AQA GCSE Maths: Higher

Your notes

Rearranging Formulas

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Simple Rearranging

What are formulas?

- A **formula** is a rule, definition or relationship between different quantities, written in shorthand using **letters** (**variables**)
  - They include an **equals** sign
- Some examples you should be **familiar** with are:
  - The equation of a straight line
    - $y = mx + c$
  - The area of a trapezium
    - $\text{Area} = \dfrac{(a + b)h}{2}$
  - Pythagoras' theorem
    - $a^2 + b^2 = c^2$

How do I rearrange formulas?

- The letter (**variable**) that is on its **own** on one side is called the **subject**
  - y is the subject of $y = mx + c$
- To make a different letter the subject, we need to **rearrange the formula**
  - This is also called **changing the subject**
- The method is as follows:
  - First, **remove** any **fractions**
    - Multiply both sides by the lowest common denominator
  - Then use **inverse** (opposite) **operations** to get the variable on its own
    - This is similar to solving equations

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- For example, make x the subject of $\frac{5x+6}{2}=y$
  - First remove fractions
    - Multiply both sides by 2
      $5x+6=2y$
  - Then get x on its own
    - Subtract 6 from both sides
      $5x=2y-6$
    - Divide both sides by 5
      $x=\frac{2y-6}{5}$
  - There may be more than one correct way to write an answer
    - The following are acceptable **alternative** forms
      $x=\frac{2y}{5}-\frac{6}{5}$
      $x=\frac{2(y-3)}{5}$
      $x=0.4(y-3)$
      $x=0.4y-1.2$

Should I expand brackets?

- **Expand brackets** if it **releases** the variable you want from **inside** the brackets
  - If not, you can leave them in
- To make x the subject of $3(1+x)=y$
  - x is **inside** the brackets, so **expand**
    - $3+3x=y$
  - Rearrange
    - $3x=y-3$
      $x=\frac{y-3}{3}$

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- To make x the subject of $(1+k)x = y$
  - x is **not inside** the brackets, so you do **not** need to expand
  - Instead, **divide** both sides by the **bracket** $(1+k)$
    - $x = \dfrac{y}{1+k}$

What if I get fractions in fractions?

- Some rearrangements can lead to **fractions in fractions**
  - $x = \dfrac{\frac{3}{t}}{2}$
- Either rewrite with a **divide sign**, $\div$, then use the method of **dividing two fractions**
  - $x = \dfrac{3}{t} \div 2$
    $x = \dfrac{3}{t} \div \dfrac{2}{1}$
    $x = \dfrac{3}{t} \times \dfrac{1}{2}$
    $x = \dfrac{3}{2t}$
- Or **multiply top and bottom** by the the **lowest common denominator** of the two fractions and **cancel**
  - $x = \dfrac{\frac{5}{y}}{\frac{t}{8}}$ becomes $x = \dfrac{\frac{5}{y} \times 8y}{\frac{t}{8} \times 8y} = \dfrac{40}{ty}$

What if I end up dividing by a negative?

- Remember that $\dfrac{a}{-b}$ (minus **below**) is the same as $\dfrac{-a}{b}$ (minus **above**) and the same as $-\dfrac{a}{b}$ (minus **outside**)

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- Though be careful, as $\frac{-a}{-b}$ is $\frac{a}{b}$
- $-2x = y - 3$ becomes $x = \frac{y-3}{-2}$ (minus **below**)
  - This is the same as $x = \frac{-(y-3)}{2}$ (minus **above**) or $-\frac{y-3}{2}$ (minus **outside**)
    - **brackets** are **required** for minus **above**
    - **brackets** are **assumed** for minus **outside**
  - You can also **expand** the brackets
    $\frac{-(y-3)}{2} = \frac{-y+3}{2} = \frac{3-y}{2}$

Examiner Tips and Tricks

- Mark schemes will accept different forms of the same answer, as long as they are correct and fully simplified

Worked Example

Make x the subject of the following.

(a) $4m + 5x = 3$

Get $5x$ on its own by subtracting $4m$ from both sides

$$5x = 3 - 4m$$

Get x on its own by dividing both sides by 5

$$\boldsymbol{x = \frac{3-4m}{5}}$$

(b) $3t = \frac{2}{x}$

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Remove fractions by multiplying both sides by the denominator, x

$$3tx = 2$$

Get x on its own by dividing both sides by $3t$

$$\boldsymbol{x = \frac{2}{3t}}$$

(c) $A = \dfrac{9(1-4x)}{2g}$

Remove fractions by multiplying both sides by the denominator, $2g$

$$2gA = 9(1-4x)$$

x is inside the brackets
Expand the brackets to release the x term

$$2gA = 9 - 36x$$

One way to get x on its own is by subtracting 9 then dividing by -36
Or you can first add $36x$ to both sides, to create positive $36x$ on the left

$$2gA + 36x = 9$$

Now get x on its own by subtracting $2gA$ then dividing by 36

$$36x = 9 - 2gA$$
$$x = \frac{9-2gA}{36}$$

$$\boldsymbol{x = \frac{9-2gA}{36}}$$

Other accepted forms of the answer are

$$\frac{2gA-9}{-36}, \quad \frac{-(2gA-9)}{36}, \quad -\frac{2gA-9}{36}, \quad \frac{1}{4}-\frac{gA}{18}$$

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Formulas where Subject Appears Twice

Subject Appears Twice

How do I rearrange formulae where the subject appears twice?

- If the **subject appears twice**, **you will need to factorise** at some point
  - E.g. When making x the subject of $x + xy = 3 - 2y$
  - **Factorise** out x on the left to get $x(1 + y) = 3 - 2y$
    - Notice that the subject **now only appears once!**
  - Then divide both sides by $(1 + y)$ to get $x = \dfrac{3 - 2y}{1 + y}$
- If the **subject appears twice**, and any of these are **inside a set of brackets**, you will need to **expand** these brackets first
  - E.g. When making x the subject of $c(x + 2) - x = f$
  - Expand the bracket first to $cx + 2c - x = f$
  - Keep the x terms on one side $cx - x = f - 2c$
  - x can then be made the subject using **factorising** as above
- If the **subject appears on two sides of a formula**, you will need to bring those terms to the **same side** before you can factorise
  - E.g. When making x the subject of $3x = y - px$
  - Add px to both sides first to form $3x + px = y$
  - x can then be made the subject using **factorising** as above

How do I factorise powers of a subject?

- If the **subject appears twice**, and **both have the same power**, you will need to **collect these terms together** before applying their inverse
  - E.g. When making x the subject of $x^2 = -px^2 + r$

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- Add px^2 to both sides first to form $x^2 + px^2 = r$
- x^2 can then be factorised out $x^2(1 + p) = r$ to give $x^2 = \frac{r}{1 + p}$
  - Now take **plus-or-minus square roots**
  - $x = \pm\sqrt{\frac{r}{1 + p}}$
- Be careful when square rooting, or cube rooting etc
  - E.g. To make x the subject of $x^3 = \frac{t^3 + 1}{t^3 + 8}$
    - The whole right hand side must be cube rooted
    - $x = \sqrt[3]{\frac{t^3 + 1}{t^3 + 8}}$
    - This cannot be simplified further
    - The right hand side is **not** equal to $\frac{t + 1}{t + 2}$, (this is a **common error**)

Worked Example

Rearrange the formula $p = \frac{2 - ax}{x - b}$ to make x the subject.

Get rid of the fraction by multiplying both sides by the expression on the denominator

$$p(x - b) = 2 - ax$$

Expand the brackets on the left hand side to 'release' the x

$$px - pb = 2 - ax$$

Bring the terms containing x to one side of the equals sign and any other terms to the other side

$$px - pb = 2 - ax$$

$$(+ax) \qquad\qquad (+ax)$$

$$px - pb + ax = 2$$

$$(+pb) \qquad\qquad (+pb)$$

$$px + ax = 2 + pb$$

Factorise the left-hand side to bring x outside of the brackets, so that it appears only once

$$x(p + a) = 2 + pb$$

Make x the subject by dividing by the whole expression $(p + a)$

$$\boldsymbol{x = \frac{2 + pb}{p + a}}$$

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