AQA GCSE Maths: Higher

Your notes

Sine, Cosine Rule & Area of Triangles

Contents

The Sine Rule

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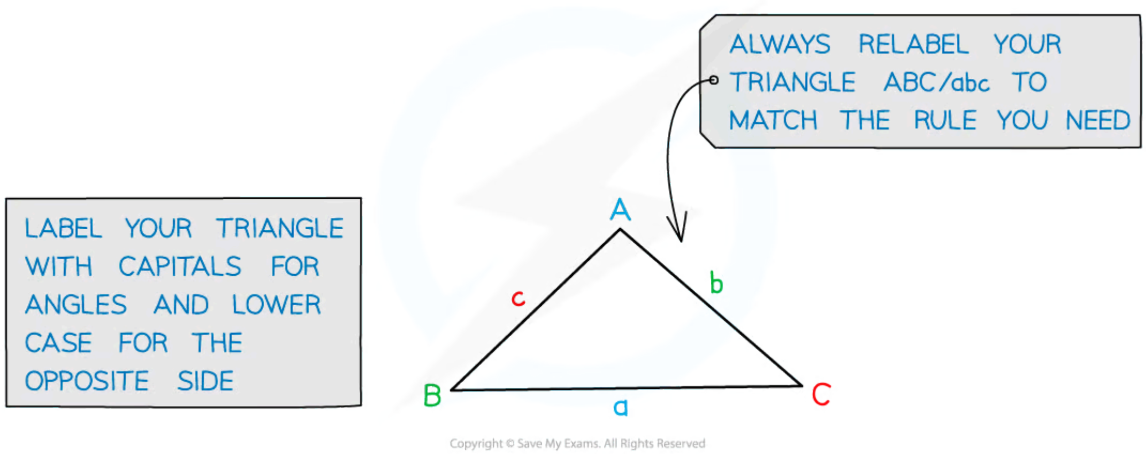
Sine Rule

What is the sine rule?

- The **sine rule** is used in **non right-angled triangles**
  - It allows us to find missing side lengths or angles
- It states that for any triangle with angles *A*, *B* and *C*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

- Where
  - a is the side **opposite** angle *A*
  - b is the side **opposite** angle *B*
  - c is the side **opposite** angle *C*



How do I use the sine rule to find missing lengths?

- Use the **sine rule**
  - when you have **opposite pairs** of sides and angles in the question

Your notes

- *a* and *A*, or *b* and *B*, or c and C
- **Start by labelling your triangle** with the angles and sides
  - Angles have upper case letters
  - Sides **opposite** the angles have the equivalent lower case letter
- To find a **missing length**, substitute numbers into the formula

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

  - You only need to have two parts equal to each other (not all three)
    - Then solve to find the side you need

How do I use the sine rule to find missing angles?

- To find a **missing angle**, it is easier to **rearrange** the formula first by flipping each part

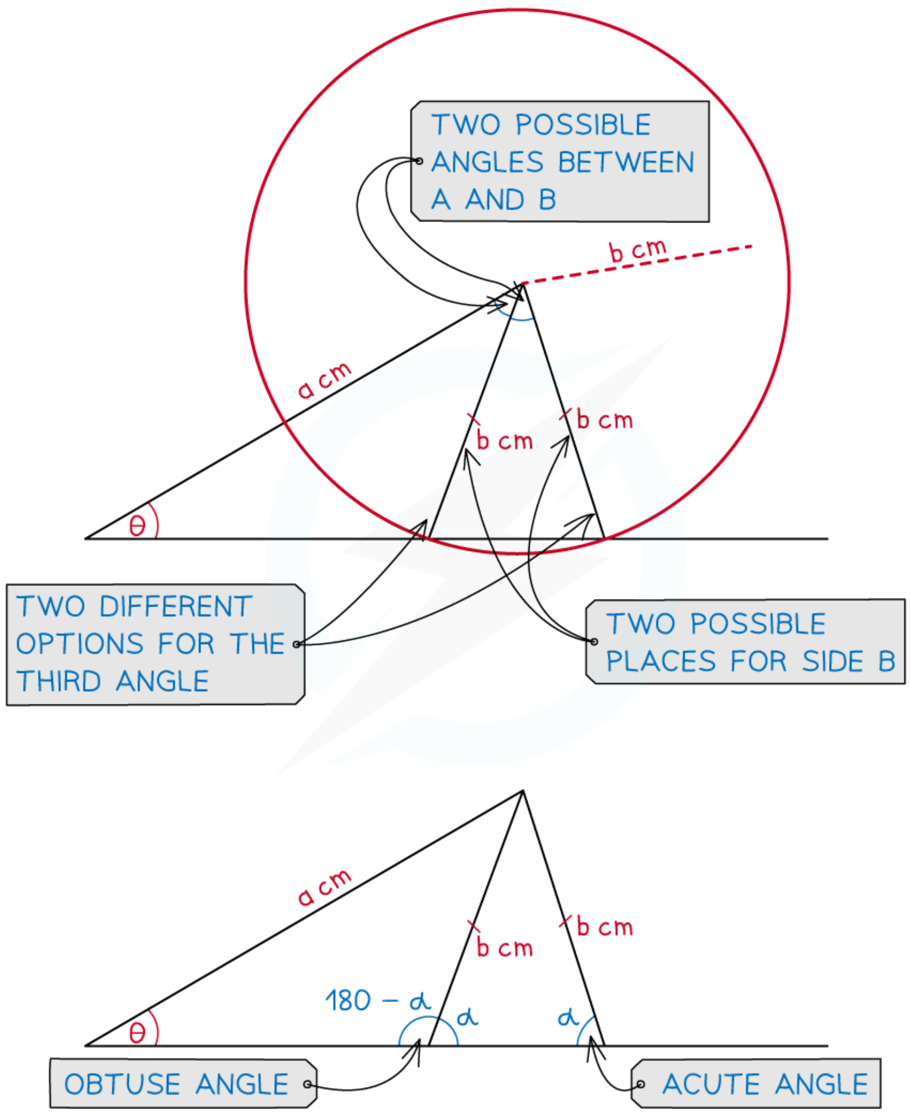
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

  - The angles are now in the numerators of the fractions
  - Substitute the values you have into the formula and solve
    - You will need to use **inverse sine** in your calculation, $\sin^{-1}(...)$

What is the ambiguous case of the sine rule?

- Given information about a triangle, there may be **two different ways** to draw it
- In the diagram below, the lengths of two sides are given, *a* and *b*
  - A base angle is also given, θ, but no angle near *b* is given
  - It turns out that there are **two possible ways** to arrange *b to* complete the triangle!
    - Both triangles have the correct values of *a*, *b* and θ
- To other base angle could either be **obtuse or acute**
  - The **sine rule** only gives the **acute** answer on your calculator
    - You need to check the diagram to see if the angle you need is actually obtuse
    - If it is, use this rule: **obtuse angle = 180 – acute angle**

Your notes



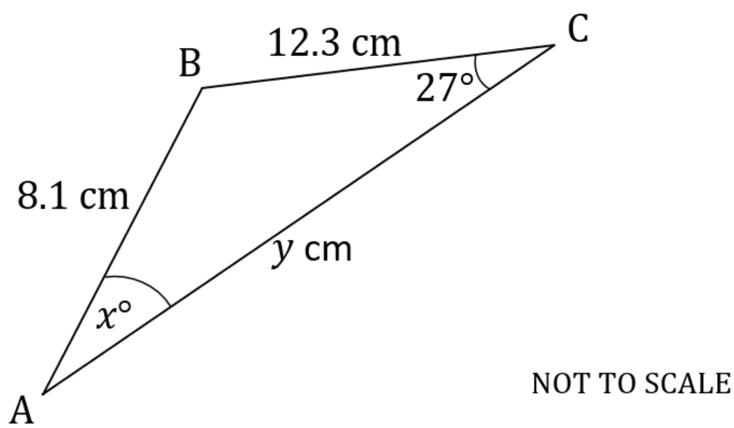
Examiner Tips and Tricks

- The sine rule is given in the formula booklet.

Worked Example

The following diagram shows triangle ABC.

$\text{AB} = 8.1 \text{ cm}$, $\text{BC} = 12.3 \text{ cm}$ and angle $\text{BCA} = 27°$.

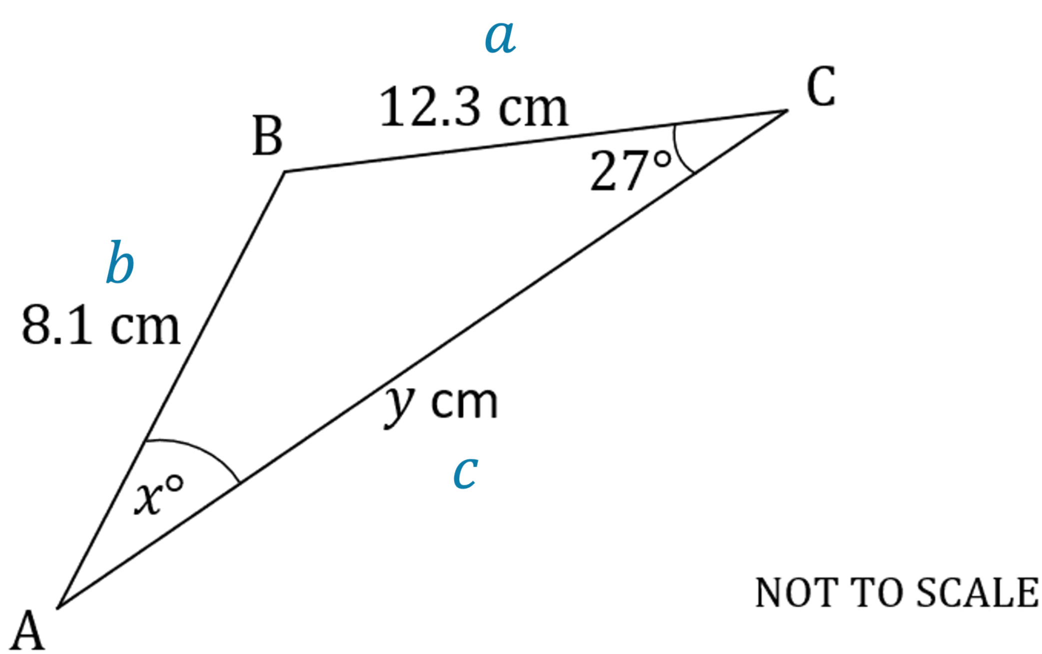


(a) Calculate the value of x.

Label the sides of the diagram

Your notes

Your notes



x is an angle so use the sine rule with the angles on top

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

In practice, you only need to equate two of these three parts

$$\frac{\sin x}{12.3} = \frac{\sin 27}{8.1}$$

$$\sin x = \frac{12.3 \sin 27}{8.1}$$

$$x = \sin^{-1}\left(\frac{12.3 \sin 27}{8.1}\right)$$

$$x = 43.58207...$$

43.6° **(to 1 d.p)**

(b) Calculate the value of y.

To find *y* you need to know the angle opposite (angle ABC)
You know 27 and *x* from above, so subtract these from 180

$$\text{Angle } ABC = 180 - 27 - 43.58207...$$
$$= 109.41792...$$

y is a length so use the sin rule with the sides on the top

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{y}{\sin(109.41792...)} = \frac{8.1}{\sin 27}$$
$$y = \frac{8.1 \sin(109.41792...)}{\sin 27}$$
$$y = 16.82691...$$

$\boldsymbol{y = 16.8}$ cm (to 3 s.f.)

Your notes

The Cosine Rule

Your notes

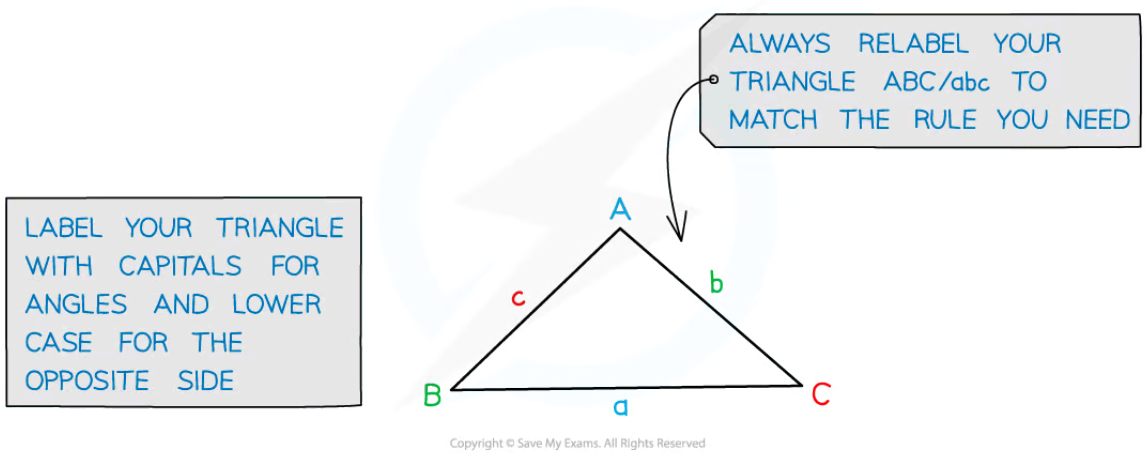
Cosine Rule

What is the cosine rule?

- The **cosine rule** is used in **non right-angled triangles**
  - It allows us to find missing side lengths or angles
- It states that for any triangle

$$a^2 = b^2 + c^2 - 2bc \cos A$$

- Where
  - a is the side **opposite** angle *A*
  - b and c are the other two sides
    - b and c are either side of angle *A*
    - *A* is the angle between them



How do I use the cosine rule to find a missing length?

- Use the **cosine rule** for lengths
  - when you have **two sides and the angle between them**

- and you want to find the **opposite side**, *a*
- **Start by labelling your triangle** with the angles and sides
  - Angles have upper case letters
  - Sides **opposite** the angles have the equivalent lower case letter
- **Substitute** values into $a^2 = b^2 + c^2 - 2bc\cos A$
  - Make *a* the subject (don't forget to **square root**)

How do I use the cosine rule to find a missing angle?

- Use the **cosine rule** for angles
  - when you have **all three sides**
  - and you want to find an angle
- It helps to **rearrange** the formula as follows, by adding $2bc\cos A$ to both sides then making $\cos A$ the subject

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc\cos A \\ a^2 + 2bc\cos A &= b^2 + c^2 \\ 2bc\cos A &= b^2 + c^2 - a^2 \\ \cos A &= \frac{b^2 + c^2 - a^2}{2bc} \end{aligned}$$

- Use the formula $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$ to find the **unknown angle** *A*
  - Remember, *A* is the angle **between** sides *b* and c
    - (you may need to relabel the triangle)
  - You will need to use **inverse cosine** at the end, $\cos^{-1}(...)$
- Unlike the sine rule, there is no ambiguous case of the cosine rule

Examiner Tips and Tricks

- You are given the cosine rule in the form $a^2 = b^2 + c^2 - 2bc\cos A$ on the formula sheet

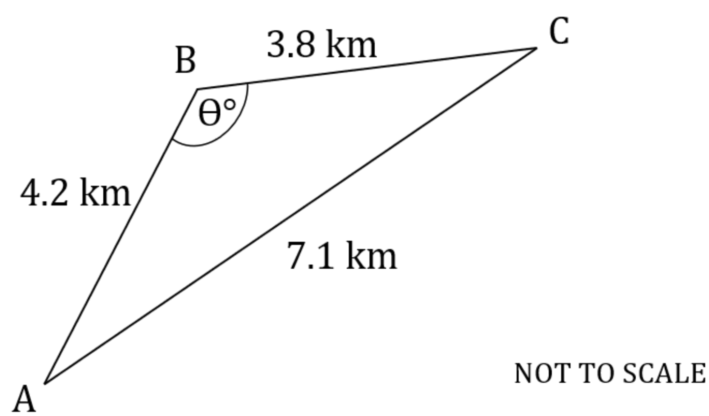
Your notes

- Getting an error on your calculator when finding an angle may mean you have rearranged the formula incorrectly

Your notes

Worked Example

The following diagram shows triangle ABC, where $\text{AB} = 4.2 \text{ km}$, $\text{BC} = 3.8 \text{ km}$ and $\text{AC} = 7.1 \text{ km}$.



Calculate the value of angle ABC.

The side opposite the angle is 7.1, so $a = 7.1$

The sides 4.2 and 3.8 are b and c (in either order)

Use the cosine rule in the rearranged form $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$$\cos \theta = \frac{4.2^2 + 3.8^2 - 7.1^2}{2(4.2)(3.8)}$$

$$\theta = \cos^{-1}\left(\frac{4.2^2 + 3.8^2 - 7.1^2}{2(4.2)(3.8)}\right)$$

$$\theta = 125.04699...$$

$\theta = 125.0°$ (to 1 d.p.)

Your notes

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Area of a Triangle

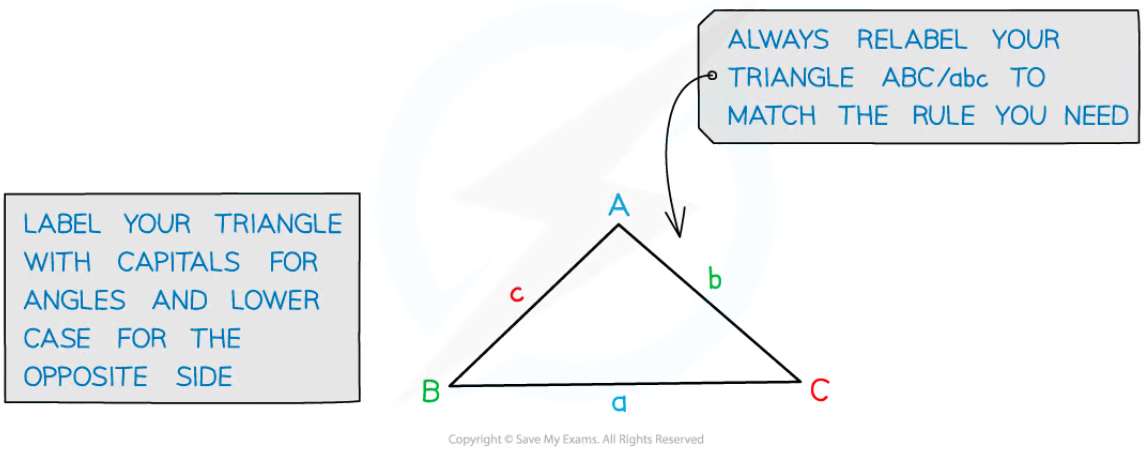
Area of a Triangle

How do I find the area of a non right-angled triangle?

- The area of **any triangle** can be found using the formula

$$\text{Area} = \frac{1}{2}ab\sin C$$

- C is the angle **between** sides a and b



- Label your triangle correctly
  - Make sure that C is always the angle **between** the two sides
- If angle C is 90°, you get a right-angled triangle
  - sin 90° = 1 so the formula becomes the familiar "Area = ½ × base × height"!

Examiner Tips and Tricks

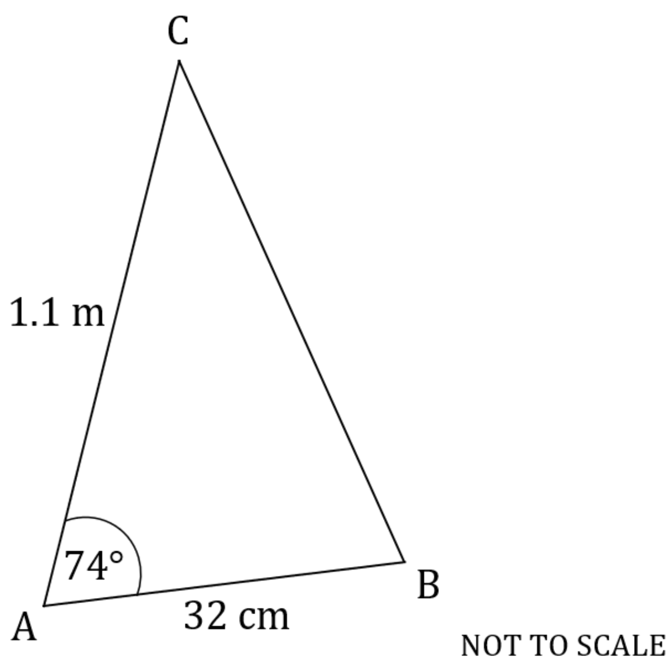
- You are given the triangle area rule on the formula sheet

Your notes

Worked Example

The following diagram shows triangle ABC.

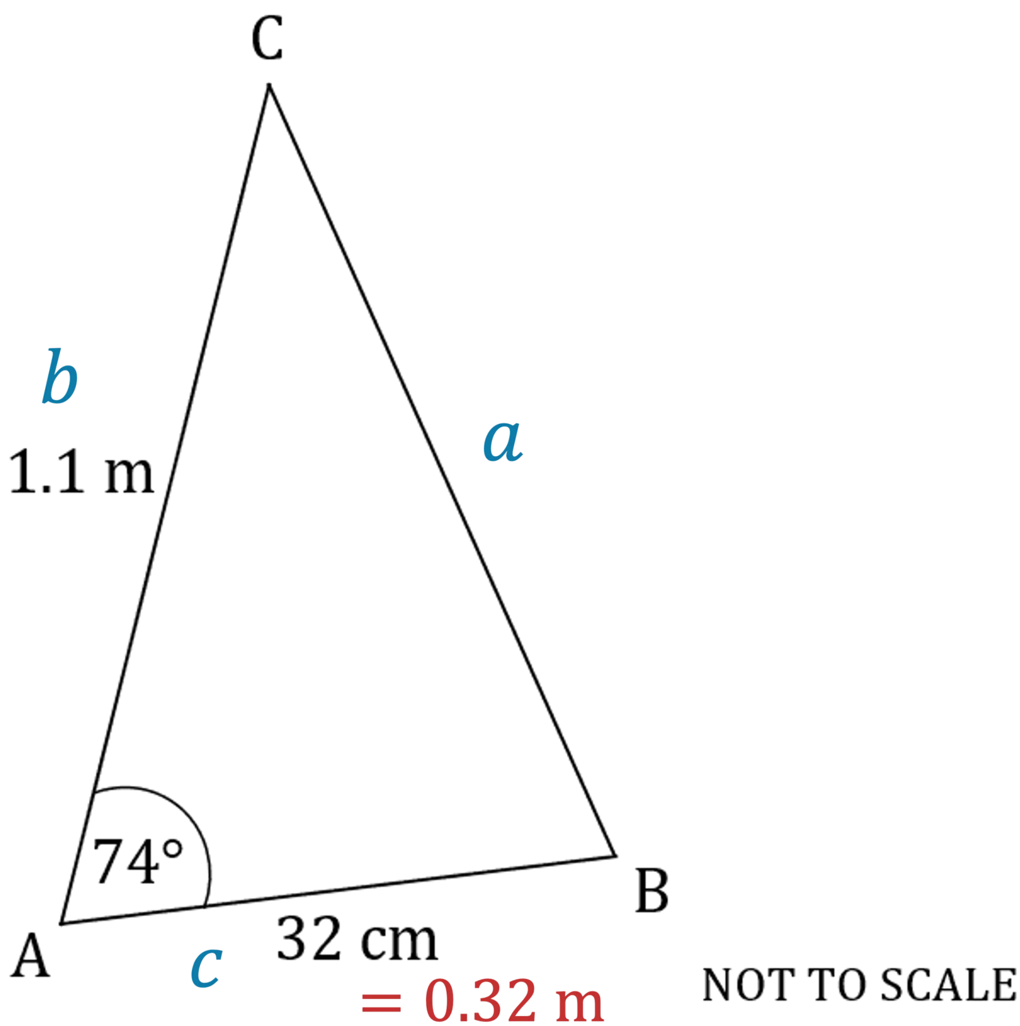
$\text{AB} = 32\ \text{cm}$, $\text{AC} = 1.1\ \text{m}$ and angle $\text{BAC} = 74°$.



Calculate the area of triangle, giving your answer in m^2.

Label the sides of the triangle
Convert the sides to be in the same units

Your notes



Use the area of a triangle formula, $\text{Area} = \frac{1}{2}bc\sin A$

$$\text{Area} = \frac{1}{2}(1.1)(0.32)\sin 74$$
$$= 0.16918...$$

The area is 0.169 m^2 (to 3 s.f.)

Deciding the Trigonometric Rule

Your notes

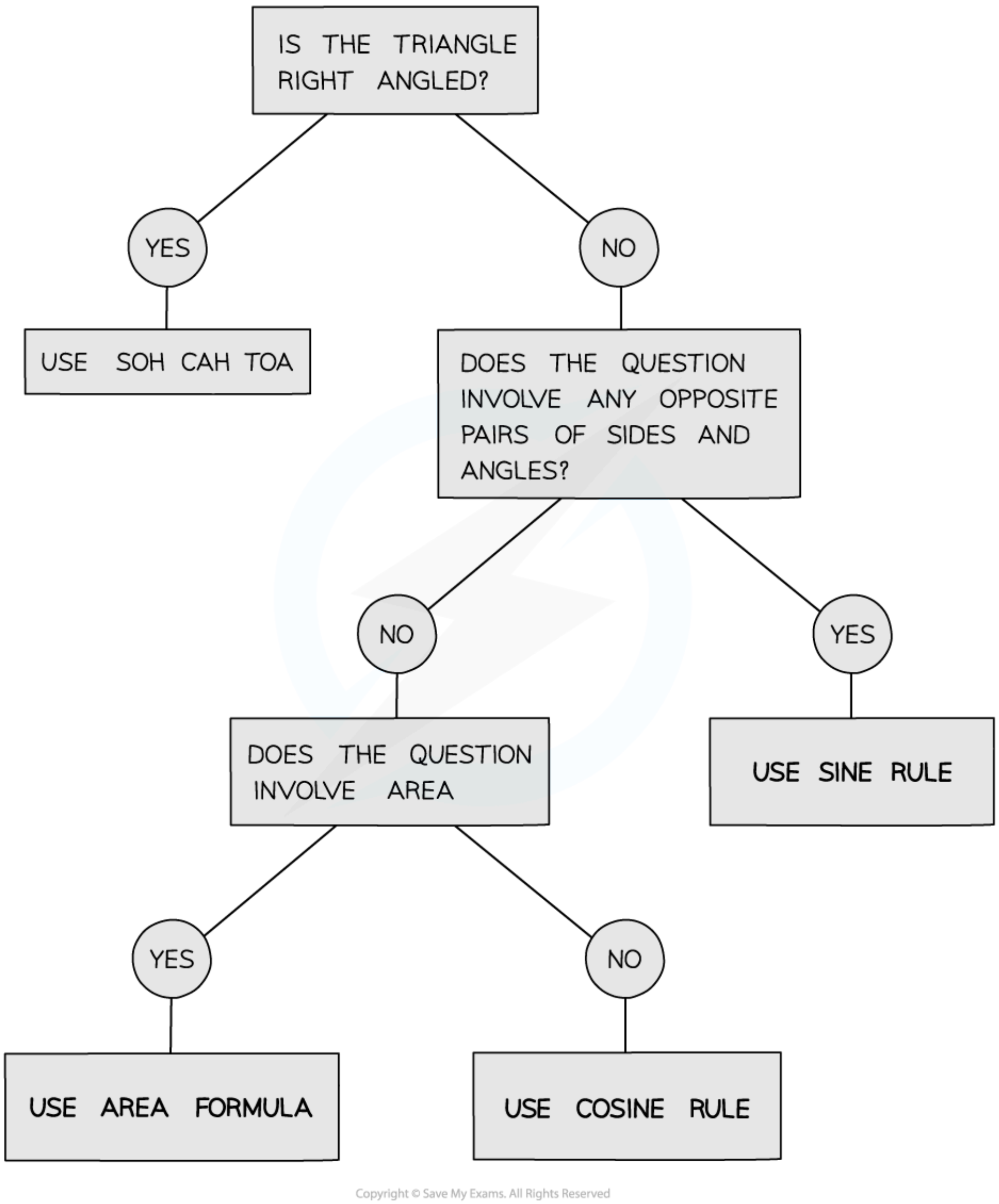
Applications of Trigonometry

How do I decide which trig rule to use?

- **Different rules** are required depending on the question
  - You need to be able to decide which is appropriate to use
  - Think about what **information** you **have** and what you want to **find**
- This table summarises the possibilities:

If you know	And you want to know	Use
Two sides and an angle opposite one of the sides	The angle opposite the other side	Sine rule
Two angles and a side opposite one of the angles	The side opposite the other angle	Sine rule
Two sides and the angle **between** them	The third side	Cosine rule
All three sides	Any angle	Cosine rule
Two sides and the angle **between** them	The area of the triangle	Area of a triangle rule

Your notes



Can I use multiple trig rules in the same question?

- Harder questions will require you to use **more than one trig rule**
  - For example, you may need the sine rule **followed by** the cosine rule
- The **area formula** only works for an angle between two sides

    - If you are **not** given this setup, you may need to use the sine or cosine rule first
- If it looks like no rule would work, remember that all **angles in a triangle sum to 180**
    - This often helps to find a missing angle

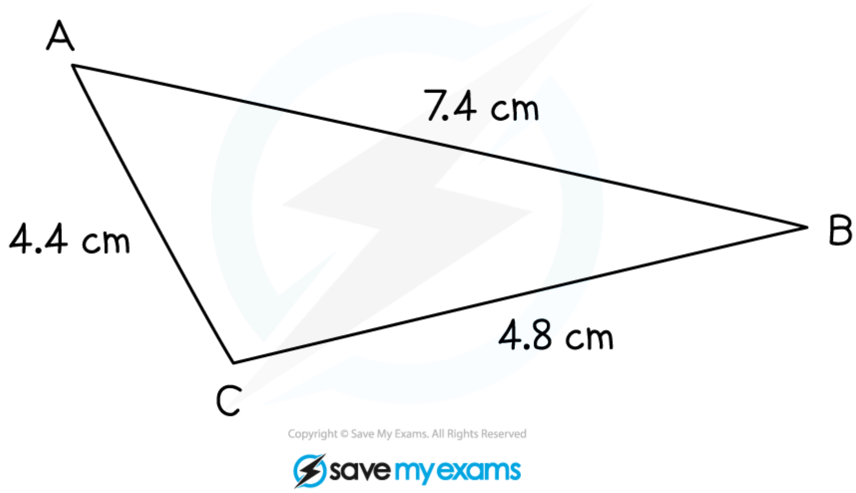
Your notes

Examiner Tips and Tricks

- Look at the number of marks for a question - if it is a lot, you are likely to need more than one trig rule!

Worked Example

Find the area of the triangle below.



The area of a triangle can be found using the formula $A = \frac{1}{2}ab \sin C$

The three side lengths are known , but we need to find an angle in order to calculate the area
Because we know all three sides, any of the angles could be found

Find angle *ABC* using the cosine rule

Your notes

Cosine Rule: $a^2 = b^2 + c^2 - 2bc\cos A$,
where A is the angle opposite side a

$$AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos ABC$$
$$4.4^2 = 7.4^2 + 4.8^2 - 2(7.4)(4.8)\cos ABC$$

Rearrange to make $\cos ABC$ the subject

$$4.4^2 - (7.4^2 + 4.8^2) = -2(7.4)(4.8)\cos ABC$$
$$\cos ABC = \frac{4.4^2 - 7.4^2 - 4.8^2}{-2(7.4)(4.8)}$$

Use your calculator to find the value of $\cos ABC$

$$\cos ABC = \frac{487}{592}$$

Use the cos^{-1} button on your calculator to find the value of ABC

$$ABC = \cos^{-1}\left(\frac{487}{592}\right)$$
$$= 34.65054...$$

Now we can find the area of the triangle using the formula and angle *ABC* as the known angle

$$\text{Area} = \frac{1}{2} \times 4.8 \times 7.4 \times \sin 34.65054...$$
$$= 10.09779...$$

$$\textbf{Area = 10.1 cm}^2\textbf{ (3 s.f.)}$$