AQA GCSE Maths: Higher

Your notes

Volume & Surface Area

Contents


Your notes

Volume

What is volume?

- The **volume** of a 3D shape is a measure of how much **space** it takes up
- You need to be able to calculate the volumes of a number of **common 3D shapes**, including:
  - Cubes and cuboids
  - Prisms
  - Pyramids
  - Cylinders
  - Spheres

How do I find the volume of a cube or a cuboid?

- A **cube** is a special **cuboid**, where the length, width and height are all of **equal length**
- A **cuboid** is another name for a rectangular-based **prism**
- To find the volume, V, of a **cube** or a **cuboid**, with length, l, width, w, and height, h, use the formula
  - $V = lwh$
  - This formula is **not** given to you in the exam

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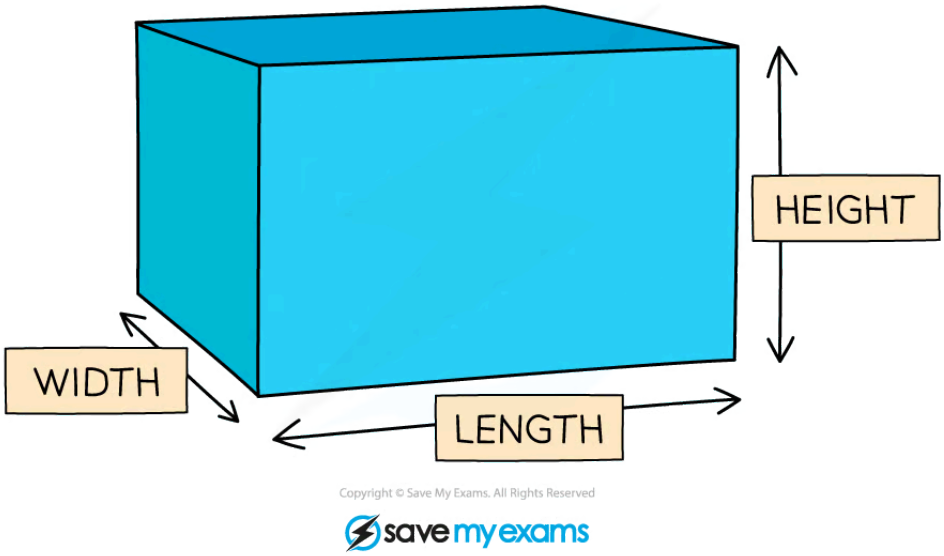

- You will sometimes see the terms 'depth' or 'breadth' instead of 'height' or 'width'

How do I find the volume of a prism?

- A **prism** is a 3D object with a **constant cross-sectional area**
- To find the volume, V, of a **prism**, with cross-sectional area, A, and length, l, use the formula
  - $V = Al$
  - This formula is **not** given to you in the exam

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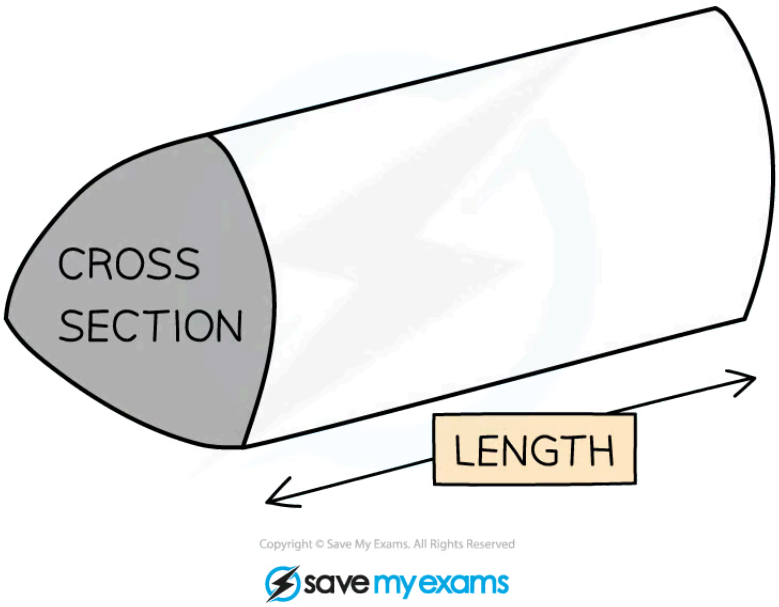
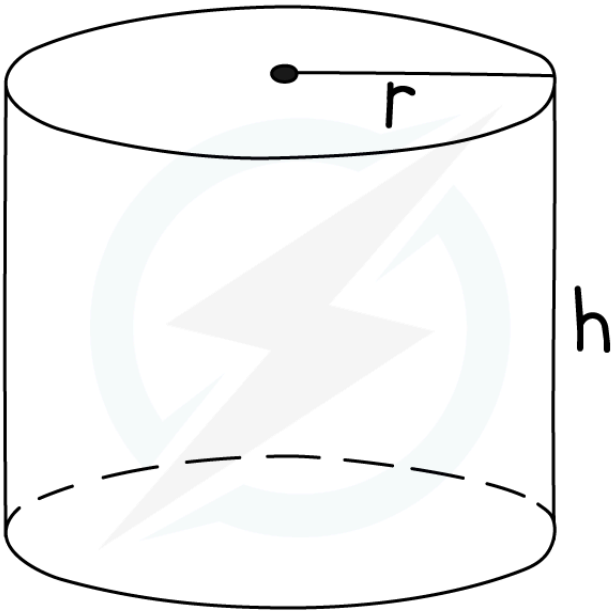


- Note that the cross-section can be **any shape**, so as long as you know its **area** and the **length** of the prism, you can calculate its **volume**
  - If you know the **volume** and **length** of the prism, you can calculate the **area** of the **cross-section**

How do I find the volume of a cylinder?

- To calculate the volume, V, of a **cylinder** with radius, r, and height, h, use the formula
  - $V = \pi r^2 h$
  - This formula is **not** given to you in the exam

Your notes

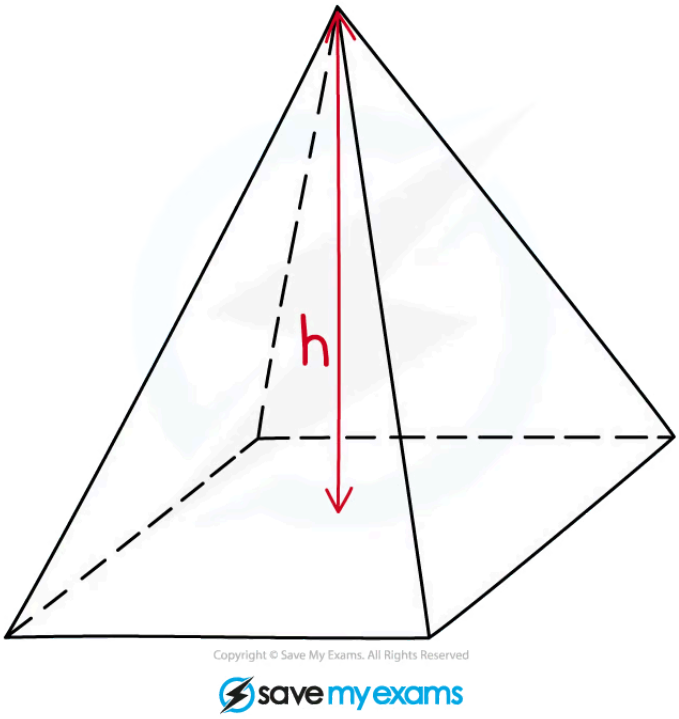


- Note that a cylinder is similar to a **prism,** its cross-section is a circle with area πr^2, and its length is *h*

How do I find the volume of a pyramid?

- To calculate the volume, *V,* of a **pyramid** with base area, *A*, and perpendicular height, *h*, use the formula
  - $V = \frac{1}{3}Ah$
  - This formula is **not** given to you in the exam

Your notes

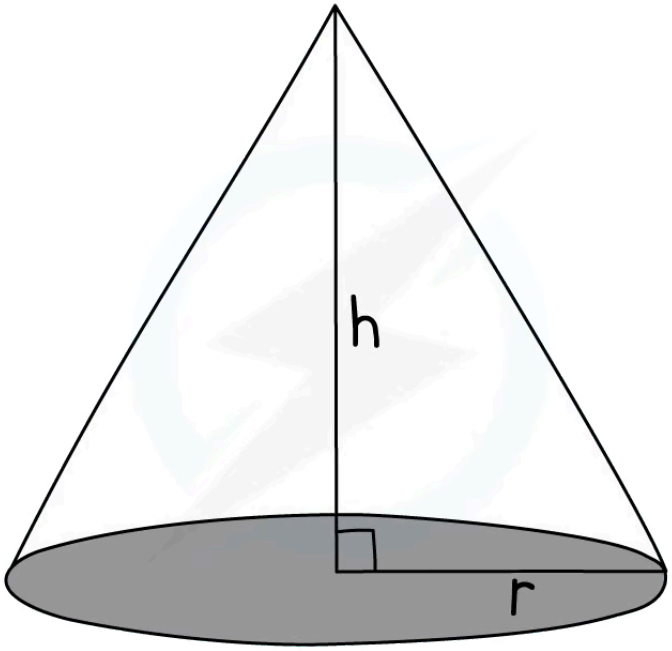


- The height must be a line from the top of the pyramid that is **perpendicular** to the base
- The **base** of a pyramid could be a square, a rectangle or a triangle

How do I find the volume of a cone?

- To calculate the volume, V, of a **cone** with base radius, r, and perpendicular height, h, use the formula
  - $V = \frac{1}{3}\pi r^2 h$
  - This formula is given to you in the exam

Your notes

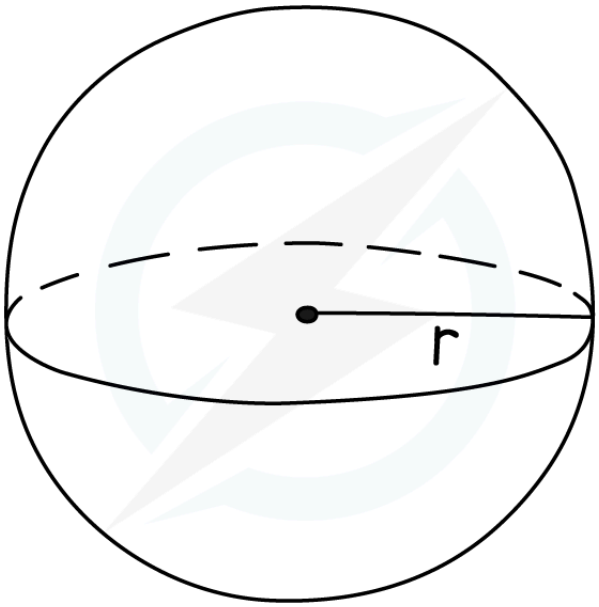


- Note that volume formula for a **cone** is similar to a **pyramid**
- The height must be a line from the top of the cone that is **perpendicular** to the base

How do I find the volume of a sphere?

- To calculate the volume, V, of a **sphere** with radius, r, use the formula
  - $V = \frac{4}{3}\pi r^3$
  - This formula is given to you in your exam

Your notes



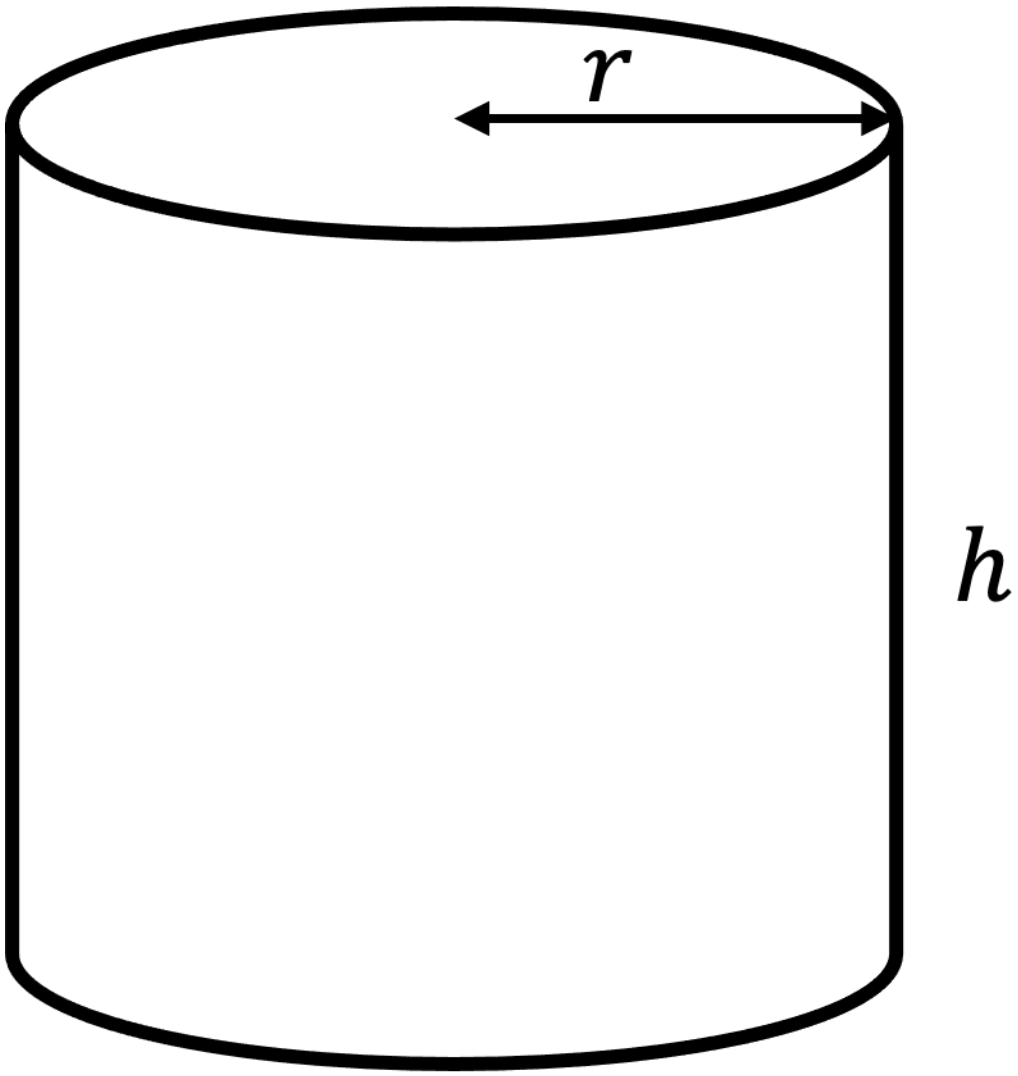
Examiner Tips and Tricks

- You only need to memorise the volume formulae for **cuboids**!

Worked Example

A cylinder is shown.

Your notes



The radius, r, is 8 cm and the height, h, is 20 cm.

Calculate the volume of the cylinder, giving your answer correct to 3 significant figures.

A cylinder is similar to a prism but with a circular base

The volume of any prism, V, is its base area × height, h, where the base area here is for a circle, πr^2

$$V = \pi r^2 h$$

Substitute $r = 8$ and $h = 20$ into the formula

$$V = \pi \times 8^2 \times 20$$

Work out this value on a calculator

4021.238...

Round the answer to 3 significant figures

4020 cm^3

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Problem Solving with Volumes

What is problem solving?

- **Problem solving**, usually has two key features:
  - A question is given as a **real-life scenario**
    - eg. The volume of water in a swimming pool...
  - There is usually **more than one topic of maths** you will need in order to answer the question
    - eg. Volume and money

What are common problems that involve volume?

- **Volume** is a commonly used topic of **'real-world'** maths
  - For example, a carton of juice in the shape of a cuboid, a cylindrical tin and a triangular prism chocolate box all involve volume
- Typically, the 'real-world' scenarios also have a **cost**
  - A lot of volume problems also involve **calculations with money**

How do I solve problems involving volume?

- Often the 3D object in a question will **not** be a standard cuboid, cone, sphere, etc.
  - It will likely either be:
    - A **prism** (3D shape with the same cross-section running through it)
    - A **portion** or **fraction** of a standard shape (a hemisphere for example)
    - A **compound object** (an object made up of two or more standard 3D objects)
- If the object is a **prism**, recall that the **volume of a prism** is the **cross-sectional area × its length**
  - The cross-sectional area may be a **compound 2D shape**
    - For example, an L-shape, or a combination of a rectangle and a triangle
- If the object is a **fraction** of a standard shape, consider the **"full" version** of the object and find the **appropriate fraction** of it
  - A hemisphere is half a sphere

- A **frustum** is a **truncated** (chopped-off) **cone** or **pyramid**
  - The volume of a frustum will be the volume of the smaller cone or pyramid subtracted from the volume of the larger cone or pyramid
- If the object is a **compound object**, find the volumes of the **individual** standard 3D objects and **add** them together
- Problem solving questions could appear on either a **non-calculator paper** or a **calculator paper**

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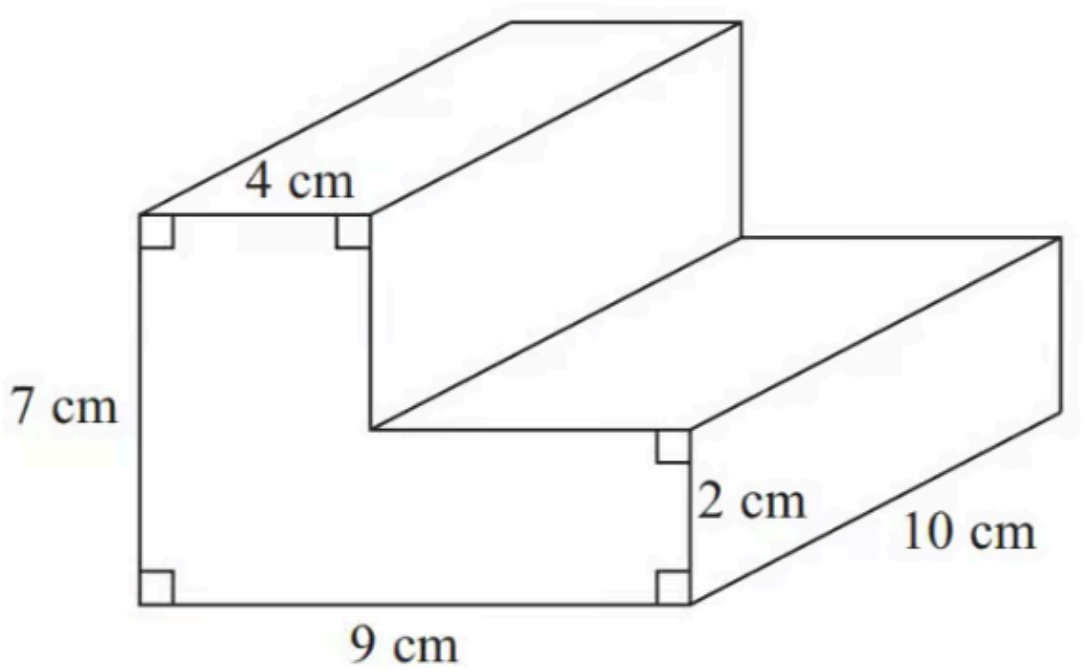
Examiner Tips and Tricks

- Before you start calculating, make a quick note of your plan to tackle the question
  - For example, "Find the area of the triangle and the rectangle, add together, multiply by the length"

Worked Example

The diagram shows a prism.



Work out the volume of the prism.

The volume is the area of the cross section × length (10 cm)
Find the area by splitting into a 7 × 4 and a (9 - 4) × 2 rectangle (or a 9 × 2 and a (7 - 2) × 4 rectangle)

$$7 \times 4 + (9 - 4) \times 2 = 38 \text{ cm}^2$$

Find the volume (by multiplying 38 by 10)

$$38 \times 10$$

380 cm^3

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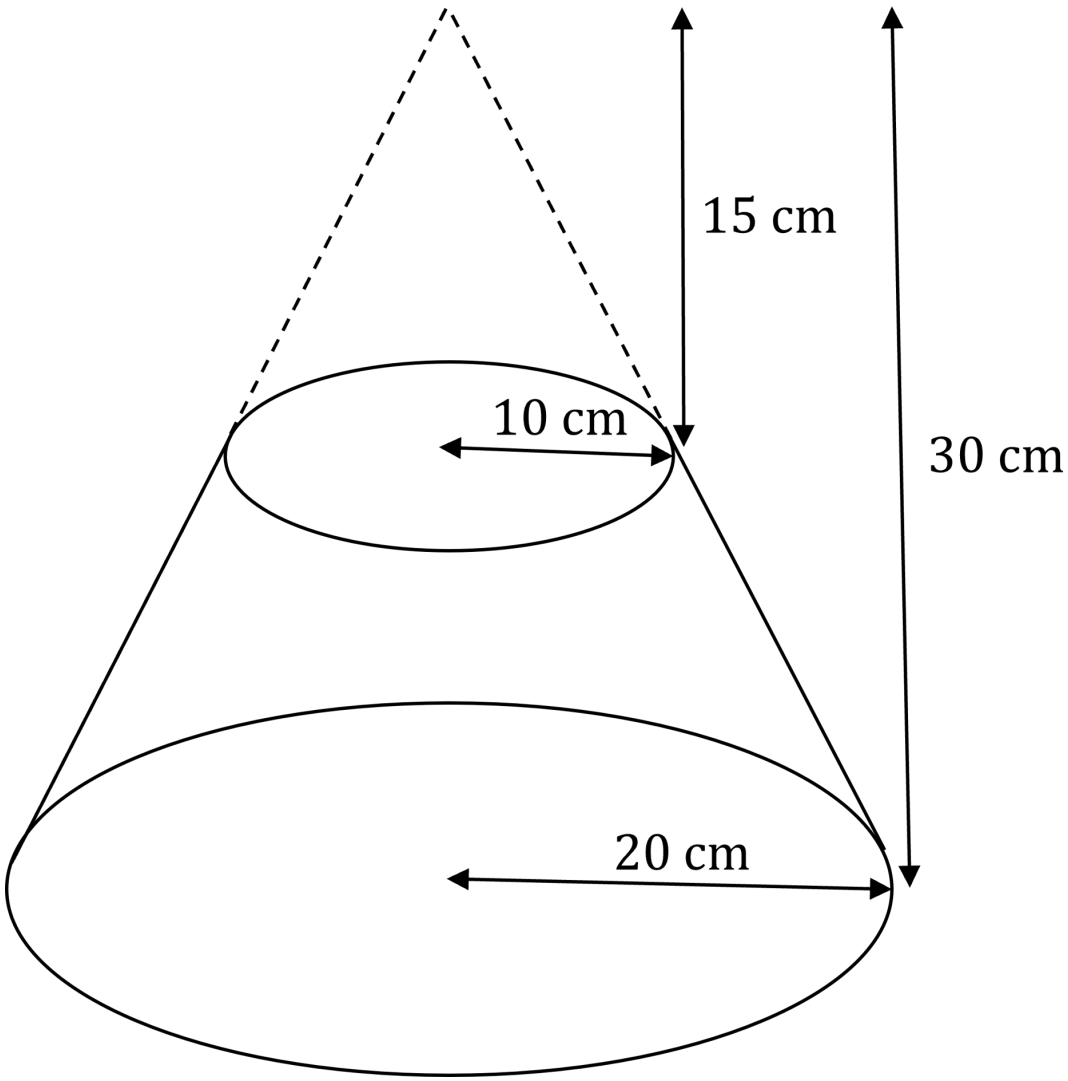
Worked Example

The diagram shows a truncated cone (a frustum).
Using the given dimensions, find the volume of the frustum to 3 significant figures.

Your notes



To find the volume of the frustum, find the volume of the larger cone (30 cm tall, with a radius of 20 cm), and subtract the volume of the smaller cone (15 cm tall, with a radius of 10 cm)

Formula for the volume of a cone: $\frac{1}{3}\pi r^2 h$

Calculate the volume of the larger cone

$$V_L = \frac{1}{3} \times \pi \times 20^2 \times 30 = 4000\pi = 12\,566.37061\ldots \text{ cm}^3$$

Calculate the volume of the smaller cone

$$V_S = \frac{1}{3} \times \pi \times 10^2 \times 15 = 500\pi = 1\,570.796327... \text{ cm}^3$$

Find the difference

$$V_L - V_S = 4000\pi - 500\pi = 3500\pi = 10\,995.57429... \text{ cm}^3$$

Round to 3 significant figures

11 000 cm^3

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Surface Area

What is surface area?

- The **surface area** of a 3D object is the **sum of the areas of all the faces** that make up the shape
  - Area is a 2D idea being applied into a 3D situation
  - A face is one of the flat or curved **surfaces** that make up a 3D object

How do I find the surface area of cubes, cuboids, pyramids, and prisms?

- In cubes, cuboids, polygonal-based pyramids, and polygonal-based prisms (ie. pyramids and prisms whose bases have straight sides), **all the faces are flat**
- The **surface area** is found by
  - calculating the area of each individual flat face
  - adding these areas together
- You should remember the formula for the **area of a rectangle**, but you are given the **area of a triangle** in your exam
- When calculating surface area, it can be helpful to **draw a 2D net for the 3D shape** in question
  - For example, consider a square-based pyramid where the top of the pyramid is directly above the centre of the base
    - Its net will consist of a square base and four identical isosceles triangular faces
    - Calculate the area of a square and the area of each triangle then add them together

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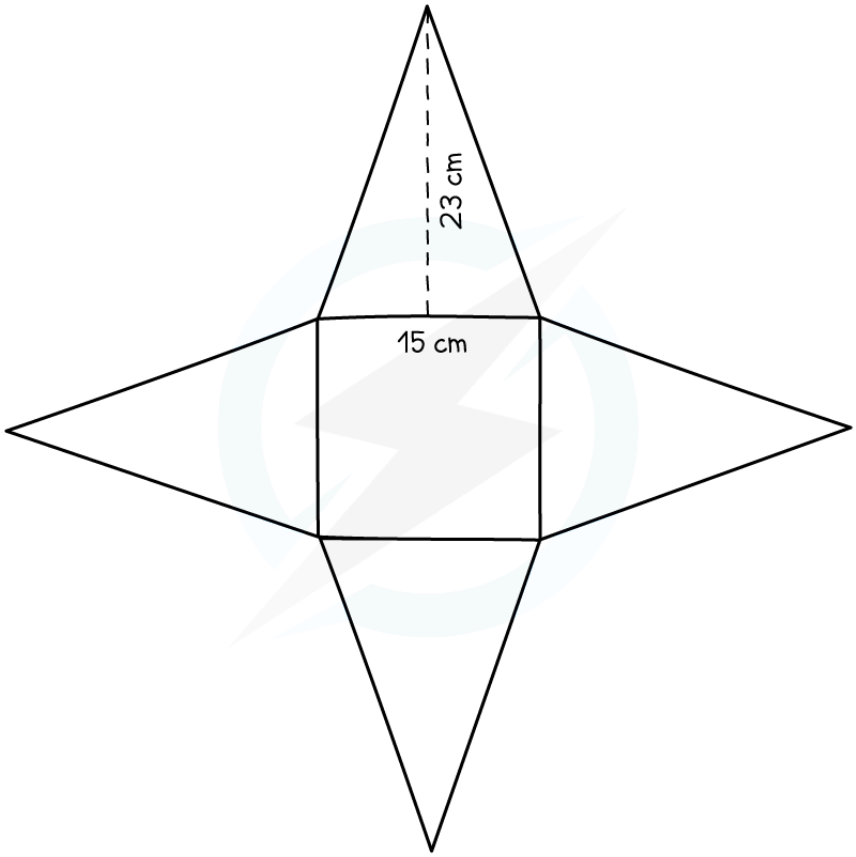
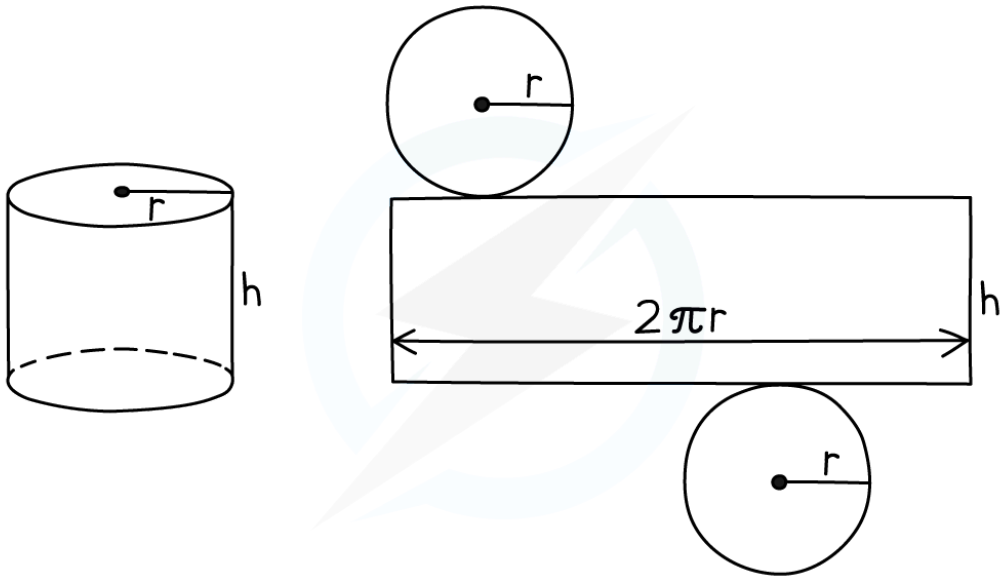


How do I find the surface area of a cylinder?

- A cylinder has **two flat surfaces** (the top and the base) and **one curved surface**
- The **net** of a cylinder consists of **two circles** and a **rectangle**

Your notes



- The **curved surface area** of a cylinder, A, with base radius, r, and height, h, is therefore given by
  - $A = 2\pi r h$
  - This formula is given to you in the exam
- The **total surface area** of a cylinder, A_{Total}, can be found using the formula
  - $A_{Total} = 2\pi r h + 2\pi r^2$
  - This formula is **not** given to you in the exam

How do I find the surface area of a cone?

- A cone has **one flat surface** (the base) and **one curved surface**
- The net of a cone, with radius, r, perpendicular height, h, and sloping edge, (slant height), l, consists of
  - A **circular base**
  - A **sector** with radius, l, and an arc length equal to the circumference of the base

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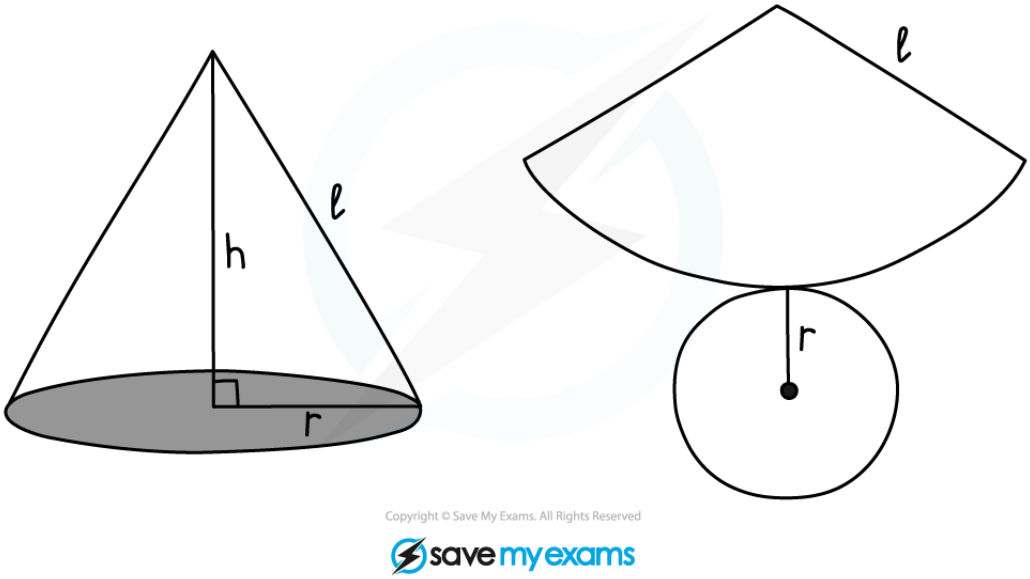
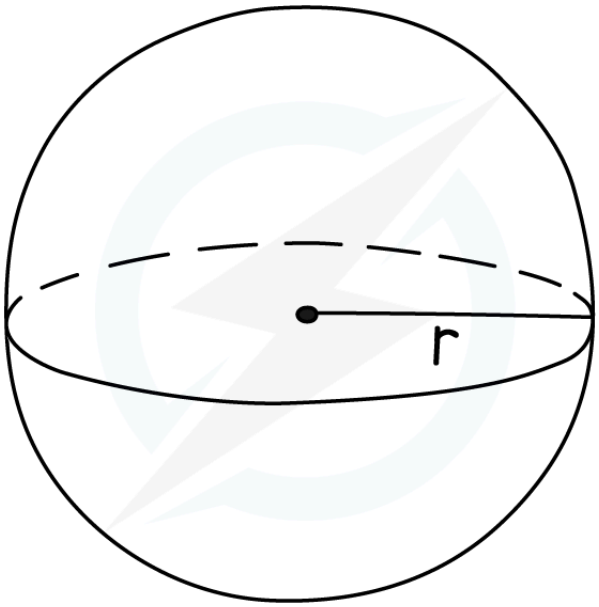


- The **curved surface area** of a cone, A, with radius, r, perpendicular height, h, and sloping edge, l, can be found using the formula
  - $A = \pi r l$
  - This formula is given to you in the exam
- The **total surface area** of a cone, A_{Total}, can be found using the formula
- $A_{Total} = \pi r l + \pi r^2$
- This formula is **not** given to you in the exam

How do I find the surface area of a sphere?

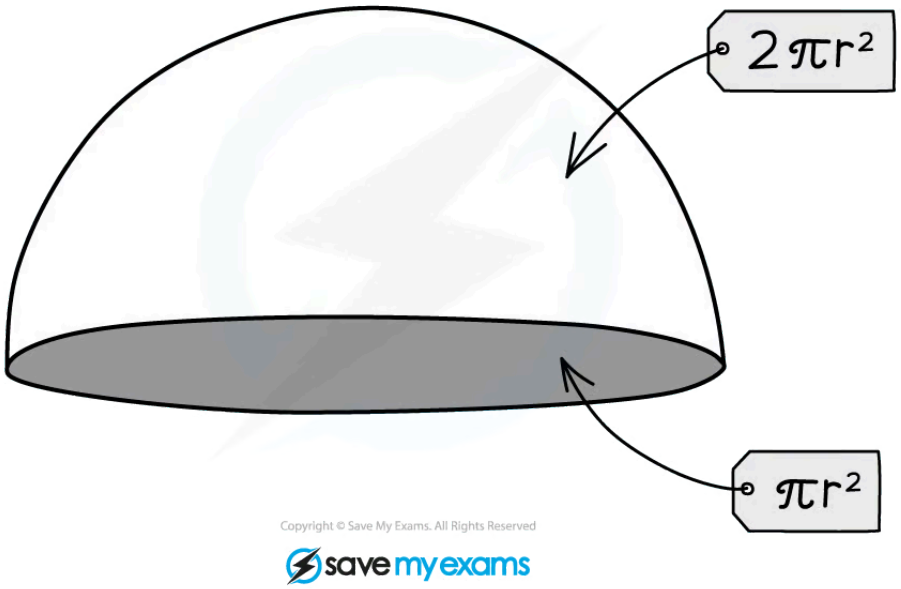
- A sphere has a **single curved surface**

Your notes



- The surface area of a sphere, A, with radius, r, can be found using the formula
  - $A = 4\pi r^2$
  - This formula is given to you in the exam
- A **hemisphere** has half the curved surface area of a sphere and the flat circular base

Your notes



- The surface area of a hemisphere, A, with radius, r, can be found using the formula
  - $A = 2\pi r^2 + \pi r^2$
  - This formula is **not** given to you in the exam

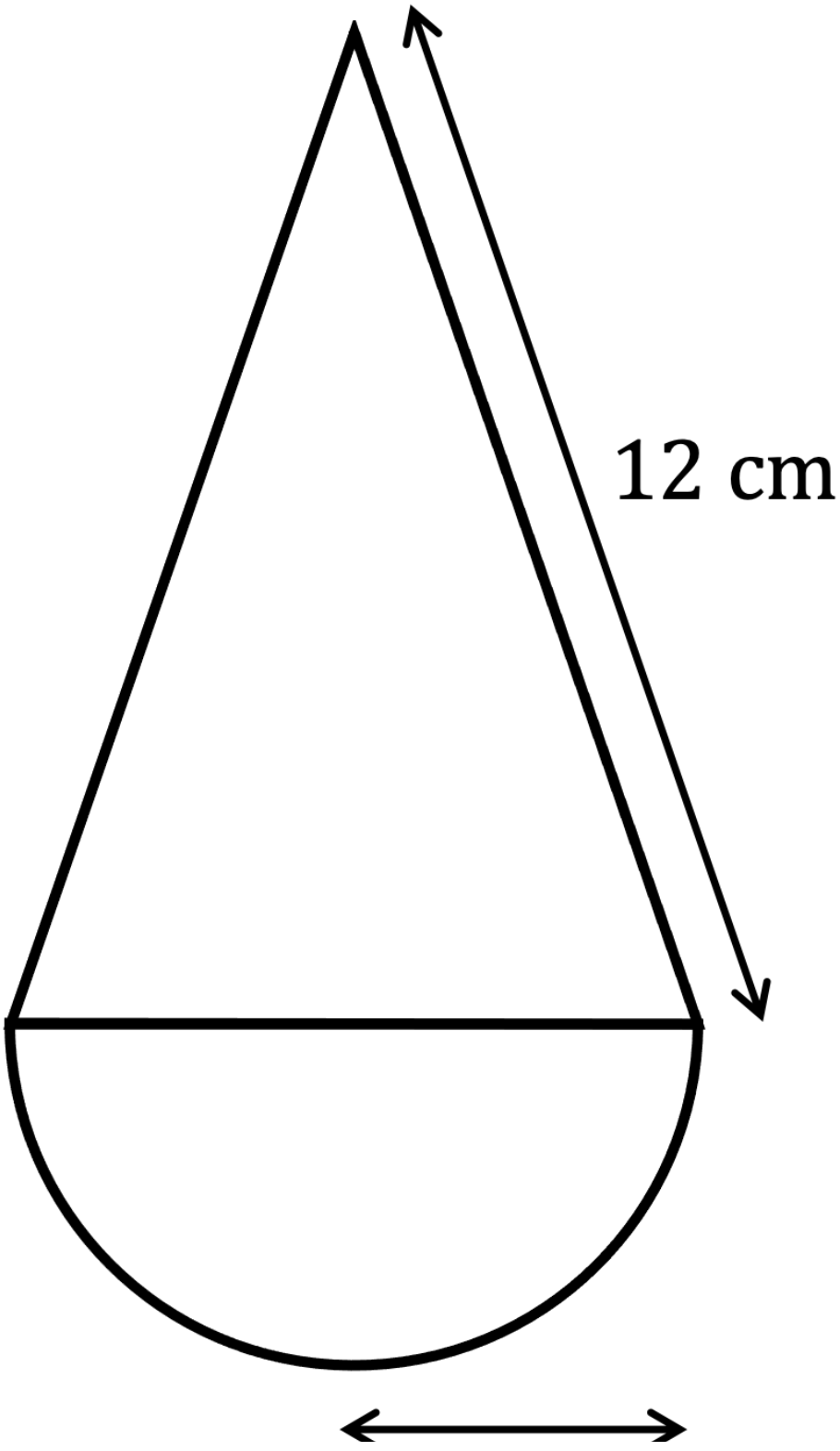
Examiner Tips and Tricks

- The **curved surface area** for a cylinder, cone and sphere are given to you in the exam
  - Read the question carefully, you may need to add additional areas, e.g. a base
  - Make you are confident in calculating the areas of rectangles, circles and triangles

Worked Example

A toy consists of a cone of radius 5 cm and slant height 12 cm placed on top of a hemisphere with the same radius. Find the total surface area of the toy. Give your answer to 3 significant figures.

Your notes



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Calculate the curved surface area of the cone using the formula, $A = \pi r l$

$$A_{cone} = \pi \times 5 \times 12 = 60\pi$$

Calculate the curved surface area of a hemisphere using the formula for the curved surface area of a sphere, $A = 4\pi r^2$ and dividing it by 2

$$A_{hemisphere} = \frac{4\pi \times 5^2}{2} = 50\pi$$

Add the two areas together to find the total surface area of the toy

$$A_{Totsl} = 60\pi + 50\pi = 110\pi$$

Evaluate on your calc and round to 3 significant figures

$$A_{Total} = 345.57519...$$

346 cm^2 (3 s.f.)