AQA GCSE Maths: Higher

Your notes

Real-Life Graphs

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Your notes

Distance-Time Graphs

How do I use a distance-time graph?

- **Distance-time graphs** show the distance travelled at different times
  - **Distance** is on the **vertical** axis
  - **Time** is on the **horizontal** axis
- The **gradient** of the graph is the **speed**
  - $\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{\text{rise}}{\text{run}}$
- The **steeper** the line the **faster** the object is moving
  - Lines with **positive** gradients represent objects **moving away** from the start point
  - Lines with **negative** gradients represent objects **moving towards** the start point
  - Lines that are **horizontal** represent **rest**
    - The object is **stationary** (not moving)

How do I work out the overall average speed?

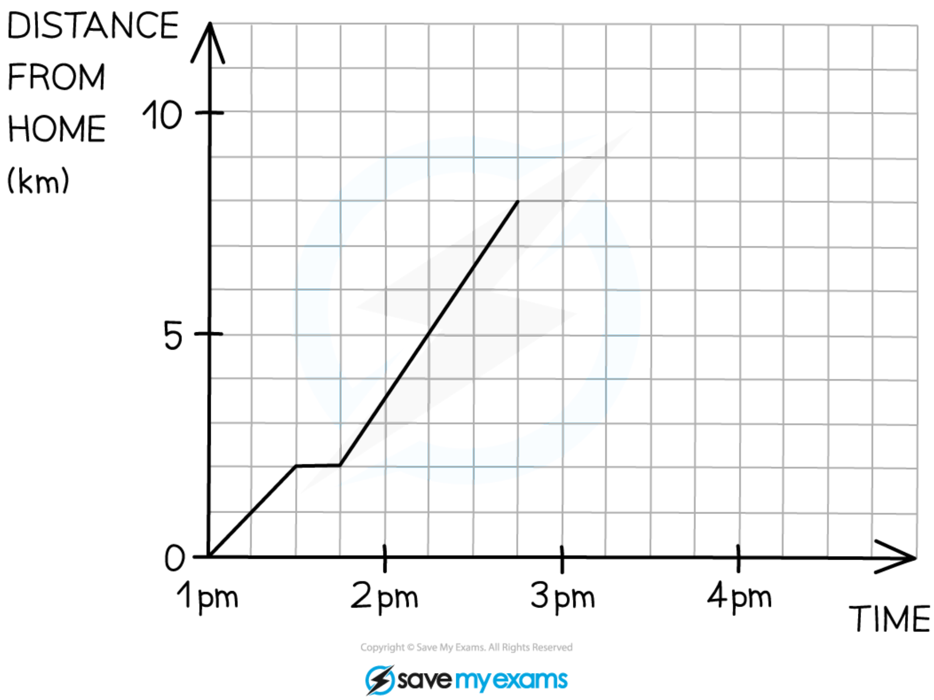
- For journeys with **multiple parts**
  - the **overall average speed** for the **whole** journey is $\frac{\text{total distance travelled}}{\text{total time}}$
  - The total time **includes** any rests

Examiner Tips and Tricks

These questions often have a lot of parts that depend on each other, so always double check each answer before continuing.

Worked Example

One afternoon, Mary cycled 8 km from her home to her grandfather's house.
Part of the travel graph for her journey is shown.



(a) Find Mary's speed at 2 pm.

The speed is the gradient of the line at that point
The gradient at 2pm is the same as the gradient for all points between 1:45pm and 2:45pm

It is easiest to use $\frac{\text{rise}}{\text{run}}$ for that bigger section

$$\frac{\text{rise}}{\text{run}} = \frac{6}{1}$$

6 km/h

(b) Calculate, in minutes, how long Mary stopped on the way to her grandfather's house.

The horizontal line is where Mary stops
The scale is 1 square = 15 minutes

15 minutes

Your notes

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(c) Calculate Mary's overall average speed from leaving her home to arriving at her grandfather's house, giving your answer correct to 3 significant figures.

Overall average speed is $\frac{\text{total distance travelled}}{\text{total time}}$

Total distance travelled is 8 km
Total time (including rests) is 1.75 hours

$$\frac{8}{1.75} = 4.57142...$$

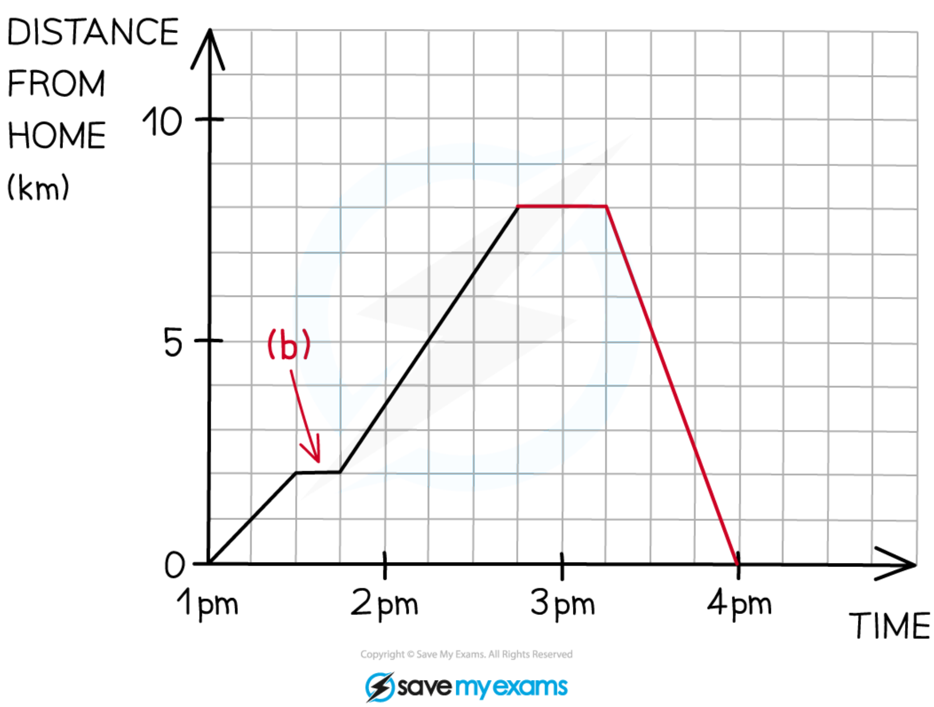
Round the answer to 3 significant figures

4.57 km/h (to 3 s.f.)

(d) Mary stayed at her grandfather's house for half an hour, then cycled home at a steady speed without stopping, arriving home at 4pm.

Complete the travel graph for Mary's journey.

Rest is a horizontal line
Returning home is a straight line with a negative gradient



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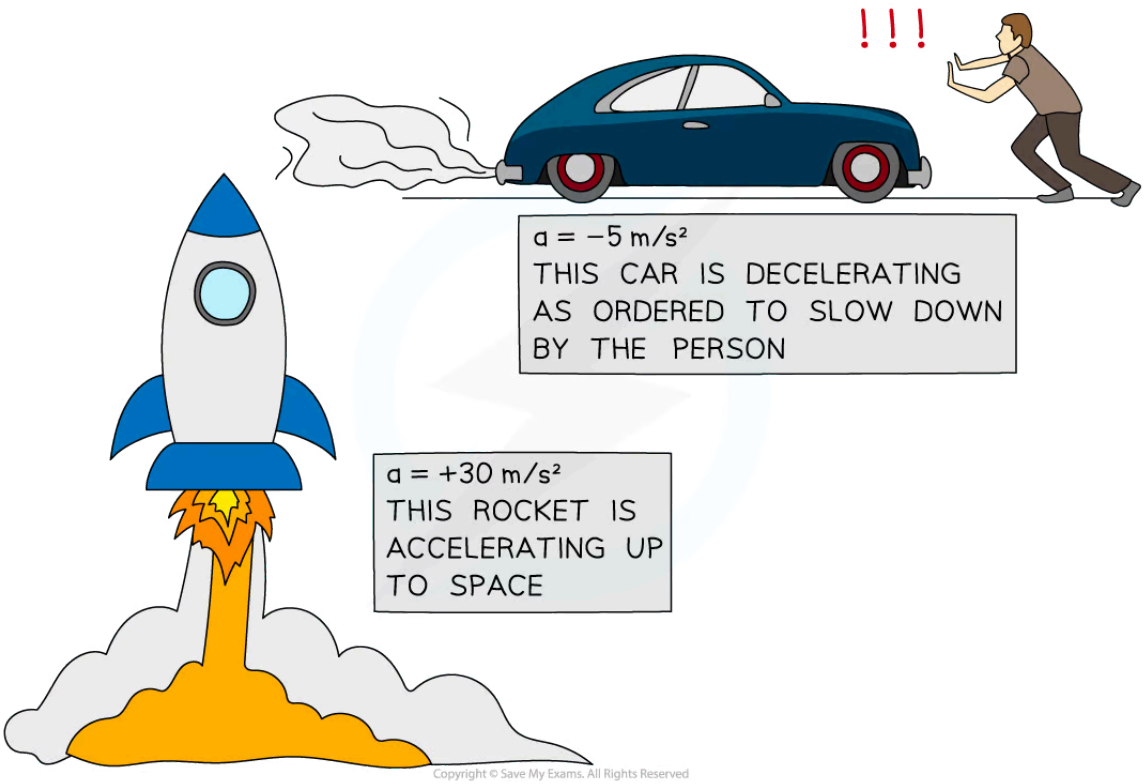
Speed-Time Graphs

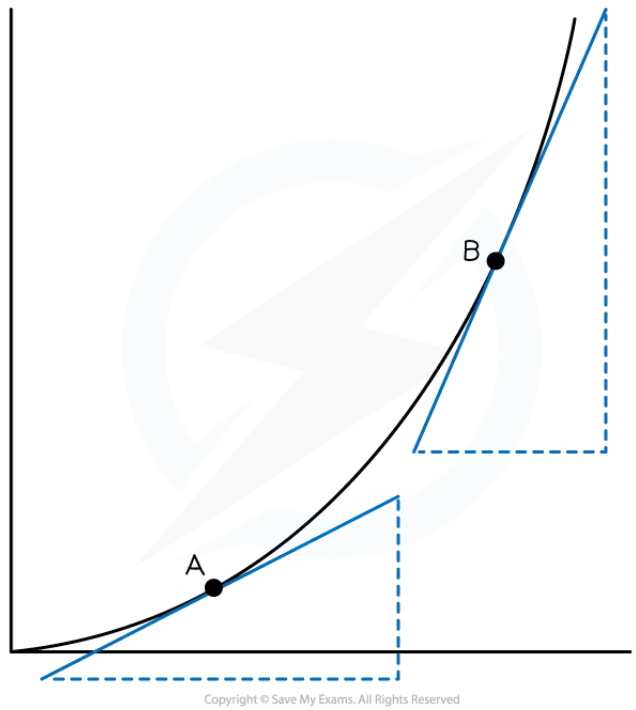
How do I use a speed-time graph?

- **Kinematics** is the study of **motion** of objects
  - It looks at how an object moves **over time**
- **Speed-time graphs** show the speed of an object at different times
  - **Speed** is on the **vertical** axis
  - **Time** is on the **horizontal** axis
- The **gradient** of the graph is the **acceleration**
  - $\text{Acceleration} = \frac{\text{speed}}{\text{time}} = \frac{\text{rise}}{\text{run}}$
- A **positive gradient** shows positive acceleration (speeding up)
- A **negative gradient** shows negative acceleration, (slowing down)
  - This is also called **deceleration**

Your notes



- **Horizontal** lines indicate moving at a **constant speed**
  - The object is neither speeding up or slowing down
  - If the constant speed is **zero**, then it is at **rest**
- A **straight line** shows **constant acceleration**
- A **curve** shows **changing acceleration**
  - To find the acceleration at a particular point on the graph
    - draw a **tangent** to the graph at this point and find its **gradient**



Your notes

- The **distance** covered by the object is the **area under the graph**
  - Split the area into simple shapes, e.g. rectangles and triangles
  - Find the area of each shape and add them together

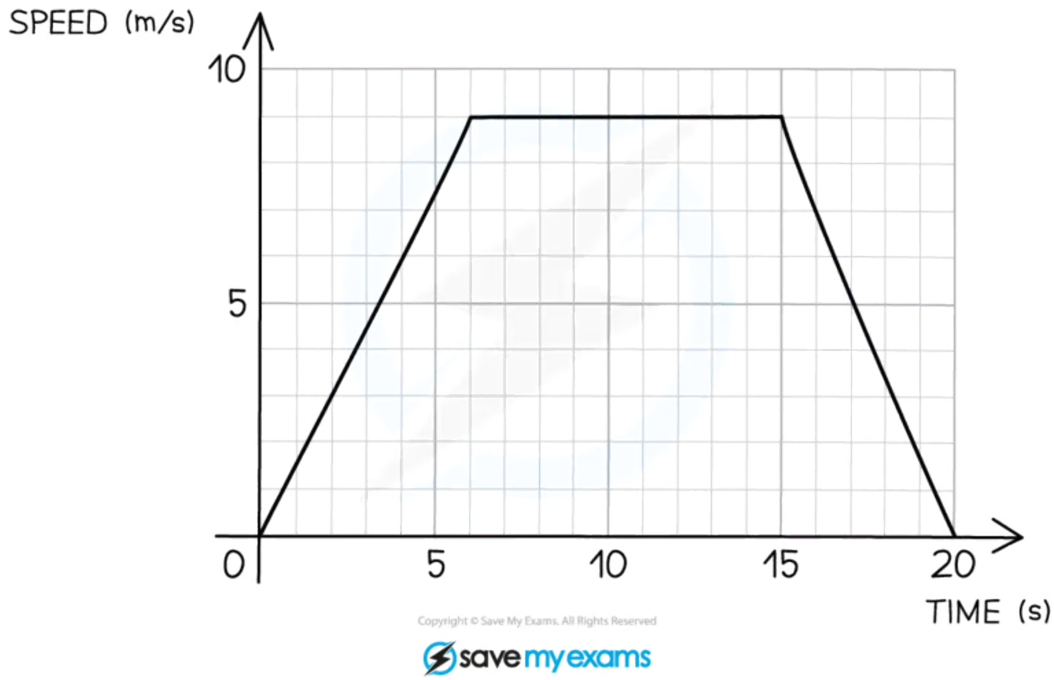
Examiner Tips and Tricks

- Always check the **vertical axis** to see if you are given a speed-time graph or a distance-time graph!

Worked Example

The speed-time graph for a car travelling between two sets of traffic lights is shown below.

Your notes



(a) For how long was the car travelling at a constant speed?

Constant speed is represented by horizontal lines

There is a horizontal line from 6 seconds to 15 seconds

$$15 - 6 = 9$$

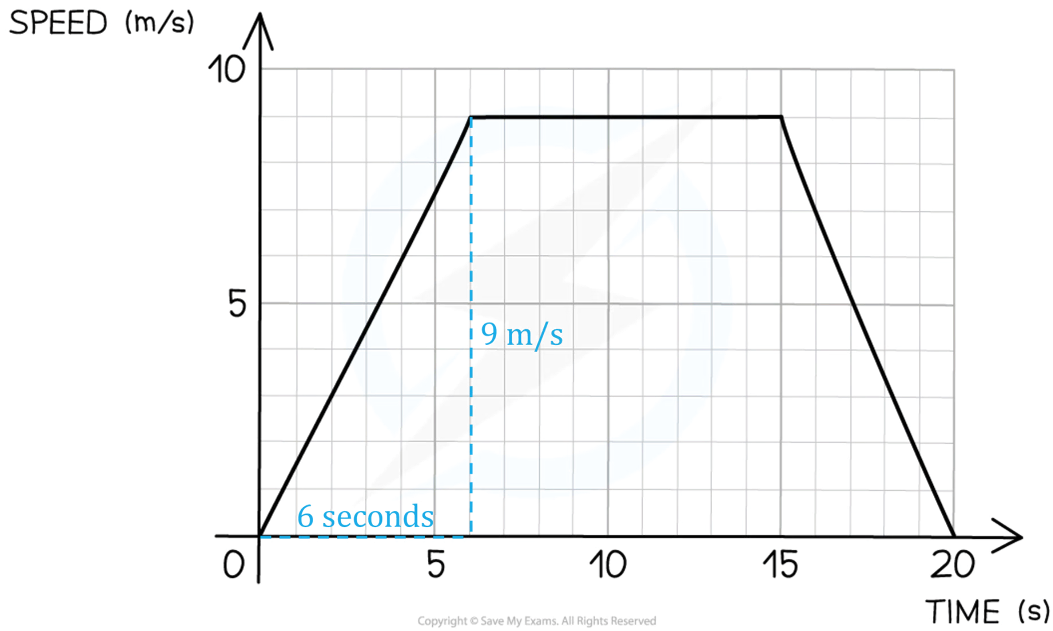
9 seconds

(b) Calculate the acceleration during the first 6 seconds.

In a speed-time graph the acceleration is the gradient of the graph

$$\text{acceleration} = \frac{\text{rise}}{\text{run}} = \frac{\text{speed}}{\text{time}}$$

Your notes



$$\text{acceleration} = \frac{9\ \text{m/s}}{6\ \text{s}} = 1.5\ \frac{\text{m/s}}{\text{s}}$$

Acceleration = 1.5 m/s^2

(c) Work out the distance covered by the car.

In a speed-time graph the distance travelled is equal to the area under the graph

The graph is a trapezium so use the formula $\text{Area} = \frac{(a + b)h}{2}$

$$\text{Area} = \frac{(9 + 20) \times 9}{2} = \frac{261}{2} = 130.5$$

Distance travelled = 130.5 m

Conversion Graphs

Your notes

Conversion Graphs

What is a conversion graph?

- A **conversion graph** is a **straight-line** graph relating **two quantities**
  - You can **convert** (change) between them by **reading values** off the graph
- Common examples include
  - **Temperature**
    - degrees Celsius (°C) and degrees Fahrenheit (°F)
  - **Currency**
    - Dollars ($) and Yen (¥)
  - **Volume**
    - Litres and gallons
  - **Prices**
    - A taxi driver charging per kilometre driven
- The **gradient** of a conversion graph represents the **rate of change**
  - If the y-axis is the **cost** of a taxi journey (£) and the x-axis is the **distance travelled** (mile) then the **gradient** represents the **cost per mile**
    - A gradient of 5 means the cost increases by £5 for each mile travelled

How do I use a conversion graph?

- Find the cost of 20kg using the conversion graph below
  - **Start** at 20kg on the **x-axis**
  - Draw a **vertical line** to the graph
  - Then a **horizontal line** across to the y-axis
  - **Read off** the value
    - $12
- Find how many kilograms can be bought with $30

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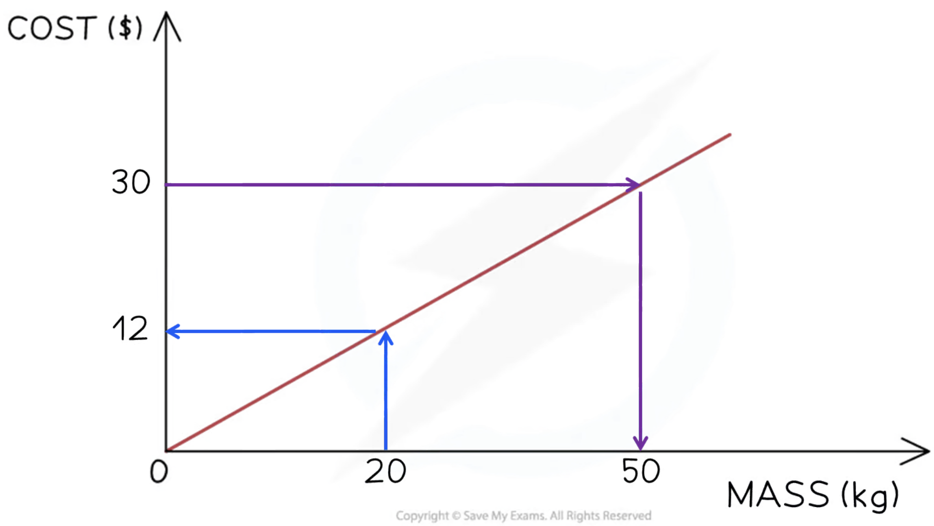
- **Start** at $30 on the ***y*-axis**
- Draw a **horizontal line** to the graph
- Then a **vertical line** down to the *x*-axis
- **Read off** the value
  - 50kg
- You can use **proportion** to find values that on **not on the axes**
  - To find the cost of 120kg
    - 120kg = 6 × 20kg costs 6 × $12 = $72
    - 120kg = 50kg + 50kg + 20kg costs $30 + $30 + $12 = $72
  - You can only do this if the **graph starts at the origin**

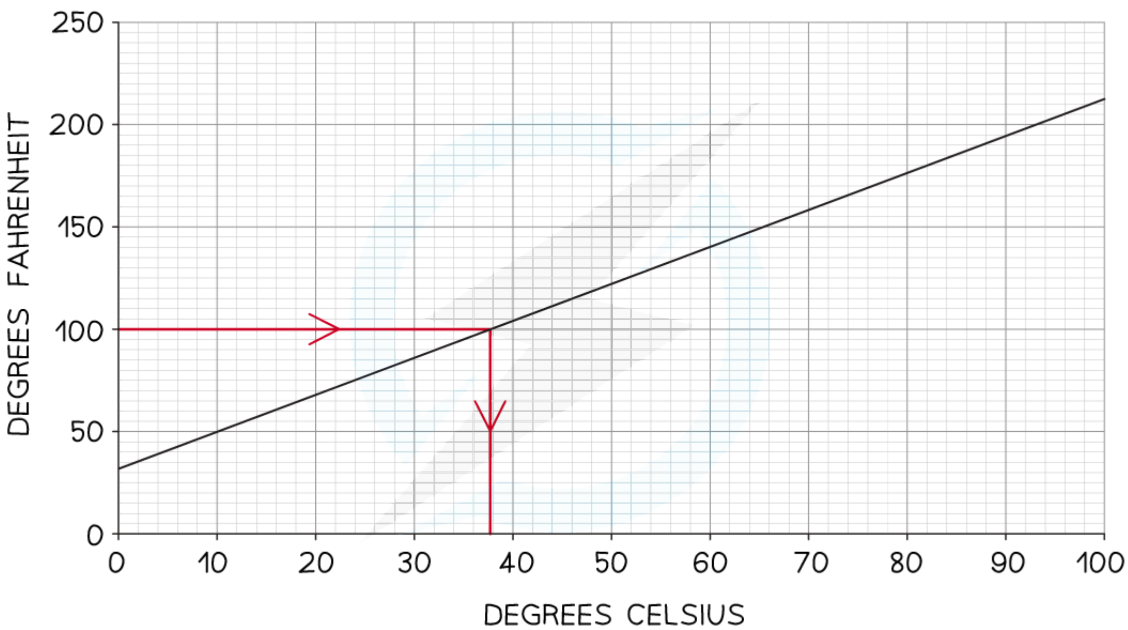


How do I use a conversion graph that does not start at the origin?

- Convert 100°F into Celsius using the conversion graph below
  - **Start** at 100°F on the ***y*-axis**
  - Draw a **horizontal line** to the graph
  - Then a **vertical line** down to the *x*-axis
  - **Read off** the value

Your notes

- $\approx$ 37.5°C
- Answers **between** 37°C and 38°C would be **accepted**
- (The true answer is 37.8°C to 1 decimal place)
- The **graph starts** at 32 on the ***y*-axis**
  - This means that 0°C is 32°F
  - This **starting value** sometimes represents a **fixed cost** when money is involved
    - It could represent the fixed charge for the cost of a taxi fare
- To convert values that are **not on the axis**
  - You would need to find an **equation for the straight-line**



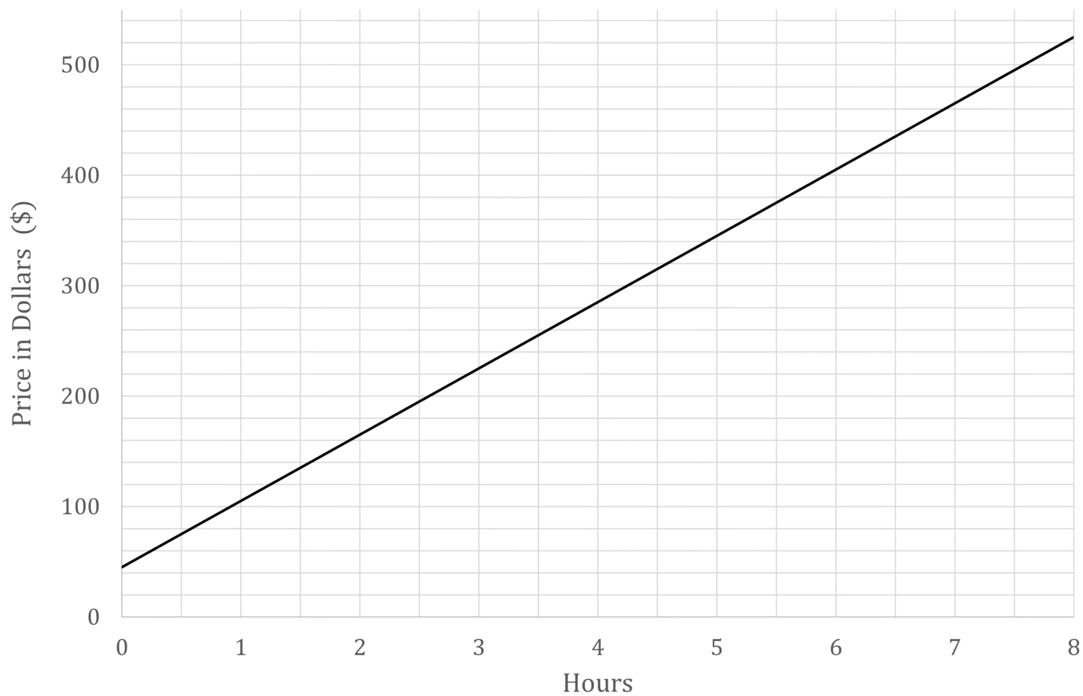
Examiner Tips and Tricks

- Always check the scales of the axes!

Your notes

Worked Example

The graph below shows the price (in dollars, $) charged by a plumber for the time spent (in hours) on a particular job.



(a) Estimate the price charged for a job that takes 3 hours.

Draw a vertical line up from the x-axis at 3 hours
Then a horizontal line across to the y-axis
Read off the value

Your notes

Approximately $225

Answers between $220 and £230 are accepted

(b) A particular job costs $320. Estimate, to the nearest half hour, how long this job took.

Draw a horizontal line across from the *y*-axis at $320
Draw a vertical line down to the *x*-axis
Read off the value to the nearest 0.5 hours

Your notes

4.5 hours (to the nearest half hour)

(c) The plumber charges a fixed callout fee for travelling to the customer and inspecting the job before starting any work.

Find the price of the callout fee.

Before starting work means 0 hours of work has been done
Find the price charged for 0 hours
This is the *y*-intercept of the graph

Approximately $45

Answers between $40 and £50 are accepted

Rates of Change of Graphs

Your notes

Rates of Change of Graphs

What is a rate-of-change graph?

- A **rate-of-change graph** usually shows how **a variable changes with time**
- The following are **examples** of rates-of-change graphs:
  - **Speed** against time
    - Speed is the rate of change of distance **as time increases**
  - **Acceleration** against time
    - Acceleration is the rate of change of velocity as time increases
  - The **depth** of water against time (e.g. in a container as it is filled with water)

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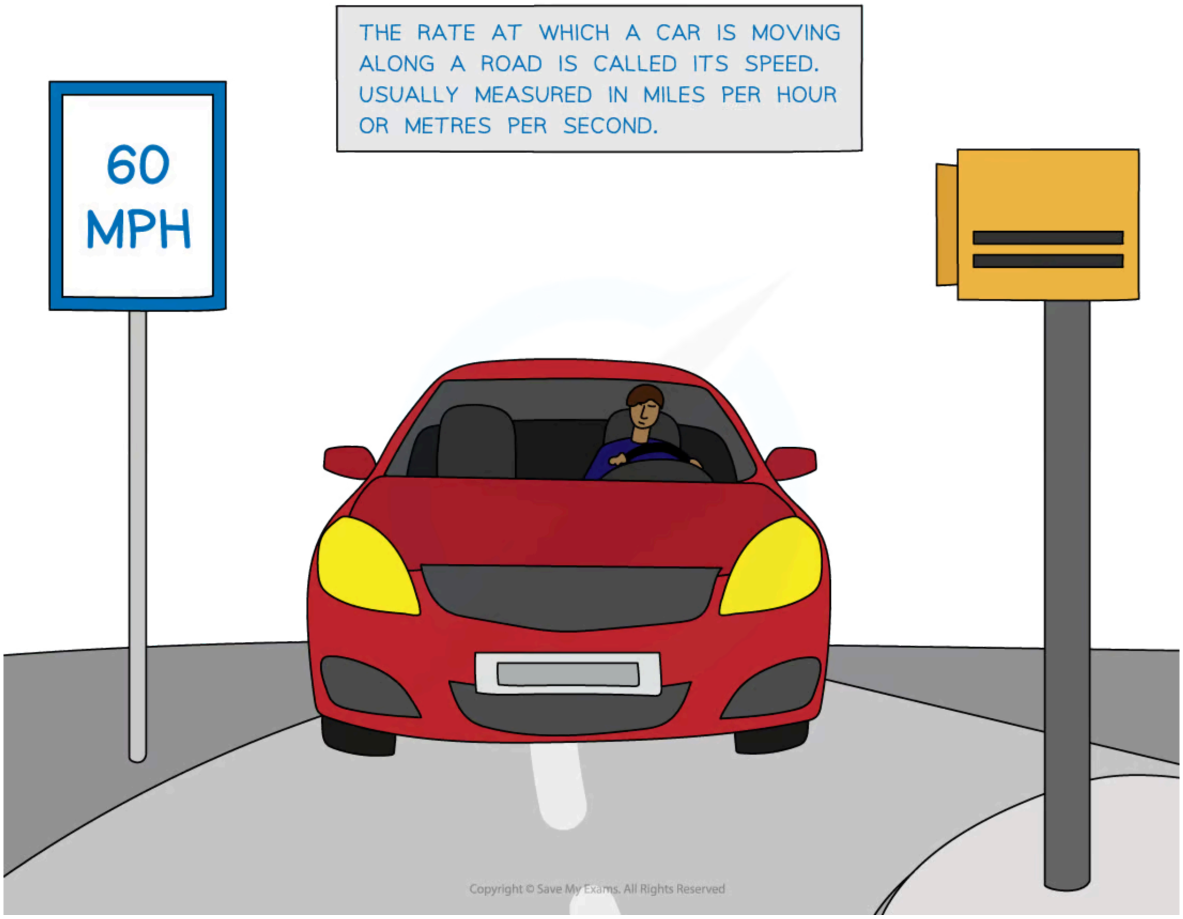

Can rates-of-change graphs not be against time?

- More generally, rate-of-change graphs can show any **two different variables** plotted against each other, **not just time**
  - E.g. the volume of air inside an inflating balloon plotted against the balloon's radius
    - This shows the rate of change of volume as **radius increases**
  - E.g. the number of ice-creams sold plotted against the weather temperature
    - This shows the rate of change of number of ice-creams **as temperature increases**

How can I use gradients to find rates of change?

- The **gradient** of the graph of *y* against *x* represents:

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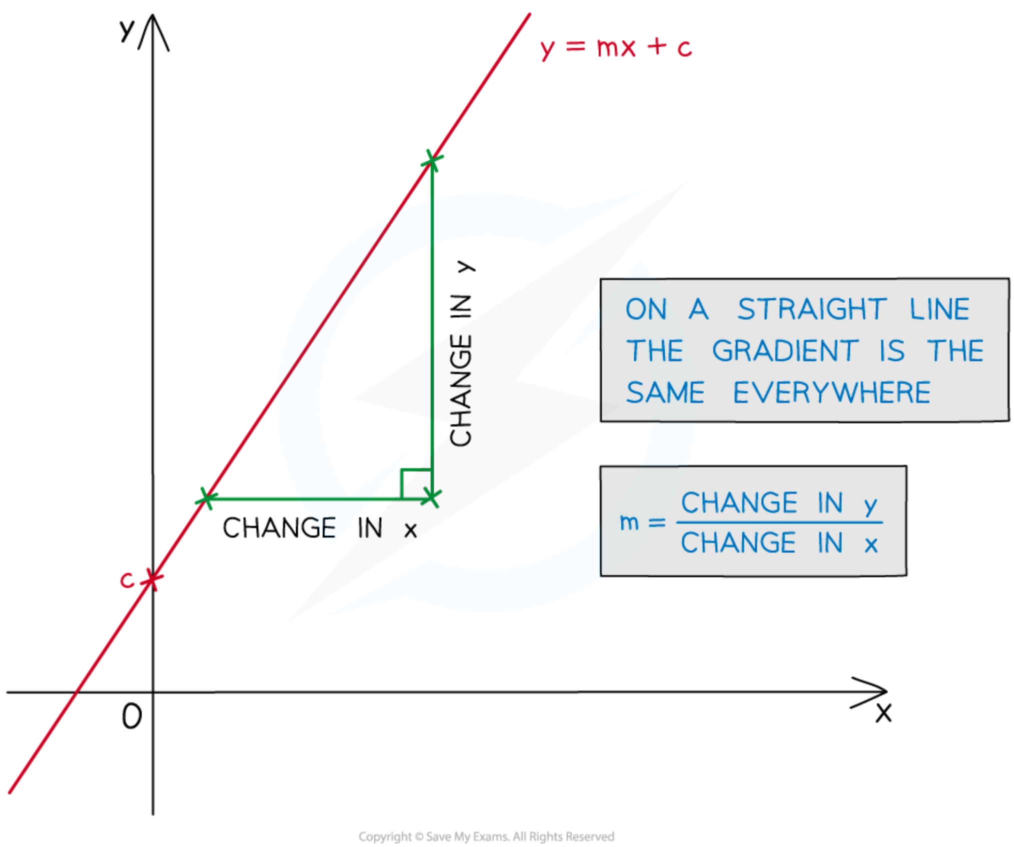
- the **amount of change in *y*** for every **1 unit of increase** in the ***x*-direction**
  - This is the amount of *y* **per** unit of *x*
- This is called the **rate of change of *y* against *x***
- The **units** of **gradients** are the units of the *y*-axis, divided by the units of the *x*-axis
  - E.g. If the graph shows volume in cm^3 on the *y*-axis and time in seconds on the *x*-axis, the rate of change is measured in cm^3/s (or cm^3s^{-1})
- If the graph is a **straight line** the rate of change is **constant**
  - If the graph is **horizontal**, the rate of change is **zero**
    - *y* is not changing as *x* changes

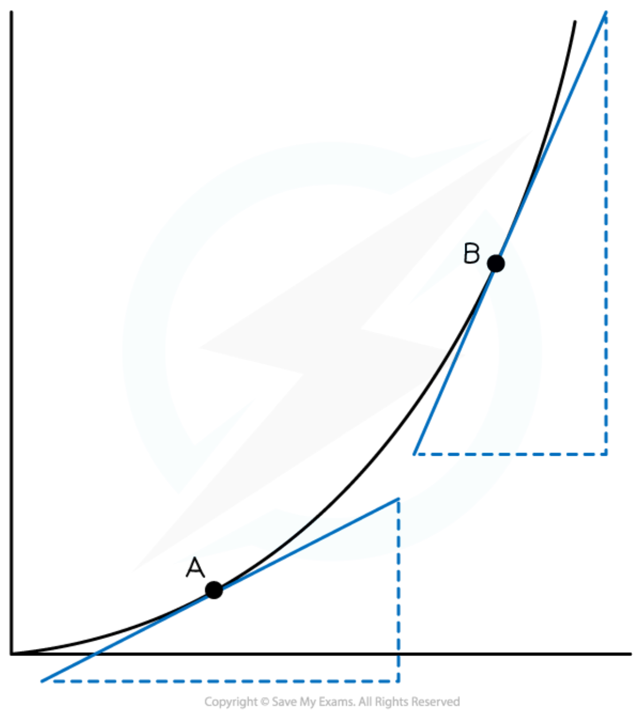


How can I use tangents to find rates of change?

- If the graph is a **curve**, you can draw a **tangent** at a point on the graph and find its **gradient**

Your notes

- This will be an **estimate** of the rate of change of *y* against *x*
- The **rate of change is greater** when the graph is **steeper**
  - In the below image
    - tangents drawn at points A and B show the graph is steeper at B
    - therefore the rate of change at B is greater



- On a **distance-time graph**, a tangent at a point on the curve can be used to estimate the **velocity** at that particular time
- On a **speed-time graph**, a tangent at a point on the curve can be used to estimate the **acceleration** at that particular time

Examiner Tips and Tricks

- The **units of the gradient** can help you understand what is happening in the context of an exam question

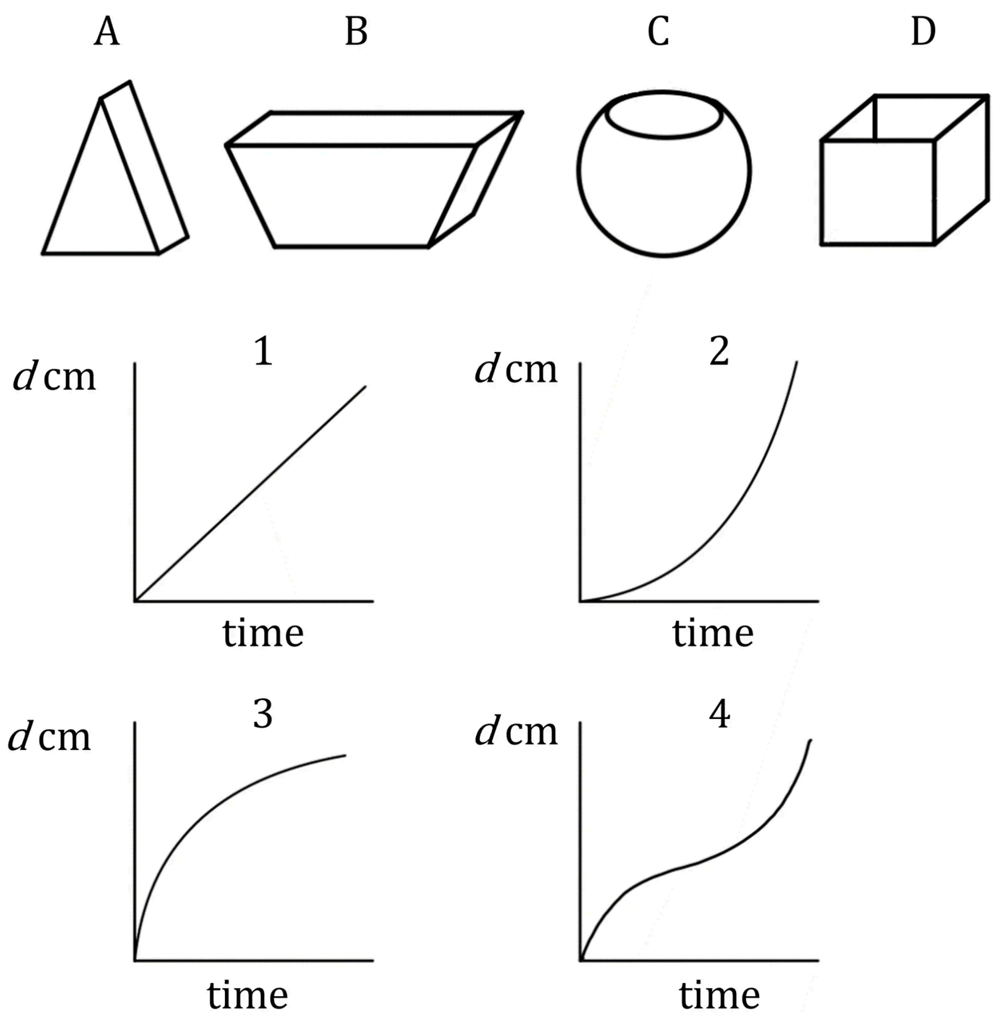
- For example, if the y-axis is in dollars and the x-axis is in hours, the gradient represents the change in dollars per hour

Your notes

Worked Example

(a) Each of the graphs below show the depth of water, d cm, in different containers that are being filled from a running tap of water.

Match each of the graphs 1, 2, 3, 4 with the containers A, B, C, D.



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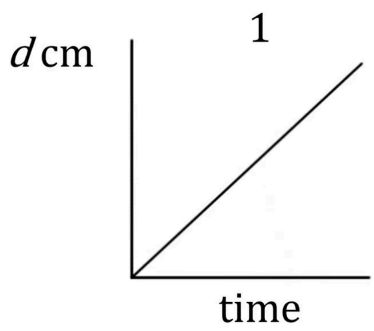
Considering graph 1: the gradient is constant
This means the rate of change is constant
So the depth increases at the same rate throughout

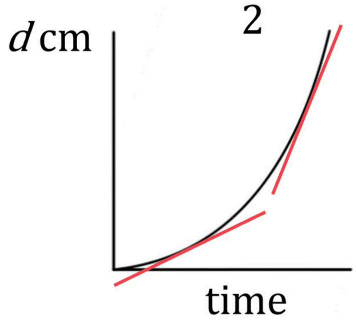
This matches container D which has vertical sides, so depth increases uniformly



Graph 1 is container D

Considering graph 2: the gradient starts shallow and becomes steeper, meaning that the depth increases faster and faster at the end

This matches container A, which gets narrower towards the top, causing the depth to increase faster at the end

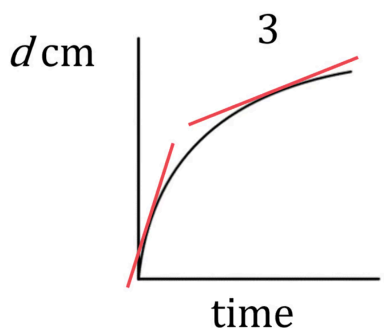


Graph 2 is container A

Considering graph 3: the gradient starts steep and becomes shallower, meaning that the depth increases at a slower and slower rate as time increases

This matches container B, which gets wider towards the top, causing the depth to increase more slowly at the end

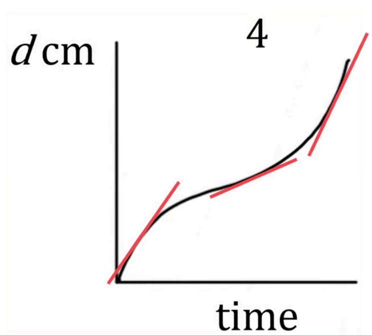
Your notes



Graph 3 is container B

Considering graph 4: the gradient starts steep, then becomes shallow, then becomes steep again. This means that the depth increases quickly, then slowly, then quickly again

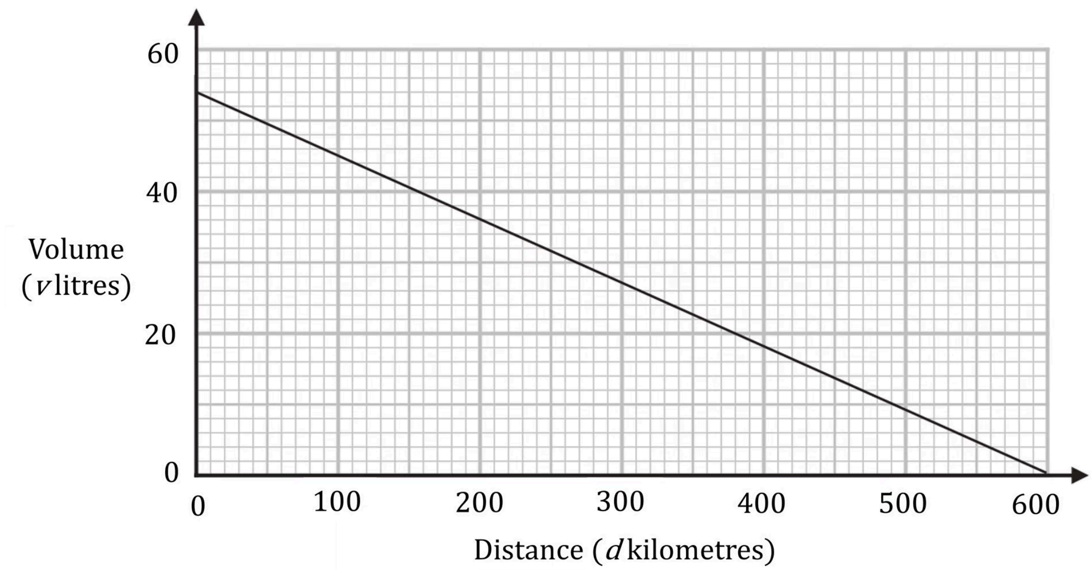
This matches container C, which is narrow at the bottom (fast depth increase), gets wider in the middle (slower depth increase) and narrow again at the top (faster depth increase)



Graph 4 is container C

(b) The graph below shows a model of the volume, *v* litres, of diesel fuel in the tank of George's truck after it has travelled a distance of *d* kilometres.

Your notes



(i) Find the gradient of the graph, stating its units clearly.

$$\text{Gradient} = \frac{\text{change in } y}{\text{change in } x}$$

$$\text{Gradient} = \frac{-54 \text{ litres}}{600 \text{ kilometres}}$$

-0.09 litres per kilometre

(ii) Interpret what the gradient of this graph represents.

Consider the units of the gradient: litres per kilometre
An increase in 1 kilometres causes a decreases of 0.09 litres in diesel fuel

The gradient represents the amount of diesel fuel used to travel one kilometre
Travelling 1 km uses up 0.09 litres of diesel fuel

(iii) Give one reason why this model may not be realistic.

Fuel may not be consumed at a constant rate
(The rate may vary as the driver accelerates)
(The rate may be different at the beginning when the truck weighs more)