AQA GCSE Maths: Higher

Your notes

Equation of a Circle

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Equation of a Circle

What is the equation of a circle?

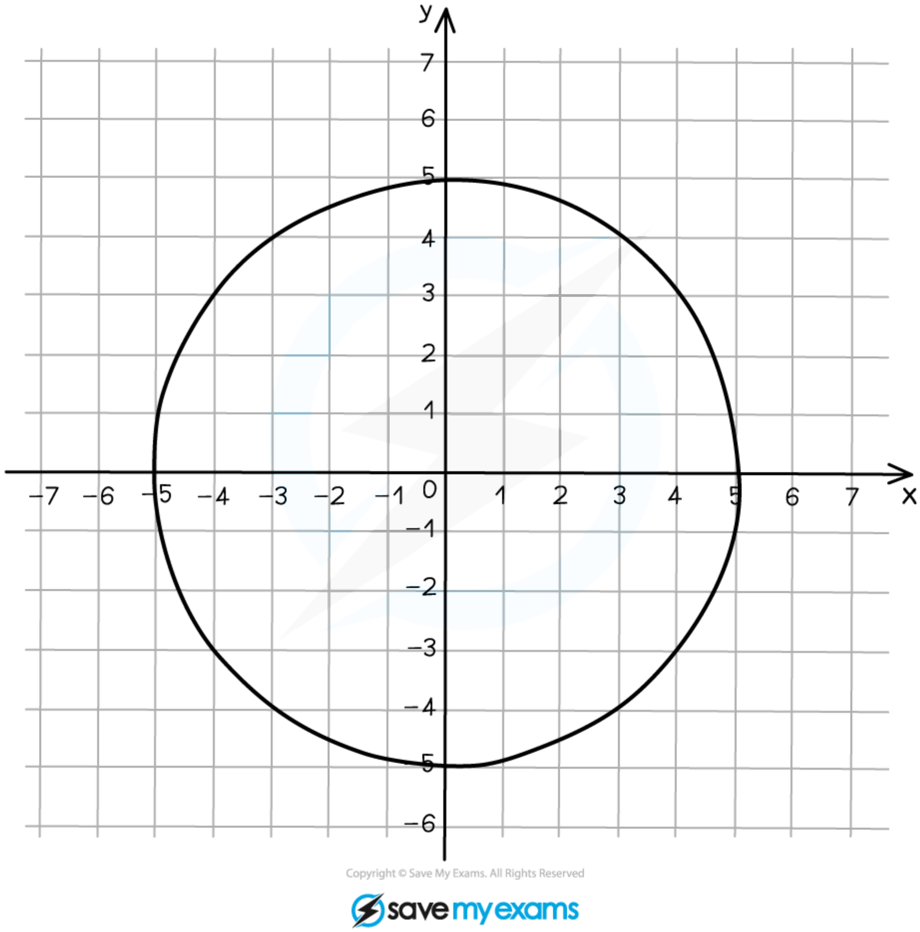
- A **circle** centered on the **origin** with radius **r** has the **equation** $\mathbf{x^2 + y^2 = r^2}$
- **(a, b)** lies on the circle if $\mathbf{a^2 + b^2 = r^2}$
  - If $\mathbf{a^2 + b^2 < r^2}$ then (a, b) lies **inside** the circle
  - If $\mathbf{a^2 + b^2 > r^2}$ then (a, b) lies **outside** the circle
- The circle cuts the **x**- and **y**-axes at **±r** ("plus or minus r")
- The diameter = 2r and the circumference = 2πr

Examiner Tips and Tricks

- if asked for the radius, don't forget to square root r^2
  - e.g. for $x^2 + y^2 = 10$, the radius is √10, not "10"

Worked Example

The diagram shows a circle, centred on the origin.



Your notes

(a) Write down the equation of the circle.

Identify the radius by checking where the circle crosses the coordinate axes

$$r = 5$$

Substitute this into $x^2 + y^2 = r^2$

$$x^2 + y^2 = 5^2$$

$$x^2 + y^2 = 25$$

(b) Does the point $P(3, 4)$ lie on, inside or outside the circle? You must show working to support your answer.

Substitute $x = 3$ and $y = 4$ into $x^2 + y^2$

$$\begin{aligned} x^2 + y^2 &= 3^2 + 4^2 \\ &= 9 + 16 \\ &= 25 \end{aligned}$$

Since $3^2 + 4^2 = 25$, the point $P(3,\ 4)$ lies on the circle

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Equation of a Tangent

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Equation of a Tangent

How do we find the equation of a tangent to a circle?

- First, make sure you are familiar with **equations of straight lines** and **perpendicular lines**
- A **tangent** just touches a circle (but does not cross it)
- The tangent at point *P* is **perpendicular** to the **radius** *OP*
  - remember, the **gradients** of perpendicular lines **multiply** to -1
    - they are **negative reciprocals**
- So if *P* is a point (a, b) on the circumference, then the gradient of the radius *OP* is $\frac{b-0}{a-0} = \frac{b}{a}$.
  - Therefore the **gradient of the tangent to the circle at *P* is** $-\frac{a}{b}$
- From here, use $\boldsymbol{y = mx + c}$ to find the equation of the tangent

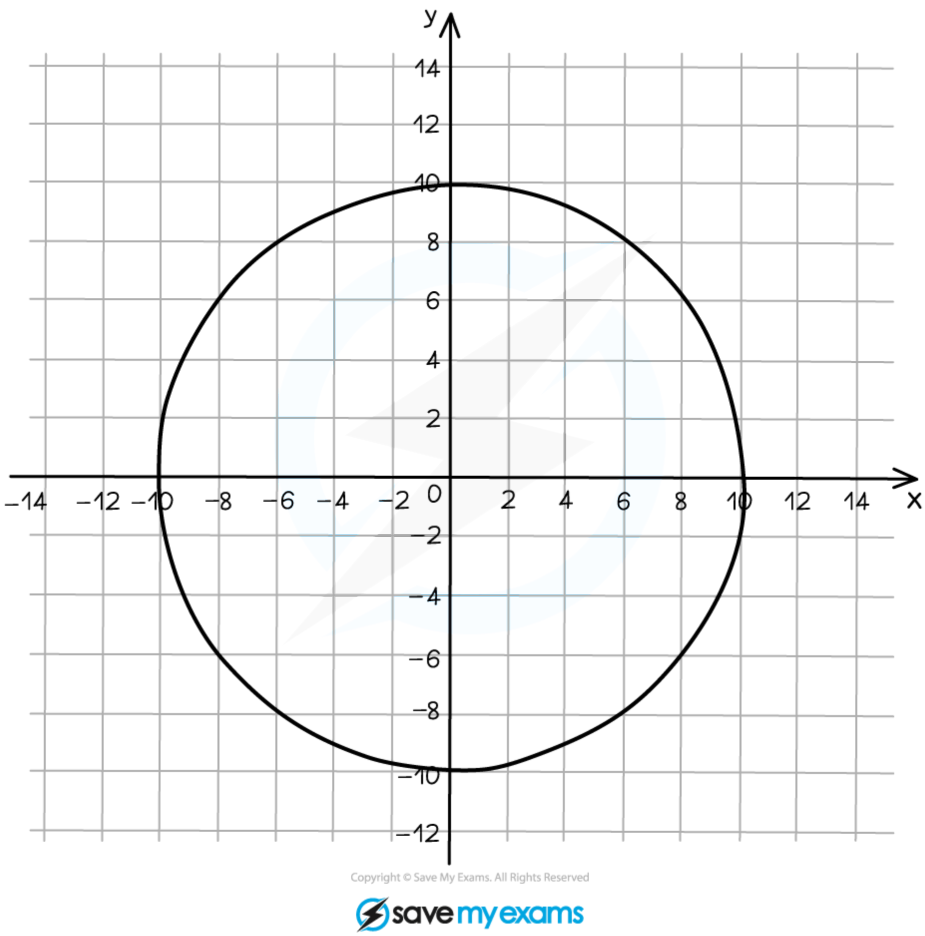
Examiner Tips and Tricks

- Solving **simultaneous equations** of circle and tangent only gives **one solution**
  - so if you are asked to show a line is a tangent, solve the simultaneous equations and show there is only one solution
- **Always draw a diagram to help!**

Worked Example

The graph shows the circle with the equation $x^2 + y^2 = 100$.

Your notes



Find an equation of the tangent to the circle at the point $P(8, 6)$.

Find the gradient of the radius by finding te gradient of the line segment form the origin to the point P

$$\text{Gradient of } OP = \frac{6-0}{8-0} = \frac{6}{8} = \frac{3}{4}$$

Find the gradient of the tangent by taking the negative reciprocal of the gradient of the radius

$$m = -\frac{4}{3}$$

Substitute $m = -\frac{4}{3}$, $x = 8$ and $y = 6$ into $y = mx + c$ to find the value of c

$$6 = -\frac{4}{3} \times 8 + c$$

$$6 = -\frac{32}{3} + c$$

$$c = 6 + \frac{32}{3}$$

$$c = \frac{50}{3}$$

The equation of the tangent is $y = -\frac{4}{3}x + \frac{50}{3}$

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