AQA GCSE Maths: Higher

Your notes

Algebraic Roots & Indices

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Algebraic Roots & Indices

What are the laws of indices?

- **Index laws** are **rules** you can use when doing **operations** with **powers**
  - They work with both **numbers** and **algebra**

Law	Description	How it works
$a^1 = a$	Anything to the power of 1 is itself	$x^1 = x$
$a^0 = 1$	Anything to the power of 0 is 1	$b^0 = 1$
$a^m \times a^n = a^{m+n}$	To multiply indices with the same base, add their powers	$c^3 \times c^2$ $= (c \times c \times c) \times (c \times c)$ $= c^5$
$a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$	To divide indices with the same base, subtract their powers	$d^5 \div d^2$ $= \frac{d \times d \times d \times \cancel{d} \times \cancel{d}}{\cancel{d} \times \cancel{d}}$ $= d^3$
$(a^m)^n = a^{mn}$	To raise indices to a new power, multiply their powers	$(e^3)^2$ $= (e \times e \times e) \times (e \times e \times e)$ $= e^6$
$(ab)^n = a^n b^n$	To raise a product to a power, apply the power to both numbers, and multiply	$(f \times g)^2$ $= f^2 \times g^2$ $= f^2 g^2$

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$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	To raise a fraction to a power, apply the power to both the numerator and denominator	$\left(\frac{h}{i}\right)^2 = \frac{h^2}{i^2}$
$a^{-1} = \frac{1}{a}$ $a^{-n} = \frac{1}{a^n}$	A negative power is the reciprocal	$j^{-1} = \frac{1}{j}$ $k^{-3} = \frac{1}{k^3}$
$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$	A fraction to a negative power, is the reciprocal of the fraction, to the positive power	$\left(\frac{l}{m}\right)^{-3} = \left(\frac{m}{l}\right)^3 = \frac{m^3}{l^3}$
$a^{\frac{1}{n}} = \sqrt[n]{a}$	The fractional power $\frac{1}{n}$ is the n^{th} root ($\sqrt[n]{\ }$)	$n^{\frac{1}{2}} = \sqrt[2]{n}$ $p^{\frac{1}{3}} = \sqrt[3]{p}$
$a^{-\frac{1}{n}} = \left(a^{\frac{1}{n}}\right)^{-1}$ $= \left(\sqrt[n]{a}\right)^{-1} = \frac{1}{\sqrt[n]{a}}$	A negative, fractional power is one over a root	$q^{-\frac{1}{2}} = \frac{1}{\sqrt[2]{q}}$ $r^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{r}}$
$a^{\frac{m}{n}} = a^{\frac{1}{n} \times m}$ $= \left(a^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{a}\right)^m$ $= \left(a^m\right)^{\frac{1}{n}} = \sqrt[n]{a^m}$	The fractional power $\frac{m}{n}$ is the n^{th} root all to the power *m*, $\left(\sqrt[n]{\ }\right)^m$, or the n^{th} root of the power *m*, $\sqrt[n]{(\ )^m}$ (both are the same)	$s^{\frac{2}{3}} = \left(s^{\frac{1}{3}}\right)^2 = \left(\sqrt[3]{s}\right)^2$ $s^{\frac{2}{3}} = \left(s^2\right)^{\frac{1}{3}} = \sqrt[3]{s^2}$

- These can be used to **simplify** expressions
    - Work out the **number** and **algebra** parts **separately**

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- $(3x^7) \times (6x^4) = (3 \times 6) \times (x^7 \times x^4) = 18x^{7+4} = 18x^{11}$
- $\dfrac{6x^7}{3x^4} = \dfrac{6}{3} \times \dfrac{x^7}{x^4} = 2x^{7-4} = 2x^3$
- $(3x^7)^2 = (3)^2 \times (x^7)^2 = 9x^{14}$

How do I find an unknown inside a power?

- A term may have a **power involving an unknown**
  - E.g. 7^{4x}
- If both sides of an equation have the **same base number**, then the powers must be equal
  - E.g. If $4^{3x} = 4^9$ then $3x = 9$
  - And $x = 3$
- You may have to do some **simplifying first** to reach this point
  - E.g. $3^{2x} \times 3^4 = 3^{18}$ simplifies to $3^{2x+4} = 3^{18}$
  - Therefore $2x + 4 = 18$
  - And $x = 7$

Worked Example

(a) Simplify $(u^5)^5$

Use $(a^m)^n = a^{mn}$

$$(u^5)^5 = u^{5 \times 5}$$

$\boldsymbol{u^{25}}$

(b) If $q^x = \dfrac{q^2 \times q^5}{q^{10}}$ find x.

Use $a^m \times a^n = a^{m+n}$ to simplify the numerator

$$q^2 \times q^5 = q^{2+5} = q^7$$

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Use $\frac{a^m}{a^n} = a^{m-n}$ to simplify the fraction

$$\frac{q^7}{q^{10}} = q^{7-10} = q^{-3}$$

Write out both sides of the equation

$$q^x = q^{-3}$$

Both sides are now over the same base of q

So x must equal the power on the right-hand side

$$\boldsymbol{x = -3}$$

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Worked Example

(a) Rewrite $\frac{1}{\sqrt[3]{x^4}}$ in the form x^n where n is a negative fraction.

Use $a^{\frac{1}{n}} = \sqrt[n]{a}$ to rewrite the cube-root as a power of $\frac{1}{3}$

$$\frac{1}{(x^4)^{\frac{1}{3}}}$$

Use $(a^m)^n = a^{mn}$ to simplify the denominator

$$\frac{1}{x^{\frac{4}{3}}}$$

Use $a^{-n} = \frac{1}{a^n}$ to rewrite as a term with a negative fraction as the power

$$\boldsymbol{x^{-\frac{4}{3}}}$$

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(b) Find the value of the constants m and a given that $\left(ax^6\right)^{\frac{1}{m}} = 8x^3$.

Use $(ab)^n = a^n b^n$ to rewrite the left hand side

Remember to apply the power to both a and x^6

$$a^{\frac{1}{m}} \times x^{\frac{6}{m}} = 8x^3$$

Both sides of the equation have a constant part, $a^{\frac{1}{m}}$ and 8

And both sides of the equation have a part in terms of x

The two sides of the equation are equal, so set the respective parts equal to one another

First,

$$x^{\frac{6}{m}} = x^3$$

The bases are the same, therefore the powers are equal

$$\frac{6}{m} = 3$$

Solve to find m

$$\boldsymbol{m = 2}$$

Then set the constant parts of both sides equal to one another

$$a^{\frac{1}{m}} = 8$$

We now know that $m = 2$, so substitute this in

$$a^{\frac{1}{2}} = 8$$

Use $a^{\frac{1}{n}} = \sqrt[n]{a}$ to rewrite as a square root

$$\sqrt[2]{a} = 8$$

Find a by squaring both sides

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$$a = 64$$

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