AQA GCSE Maths: Higher

Your notes

Direct & Inverse Proportion

Contents

Direct Proportion

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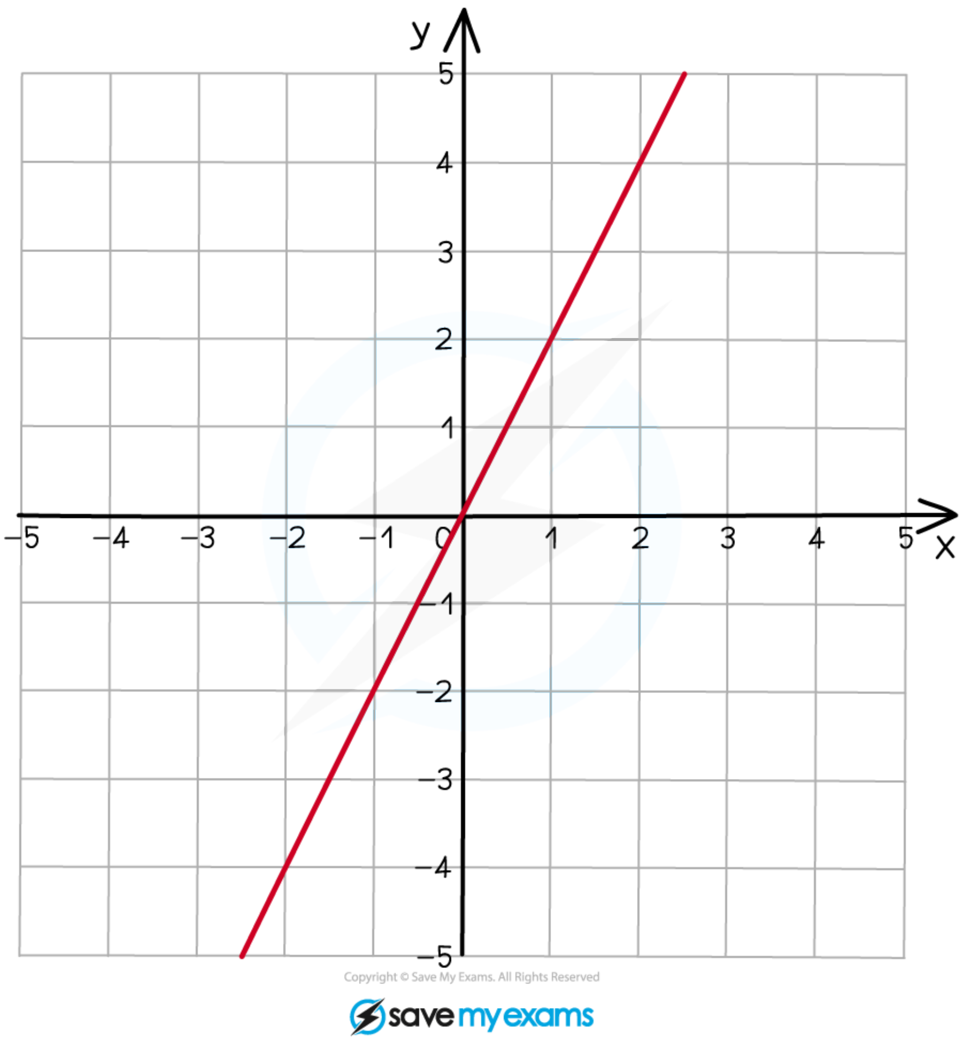
Direct Proportion

What is direct proportion?

- **Proportion** is a way of talking about how two **variables** are related to each other
- **Direct** proportion means that as one variable goes **up** the other goes up by the same **factor**
  - The **ratio** between the two amounts will always stay the **same**
- The symbol $\propto$ means "proportional to"
  - E.g. y is directly proportional to x, $y \propto x$
- If x and y are directly proportional, then
  - $x : y$ will always be the same
  - there will be some value, k, such that $\boldsymbol{y = kx}$
  - the graph relating x and y is a linear graph, with **gradient** $\boldsymbol{k}$
- k is called the **constant of proportionality**

Your notes



How do I use direct proportion with powers and roots?

- Problems may involve a variable being **directly proportional** to a **power or root** of another variable
- For example
  - y is directly proportional to the **square of *x***
    - $y \propto x^2$
    - means that $y = kx^2$
  - y is directly proportional to the **square root of *x***

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- $y \propto \sqrt{x}$
- means that $y = k\sqrt{x}$
- *y* is directly proportional to the **cube of *x***
  - $y \propto x^3$
  - means that $y = kx^3$
- *y* is directly proportional to the **cube root of *x***
  - $y \propto \sqrt[3]{x}$
  - means that $y = k\sqrt[3]{x}$
- Each of these would have a **different type of graph**, depending on the power or root

How do I find the equation between two directly proportional variables?

- Direct proportion questions always have the same process:
  - **STEP 1**
    Identify the two variables and write down the **formula in terms of *k***
    - E.g. ***y*** is **directly** proportional to ***x***
    - write down the formula $y = kx$
  - **STEP 2**
    Find *k* by substituting any given values **from the question** into your formula, then solving to get *k*
    - E.g. if you are told $y = 6$ when $x = 2$
    - then $6 = k \times 2$ giving $k = 3$
  - **STEP 3**
    Rewrite the formula with the **known value of *k*** from above (substitute it in)
    - $y = 3x$
    - This is the **equation** relating the two variables
  - **STEP 4**
    Use the equation to answer other parts of the question
    - E.g. find y when $x = 10$

- $y = 3x$ gives $y = 3 \times 10 = 30$

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Examiner Tips and Tricks

- Some harder exam questions do not tell you to work out the equation
    - You are expected to do it on your own

Worked Example

It is known that y is directly proportional to the square of x.

When $x = 3$, $y = 18$.

Find the value of y when $x = 4$.

Identify the two variables

$$y,\ x^2$$

We are told this is **direct** proportion
Write down the formula involving *k*

$$y = kx^2$$

Find *k* by substituting in $y = 18$ when $x = 3$
Then solve the equation for *k*

$$18 = k(3)^2$$
$$18 = 9k$$
$$\frac{18}{9} = k$$
$$2 = k$$

Substitute this value of *k* back into the formula to get the full equation

$$y = 2x^2$$

Use this formula to find *y* when $x = 4$

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$$y = 2 \times 4^2$$
$$y = 2 \times 16$$
$$y = 32$$

$$\boldsymbol{y = 32}$$

Inverse Proportion

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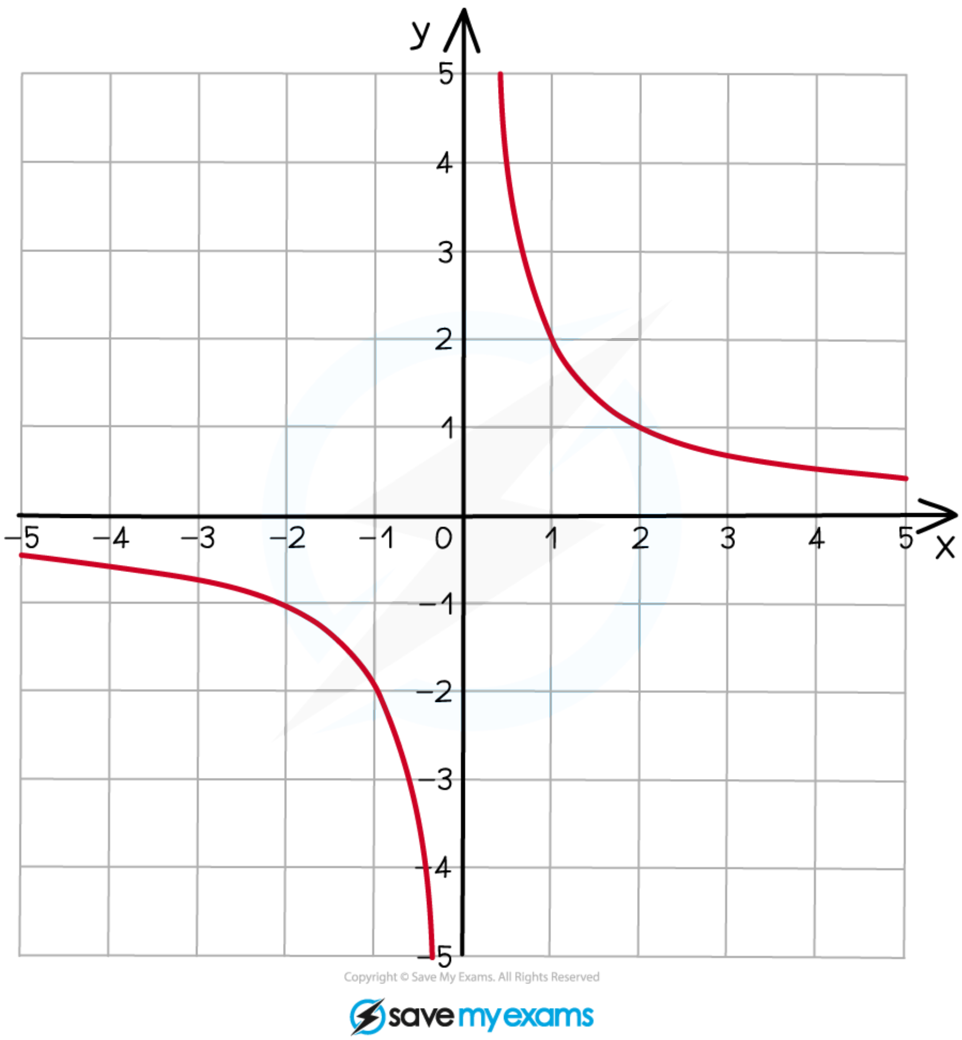
Inverse Proportion

What is inverse proportion?

- **Inverse** proportion means as **one variable goes up** the **other goes down** by the same **factor**
  - If two quantities are **inversely proportional**, then we can say that one is **directly proportional** to the **reciprocal** of the other
- The symbol, $\propto$, is used to show that one quantity is "directly proportional to the reciprocal of" (inversely proportional to) another quantity
  - "*y* is inversely proportional to *x*" is written $y \propto \frac{1}{x}$
- If *x* and *y* are inversely proportional then
  - $\frac{1}{x} : y$ will always be the same
  - there will be some value of *k* such that $y = \frac{k}{x}$
- The **graph** of $y = \frac{k}{x}$ is shown below

Your notes



How do I use inverse proportion with powers and roots?

- Problems may involve a variable being **inversely proportional** to a **power or root** of another variable
- For example
  - *y* is **inversely** proportional to the **square of *x***
    - $y \propto \frac{1}{x^2}$

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- means that $y = \frac{k}{x^2}$
- *y* is **inversely** proportional to the **square root of *x***
  - $y \propto \frac{1}{\sqrt{x}}$
  - means that $y = \frac{k}{\sqrt{x}}$
- *y* is **inversely** proportional to the **cube of *x***
  - $y \propto \frac{1}{x^3}$
  - means that $y = \frac{k}{x^3}$
- *y* is **inversely** proportional to the **cube root of *x***
  - $y \propto \frac{1}{\sqrt[3]{x}}$
  - means that $y = \frac{k}{\sqrt[3]{x}}$
- Each of these would have a **different type of graph**, depending on the power or root

How do I find the equation between two inversely proportional variables?

- Inverse proportion questions always have the same process:
  - **STEP 1**
    Identify the two variables and write down the **formula in terms of *k***
    - E.g. ***y*** is **inversely** proportional to ***x***
    - write down the formula $y = \frac{k}{x}$
  - **STEP 2**
    Find *k* by substituting any given values **from the question** into your formula, then solving to get *k*

Your notes

- E.g. if you are told $y = 5$ when $x = 6$
- then $5 = \frac{k}{6}$ giving $k = 30$

- **STEP 3**
Rewrite the formula with the **known value of *k*** from above (substitute it in)
  - $y = \frac{30}{x}$
  - This is the **equation** relating the two variables
- **STEP 4**
Use the equation to answer other parts of the question
  - E.g. find y when $x = 2$
  - $y = \frac{30}{x}$ gives $y = \frac{30}{2} = 15$

Worked Example

The time, t hours, it takes to complete a project is inversely proportional to the cube root of the number, n, of people working on it.

If 27 people work on the project, it takes 50 hours to complete.

(a) Find an equation connecting t and n.

Identify the two variables

$$t, n$$

We are told this is **inverse** proportion to the cube root of n
Write down the formula involving k

$$t = \frac{k}{\sqrt[3]{n}}$$

Find k by substituting in $n = 27$ and $t = 50$ (from the words in the question)

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$$50 = \frac{k}{\sqrt[3]{27}}$$
$$50 = \frac{k}{3}$$
$$50 \times 3 = k$$
$$150 = k$$

Substitute this value of *k* back into the formula to get the full equation

$$\boldsymbol{t = \frac{150}{\sqrt[3]{n}}}$$

(b) Given that the project needs to be completed within 60 hours, find the minimum number of people needed to work on it.

Use the formula to find n when $t = 60$

$$60 = \frac{150}{\sqrt[3]{n}}$$
$$\sqrt[3]{n} = \frac{150}{60} = 2.5$$
$$n = 2.5^3 = 15.625$$

A sensible answer here is a whole number (as n is the number of people)
15 people would not complete it in time, but 16 would

16 people