AQA GCSE Maths: Higher

Your notes

Algebraic Fractions

Contents

Simplifying Algebraic Fractions

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Simplifying Algebraic Fractions

What is an algebraic fraction?

- An **algebraic fraction** is a fraction with an **algebraic expression** on the top (numerator) and/or the bottom (denominator)

How do you simplify an algebraic fraction?

- If possible, **factorise fully** the top and bottom
  - E.g. $\frac{2x}{x^2+3x} = \frac{2x}{x(x+3)}$
- **Cancel** common factors
  - This factor may be a **single term**
    - E.g. $\frac{\cancel{x}(5x-1)}{4\cancel{x}} = \frac{5x-1}{4}$
  - It could also be a **common bracket**
    - E.g. $\frac{x\cancel{(x+2)}}{\cancel{(x+2)}(x-1)} = \frac{x}{x-1}$
- A **common mistake** is to cancel a factor that is not common to **all terms** in either the top or the bottom of a fraction
  - E.g. The fraction $\frac{6x}{x+1}$ cannot be simplified
    - x is not common to **all** terms in the bottom of the fraction
    - and the expression on the bottom cannot be factorised

Examiner Tips and Tricks

- When asked to **simplify** an algebraic fraction, **factorise** top and bottom

- It is very likely that one of the factors will be the same on the top and the bottom
- Use this fact to help you, if one of the expressions is difficult to factorise!

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Worked Example

Simplify $\dfrac{4x+6}{2x^2-7x-15}$

Factorise the top, by using 2 as a common factor

$$\frac{2(2x+3)}{2x^2-7x-15}$$

Factorise the bottom using your preferred method
Using the fact that the top factorised to $(2x+3)$ may help!

$$\frac{2(2x+3)}{(2x+3)(x-5)}$$

The common factors on the top and bottom reduce to 1 (cancel out)

$$\frac{2\cancel{(2x+3)}}{\cancel{(2x+3)}(x-5)}$$

$$\mathbf{\frac{2}{(x-5)}}$$

Test yourself Next topic

Adding & Subtracting Algebraic Fractions

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Adding & Subtracting Algebraic Fractions

How do I add (or subtract) two algebraic fractions?

- The **rules** for adding and subtracting **algebraic fractions** are the **same** as they are for **fractions with numbers**
- **STEP 1**
  Find the **lowest common denominator** (LCD)
  - Sometimes the LCD can be found by **multiplying** the denominators together
    - E.g. The LCD for the fractions $\frac{1}{x+2}$ and $\frac{1}{x+5}$ is $(x+2)(x+5)$
    - Similarly, with numbers, the LCD of $\frac{1}{2}$ and $\frac{1}{5}$ is 2 × 5 = 10
  - Although multiplying the denominators will always give you a multiple, it is **not necessarily the lowest** multiple
    - E.g. The LCD for the fractions $\frac{1}{x}$ and $\frac{1}{2x}$ is $2x$ (not $2x^2$) as both terms already include an x
    - Similarly, with numbers, the LCD of $\frac{1}{2}$ and $\frac{1}{4}$ is just 4, not 2 × 4 = 8
  - Other examples include:
    - The LCD of $\frac{1}{x+2}$ and $\frac{1}{(x+2)(x-1)}$ is $(x+2)(x-1)$
    - The LCD of $\frac{1}{x+1}$ and $\frac{1}{(x+1)^2}$ is $(x+1)^2$
    - The LCD of $\frac{1}{(x+3)(x-1)}$ and $\frac{1}{(x+4)(x-1)}$ is $(x+3)(x-1)(x+4)$
- **STEP 2**

  Write each fraction over the **lowest common denominator**

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Multiply the numerator of each fraction by the **same amount** as the denominator

- E.g. $\frac{x}{x-4}+\frac{1}{x+2}=\frac{x(x+2)}{(x-4)(x+2)}+\frac{(x-4)}{(x-4)(x+2)}$

- **STEP 3**

  Write as a **single fraction** over the lowest common denominator and **simplify the numerator**

  - Do this by **adding** or **subtracting** the **numerators**
  - Take particular care if subtracting
  - E.g. $\frac{x(x+2)+(x-4)}{(x-4)(x+2)}=\frac{x^2+2x+x-4}{(x-4)(x+2)}=\frac{x^2+3x-4}{(x-4)(x+2)}$

- **STEP 4**

  Check at the end to see if the top **factorises** and the fraction can be **simplified**

  - E.g. $\frac{(x+4)(x-1)}{(x-4)(x+2)}$, the top factorises but there are no common factors so it is in its most simple form

Examiner Tips and Tricks

- Leaving the top and bottom of your answer in **factorised form** will help you see if anything cancels at the end

Worked Example

(a) Express $\frac{x}{x+4}-\frac{3}{x-1}$ as a single fraction.

The lowest common denominator is $(x+4)(x-1)$
Write each fraction over this common denominator, remember to multiply the top of the fractions too

$$\frac{x(x-1)}{(x+4)(x-1)}-\frac{3(x+4)}{(x-1)(x+4)}$$

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Combine the fractions, as they now have the same denominator

$$\frac{x(x-1)-3(x+4)}{(x+4)(x-1)}$$

Simplify the numerator
Be careful expanding with the negative signs

$$\frac{x^2-x-3x-12}{(x+4)(x-1)}=\frac{x^2-4x-12}{(x+4)(x-1)}$$

Factorise the top

$$\frac{(x+2)(x-6)}{(x+4)(x-1)}$$

There are no terms which would cancel here, so this is the final answer

$$\boldsymbol{\frac{(x+2)(x-6)}{(x+4)(x-1)}}$$

(b) Express $\frac{x-4}{2(x-3)}-\frac{x-1}{2x}$ as a single fraction.

The lowest common denominator is $2x(x-3)$
(You could also use $4x(x-3)$ but this wouldn't be the *lowest* common denominator)

Write each fraction over this common denominator, remember to multiply the top of the fractions too

$$\frac{x(x-4)}{2x(x-3)}-\frac{(x-1)(x-3)}{2x(x-3)}$$

Combine the fractions, as they now have the same denominator

$$\frac{x(x-4)-(x-1)(x-3)}{2x(x-3)}$$

Simplify the numerator
Be careful expanding with negative signs

$$\frac{(x^2-4x)-(x^2-4x+3)}{2x(x-3)}=\frac{x^2-4x-x^2+4x-3}{2x(x-3)}=\frac{-3}{2x(x-3)}$$

There is nothing else that can be factorised on the numerator, so this is the final answer

$$\mathbf{\frac{-3}{2x(x-3)}}$$

There are other accepted answers, e.g. $\frac{3}{2x(3-x)}$

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Multiplying & Dividing Algebraic Fractions

Multiplying & Dividing Algebraic Fractions

How do I multiply algebraic fractions?

- **STEP 1**

  Simplify both fractions first by **fully factorising**

  - E.g. $\frac{x}{3x+6} \times \frac{2x+4}{x+7} = \frac{x}{3(x+2)} \times \frac{2(x+2)}{x+7}$

- **STEP 2**

  Cancel any common factors on top and bottom (from either fraction)

  - E.g. $\frac{x}{3\cancel{(x+2)}} \times \frac{2\cancel{(x+2)}}{x+7} = \frac{x}{3} \times \frac{2}{x+7}$

- **STEP 3**
  Multiply the **tops** together
  Multiply the **bottoms** together

  - E.g. $\frac{2x}{3(x+7)}$

- **STEP 4**

  Check for any further factorising and cancelling

  - E.g. $\frac{2x}{3(x+7)}$ has no common factors so is in its simplest form

How do I divide algebraic fractions?

- **Flip** (find the reciprocal of) the **second** fraction and replace ÷ with ×

  - So $\div \frac{a}{b}$ becomes $\times \frac{b}{a}$

  - E.g. $\frac{3x-12}{x} \div \frac{2x+8}{x+3} = \frac{3x-12}{x} \times \frac{x+3}{2x+8}$

- Then follow the same rules for **multiplying** two fractions

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Worked Example

Divide $\frac{x+3}{x-4}$ by $\frac{2x+6}{x^2-16}$, giving your answer as a simplified fraction.

Division is the same as multiplying by the reciprocal (the fraction flipped)

$$\frac{x+3}{x-4} \div \frac{2x+6}{x^2-16} = \frac{x+3}{x-4} \times \frac{x^2-16}{2x+6}$$

Factorise all numerators and denominators to see which factors cancel out

You need to use the difference of two squares, $x^2 - 4^2 = (x-4)(x+4)$

$$\frac{x+3}{x-4} \times \frac{x^2-16}{2x+6} = \frac{\cancel{x+3}}{\cancel{x-4}} \times \frac{\cancel{(x-4)}(x+4)}{2\cancel{(x+3)}}$$

Multiply the remaining numerators and denominators together

$$\frac{1 \times (x+4)}{1 \times 2} = \frac{x+4}{2}$$

Check to see if you missed any terms that are the same on the top and bottom that could be cancelled

$\frac{x+4}{2}$ is already in its simplest form

$$\boldsymbol{\frac{x+4}{2}}$$

Solving Equations with Algebraic Fractions

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Solving Algebraic Fractions

How do I solve an equation that contains algebraic fractions?

- There are **two methods** for **solving equations** that contain algebraic fractions
- One method is to **add** or **subtract** the **algebraic fractions first** and then solve as usual
  - For example, to solve $\frac{8}{x+1} - \frac{5}{x+2} = 1$
  - First subtract the fractions and simplify, $\frac{3x+11}{(x+1)(x+2)} = 1$
  - Then cross-multiply, expand and solve

$$\begin{aligned} 3x + 11 &= 1(x+1)(x+2) \\ 3x + 11 &= x^2 + 3x + 2 \\ 0 &= x^2 - 9 \\ 0 &= (x-3)(x+3) \\ x &= 3 \text{ or } x = -3 \end{aligned}$$

- Alternatively, you can **remove the fractions first** by multiplying everything **on both sides** of the equation by each expression in the denominators and then solve
  - For example, to solve the equation $\frac{4}{x-3} + \frac{5}{x+1} = 5$
  - First multiply every term in the equation by both $(x-3)$ and $(x+1)$ and cancel common factors where possible
    - Multiply every term by $(x-3)$ (this bracket goes in the **numerator** of any fractions)

$$\frac{4}{\cancel{(x-3)}}\cancel{(x-3)} + \frac{5(x-3)}{x+1} = 5(x-3)$$

$$4 + \frac{5(x-3)}{x+1} = 5(x-3)$$

    - Then multiply every term by $(x+1)$

$$4(x+1)+\frac{5(x-3)}{\cancel{(x+1)}}\cancel{(x+1)}=5(x-3)(x+1)$$
$$4(x+1)+5(x-3)=5(x-3)(x+1)$$

- Then solve

$$4x+4+5x-15=5(x^2-2x-3)$$
$$9x-11=5x^2-10x-15$$
$$0=5x^2-19x-4$$
$$0=(5x+1)(x-4)$$
$$x=-\frac{1}{5}\text{ or }x=4$$

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Examiner Tips and Tricks

- When multiplying by an algebraic expression, use **brackets** around the expression, e.g. $(2x+3)$
- Multiplying by both denominators at once can speed up the process, but take care if choosing this technique in the exam!
  - and remember to multiply **all terms** on either side of the equation

Worked Example

$$\frac{2}{p+3}-\frac{5}{p}=6p$$

Show that this equation can be written as $6p^3+18p^2+3p+15=0$.

To clear the fractions, we multiply both sides of the equation by each denominator

Start by multiplying all terms in the equation by the denominator $(p+3)$
The $(p+3)$ on top and bottom will cancel in the first term

$$2-\frac{5(p+3)}{p}=6p(p+3)$$

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Now multiply all terms on both sides by the next denominator, p

The p on top and bottom will cancel in the second term

$$2(p) - 5(p+3) = 6p(p+3)(p)$$

Expand brackets

Be careful with negative signs

$$2p - 5(p+3) = 6p^2(p+3)$$

$$2p - 5p - 15 = 6p^3 + 18p^2$$

Collect like terms

$$-3p - 15 = 6p^3 + 18p^2$$

Add $3p$ and 15 to both sides of the equation

$$0 = 6p^3 + 18p^2 + 3p + 15$$

$$\mathbf{6p^3 + 18p^2 + 3p + 15 = 0}$$