AQA GCSE Maths: Higher

Your notes

Estimating Gradients & Areas under Graphs

Contents

Your notes

Finding Gradients of Tangents

How are the gradients of graphs and tangents related?

- The **gradient of a graph at a point** is **equal** to the **gradient of the tangent** to the curve **at that point**
  - A tangent is a line that touches a curve, but does not cross it

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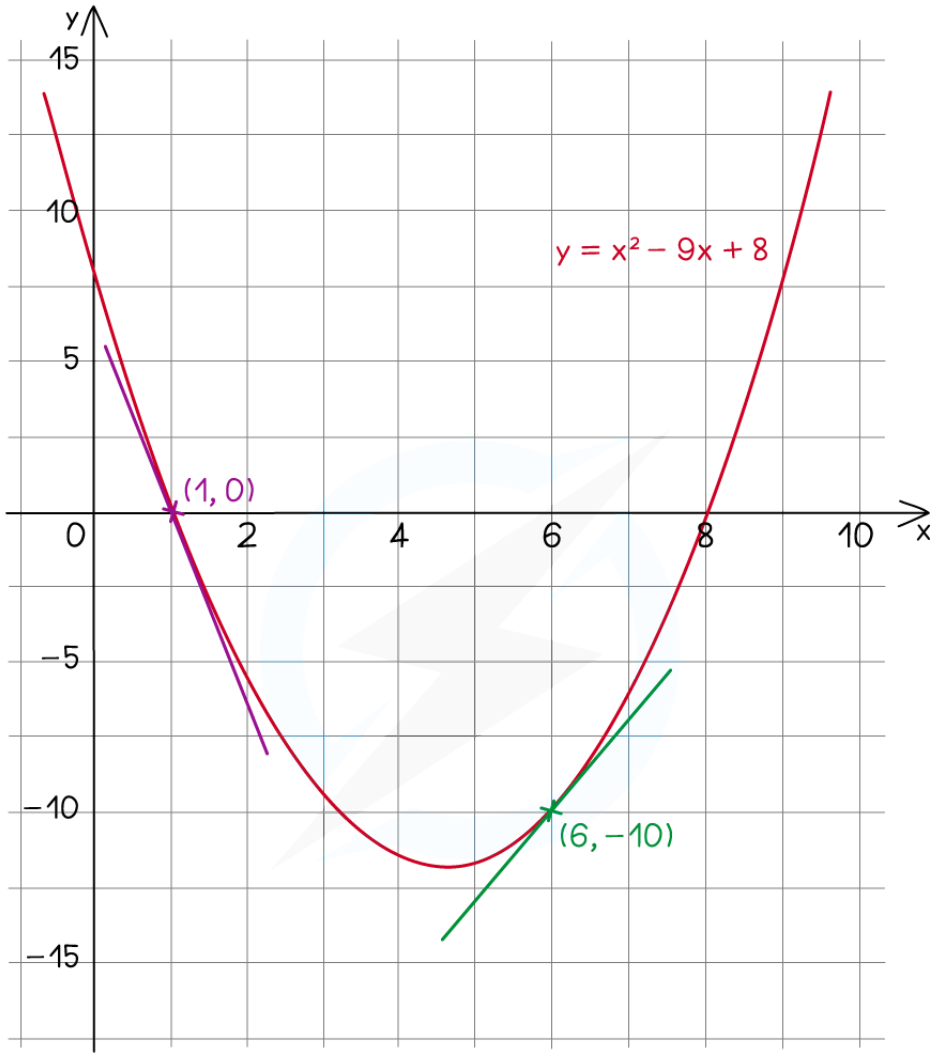

THE GRADIENT OF THE CURVE AT THE POINT x = 1 WILL BE EQUAL TO THE GRADIENT OF THE PURPLE TANGENT.
THE GRADIENT OF THE CURVE AT THE POINT x = 6 WILL BE EQUAL TO THE GRADIENT OF THE GREEN TANGENT.

How do I estimate the gradient of a curve using a tangent?

- To find an **estimate** for the **gradient** of a curve **at a point**:
  - **Draw** a **tangent** to the curve at the **point**

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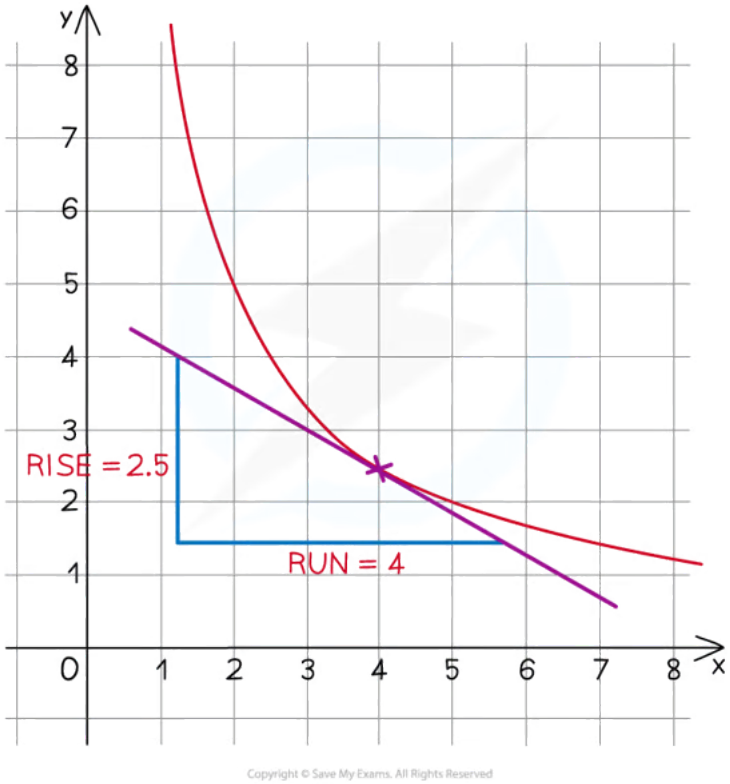
- Find the **gradient of the tangent** using
  - **Gradient = rise ÷ run**
  - or difference in *y* ÷ difference in *x*
  - In the example below, the **gradient of the tangent** at *x* = 4 would be $\frac{-2.5}{4} = -0.625$
  - Remember that the rise is **negative** if it is going down
  - This means the **gradient of the curve** at *x* = 4 is **also -0.625**



- It is an **estimate** because the **tangent** has been drawn by eye and is not exact
  - To find the exact gradient we would need to use **differentiation**

What does the gradient represent?

- The gradient represents the **rate of change** of *y* with *x*
  - I.e. For every increase in *x* by 1, how much does *y* increase?
- Consider the quantities used for the axes to determine the meaning of the gradient

- In a **distance-time graph**, the gradient is the rate of change of distance with time
  - This is the **speed**
- In a **speed-time graph**, the gradient is the rate of change of speed with time
  - This is the **acceleration**
- In a **graph of volume against radius**, e.g. as a balloon is inflated, the gradient is the rate of change of volume as the radius increases

Your notes

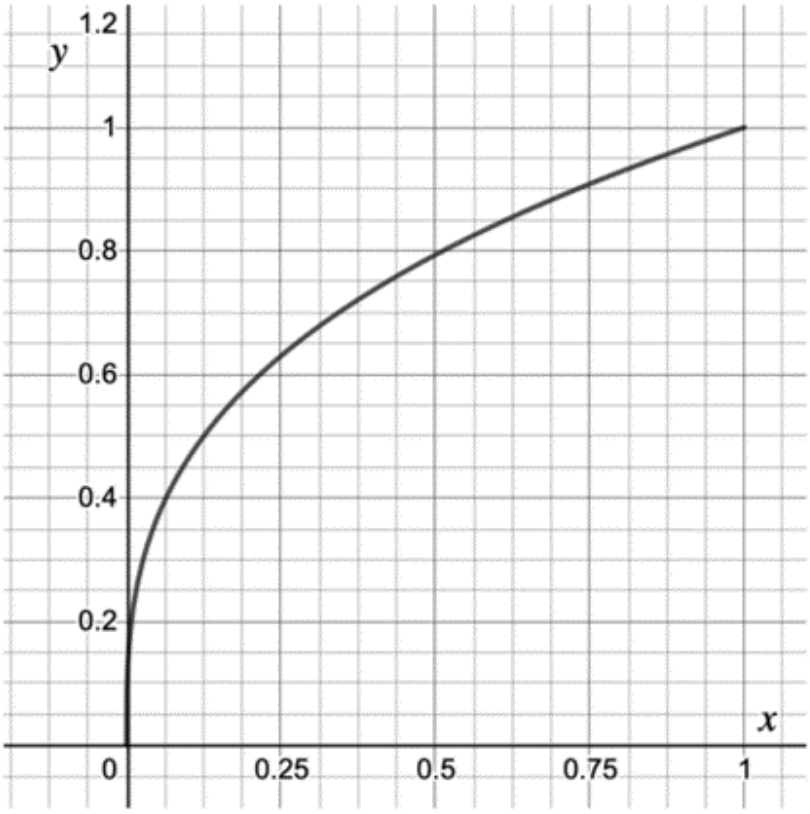
Examiner Tips and Tricks

- When drawing a tangent by hand:
  - Use a ruler
  - Draw the line as long as you can
- When finding the gradient of the tangent:
  - Pick two points that are far away from one another
  - This will reduce the effect of any inaccuracy

Worked Example

The graph below shows $y = \sqrt[3]{x}$ for $0 \leq x \leq 1$.

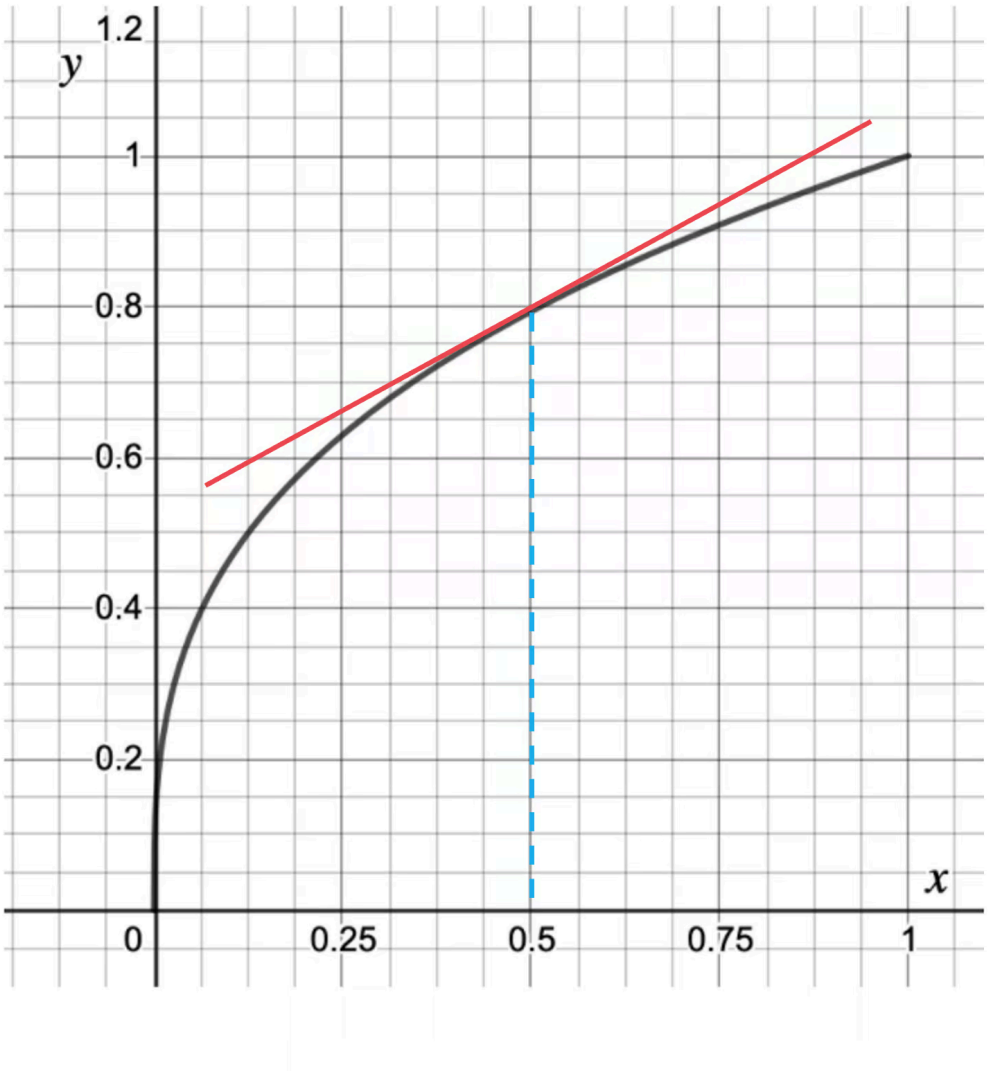
Find an estimate of the gradient of the curve at the point where $x = 0.5$.



Draw a tangent to the curve at the point where $x = 0.5$

Your notes

Your notes



Find suitable, easy to read coordinates as far apart as possible and draw a right-angled triangle between them

Find the difference in the *y* coordinates (rise) and the difference in the *x* coordinates (run).

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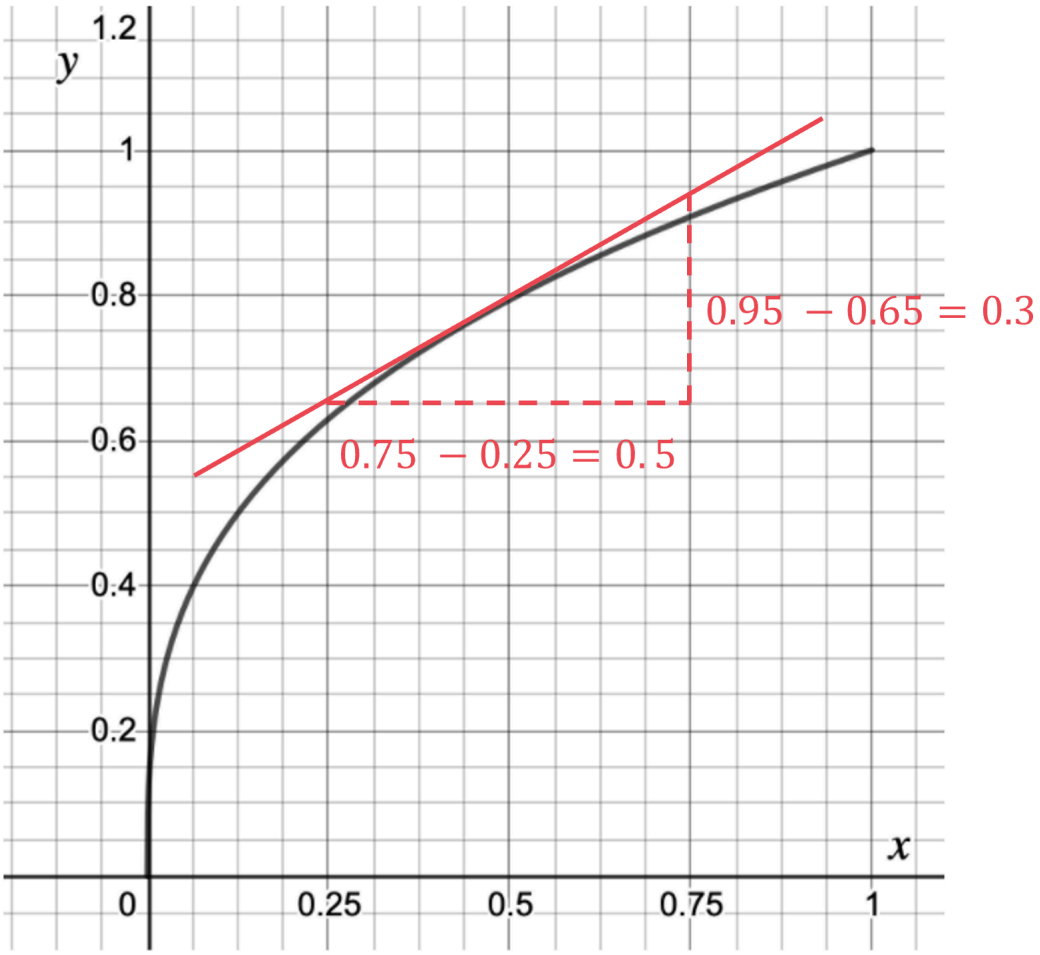


Find the gradient by dividing the difference in y (rise) by the difference in x

$$\text{Gradient} = \frac{0.3}{0.5} = \frac{3}{5} = 0.6$$

The gradient of the tangent at $x = 0.5$ is equal to the gradient of the curve at $x = 0.5$

Estimate of gradient = 0.6

Your notes

Finding Areas under Graphs

How do I estimate the area under a graph?

- To find an estimate for the **area**:
  - Split area into vertical **strips**
  - Draw **straight lines** between the tops of the strips
  - Find area of strips (trapeziums) using **Area = ½(a + b)h**
  - Add the areas

How do I know if my answer is an underestimate or an overestimate?

- A common exam question is to ask if your estimate of the area is an underestimate or an overestimate
- To answer this, simply look at the straight lines joining the tops of your strips
  - If the straight lines are **below** the curve, it is an **underestimate**
  - If the straight lines are **above** the curve, it is an **overestimate**
- In your exam, the lines will **all** be **below** or **all** be **above** the curve- though it may be difficult to tell which for some strips

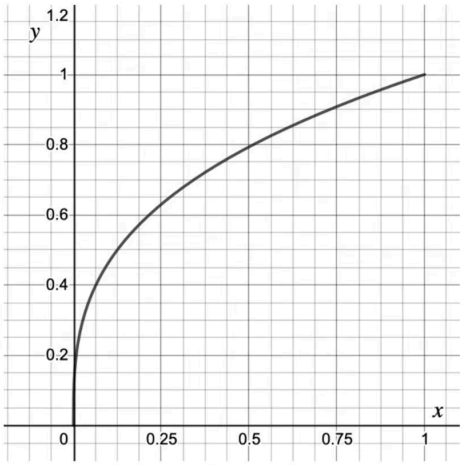
Examiner Tips and Tricks

- This is particularly useful when working with speed-time and distance-time graphs if they are curves and not straight lines.

Worked Example

The graph below shows $y = \sqrt[3]{x}$ for $0 \leq x \leq 1$

Your notes

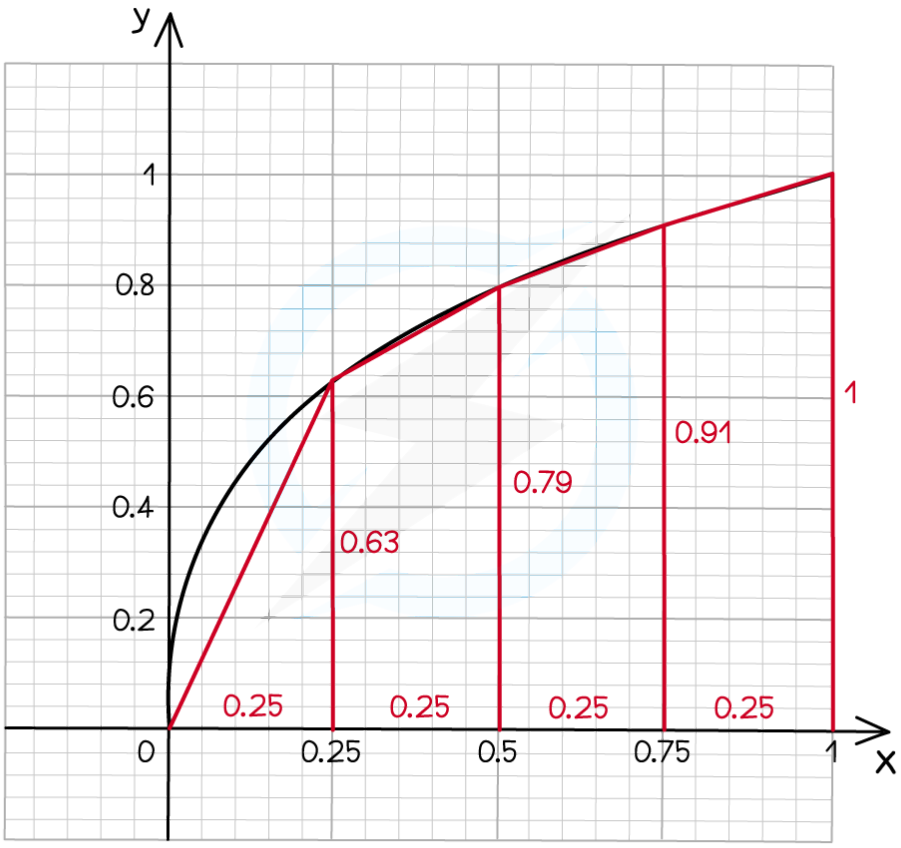


Find an estimate for the area between the curve, the x axis and the line $x = 1$. Use four strips of equal width.

Split the area into four strips using the width of 0.25 for each one

Find the y coordinate at the end of each strip by reading the value from the graph or substituting the x coordinate into $y = \sqrt[3]{x}$

Your notes



Find the area of each strip by using the formula for the area of a trapezium

$$\frac{1}{2}(0+0.63)(0.25)+\frac{1}{2}(0.63+0.79)(0.25)+\frac{1}{2}(0.79+0.91)(0.25)+\frac{1}{2}(0.91+1)(0.25)=0.7075$$

$$\text{Area}\approx 0.708$$