



AQA GCSE Maths: Higher



Your notes

Iteration

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What is iteration?

- Some equations do not have “nice” solutions
 - They are **not** integers (whole numbers), fractions or simple decimals
 - Instead, they can be irrational decimal solutions that go on forever with no pattern
- Iteration** is a repeated process used to solve such equations
 - the process starts with an **initial value** (starting value)
 - after each stage of the process (after each “iteration”), a solution is produced
 - the solutions get more and more accurate as more and more iterations are performed
 - these solutions are called **estimates**
- Scientific calculators allow us to perform iterations very quickly using the ANS button
 - Iteration questions will only be asked in the calculator exam

ANS BUTTON



THE ANS BUTTON IS USUALLY LOCATED NEAR THE =/EXE BUTTON ON MODERN CALCULATORS

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Your notes

How do I make an iterative formula?

- Find the equation you would like to solve using iteration
 - for example, $x^3 + x = 7$
- Rearrange this equation into the form $x = f(x)$ by making any x the subject of the equation
 - for example, $x = \sqrt[3]{7 - x}$
- Replace the x on the left with x_{n+1} (meaning the "next" value of x) and any x 's on the right with x_n (meaning the "current" value of x)
 - $x_{n+1} = \sqrt[3]{7 - x_n}$
 - This called the **iterative formula**
 - n and $n+1$ are just counters: $n+1$ is simply one more than n
 - n starts at **0** so the process starts with x_0 , the initial value
 - x_1 is your first estimate, x_2 is your second estimate, etc

How do I use my calculator to do iteration?

- Find a good **initial (starting) value (x_0)** near to the solution
 - This is often given in the question, for example $x_0 = 2$
- Store $x_0 = 2$ into your calculator (by typing 2 and pressing the "=" button)
 - 2 is now stored under the "Ans" button
- Type in the right-hand side of the iteration formula with "Ans" instead of x_n
 - $\sqrt[3]{7 - \text{Ans}}$
- Press "=" to find x_1 (be careful to only press "=" once)
 - $x_1 = 1.709975...$
- Without pressing any other button, press "=" again to find x_2
 - $x_2 = 1.742418...$
- Press "=" again to find x_3
 - $x_3 = 1.738849...$
- Repeat as many times as required

- the more you do, the closer the estimates get to the true solution



Your notes

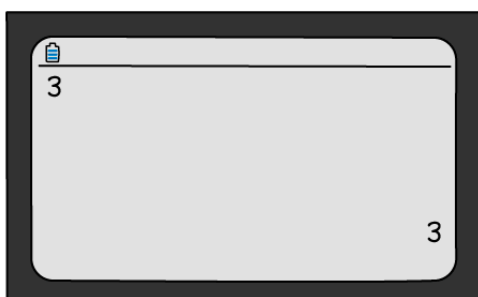


Your notes

e.g. USE THE ITERATIVE FORMULA $x_{n+1} = 5 - 2\sqrt[3]{x}$
WITH $x_0 = 3$ TO FIND x_1, x_2, x_3 AND x_4 .
GIVE YOUR ANSWERS TO 3 DECIMAL PLACES.

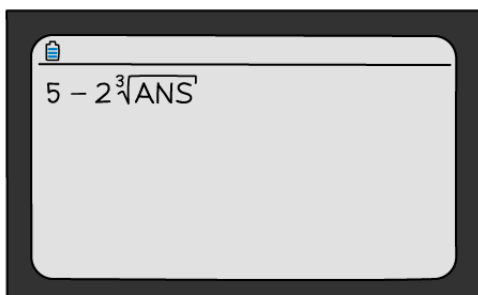
STEP 1:

ENTER INITIAL VALUE x_0 AND PRESS =/EXE



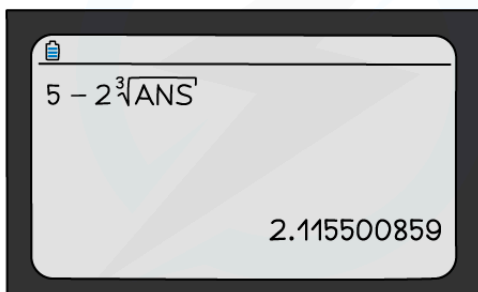
STEP 2:

ENTER (THE RIGHT HAND SIDE OF) THE
ITERATIVE FORMULA USING ANS INSTEAD
OF x_n .



STEP 3:

PRESS =/EXE TO OBTAIN x_1



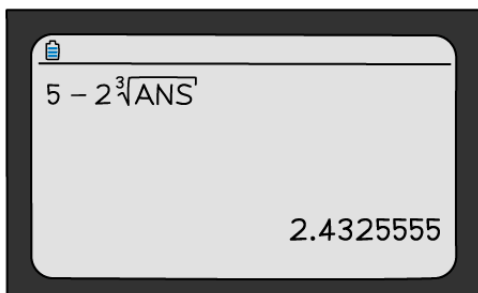
$x_1 = 2.116$



Your notes

STEP 4:

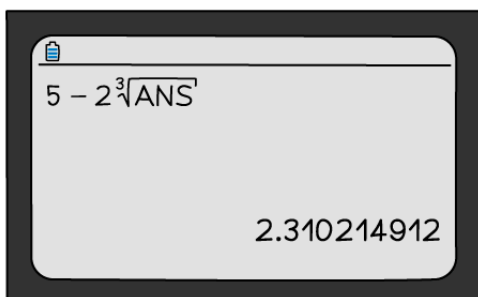
REPEAT STEP 3 AS NECESSARY



$$5 - 2\sqrt[3]{\text{ANS}}$$

$$2.432555$$

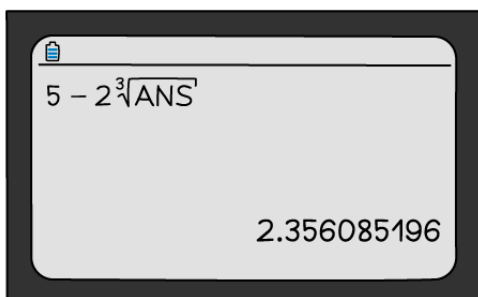
$$x_2 = 2.433$$



$$5 - 2\sqrt[3]{\text{ANS}}$$

$$2.310214912$$

$$x_3 = 2.310$$



$$5 - 2\sqrt[3]{\text{ANS}}$$

$$2.356085196$$

$$x_4 = 2.356$$

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What do x_1, x_2, x_3, \dots represent?

- $x_1, x_2, x_3 \dots$ etc are **estimates** to the **solution** of $x = f(x)$
 - for example, $x_1 = 1.709975\dots, x_2 = 1.742418\dots, x_3 = 1.738849\dots$ are estimates to $x = \sqrt[3]{7-x}$
- They are also estimates of solutions to any **rearrangements** of $x = f(x)$
 - such as the original equation trying to be solved, $x^3 + x = 7$
- This makes $x_1, x_2, x_3 \dots$ estimates to the solution of the original equation
- The more times you perform the iteration, the better the estimates get to the real solution



Your notes

How do you show that there is a solution in a given interval?

- To find x_0 (the initial / starting value), you are often asked to show that there is a solution in an interval
- For example, show that there is a solution to $x^3 + x = 7$ between 1 and 2
 - Method 1: Leave a constant term (e.g. the 7) on the right, substitute $x = 1$ and $x = 2$ into the left and show that this gives values **below and above** 7
 - $1^3 + 1 = 2$ and $2^3 + 2 = 10$ which are below and above 7
 - A solution therefore lies between 1 and 2
 - Method 2: Use "0" as your constant term on the right (by rearranging the equation into "... = 0"), then substitute in $x = 1$ and $x = 2$, showing this gives values below and above 0, i.e. **negative and positive**
 - this is called a **change of sign** between 1 and 2
 - $x^3 + x - 7 = 0$
 - Substitute $x = 1$ into the left-hand side: $1^3 + 1 - 7 = -5$ (negative)
 - Substitute $x = 2$ into the left-hand side: $2^3 + 2 - 7 = 3$ (positive)
 - A solution lies between 1 and 2 as there is a **change of sign**

- Knowing an interval that contains the solution helps to find x_0
 - If the solution is between 1 and 2 then you could choose either $x_0 = 1$ or $x_0 = 2$



Examiner Tips and Tricks

- Be careful to not press \div /EXE or "Ans" more than once at a time. If you do the best thing to do is to restart from the beginning.
- Iteration questions always require working with a lot of decimal places, so write down all digits from your calculator display for x_1, x_2 , etc. and round them at the end if necessary



Worked Example



Your notes



Your notes

? Use the iterative formula

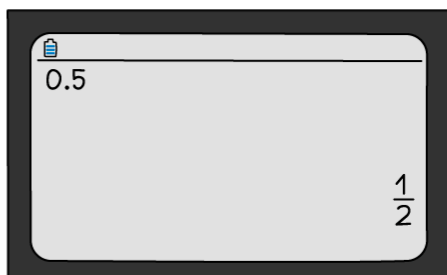
$$x_{n+1} = \frac{\sqrt{x_n^5 - x_n + 2}}{3}$$

with $x_0 = 0.5$ to find values for x_1, x_2 , and x_3 .

Give your answers to 5 decimal places.

STEP 1:

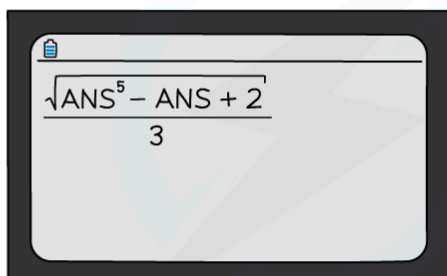
ENTER INITIAL VALUE INTO CALCULATOR
AND PRESS =

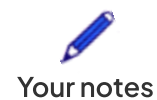


YOU MAY HAVE
TO PRESS $S \leftrightarrow D$
BUTTON TO SEE
AS A DECIMAL

STEP 2:

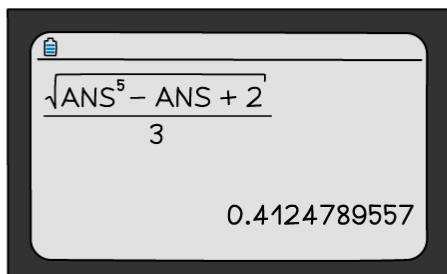
ENTER ITERATIVE FORMULA USING
ANS INSTEAD OF x_n





STEP 3:

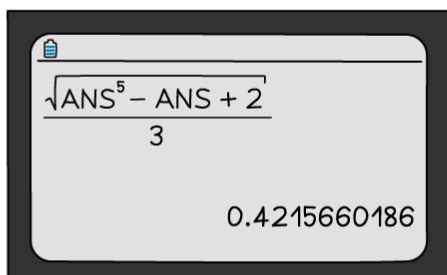
PRESS =/EXE TO CALCULATE x_1



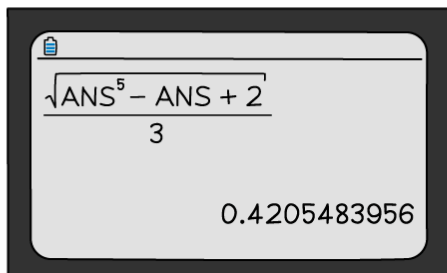
$$x_1 = 0.41248$$

STEP 4:

REPEAT STEP 3 AS MANY TIMES AS NECESSARY



$$x_2 = 0.42157$$



$$x_3 = 0.42055$$

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