AQA GCSE Maths: Higher

Your notes

Algebraic Proof

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What is algebraic proof?

- **Algebraic proof** means **proving a result** using **algebra**
  - This is different to proving a result by individually testing all possible values
- The proofs may require **algebraic skills** such as
  - expanding brackets
  - factorising
  - collecting like terms
    - The **difference of two squares** factorisation can also be helpful

How do I prove results about integers?

- To prove results about **integers** (whole numbers), you need to first represent the integers as algebraic **letters** or **terms**
  - The following table shows the most commonly used algebraic terms

Type of integer	Term	Comment
Any integer	n	
Consecutive integers	$n,\ n+1$	This means one after the other. Could also use $n-1,\ n$
Any two integers	$n,\ m$	A different letter is used (to show it is not necessarily consecutive)
An **even** integer	$2n$	
Consecutive even integers	$2n,\ 2n+2$	Could also use $2n-2,\ 2n$
Any two even integers	$2n,\ 2m$	

An **odd** integer	$2n+1$	Could also use $2n-1$
A **multiple** of 5	$5n$	
A multiple of k	kn	
One more than a multiple of 3	$3n+1$	
A **square** number	n^2	
A **cube** number	n^3	
A **rational** number	$\frac{a}{b}$	Where a and b are integers and $b \neq 0$

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- You then need to be able to apply **operations** to the terms above
  - Common operations are the
    - **sum** ($+$)
    - **difference** ($-$)
    - **product** ($\times$)
    - **square** $(\ldots)^2$

How do I show that a result is odd or even?

- To prove an expression is **even**, show that it can be written as $2 \times (\text{integer})$
  - For example, $2(n^2 - 3n)$ is even
    - This may require **factorising out a 2**
- To prove something is **odd,** show that it can be written as $2 \times (\text{integer}) + 1$
  - For example, $2(n+m)+1$ is odd
- Make sure the part **inside the brackets** is an **integer**

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- For example, $2\left(n+\frac{1}{3}\right)$ is **not** even as $\frac{1}{3}$ is not an integer
- You can apply similar ideas to prove expressions are **multiples** of other numbers
    - For example, $7(n^2+2n)$ is a multiple of 7

How do I prove results with prime numbers?

- When proving results with **prime numbers**, remember that primes **only have two factors**: 1 and themselves
    - If p is prime then $1 \times p$ or $p \times 1$ are the only ways to write it as a product of two integers

Examiner Tips and Tricks

- At the end of an algebraic proof, you need to write a **conclusion** in full sentences
    - A good trick is to copy word-for-word the phrases used in the question
        - for example, "this proves that all squares of odd numbers are odd"

Worked Example

Prove that the difference of the squares of two consecutive even numbers is divisible by 4.

Break down the question into smaller parts
First find expressions for two consecutive even numbers
The first even number can be written as follows:

$$2n$$

Write down an expression for the next consecutive even number after $2n$

$$2n+2$$

Now square the two consecutive even numbers
Then write down the difference of these squares
Write the larger value subtract the smaller value

$$(2n+2)^2-(2n)^2$$

Method 1

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Expand the brackets (using a double-bracket expansion)
Collect any like terms (the $4n^2$ and the $-4n^2$ cancel out)

$$\begin{aligned}&(2n+2)(2n+2)-4n^2\\&=4n^2+4n+4n+4-4n^2\\&=8n+4\end{aligned}$$

Show that the final answer is divisible by 4 (a multiple of 4)
Do this by writing it as $4\times(\text{integer})$

$$=4(2n+1)$$

Write a conclusion that copies the wording in the question

$4\times(2n+1)$ is a multiple of 4, so the difference between the squares of two consecutive even numbers is divisible by 4

Method 2

Use the difference of two squares to factorise, $a^2-b^2=(a-b)(a+b)$

$$(2n+2)^2-(2n)^2=(2n+2-2n)(2n+2+2n)$$

Simplify inside both brackets

$$\begin{aligned}&=(2)(4n+2)\\&=2(4n+2)\end{aligned}$$

Factorise out a 2 from the second bracket

$$\begin{aligned}&=2\times2(2n+1)\\&=4(2n+1)\end{aligned}$$

This has the form $4\times(\text{integer})$ so it is divisible by 4 (a multiple of 4)
Write a conclusion that copies the wording in the question

$4\times(2n+1)$ is a multiple of 4, so the difference between the squares of two consecutive even numbers is divisible by 4