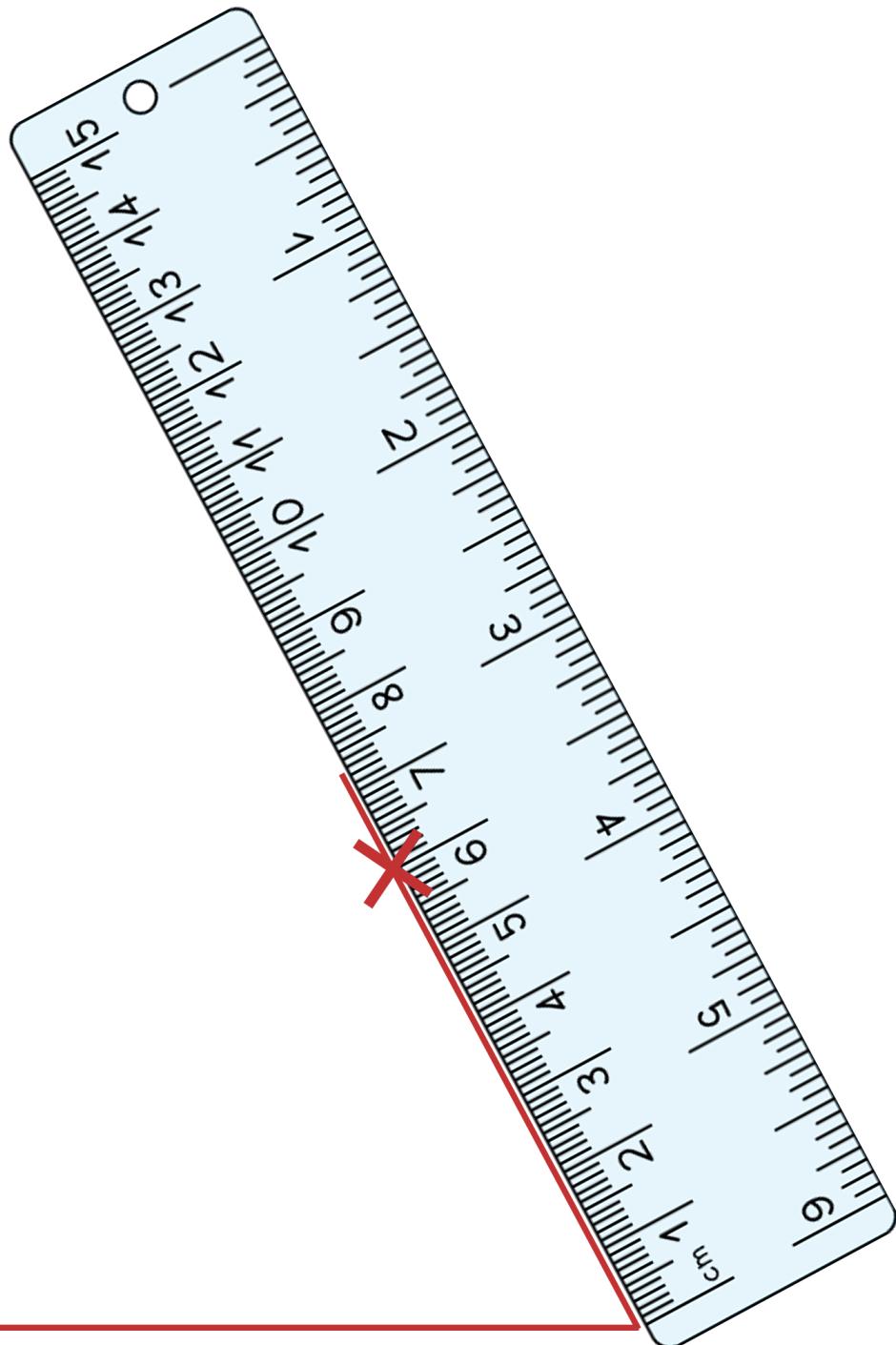
Place the centre of the protractor at one end of the horizontal line and measure 62°



Measure 6 cm along the new line and indicate it with a mark

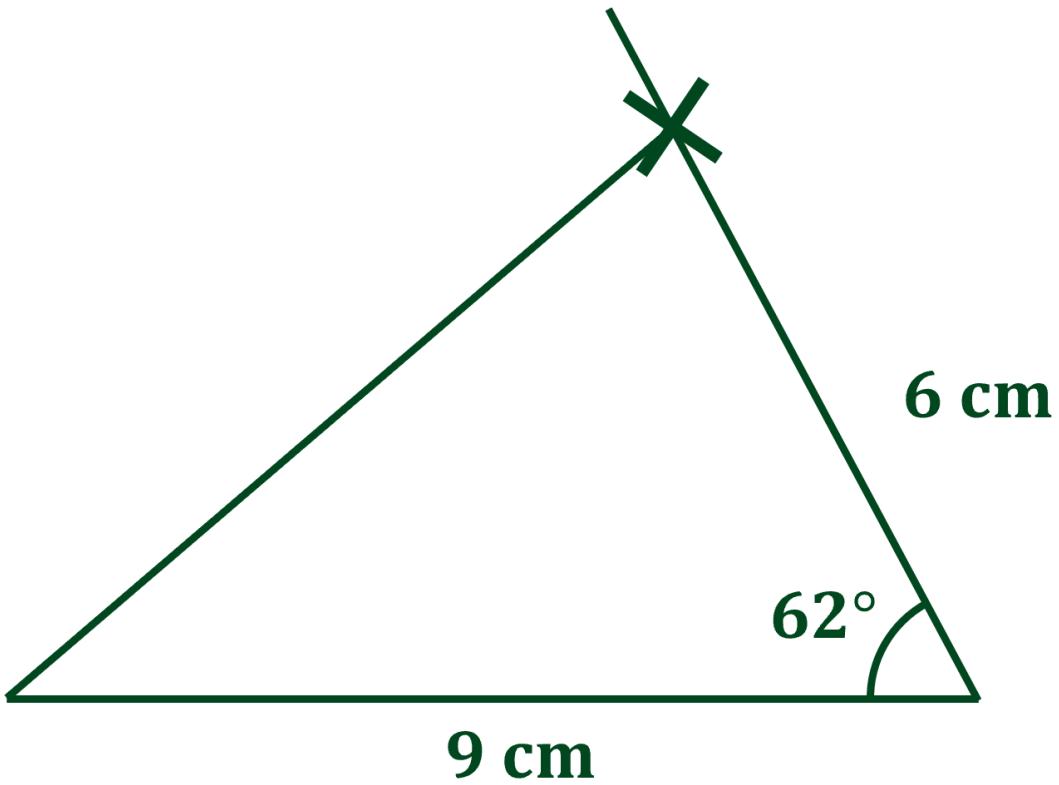


Your notes



Use your ruler to draw a straight line connecting the other end of the horizontal line to the mark

Label the lengths of the sides and the angle that you are given in the question



How do I construct an ASA triangle?

▪ STEP 1

Use a **ruler** to draw the given side as the horizontal base

- This needs to be accurate, measure it carefully with your ruler
- Write its length (with units) just underneath

▪ STEP 2

Measure **one angle** from one of the ends of the line

- Place the centre point of the **protractor** on one end of the side that you have just drawn, measure the given angle from the side and make a mark to indicate where it is
- Draw a straight line from where you placed the protractor, through the mark and extend it further

▪ STEP 3

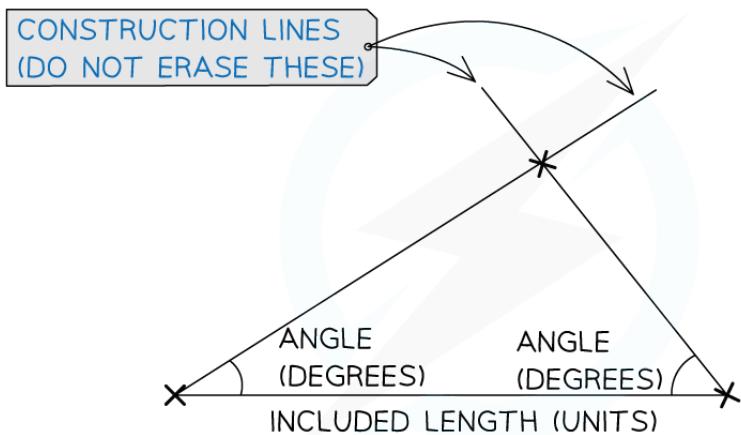
Measure the **second angle** from the other end of the line

- Repeat STEP 2 for the second angle, but from the other end of the horizontal line



Your notes

- This line should cross the line you drew in STEP 2, if it doesn't, extend both lines further so that they do intersect
- **STEP 4**
Use your **protractor** to **check** that the two measured angles are exactly equal to the sizes given in the question
 - When you are confident that they are accurate, label the angles
- It is important that you **do not rub out your construction lines** as the examiner will use these to check your work



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Examiner Tips and Tricks

- To ensure you get full marks in constructions questions:
 - Make sure you are confident using your compasses
 - Make sure that your compasses are not loose
 - Do not erase the construction arcs from your diagram



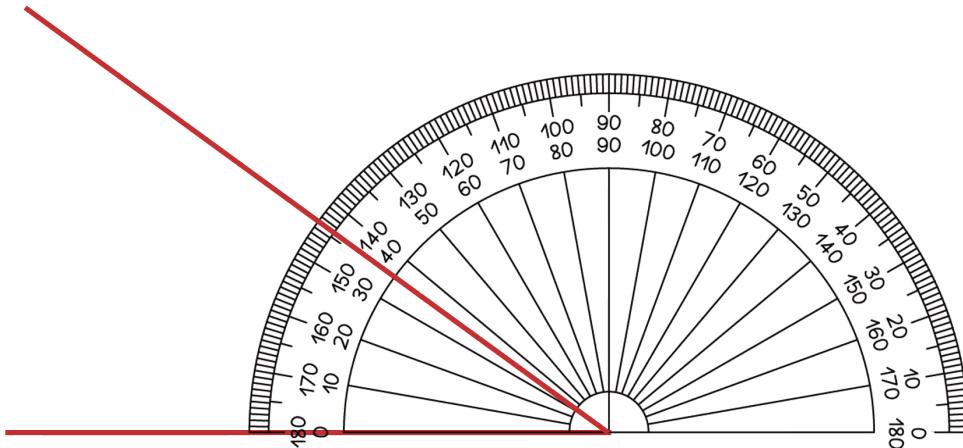
Worked Example

Using a **ruler** and a **protractor** only, construct a triangle with angles 36° and 59° and an included side of length 8 cm.

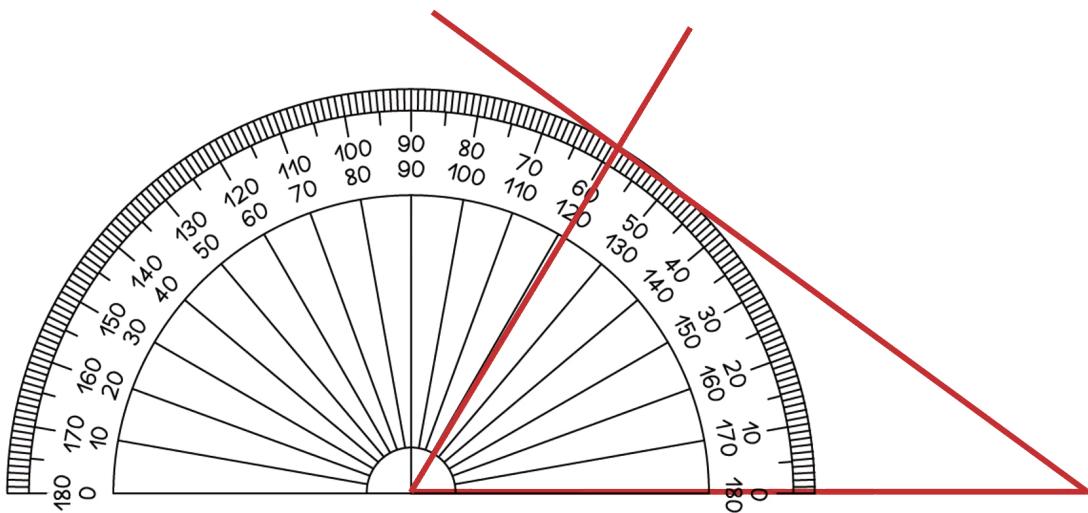
Draw the 8 cm line as the horizontal base



Place the centre of the protractor at one end of the horizontal line and measure 36°



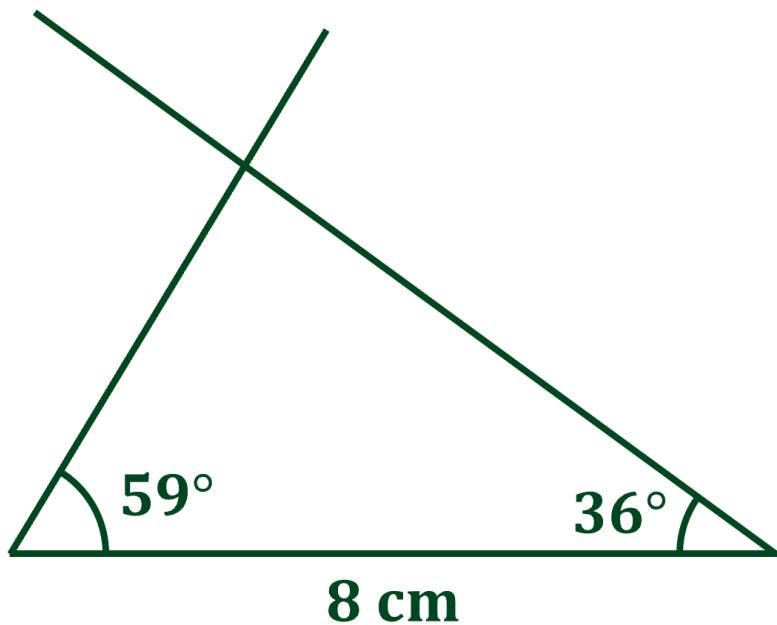
Put the centre of the protractor over the other end of the horizontal line and measure 59°



Using your ruler, join each end of the horizontal line to the point where the other two lines intersect
Label the size of the angles and the length of the side that you were given in the question



Your notes



Constructions



Your notes

Constructions

What are constructions?

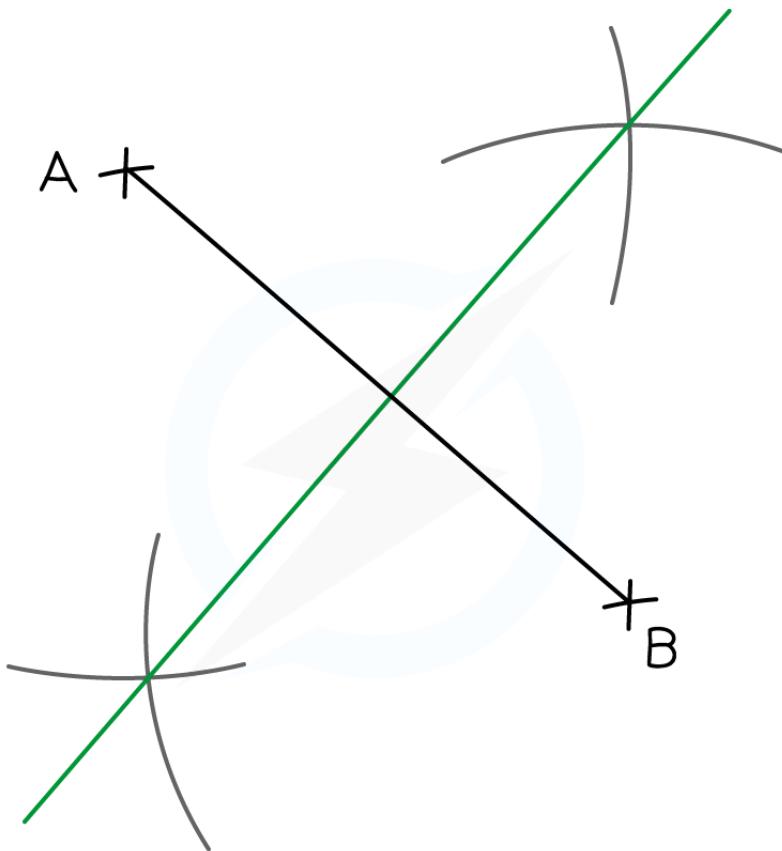
- A **construction** is a process where you create an accurate geometric object using a **pair of compasses** and a **straight edge** (and a pencil)
- There are several types of construction you must be able to carry out:
 - A **perpendicular bisector** of a line
 - This is a line that cuts another one exactly in half (**bisects**) but also crosses it at a right angle (**perpendicular**)
 - It shows a path that is **equidistant (equal distance)** between the two endpoints of the line
 - A **perpendicular from a point to a line**
 - This is the **shortest path** between the point and the line
 - It will meet the line at a right angle
 - An **angle bisector**
 - This is a line that **cuts an angle exactly in half (bisects)**
 - It shows a path that is **equidistant (equal distance)** between the two lines that form the angle

How do I construct a perpendicular bisector of a line?

- **STEP 1**
Set the **distance** between the point of the **compasses** and the pencil to be **more than half the length** of the line
- **STEP 2**
Place the point of the compasses **on one end** of the line and sketch an **arc** above and below the line
- **STEP 3**
Keeping your compasses **set to the same distance**, place the point of the compasses on **the other end of the line** and sketch an **arc** above and below the line
 - The **arcs should intersect** each other both above and below the line
- **STEP 4**
Connect the points where the arcs intersect with a **straight line**



Your notes



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How do I construct a perpendicular from a point to a line?

- **STEP 1**

Set the distance between the point of your compasses and the pencil to be **greater than the distance between the point P and the line**

- **STEP 2**

Placing the point of the compasses on the point P, draw an **arc** that **intersects the line in two places**

- **STEP 3**

Set the **distance** between the point of the **compasses** and the pencil to be **more than half the distance** between the two points of intersection on the line

- **STEP 4**

Place the point of the compasses **on one point of intersection** and sketch an **arc** on the opposite side

of the line to P

- **STEP 5**

Keeping your compasses set to the same distance, place the point of the **compasses** on the other **point of intersection** and sketch an **another arc**

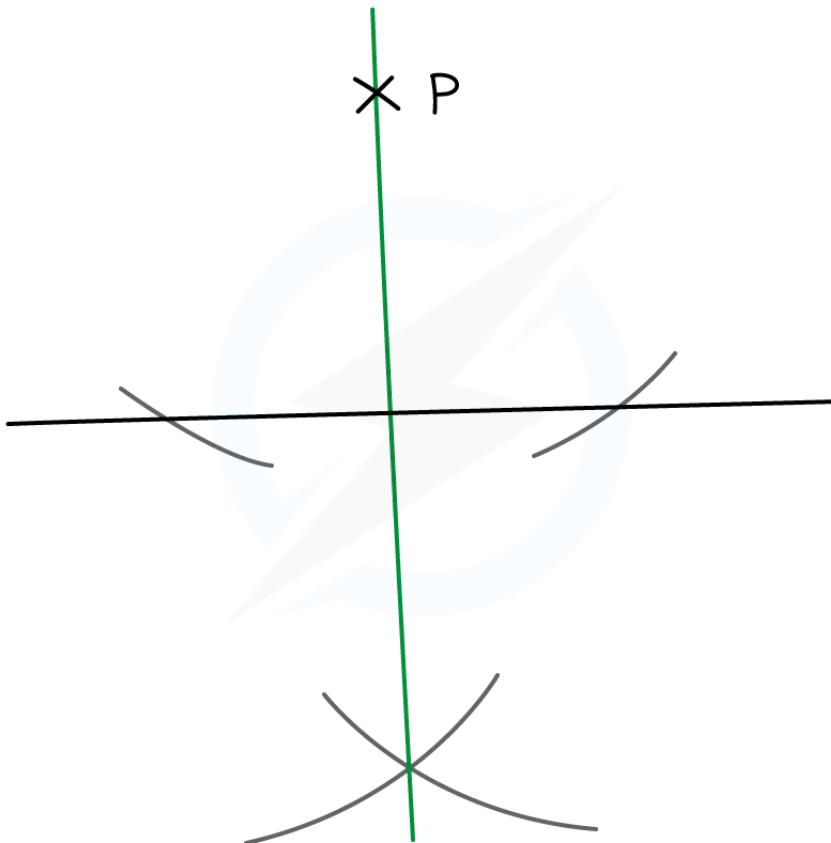


Your notes

- The **arcs should intersect**

- **STEP 6**

Connect the point where the arcs intersect to point P with a **straight line**



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How do I construct an angle bisector?

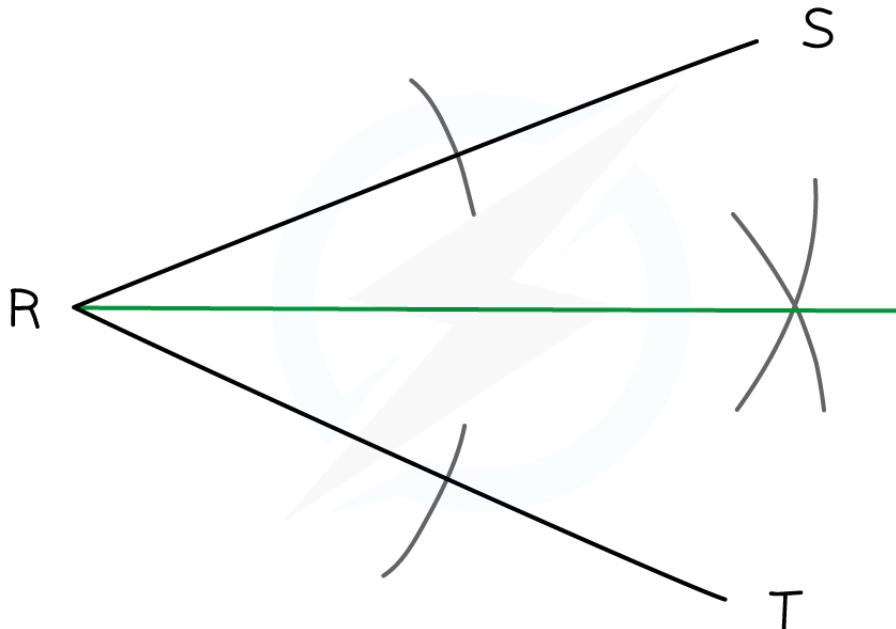
- **STEP 1**

Set the **distance** between the point of your **compasses** and the pencil to be **about half** the distance of the smallest line that makes the angle



Your notes

- The precise distance is not important
- **STEP 2**
Place the **point** of the compasses where the **lines meet** and sketch an **arc** that **intersects both of the lines** that form the angle
- **STEP 3**
Keeping your compasses set to the same distance, place the point of the compasses on one of the **points of intersection** and sketch an **arc**
- **STEP 4**
Keeping your compasses set to the same distance, place the point of the compasses on the **other point of intersection** and sketch an **arc**
 - This **should intersect** the last arc you drew
- **STEP 5**
Join the **point of the angle** to the **point of intersection** with a **straight line**



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Examiner Tips and Tricks

- Make sure you have all the equipment you need for the exam; pen, pencil, ruler, compasses, protractor, calculator
- An eraser and a pencil sharpener can be helpful on these questions as they are all about **accuracy**
 - But do not erase your construction lines
- Make sure your compasses aren't loose and wobbly
 - They can usually be tightened with a screwdriver
- Make sure you can see and read the markings on your ruler and protractor



Your notes

Loci



Your notes

Loci

What are loci?

- A **locus** (**loci** is plural) is a line, shape, or path that is determined by following a restriction
 - e.g. always being 2 m away from a point would form a circular locus
- You may be asked to **construct** a locus, although the exam question won't always use these words as questions are often based on real-world situations
- You may be expected to use some of the **constructions** mentioned above
- You may need to use a **ruler**, and a **protractor** or a **pair of compasses**

What are the common types of loci?

- A **fixed distance** from a point
 - This locus will be a **circle** around the point
 - Use a pair of compasses
- A **fixed distance** from a line
 - This locus will be a running track shape
 - A pair of **parallel lines**
 - A **semi-circle** at each end
- **Equidistant** from **two points**
 - This locus will be the **perpendicular bisector** of the line segment connecting the two points
- **Equidistant** from **two lines**
 - This locus will be the **angle bisector** of the angle between the two lines

How do I know which region to shade?

- To find the **region** that is (or is not) **within a given distance** from a point
 - Draw a **circle** around that point
 - The radius will be the given distance



Your notes

- The region **inside the circle** is **closer** to the point than the given distance
- The region **outside the circle** is **further away** from the point than the given distance
- To find the **region** that is **closer to point A than point B**
 - Draw a **straight line** that joins A to B
 - Draw the **perpendicular bisector** of the line
 - The region that is **closer to A** is the side of the perpendicular bisector that **contains A**
- To find the region that is closer to line AB than line AC
 - Draw the **angle bisector** at point A
 - This is the point where the two lines meet
 - The region that is **closer to the AB** is the side of the angle bisector that **contains line AB**
- You might have to share the **region that satisfies multiple conditions**
 - Deal with **one condition at a time**
 - Put a **tick** in the region(s) that **satisfy that condition**
 - Put a **cross** in the region(s) that **do not**
 - Once all conditions have been dealt with, **shade the region** that **only contains ticks** and no crosses

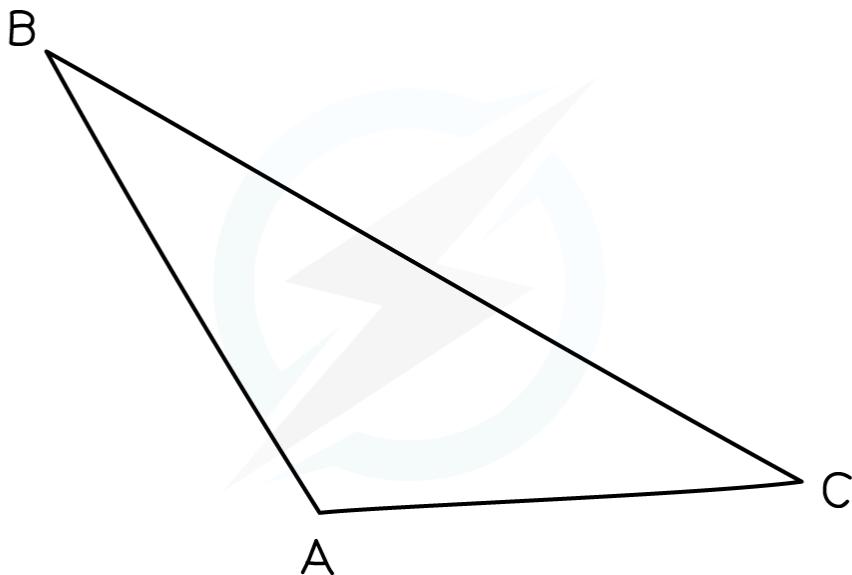


Worked Example

On triangle ABC below, indicate the region that is closer to the side AC than the side BC.



Your notes



This question is asking for the region that is closer to one side of an angle than the other, so an **angle bisector** is needed

Open your set of compasses to a distance that is approximately half the length of the sides AB and AC.

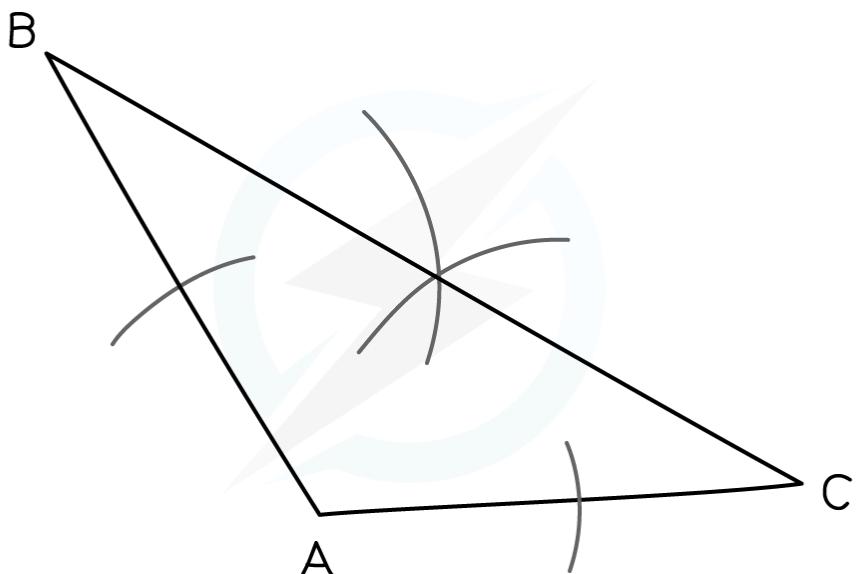
This distance is not too important, but keeping it the same length throughout the question is **very important**

Place the point of the compasses at A and draw arcs across the lines AB and AC. Be very careful not to change the length of the compasses as you draw the arcs

Leaving the compasses open at the same length, put the point at each of the places where the arcs cross the sides AB and AC and draw new arcs which cross over each other in the middle



Your notes

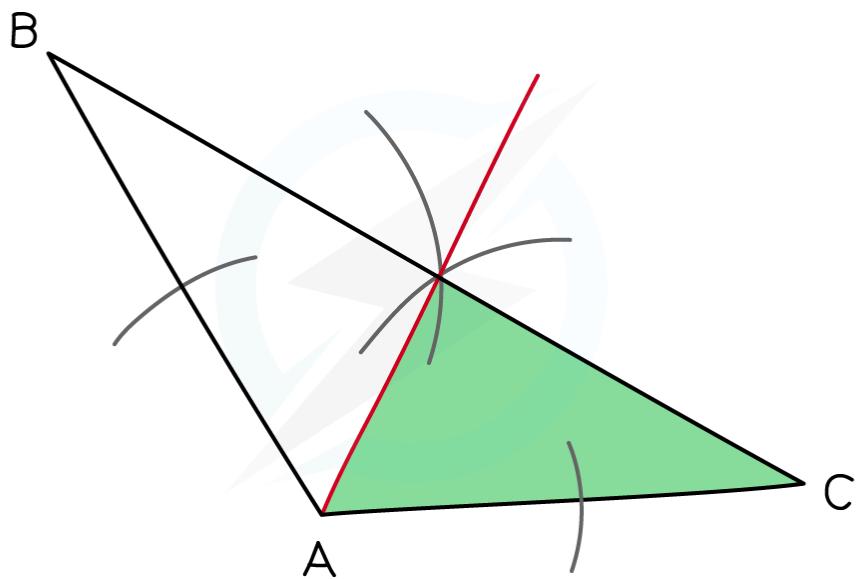


Draw a line from A to the point where the arcs cross over each other (this won't necessarily be directly on the third side of the triangle!)

Shade the region between the angle bisector (the line you have drawn) and the side AC



Your notes



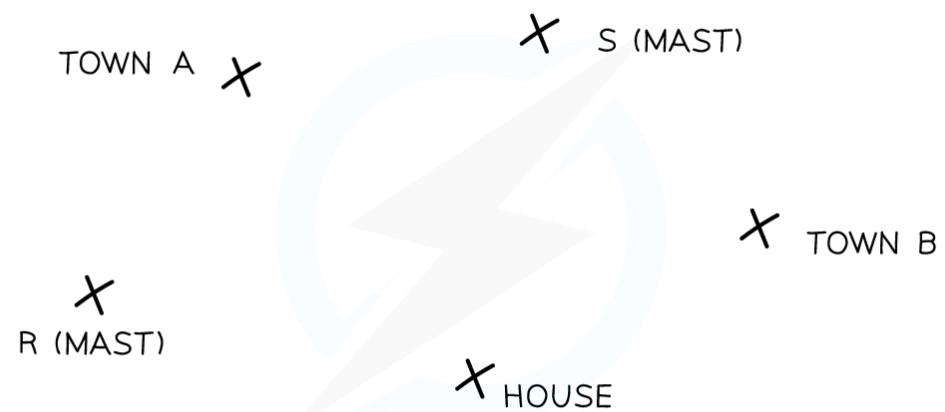
Worked Example

A house lies between Town A and Town B as shown on the scale diagram below.



Your notes

SCALE: 1cm = 1 mile



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Two masts, located at the points R and S, provide the area shown on the map with radio signals.

The house will receive its radio signal for the mast located at point R if it is either...

... closer to Town A than Town B, or...

... outside a region 5 miles from the mast at point S.

Showing your working carefully on the scale diagram below, determine whether the house receives radio signals from the mast at point R or the mast at point S.

Begin by finding the region satisfying the first condition; that the house is closer to Town A than Town B

This is found by constructing the perpendicular bisector of the line segment that joins Town A and Town B

You will need to add this line segment in yourself before starting

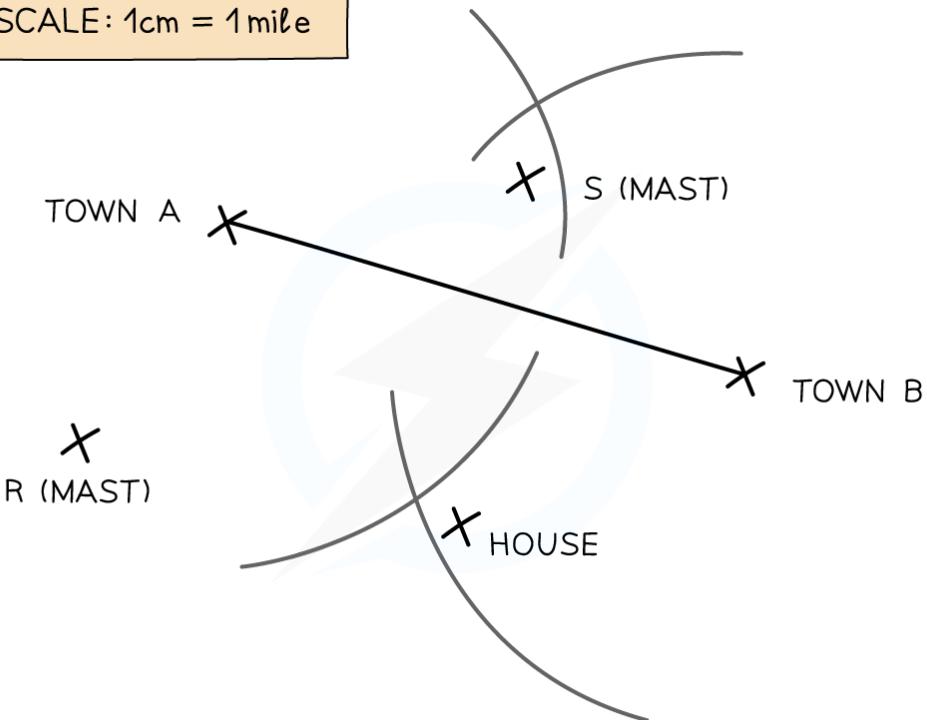
Open your compasses to more than half of the distance from Town A to Town B and draw arcs both above and below the line joining Town A to Town B

Do this from both the point at Town A and the point at Town B. The arcs should cross over each other



Your notes

SCALE: 1cm = 1 mile



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The perpendicular bisector is the line that passes through both of the points where the arcs cross over each other

The house is on the same side of the perpendicular bisector as Town B is

The house is closer to Town B than Town A.

Now check the second condition; to see if the house is further than 5 miles from the mast at point S. To find the locus of points exactly 5 miles from point S, first consider the scale given on the scale drawing.

$$1\text{ cm} = 1\text{ mile}$$

$$5\text{ cm} = 5\text{ miles}$$

Open your set of compasses to exactly 5 cm. Measure this carefully using a ruler.

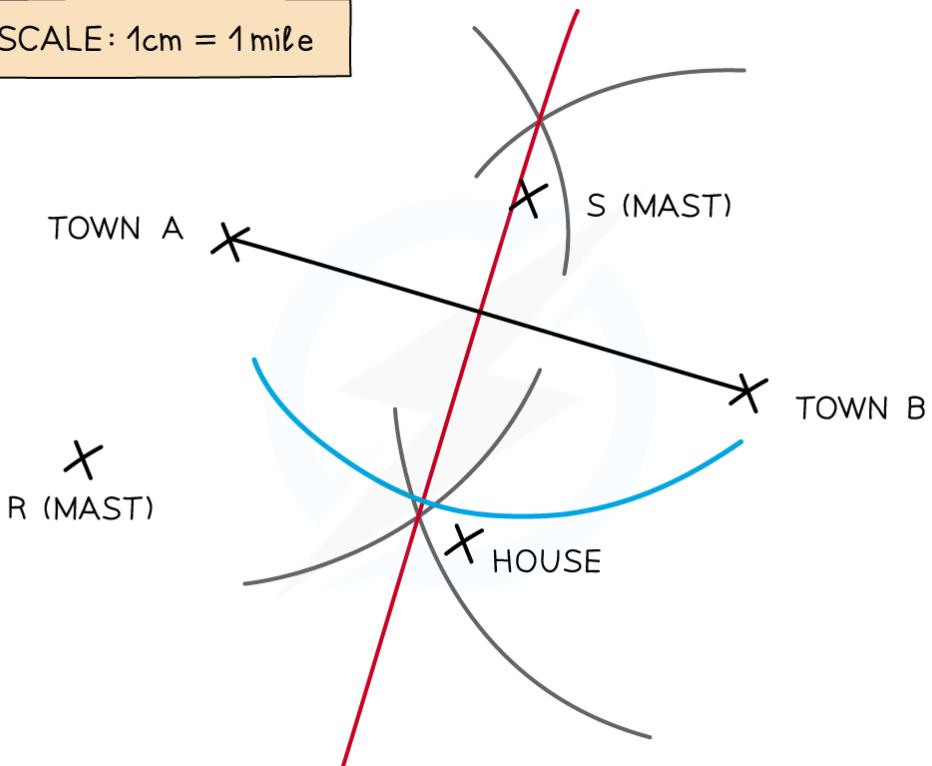


Your notes

Being extra careful not to change the length of your compasses, put the point at S and draw a circle around S with a radius of 5 cm

You may not be able to draw the full circle, but make sure you have the part that is near the point where the house is located

SCALE: 1cm = 1 mile



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The house is located outside of this region, so it is more than 5 miles from the mast at the point S

The house satisfies one of the conditions given to receive its signal from the mast at point R

The house receives its radio signal from the mast at point R.