AQA GCSE Maths: Higher

Your notes

Completing the Square

Contents

Completing the Square

Your notes

Completing the Square

How can I rewrite the first two terms of a quadratic expression as the difference of two squares?

- Look at the quadratic expression $x^2 + bx + c$
- The first **two terms** can be written as the **difference of two squares** using the following rule

$x^2 + bx$ is the same as $(x + p)^2 - p^2$ where p is **half** of b

- Check this is true by expanding the right-hand side
  - Is $x^2 + 2x$ the same as $(x + 1)^2 - 1^2$?
    - Yes: $(x + 1)(x + 1) - 1^2 = x^2 + 2x + 1 - 1 = x^2 + 2x$
- This works for **negative** values of b too
  - $x^2 - 20x$ can be written as $(x - 10)^2 - (-10)^2$ which is $(x - 10)^2 - 100$
  - A negative b does not change the sign at the end

How do I complete the square?

- **Completing the square** is a way to rewrite a quadratic expression in a form containing a **squared bracket**
- To complete the square on $x^2 + 10x + 9$
  - Use the rule above to replace the first two terms, $x^2 + 10x$, with $(x + 5)^2 - 5^2$
  - then add 9: $(x + 5)^2 - 5^2 + 9$
  - **simplify** the **numbers**: $(x + 5)^2 - 25 + 9$
  - answer: $(x + 5)^2 - 16$

How do I complete the square when there is a coefficient in front of the x^2 term?

- You first need to take a out as a **factor** of the x^2 and x terms only
  - **Factorise** the **first two terms**

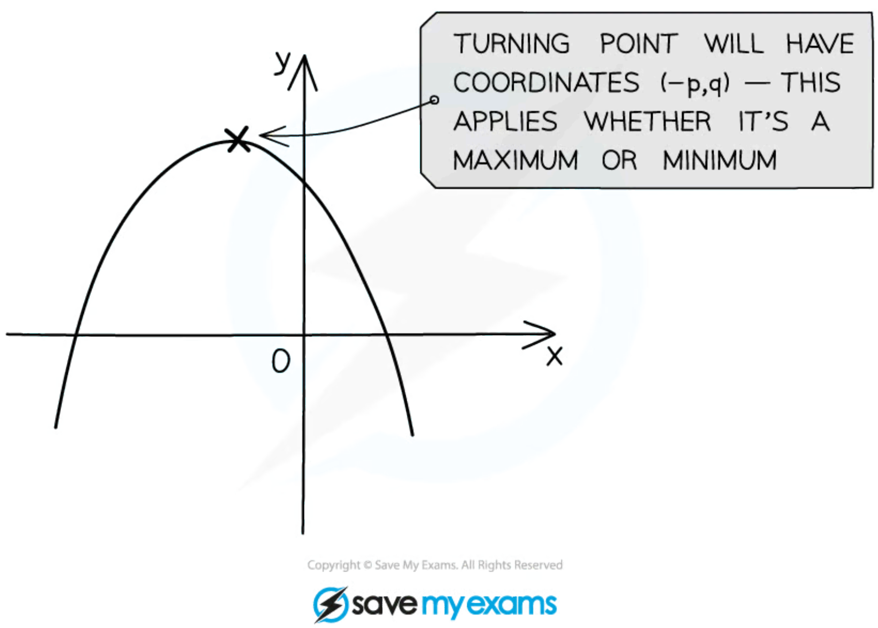
Your notes

- $ax^2 + bx + c = a\left[x^2 + \frac{b}{a}x\right] + c$
  - Use **square-shaped brackets** here to avoid confusion with round brackets later
- Then complete the square on the bit **inside** the brackets: $x^2 + \frac{b}{a}x$
  - This gives $a[(x+p)^2 - p^2] + c$
    - where p is half of $\frac{b}{a}$
- Finally **multiply** this expression through by a (from outside the square brackets) and **add the c** on to the end
  - $a(x+p)^2 - ap^2 + c$
    - This looks far more complicated than it is in practice!
  - Usually you are asked to give your final answer in the form $a(x+p)^2 + q$
  - For example, $y = 4x^2 + 16x + 5$
    - Factorise out 'a' on the right-hand side (use square brackets)
      - $y = 4[x^2 + 4x] + 5$
    - Replace $x^2 + 4x$ with $(x + 2)^2 - 2^2$ (because $p = \frac{4}{2} = 2$)
      - $y = 4[(x + 2)^2 - 2^2] + 5$
    - Simplify the terms inside the square brackets
      - $y = 4[(x + 2)^2 - 4] + 5$
    - Multiply everything inside the square brackets by 4
      - $y = 4(x + 2)^2 - 16 + 5$
    - Simplify to get the final answer
      - $y = 4(x + 2)^2 - 11$
- For quadratics like $-x^2 + bx + c$, do the above but with $a = -1$

How do I find the turning point by completing the square?

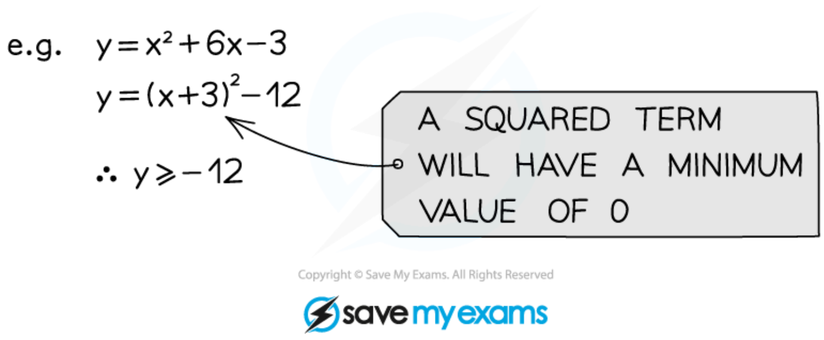
Your notes

- Completing the square helps us find the **turning point** on a quadratic graph
  - If $y = (x+p)^2 + q$ then the turning point is at $(-p, q)$
    - Notice the negative sign in the *x*-coordinate
    - This links to transformations of graphs
    - A translation of $y = x^2$ by *p* to the left and *q* up
  - If $y = a(x+p)^2 + q$ then the turning point is still at $(-p, q)$
    - The *a* does **not change** the coordinates
    - The turning point is a minimum point if $a > 0$
    - or a maximum point if $a < 0$
- This can also help you **create the equation** of a quadratic when given the turning point



- It can also be used to **prove** or **show** results using the fact that any **squared term**, such as the squared bracket $(x \pm p)^2$, will always be **greater than or equal to 0**
  - You cannot square a number and get a negative value
  - The smallest a squared term can be is 0

Your notes



Examiner Tips and Tricks

- To know if you have completed the square correctly, expand your answer to check

Worked Example

(a) By completing the square, find the coordinates of the turning point on the graph of $y = x^2 + 6x - 11$.

Find half of +6 (call this *p*)

$$p = \frac{6}{2} = 3$$

Write $x^2 + 6x$ in the form $(x + p)^2 - p^2$

$x^2 + 6x$ is the same as $(x + 3)^2 - 3^2$

Put this result into the equation of the curve

$$y = (x + 3)^2 - 3^2 - 11$$

Simplify the numbers

$$y = (x + 3)^2 - 20$$

Your notes

Use the fact that the turning point of $y=(x+p)^2+q$ is at $(-p,q)$
Here p = 3 and q = -20

turning point at (-3, -20)

(b) Write $-3x^2+12x+24$ in the form $a(x+p)^2+q$.

Factorise -3 out of the first two terms only
Use square-shaped brackets

$$-3[x^2-4x]+24$$

Complete the square on the x^2 - 4x inside the brackets
Write in the form $(x+p)^2 - p^2$ where p is half of -4

$$-3[(x-2)^2-(-2)^2]+24$$

Simplify the numbers inside the brackets
$(-2)^2$ is 4

$$-3[(x-2)^2-4]+24$$

Multiply -3 by all the terms inside the square brackets
(You do not multiply -3 by the 24)

$$-3(x-2)^2+12+24$$

Simplify the numbers

$$-3(x-2)^2+36$$

This is now in the form $a(x+p)^2+q$ where a = -3, p = -2 and q = 36

$$-3(x-2)^2+36$$