AQA GCSE Maths: Higher

Your notes

Graphs of Functions

Contents

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Types of Graphs

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Types of Graphs

What types of graphs do I need to know?

- You need to be able to **recognise, sketch, and interpret** the following types of graph:
  - **Linear** ($y = \pm x$)
    - $y = mx + c$ or $ax + by = c$
  - **Quadratic** ($y = \pm x^2$)
    - $y = ax^2 + bx + c$
  - **Cubic** ($y = \pm x^3$)
    - $y = ax^3 + b$ or $y = ax^3 + bx^2 + cx$
  - **Reciprocal** ($y = \pm \frac{1}{x}$)
    - $y = \frac{a}{x} + b$
  - **Exponential** ($y = k^{\pm x}$)
    - $y = ak^x + b$

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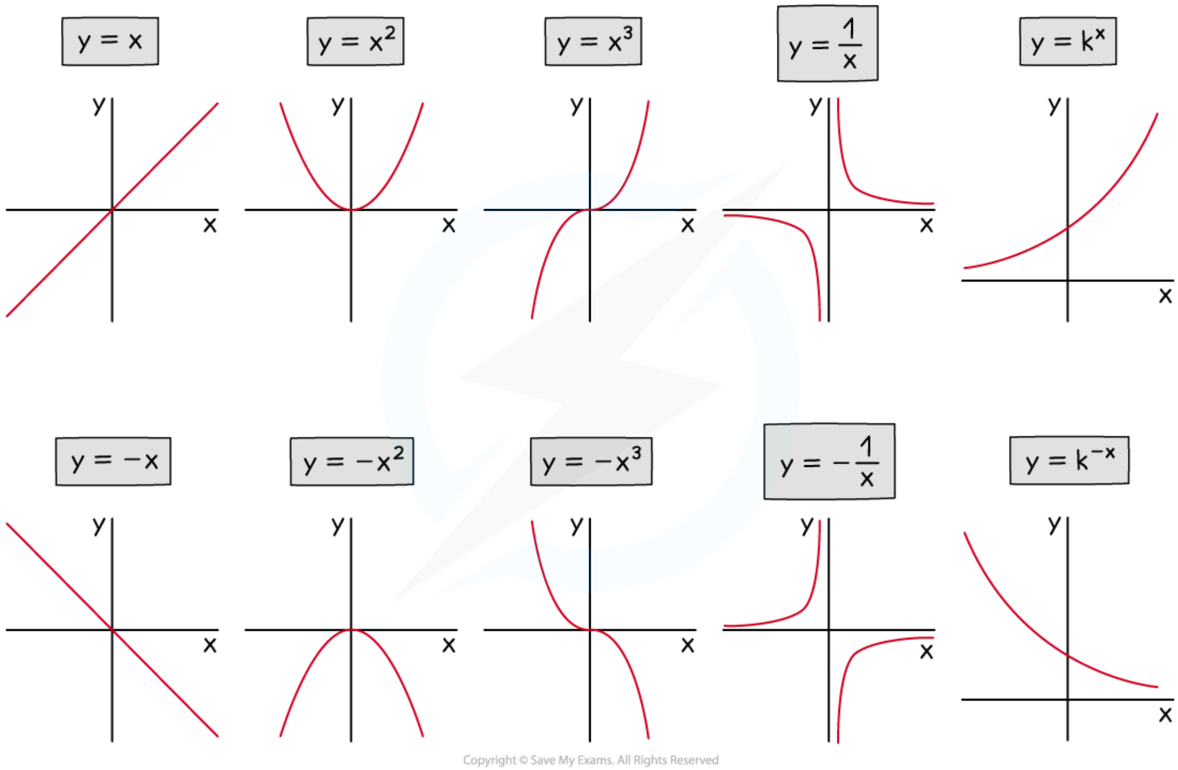

- You must also be able to recognise the three basic **trigonometric graphs**, covered in the Trigonometry section

Where are the asymptotes on reciprocal graphs?

- An **asymptote** is a line on a graph that a curve becomes closer to but never touches
  - These may be horizontal or vertical
- The **reciprocal** graph, $y = \frac{a}{x}$ (where a is a constant)
  - **does not have a** ***y*****-intercept**
  - **and does not have any roots**
- This graph has **two asymptotes**
  - A **horizontal** asymptote at the *x*-axis: $y = 0$
    - This is the **limiting value** when the value of *x* gets very large (or very negative)

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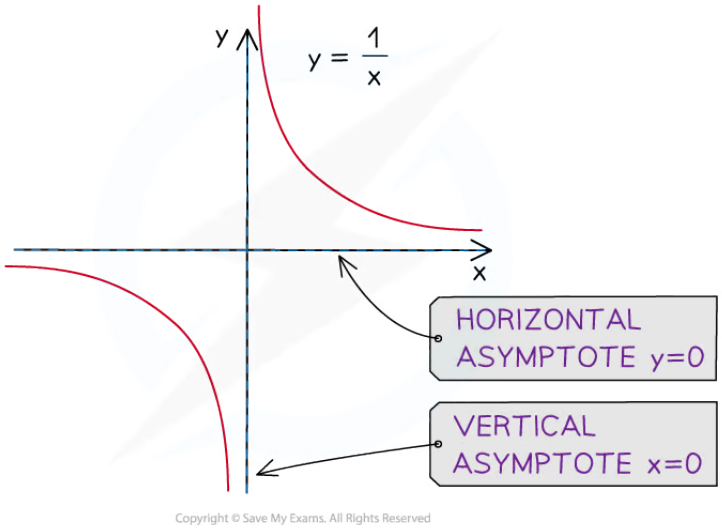
- A **vertical** asymptote at the y-axis: $x = 0$
  - This is the value that causes the **denominator to be zero**



- The reciprocal graph, $y = \frac{a}{x} + b$ (where a and b are both constants)
  - is the **same shape** as $y = \frac{a}{x}$
  - but is **shifted upwards** by b units
    - $y = \frac{a}{x} - 3$ would be $y = \frac{a}{x}$ shifted **down** by 3 units
  - This means the **horizontal asymptote** also **shifts up** by b units
    - The **vertical asymptote** remains on the ***y*-axis**

How do I draw exponential growth and decay?

- The equation $y = k^x$ represents **exponential growth** when $k > 1$
  - $y = k^x$ represents **exponential decay** when $0 < k < 1$
    - k is positive but **less than 1**

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- Both of these graphs:
  - have a **horizontal asymptote** at $y = 0$
  - **do not** have a **vertical asymptote**
  - have a y-intercept of $(0,\ 1)$
- The graph of $y = ak^x + b$ is a similar shape to $y = k^x$, but there are some differences
  - It is first **stretched** vertically by a
  - It is then **shifted** b units upwards
    - Therefore it has a **horizontal asymptote** at $y = b$
    - and a y-intercept of $(0,\ a + b)$
- For example, a population may be modelled as $y = 400 \times \left(\frac{1}{2}\right)^x + 100$, where y is the population and x represents time
  - This is an exponential decay as $0 < k < 1$
  - The initial population (when $x = 0$) will be 400 + 100 = 500
    - The y-intercept is (0, 500)
  - Over a long period of time (large x-value) the population will settle to 100
    - The asymptote is at $y = 100$
- **Exponential decay** can also be identified by a **negative power** using **index laws**
  - $\left(\frac{1}{2}\right)^x = (2^{-1})^x = 2^{-x}$ so $y = 400 \times 2^{-x} + 100$ is the model above
  - This has the form $y = k^{-x}$ where $k > 1$

Worked Example

Match the graphs to the equations.

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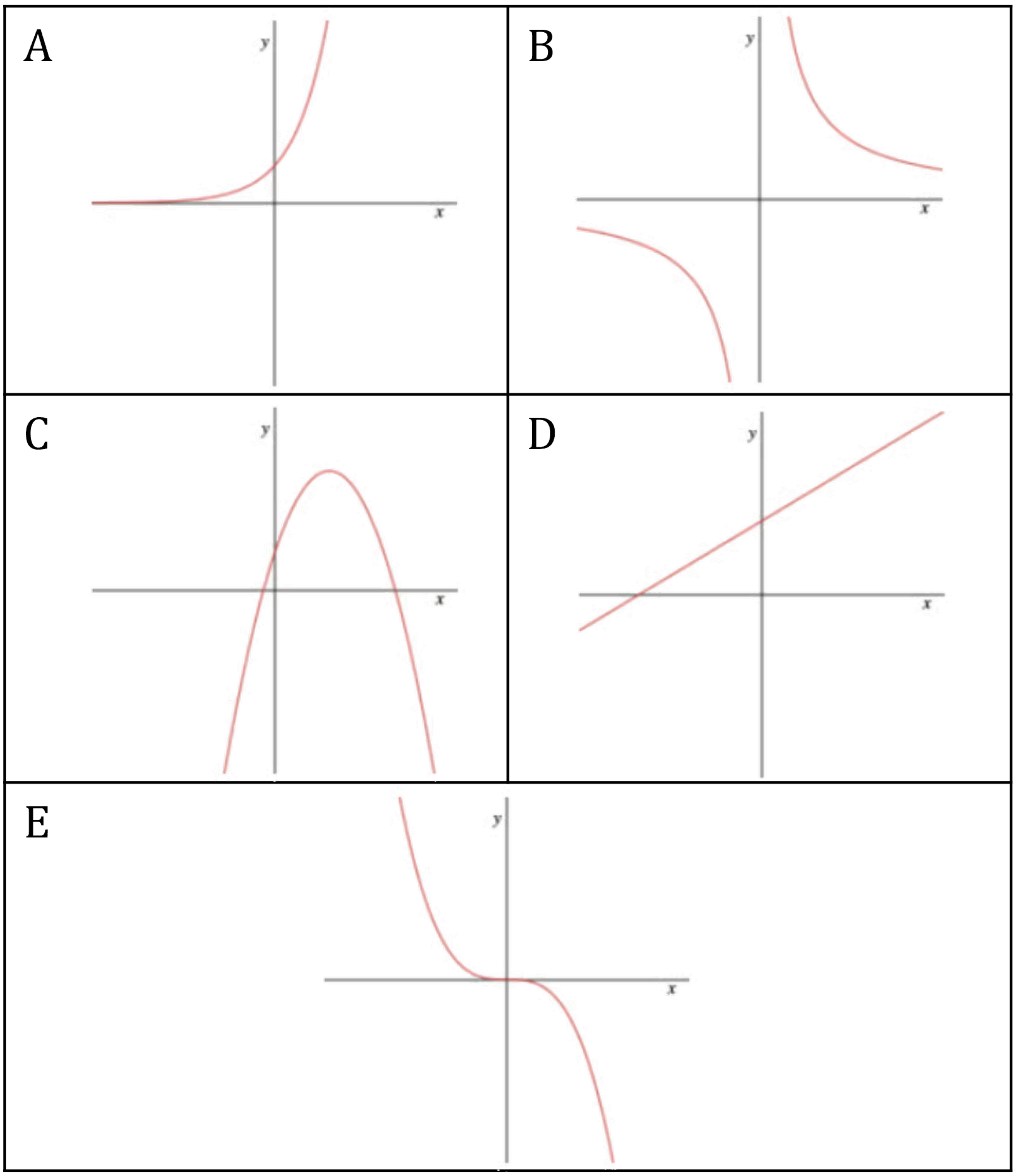

(1) $y = 0.6x + 2$, (2) $y = 3^x$, (3) $y = -0.7x^3$, (4) $y = \frac{4}{x}$, (5) $y = -x^2 + 3x + 2$

Starting with the equations,

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(1) is a linear equation ($y = mx + c$) so matches the only straight line, graph **D**

(2) is an exponential equation with a positive coefficient so matches graph **A**

(3) is a cubic equation with a negative coefficient so matches graph **E**

(4) is a reciprocal equation with a positive coefficient so matches graph **B**

(5) is a quadratic equation with a negative coefficient so matches graph C

Graph A → Equation 2

Graph B → Equation 4

Graph C → Equation 5

Graph D → Equation 1

Graph E → Equation 3

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Quadratic Graphs

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Quadratic Graphs

What is a quadratic graph?

- A **quadratic graph** has the form $y = ax^2 + bx + c$
  - where a is not zero

What does a quadratic graph look like?

- A quadratic graph is a smooth curve with a **vertical line of symmetry**
  - A **positive** number in front of x^2 gives a **u-shaped curve**
  - A **negative** number in front of x^2 gives an **n-shaped curve**
- The shape made by a quadratic graph is known as a **parabola**
- A quadratic graph will **always** cross the y**-axis**
- A quadratic graph crosses the x**-axis twice, once, or not at all**
  - The points where the graph crosses the x-axis are called the **roots**
- If the graph is a **u-shape**, it has a **minimum point**
- If the graph is an **n-shape**, it has a **maximum point**
- Minimum and maximum points are both examples of **turning points**
  - A turning point can also be called a **vertex**

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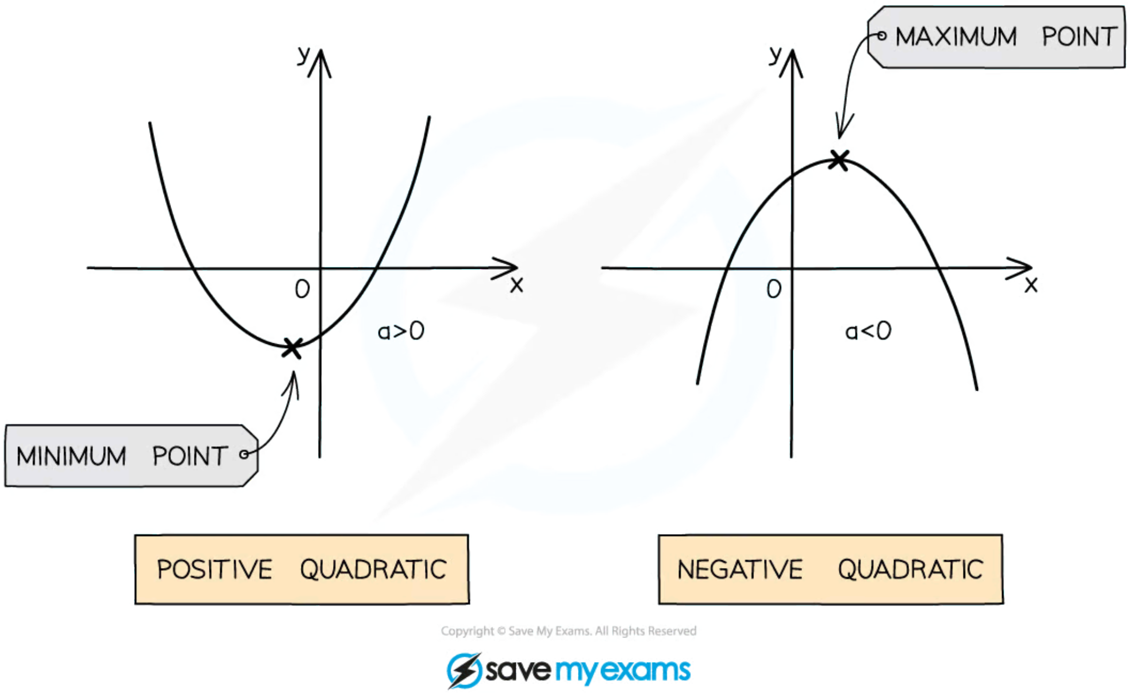

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How do I sketch a quadratic graph?

- It is important to know how to **sketch** a quadratic curve
  - A simple drawing showing the **key features** is often sufficient
  - (For a more accurate graph, create a table of values and plot the points)
- To **sketch a quadratic graph**:
  - First sketch the x and y-axes
  - Identify the y-intercept and mark it on the y-axis
    - The y-intercept of $y = ax^2 + bx + c$ will be $(0,\ c)$
    - It can also be found by substituting in $x = 0$
  - Find all root(s) (0, 1 or 2) of the equation and mark them on the x-axis
    - The roots will be the solutions to $y = 0$; $ax^2 + bx + c = 0$
    - You can find the solutions by factorising, completing the square or using the quadratic formula

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- Identify if the number a in $ax^2 + bx + c$ is positive or negative
  - A positive value will result in a u-shape
  - A negative value will result in an n-shape
- Sketch a smooth curve through the x and y-intercepts
  - Mark on any axes intercepts
  - Mark on the coordinates of the maximum/minimum point if you know it

How do I find the coordinates of the turning point by completing the square?

- The coordinates of the **turning point** (vertex) of a quadratic graph can be found by **completing the square**
- For a quadratic graph written in the form $y = a(x - p)^2 + q$
  - the minimum or maximum point has **coordinates** (p, q)
- Beware: there is a **sign change** for the x-coordinate
  - A curve with equation $y = (x - 3)^2 + 2$, has a minimum point at $(3, 2)$
  - A curve with equation $y = (x + 3)^2 + 2$, has a minimum point at $(-3, 2)$
- The value of a does **not affect the coordinates** of the turning point but it **will change the shape** of the graph
  - If it is positive, the graph will be a u-shape
    - The curve $y = 5(x - 3)^2 + 2$ has a **minimum** point at $(3, 2)$
  - Of it is negative, the graph will be an n-shape
    - The curve $y = -8(x - 3)^2 + 2$ has a **maximum** point at $(3, 2)$

Worked Example

(a) Sketch the graph of $y = x^2 - 5x + 6$ showing the x and y intercepts clearly.

The $+c$ at the end is the y-intercept

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y-intercept: (0, 6)

Factorise the quadratic expression

$$y = (x-2)(x-3)$$

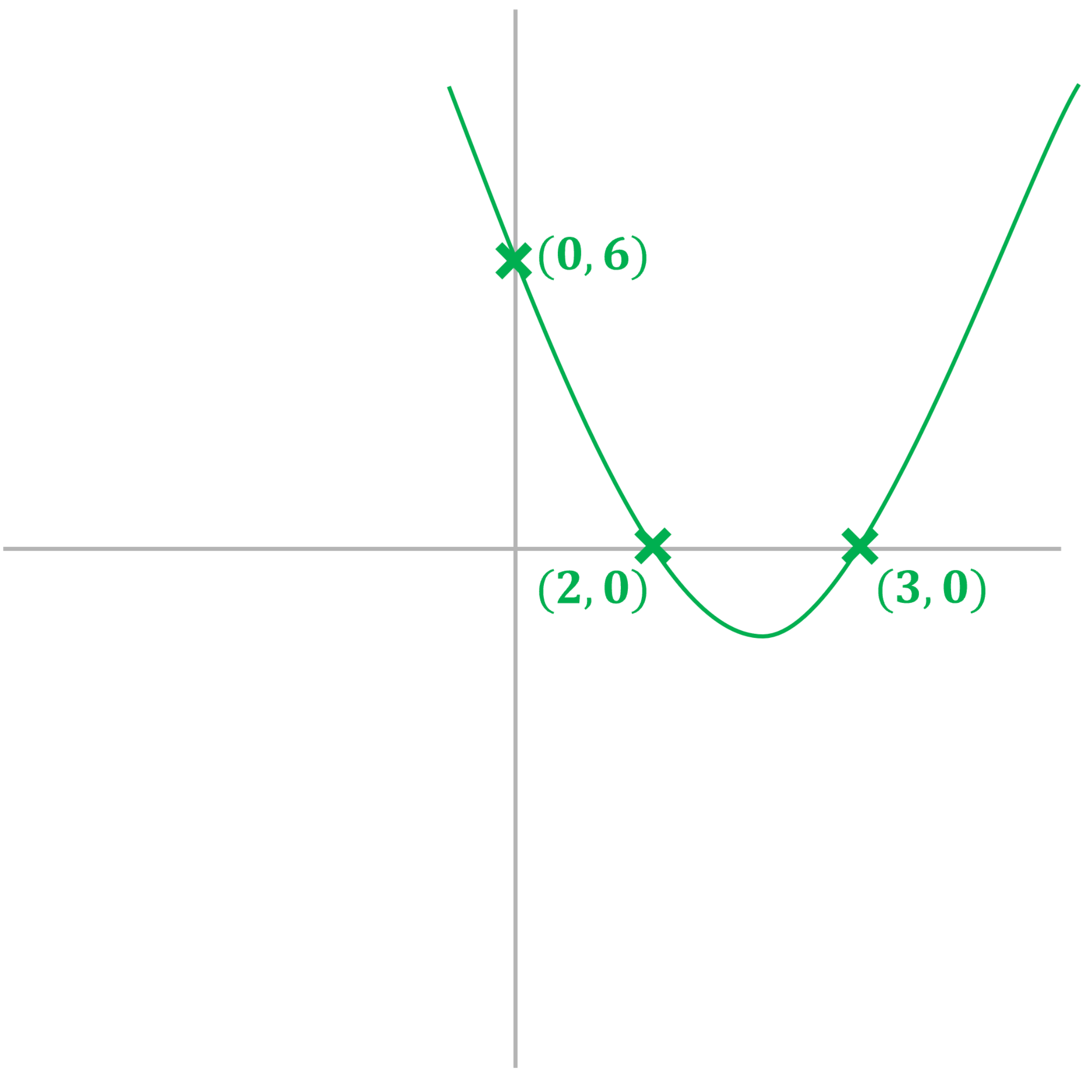
Solve $y = 0$

$$(x-2)(x-3) = 0, \text{ so } x = 2 \text{ or } x = 3$$

So the *x*-intercepts are given by the coordinates

(2, 0) and (3, 0)

It is a positive quadratic graph, so will be a u-shape



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(b) Sketch the graph of $y = x^2 - 6x + 13$ showing the y-intercept and the coordinates of the turning point.

It is a positive quadratic, so will be a u-shape
The turning point will therefore be a minimum

The $+c$ at the end is the y-intercept

y-intercept: (0, 13)

Find the minimum point by completing the square

For example, complete the square by writing the equation in the form $a(x-p)^2 + q$ (you may need to look this method up)

$$\begin{aligned} x^2 - 6x + 13 &= (x-3)^2 - 9 + 13 \\ &= (x-3)^2 + 4 \end{aligned}$$

The turning point of $y = a(x-p)^2 + q$ has coordinates $(p,\ q)$
The minimum point is therefore

(3, 4)

As the **minimum** point is above the x-axis, and the curve is a u-shape, this means the graph will not cross the x-axis (it has no roots)

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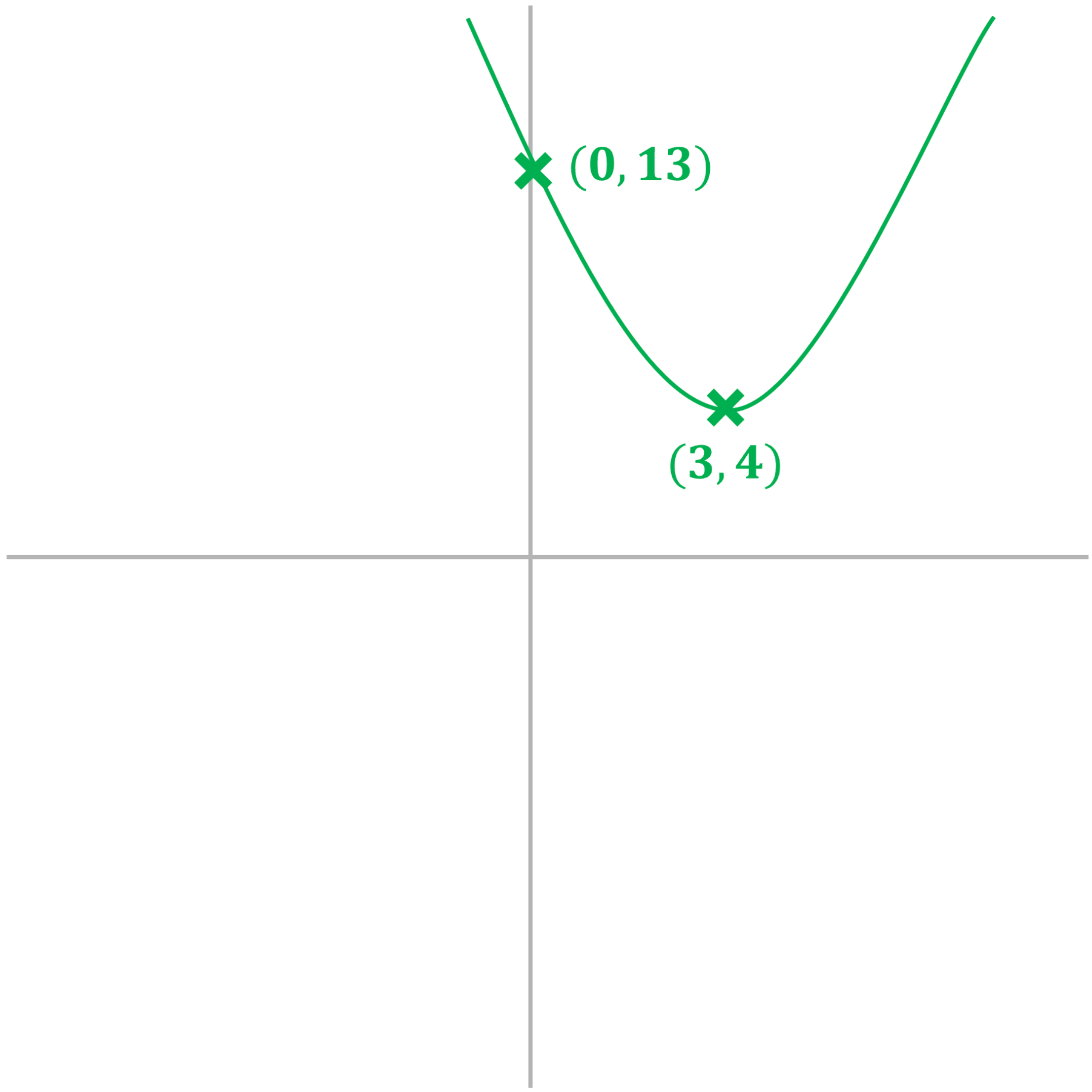

(c) Sketch the graph of $y = -x^2 - 4x - 4$ showing the root(s), y-intercept, and the coordinates of the turning point.

It is a negative quadratic, so will be an n-shape
The turning point will therefore be a maximum

The $+c$ at the end is the y-intercept

y-intercept: (0, -4)

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Find the minimum point by completing the square

$$y = -(x^2 + 4x) - 4$$
$$y = -[(x+2)^2 - 2^2] - 4$$
$$y = -[(x+2)^2 - 4] - 4$$
$$y = -(x+2)^2 + 4 - 4$$
$$y = -(x+2)^2$$

The turning point of $y = a(x-p)^2 + q$ has coordinates (p, q)
The minimum point is therefore

Minimum = (-2, 0)

As the maximum is on the x-axis, there is **only one root**

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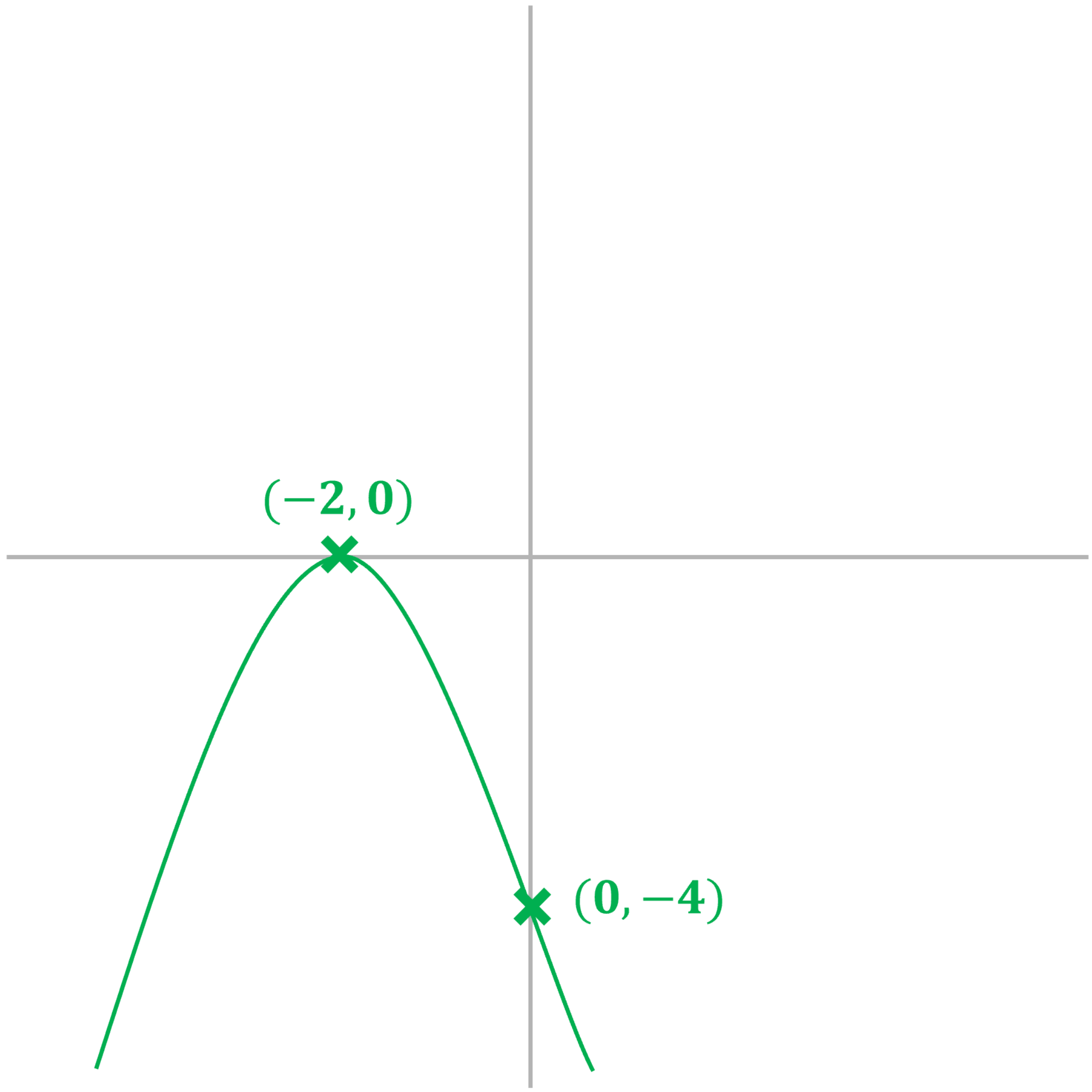

How do I find the equation of a quadratic from its graph?

- If the **vertex** and **one other point** are known
  - Use the form $y = a(x - p)^2 + q$ to fill in p and q
    - The vertex is at (p, q)
  - Then substitute in the other known point (x, y) to find a
- If the **roots** (x-intercepts) and **one other point** are known

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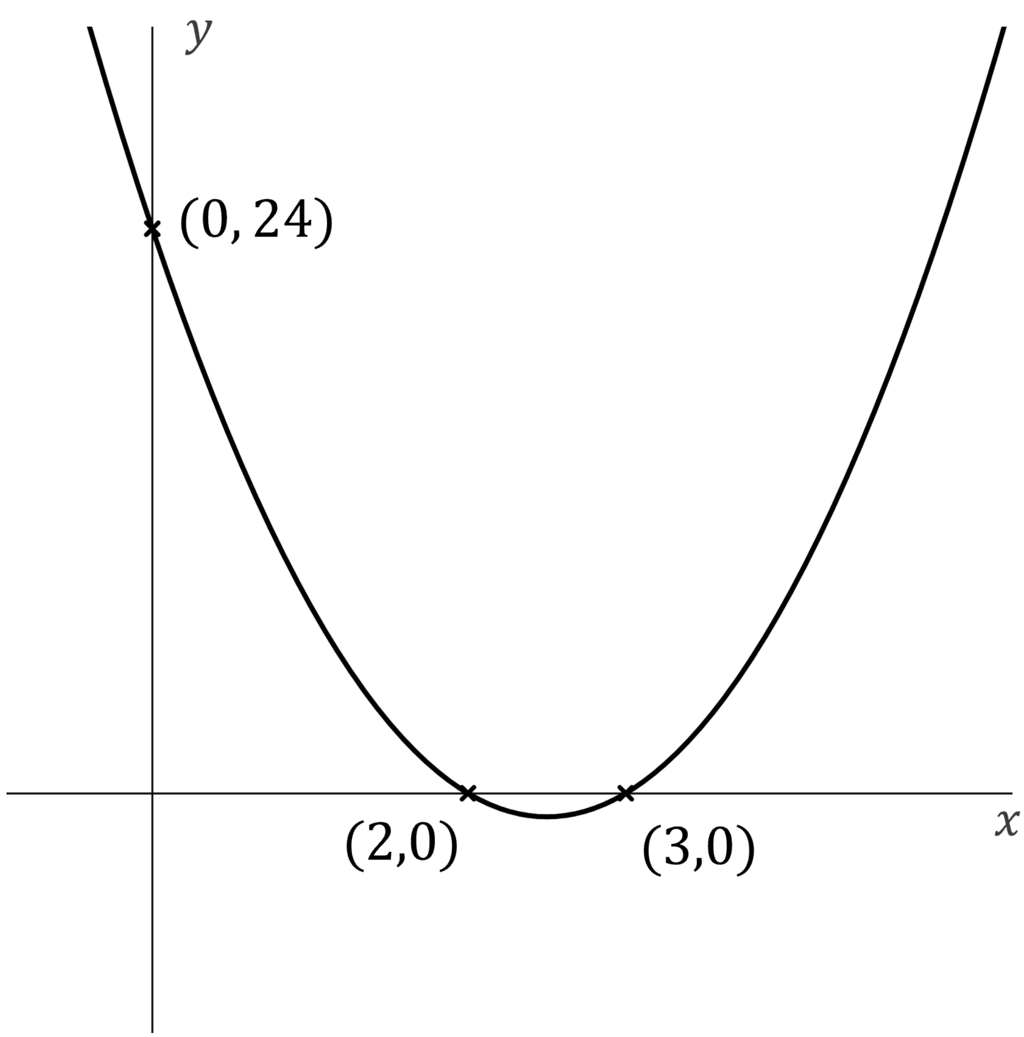
- Use the form $y = a(x - x_1)(x - x_2)$ to fill in x_1 and x_2
  - The roots are at $(x_1, 0)$ and $(x_2, 0)$
- Then substitute in the other known point (x, y) to find a
- If $a = 1$ then you only need either the **vertex** or the **roots**

Worked Example

(a) Find the equation of the graph below.



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The graph shows the roots and a point on the curve (in this case the y-intercept)

Use the form $y = a(x - x_1)(x - x_2)$ to fill in x_1 and x_2 by inspection

The roots are at $(x_1, 0)$ and $(x_2, 0)$

$$y = a(x-2)(x-3)$$

Substitute in the other known point (0, 24) to find a

$$24 = a(0-2)(0-3)$$
$$24 = a(-2)(-3)$$
$$24 = 6a$$
$$4 = a$$

Write the full equation

$$\mathbf{y = 4(x-2)(x-3)}$$

You could also write this in expanded form: $y = 4x^2 - 20x + 24$

(b) Find the equation of the graph below.

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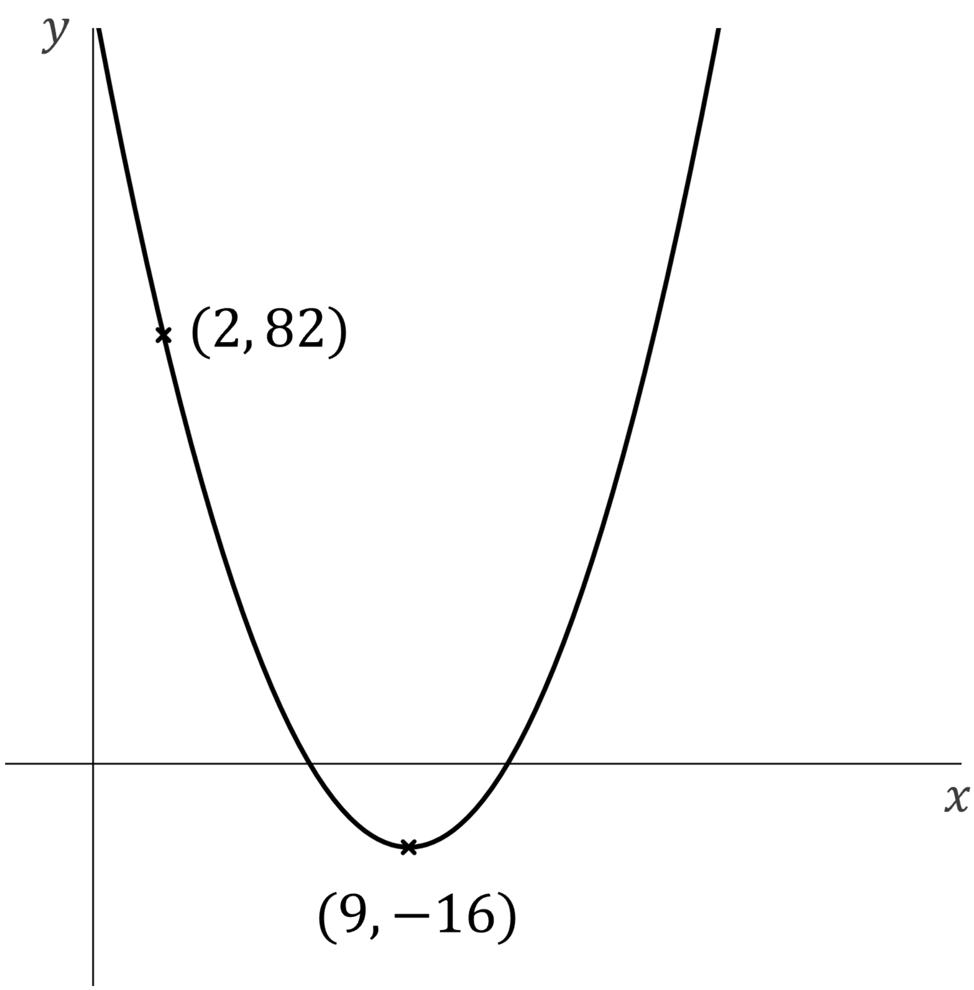

The graph shows the vertex and a point on the curve

Use the form $y = a(x - p)^2 + q$ to fill in p and q by inspection

The vertex is at $(p,\ q)$

$$y = a(x - 9)^2 - 16$$

Substitute in the other known point (2, 82) to find a

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$$82 = a(2-9)^2 - 16$$
$$82 = a(-7)^2 - 16$$
$$82 = 49a - 16$$
$$98 = 49a$$
$$2 = a$$

Write the full equation

$$\boldsymbol{y = 2(x-9)^2 - 16}$$

You could also write this in expanded form: $y = 2x^2 - 36x + 146$

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Drawing Graphs from Tables

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Drawing Graphs Using a Table

How do I draw a graph using a table of values?

- To create a **table of values**
  - **substitute** different ***x*-values** into the **equation**
    - This gives the ***y*-values**
- To plot the points
  - use the *x* and *y*-values to mark **crosses** on the grid at the **coordinates (*x* , *y*)**
    - Each point is expected to be plotted to **an accuracy within half of the smallest square** on the grid
- **Draw** a **single smooth freehand** curve
  - Go through **all** the plotted **points**
  - Make it the **shape** you would **expect**
    - For example, **quadratic** curves have a vertical line of **symmetry**
  - Do **not** use a ruler for curves!

Which numbers should I be careful with?

- For **quadratic** graphs, be careful substituting in **negative** numbers
  - Always put **brackets** around negative values and use **BIDMAS**
    - For example, substitute $x = -3$ into $y = -x^2 + 8x$
    - This becomes $y = -(-3)^2 + 8(-3)$
    - which simplifies to -9 - 24
    - so $y = -33$
- For **reciprocal** graphs like $y = \frac{1}{x}$, or $y = \frac{1}{x^2}$, do **not include x = 0**
  - You **cannot divide** by **zero**
    - You get an **error** on your **calculator**

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- There is **no value** at $x = 0$
  - The L-shaped branches **can't cross** the ***y*-axis**
  - There will be a **vertical asymptote** at $x = 0$
- An example is given below with $y = \frac{1}{x}$

x	-3	-2	-1	0	1	2	3
y	$-\frac{1}{3}$	$-\frac{1}{2}$	-1	No value	1	$\frac{1}{2}$	$\frac{1}{3}$

- You should also be careful when there is a **combination of different types of function**
  - E.g. $y = x^2 - \frac{1}{x} + 4$ has a quadratic term and a reciprocal term
  - This makes it **harder to know the shape** of the graph
  - But you can still **use a table of values** to plot them
  - Just be aware of points like $x = 0$ as described above, where there will be no value

How do I use the table function on my calculator?

- **Calculators** can create **tables of values** for you
- Find the **table** function
  - Type in the **graph** equation (called the **function**, f(x))
    - Use the alpha button then *X* or *x*
    - Press = when finished
  - If you are asked for another function, g(x), press = to ignore it
- Enter the **start** value
  - The first *x*-value in the table
  - Press =
- Enter the **end** value
  - The last *x*-value in the table

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- Enter the **step size**
  - How big the steps (gaps) are from one *x*-value to the next
  - Press =
- Then scroll up and down to see all the ***y*-values**

Examiner Tips and Tricks

- If you find a point that doesn't seem to fit the shape of the curve, check your working!
- If any *y*-values are given in the question, check that your calculations agrees with them

Worked Example

(a) Complete the table of values for the graph of $y = 10 - 8x^2$.

x	-1.5	-1	-0.5	0	0.5	1	1.5
y		2					-8

Use the table function on your calculator for $\mathrm{f}(x) = 10 - 8x^2$
Start at -1.5, end at 1.5 and use steps of 0.5

Alternatively, substitute the *x*-values into the equation, for example *x* = -1.5

$$\begin{aligned} y &= 10 - 8(-1.5)^2 \\ &= 10 - 8 \times 2.25 \\ &= 10 - 18 \\ &= -8 \end{aligned}$$

x	-1.5	-1	-0.5	0	0.5	1	1.5
y	-8	2	8	10	8	2	-8

(b) Plot the graph of $y = 10 - 8x^2$ on the axes below, for values of x from -1.5 to 1.5.

Carefully plot the points from your table on to the grid
Note the different scales on the axes

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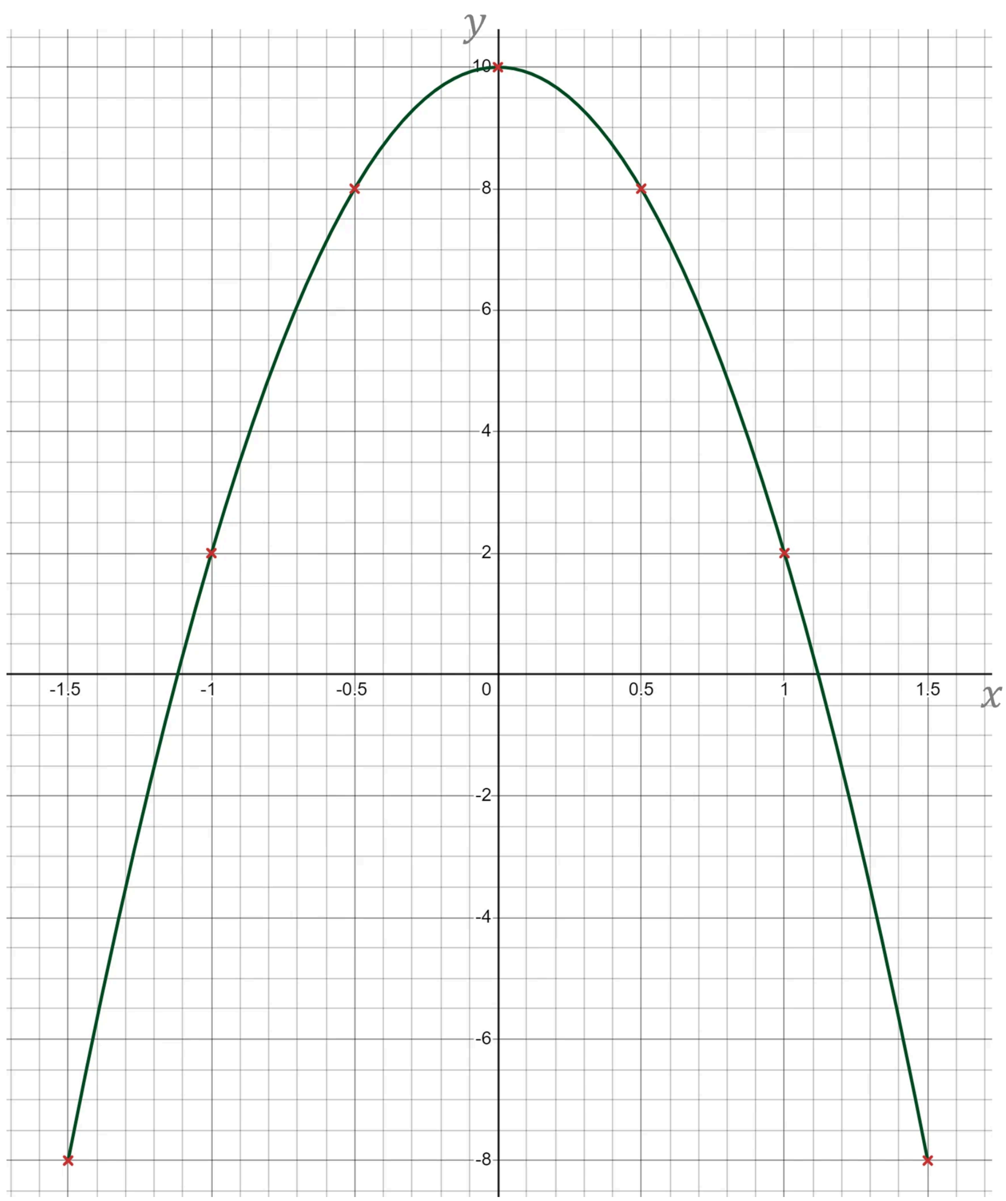
Join the points with a smooth curve (do not use a ruler)



(c) Write down the equation of the line of symmetry of the curve.

There is a vertical line of symmetry about the y-axis

The equation of the y-axis is $x = 0$

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$x = 0$

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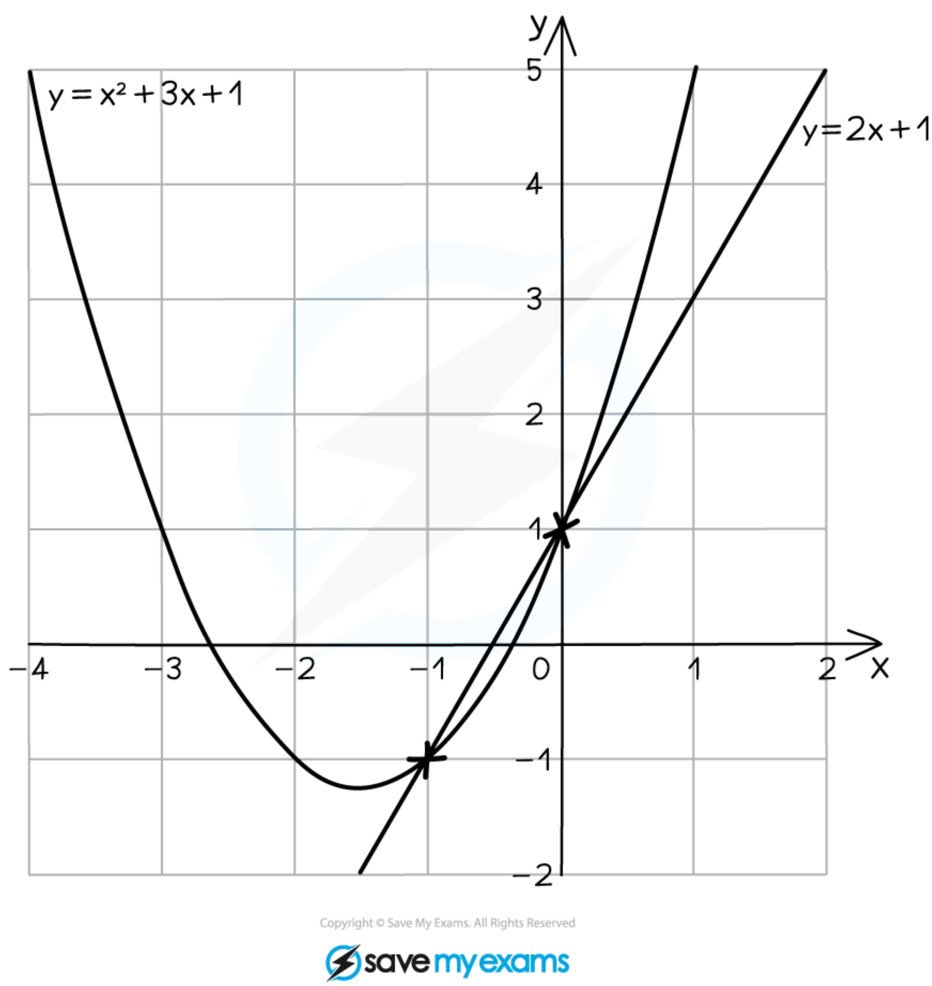
Solving Equations Using Graphs

Solving Equations Using Graphs

How do I find the coordinates of points of intersection?

- Plot **two graphs** on the **same** set of **axes**
  - The **points of intersection** are where the two lines **meet**
- For example, plot $y = x^2 + 3x + 1$ and $y = 2x + 1$ on the same axes
  - They meet twice, as shown
  - The **coordinates** of **intersection** are (-1, -1) and (0, 1)



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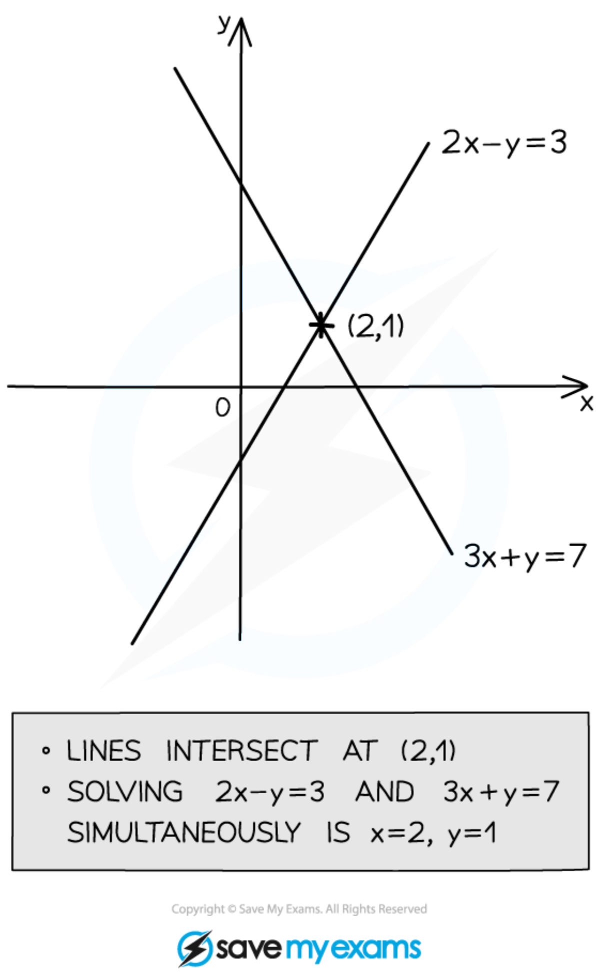
How do I solve simultaneous equations graphically?

- The x and y **solutions** to **simultaneous equations** are the x and y **coordinates** of the **point of intersection**
- For example, to solve $2x - y = 3$ and $3x + y = 7$ simultaneously
  - **Rearrange** them into the form $y = mx + c$
    - $y = 2x - 3$ and $y = -3x + 7$
  - Use a **table of values** to plot each line
  - Find the **point of intersection**, (2, 1)
  - The **solutions** are therefore $x = 2$ and $y = 1$



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How do I use graphs to solve equations?

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- This is easiest explained through an example
- You can use the **graph** of $y = x^2 - 4x - 2$ to **solve** the following **equations**
  - $x^2 - 4x - 2 = 0$
    - The **solutions** are the two ***x*-intercepts**
    - This is where the curve cuts the *x*-axis (also called **roots**)
  - $x^2 - 4x - 2 = 5$
    - The **solutions** are the two ***x*-coordinates** where the curve **intersects** the **horizontal line** $y = 5$
  - $x^2 - 4x - 2 = x + 1$
    - The **solutions** are the two ***x*-coordinates** where the curve **intersects** the **straight line** $y = x + 1$
    - The straight line must be **plotted** on the same axes first
- To solve a **different** equation like $x^2 - 4x + 3 = 1$, if you are **already given** the graph of an equation, e.g. $y = x^2 - 4x - 2$
  - **add / subtract terms** to both sides to get "given graph = ..."
    - For example, subtract 5 from both sides
      - $x^2 - 4x - 2 = -4$
      - You can now draw on the horizontal line $y = -4$ and find the *x*-coordinates of the points of intersection

Examiner Tips and Tricks

- When solving equations in *x*, only give ***x*-coordinates** as final answers
  - Include the *y*-coordinates if solving simultaneous equations

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Worked Example

Use the graph of $y = 10 - 8x^2$ shown to estimate the solutions of each equation given below.

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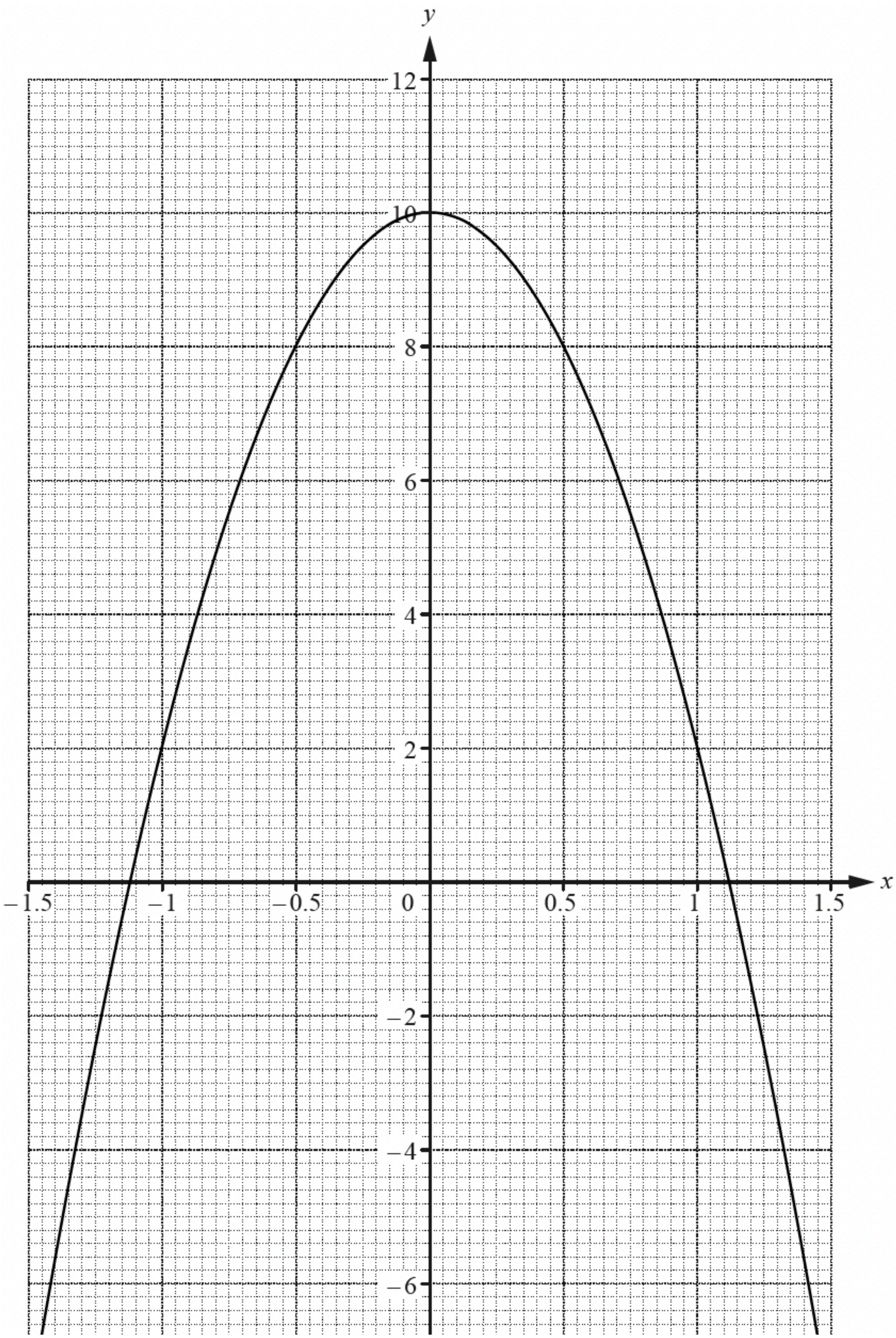

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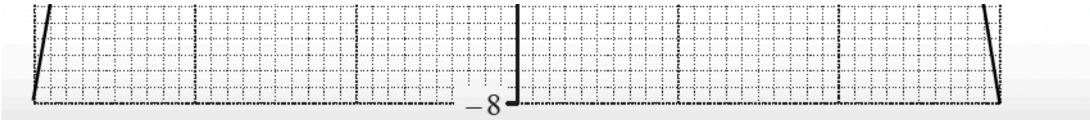

(a) $10 - 8x^2 = 0$

This equals zero, so the *x*-intercepts are the solutions
Read off the values where the curve cuts the *x*-axis
Use a suitable level of accuracy (no more than 2 decimal places from the scale of this graph)

-1.12 and 1.12

These are the two solutions to the equation

***x* = -1.12 and *x* = 1.12**

A range of solutions are accepted, such as "between 1.1 and 1.2"
Solutions must be ± of each other (due to the symmetry of quadratics)

(b) $10 - 8x^2 = 8$

This equals 8, so draw the horizontal line *y* = 8
Find the *x*-coordinates where this cuts the graph

-0.5 and 0.5

These are the two solutions to the original equation

***x* = -0.5 and *x* = 0.5**

The solutions here are exact

Worked Example

The graph of $y = x^3 + x^2 - 3x - 1$ is shown below.

Use the graph to estimate the solutions of the equation $x^3 + x^2 - 4x = 0$.

Give your answers to 1 decimal place.

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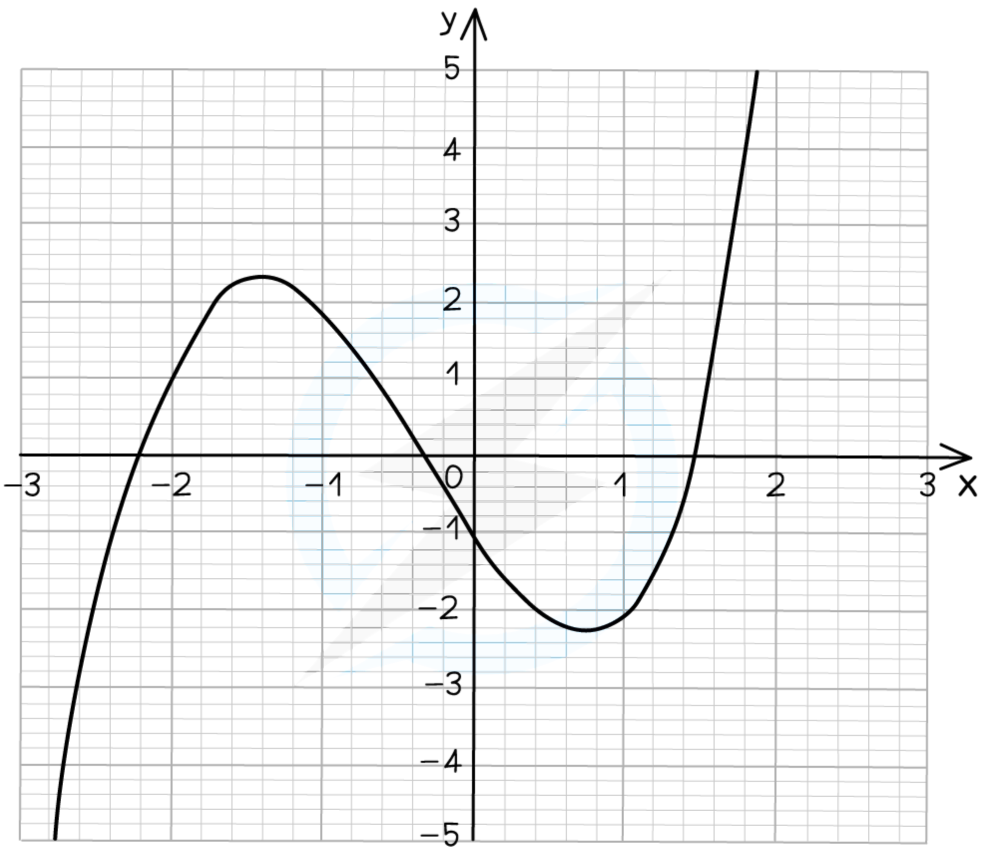

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We are given a different equation to the one plotted so we must rearrange it to $\text{graph} = mx + c$, in this case $x^3 + x^2 - 3x - 1 = mx + c$

$$x^3 + x^2 - 4x = 0$$

$$+x - 1 \qquad\qquad +x - 1$$

$$x^3 + x^2 - 3x + 1 = x - 1$$

Now plot $y = x - 1$ on the same axes

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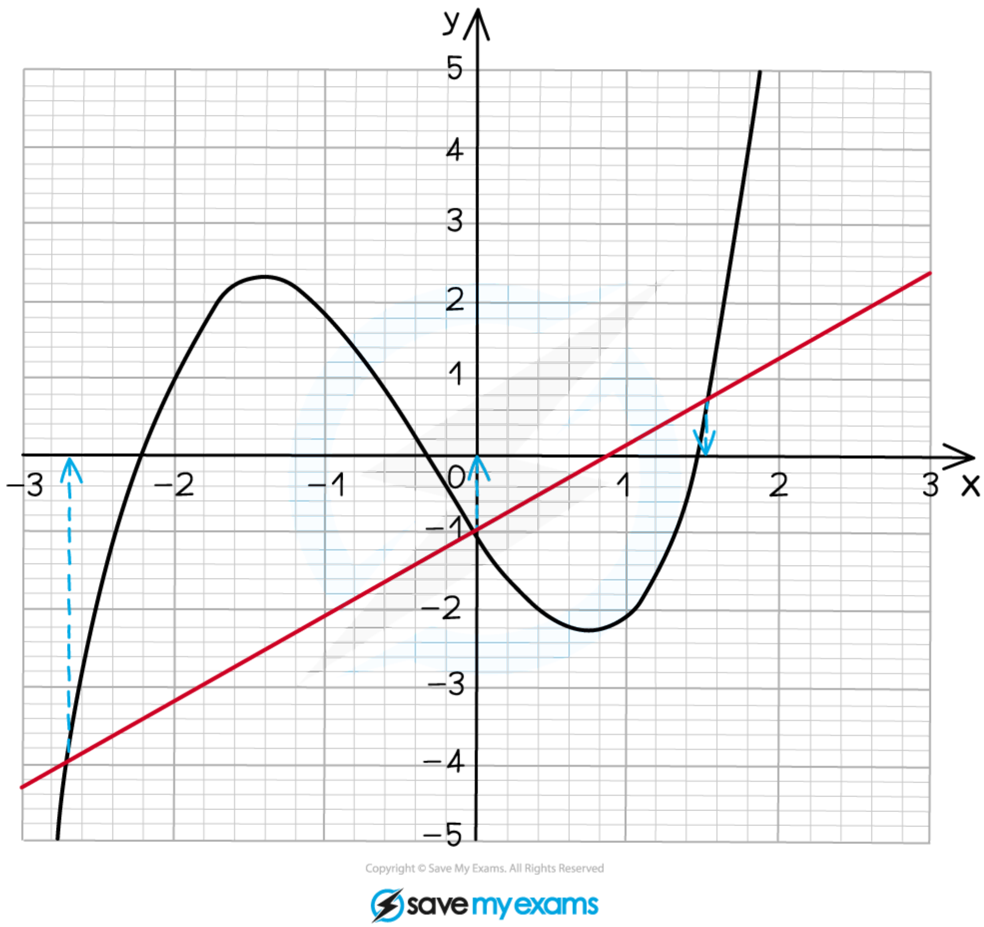

The solutions are the x-coordinates of where the curve and the straight line intersect

$$x = -2.6, \quad x = 0, \quad x = 1.6$$

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Trigonometric Graphs

Drawing Trig Graphs

What are trig graphs?

- **Trigonometric (trig) graphs** are the graphs of
  - $y = \sin x$
  - $y = \cos x$
  - $y = \tan x$
- The variable x is like an angle
  - but the angle can now go **beyond acute** to become **obtuse** and **reflex**
    - $0° \le x \le 360°$
- Trig graphs have **repeating** (periodic) shapes and **symmetries** that you need to know

How do I draw the graph of y = sin x?

- The graph of $y = \sin x$ is a **wave** that oscillates between **heights of 1 and -1** and **repeats every 360°** (its **period** is 360°)
  - It goes through the **origin**, (0, 0)
  - Then every 90° it cycles through the heights 1, 0, -1, 0, ...

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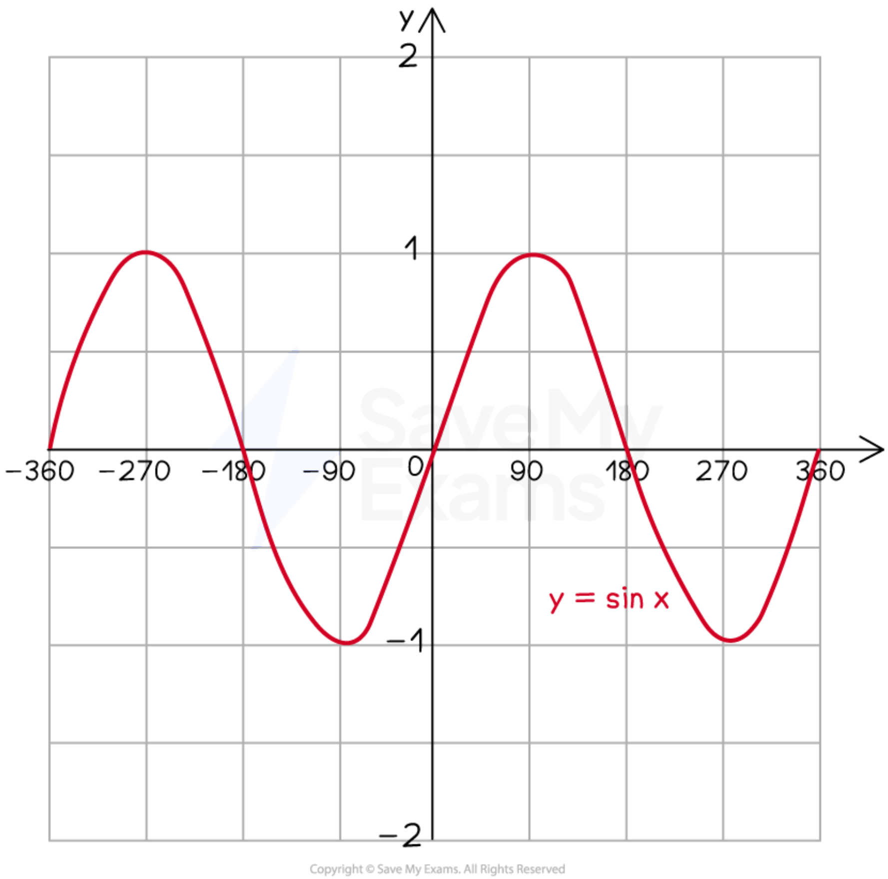

How do I draw the graph of y = cos x?

- The graph of $y = \cos\ x$ is a **wave** that oscillates between **heights of 1 and -1** and **repeats every 360°** (its **period** is 360°)
  - It has a **y-intercept of 1**, coordinates (0, 1)
  - Then every 90° it cycles through the heights 0, -1, 0, 1, ...
- $y = \cos\ x$ is the same as translating $y = \sin\ x$ by 90 to the left

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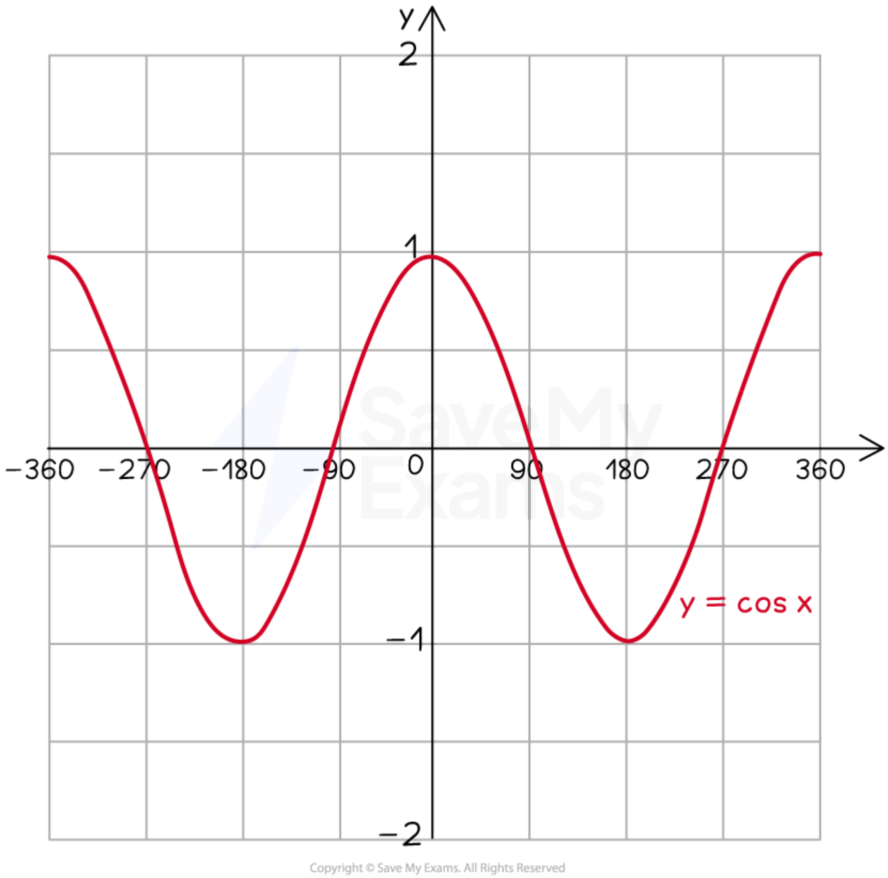

How do I draw the graph of y = tan x?

- The graph of $y = \tan x$ is **not a wave** but consists of **branches** that **repeat every 180°** (its **period** is 180°)
  - This is **half the period** of $\sin x$ and $\cos x$
- There are **dotted vertical lines** that separate the branches called **asymptotes**
  - These are every 180° at $x = 90°$, $x = 270°$, ...
  - The curve cannot touch these, but get **closer and closer** to them
  - A branch starts down at a height of $-\infty$ and goes up to a height of $+\infty$
- $y = \tan x$ goes through the **origin**, (0, 0)

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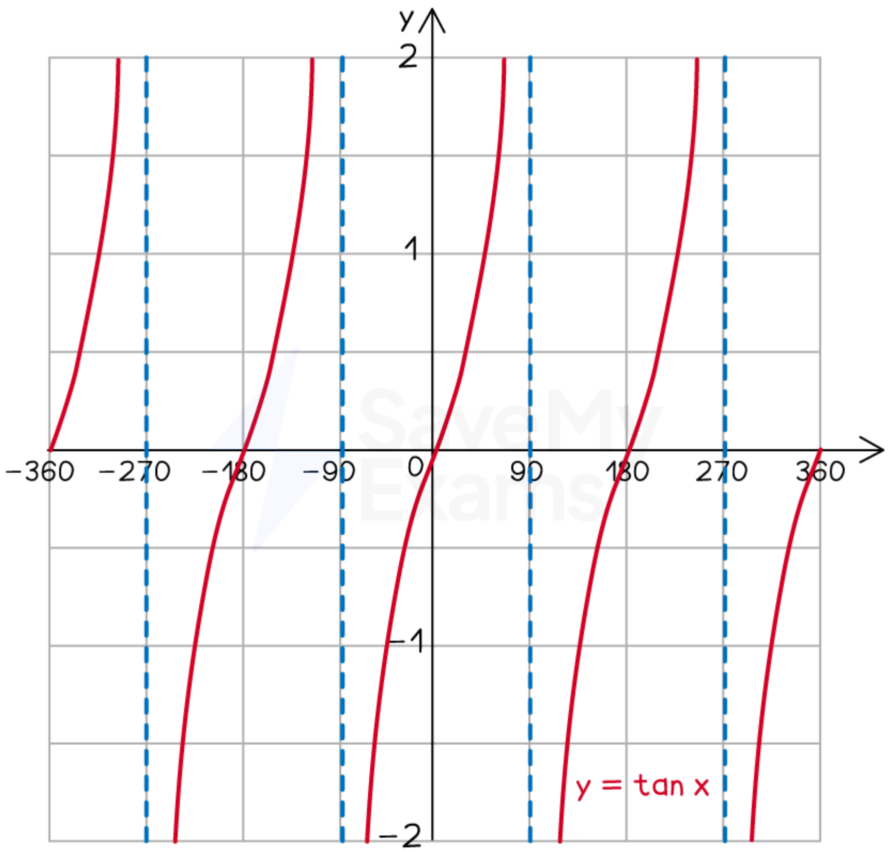

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Worked Example

On the axes provided, sketch the graph of $y = \sin x°$ for $0 \leq x \leq 360$.

Mark 1 and –1 on the *y*-axis
Mark 0, 90, 180, 270 and 360 on the *x*-axis (try to space them evenly apart)

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$y = \sin x$ starts at (0, 0) then every 90° it cycles though heights of 1, 0 , –1, 0, ...

Mark these points on the axes

Join the points with a smooth line

Label the curve with its equation

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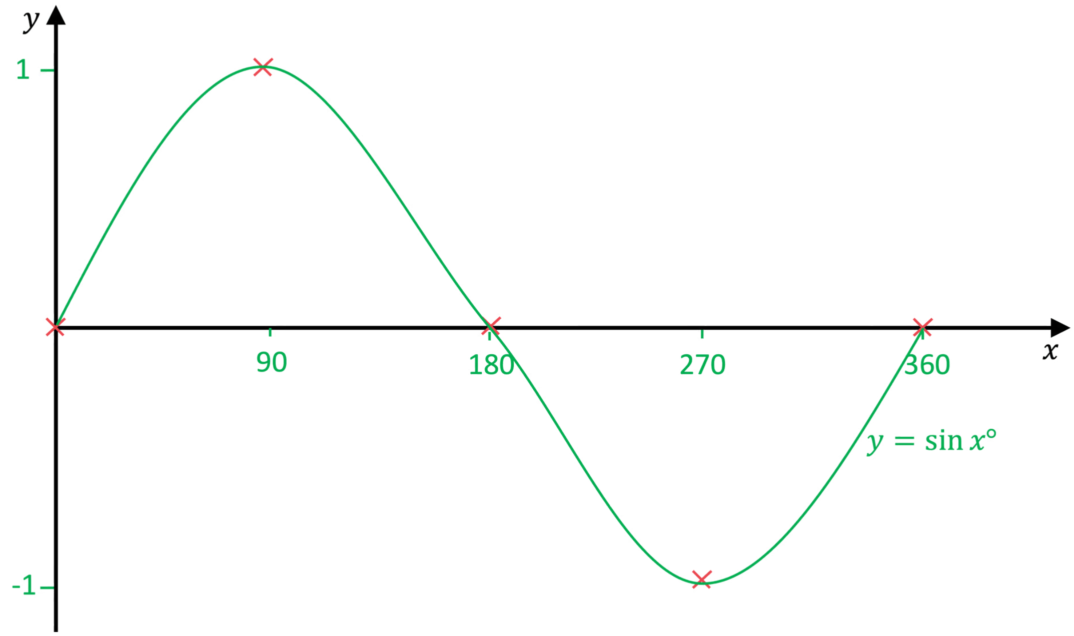

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Solving Trig Equations

Solving Trig Equations

What are trig equations?

- **Trig equations** are equations involving $\sin x$, $\cos x$ and $\tan x$
- They often have **multiple solutions**
  - A **calculator** gives the **first solution**
  - You need to use **trig graphs** to find the others
  - The solutions must lie in the **interval** (range) of x given in the question, e.g. $0° \le x \le 360°$

How do I solve sin x = ...?

- Find the **first solution** of the equation by taking the **inverse sin function** on your **calculator** (or using an exact trig value)
  - E.g. For the first solution of the equation $\sin x = 0.5$ for $0° \le x \le 360°$
    - This gives $x = \sin^{-1}(0.5) = 30°$
- Then **sketch** the **sine graph** for the **given interval**
  - Identify the **first solution** on the graph
  - Use the **symmetry** of the graph to find **additional solutions**
  - E.g. For the equation $\sin x = 0.5$ for $0° \le x \le 360°$
    - Sketch the graph $y = \sin x$ for $0° \le x \le 360°$
    - Draw on $\sin(30) = 0.5$
    - By the symmetry, the new value of x is $180° - 30° = 150°$
    - The solutions are $30°$ or $150°$

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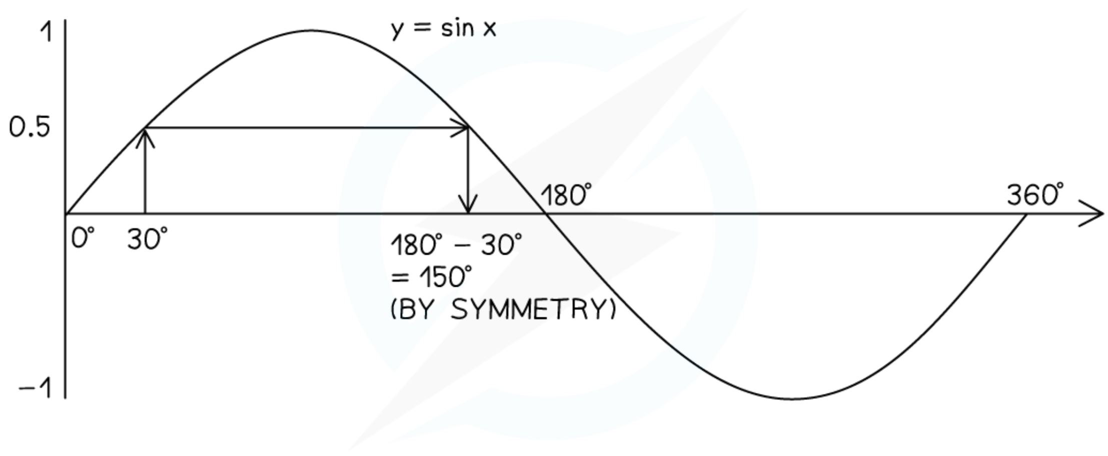

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- **Check** the solutions
  - E.g. For the equation $\sin x = 0.5$ for $0° \le x \le 360°$
    - Substitute $x = 30°$ and $x = 150°$ in to the calculator
    - $\sin(30)$ and $\sin(150)$ both give a value of 0.5, so are correct
- In general, if x is an **acute** solution to $\sin x = \ldots$
  - Then $180 - x$ is an **obtuse** solution to the same equation

How do I solve cos x = ...?

- Find the **first solution** of the equation by taking the **inverse cos function** (or using an exact trig value)
  - E.g. For the first solution of the equation $\cos x = 0.5$ for $0° \le x \le 360°$
    - This gives $x = \cos^{-1}(0.5) = 60°$
- Then **sketch** the **cosine graph** for the **given interval**
  - Identify the **first solution** on the graph
  - Use the **symmetry** of the graph to find **additional solutions**
  - E.g. For the equation $\cos x = 0.5$ for $0° \le x \le 360°$
    - Sketch the graph $y = \cos x$ for $0° \le x \le 360°$

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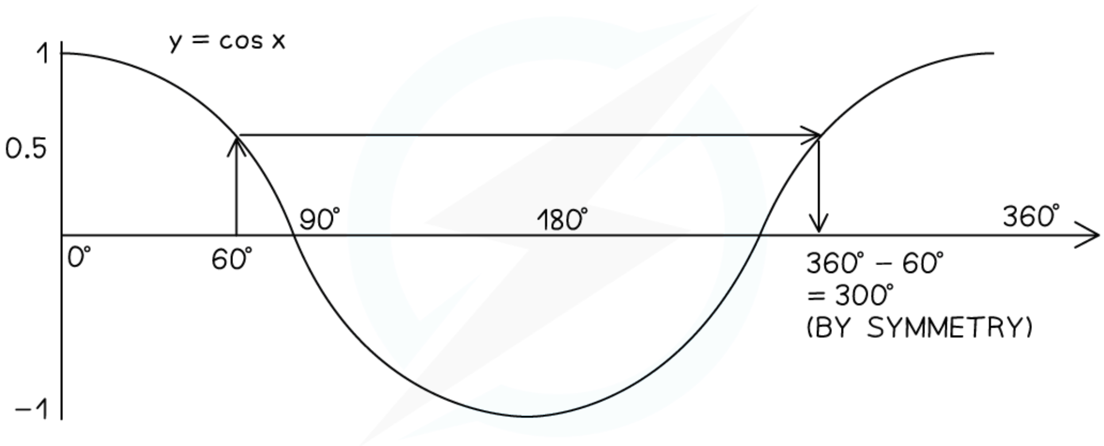
- By the symmetry, the new value of x is $360° - 60° = 300°$
- The solutions are $60°$ or $300°$



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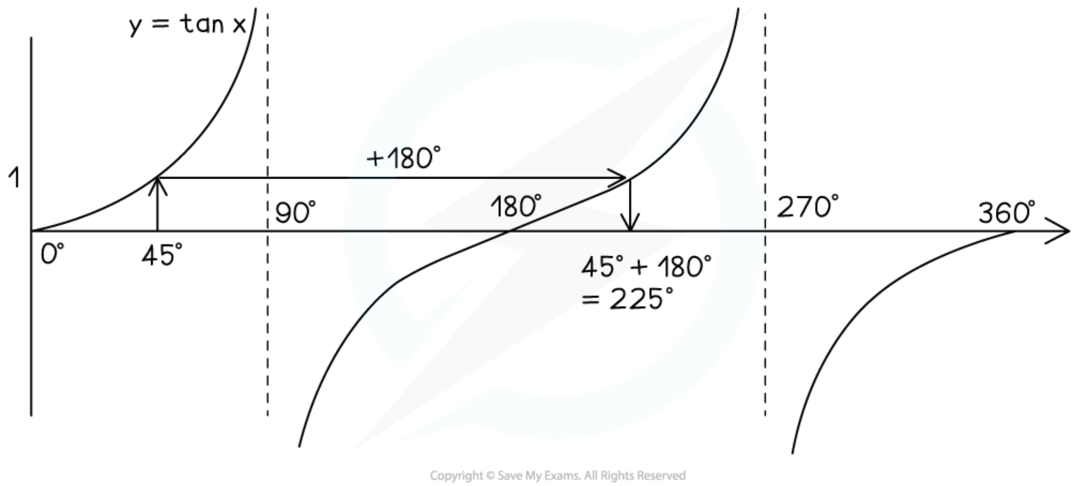
- **Check** the solutions
  - E.g. For the equation $\cos x = 0.5$ for $0° \leq x \leq 360°$
    - Substitute $x = 60°$ and $x = 300°$ in to the calculator
    - $\cos(60)$ and $\cos(300)$ both give a value of 0.5 so are correct
- In general, if x is a **solution** to $\cos x = \ldots$
  - Then $360 - x$ is **another solution** to the same equation

How do I solve tan x = …?

- Find the **first solution** of the equation by taking the **inverse tan function** (or using an exact trig value)
  - E.g. For the first solution of the equation $\tan x = 1$ for $0° \leq x \leq 360°$
    - This gives $x = \tan^{-1}(1) = 45°$
- Then **sketch** the **tangent graph** for the **given interval**
  - Identify the **first solution** on the graph
  - Use the **periodic nature** of the graph to find **additional solutions**

- E.g. For the equation $\tan x = 1$ for $0° \le x \le 360°$
  - Sketch the graph $y = \tan x$ for $0° \le x \le 360°$
  - By the periodic nature, the new value of x is $45° + 180° = 225°$



- **Check** the solutions
  - E.g. For the equation $\tan x = 0.5$ for $0° \le x \le 360°$
    - Substitute $x = 45°$ and $x = 225°$ in to the calculator
    - $\tan(45)$ and $\tan(225)$ both give a value of 1 so are correct
- In general, if x is a solution to $\tan x = ...$
  - Then $x + 180$ is **another solution** to the same equation

How do I rearrange trig equations?

- Trig equations may be given in a **different form**
  - Equations may require **rearranging** first
    - E.g. $2 \sin x - 1 = 0$ can be rearranged to $\sin x = \frac{1}{2}$
  - They can then be solved as usual

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What do I do if the first solution from my calculator is negative?

- Sometimes the first solution given by the calculator for x will be **negative**
  - Continue sketching the graph to the **left** of the x-axis to help
  - Then find **solutions** that lie in the **interval** given in the question

Examiner Tips and Tricks

- Know how to use the **inverse functions** on your calculator
  - It may involve **exact trig values** which do not need a calculator
- **Check** your solutions by substituting them back into the original equation

Worked Example

Use the graph of $y = \sin x$ to solve the equation $\sin x = 0.25$ for $0° \le x \le 360°$.

Give your answers correct to 1 decimal place.

Use a calculator to find the first solution
Take the inverse sin of both sides

$$x = \sin^{-1}(0.25) = 14.47751...$$

Sketch the graph of $y = \sin x$

Mark on (roughly) where $x = 14.48$ and $y = 0.25$ would be

Draw a vertical line up to the curve
Draw another line horizontally across to the next point on the curve
Bring a line vertically back down to the x-axis

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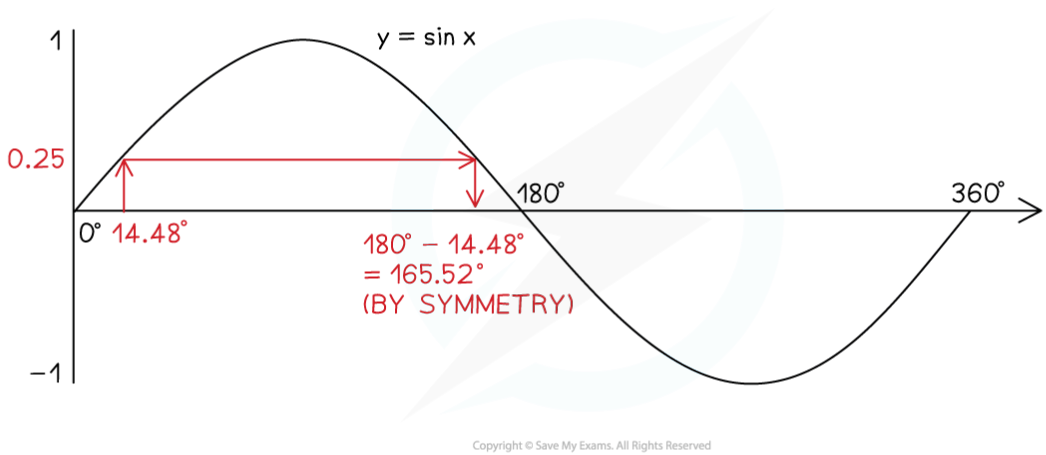

Find this value using the symmetry of the curve
Subtract 14.48 from 180

$$180 - 14.48 = 165.52$$

Give both answers correct to 1 decimal place

$$\boldsymbol{x = 14.5°} \text{ or } \boldsymbol{x = 165.5°}$$

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