AQA GCSE Maths: Higher

Your notes

Surds

Contents

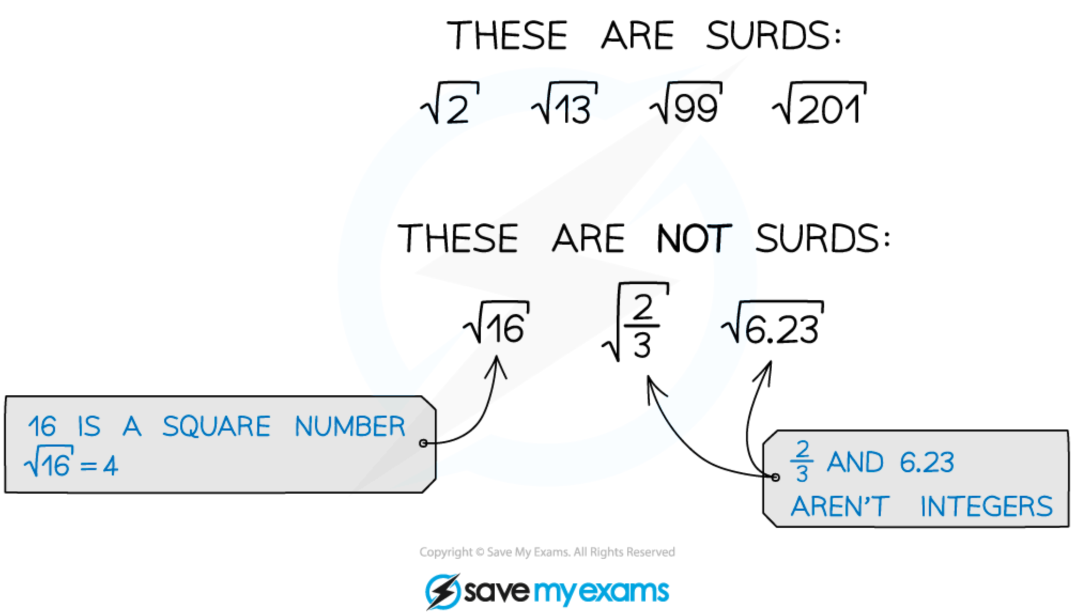
Simplifying Surds

Your notes

Surds & Exact Values

What is a surd?

- A surd is the square root of a non-square integer
- Using surds lets you leave answers in exact form
  - e.g. $5\sqrt{2}$ rather than $7.071067812...$



How do I do calculations with surds?

- **Multiplying surds**
  - You can multiply numbers under square roots together
  - $\sqrt{3} \times \sqrt{5} = \sqrt{3 \times 5} = \sqrt{15}$
- **Dividing surds**
  - You can divide numbers under square roots

Your notes

- $\frac{\sqrt{21}}{\sqrt{7}} = \sqrt{21} \div \sqrt{7} = \sqrt{21 \div 7} = \sqrt{3}$
- **Factorising surds**
  - You can factorise numbers under square roots
  - $\sqrt{35} = \sqrt{5 \times 7} = \sqrt{5} \times \sqrt{7}$
- **Adding or subtracting surds**
  - You can only add or subtract multiples of "like" surds
    - This is similar to collecting like terms when simplifying algebra
  - $3\sqrt{5} + 8\sqrt{5} = 11\sqrt{5}$
  - $7\sqrt{3} - 4\sqrt{3} = 3\sqrt{3}$
    - However $2\sqrt{3} + 4\sqrt{6}$ cannot be simplified
  - You cannot add or subtract numbers under square roots
  - Consider $\sqrt{9} + \sqrt{4} = 3 + 2 = 5$
    - This is not equal to $\sqrt{9+4} = \sqrt{13} = 3.60555...$

Examiner Tips and Tricks

If your calculator gives an answer as a surd, leave the value as a surd throughout the rest of your working.

This will ensure you do not lose accuracy throughout your working.

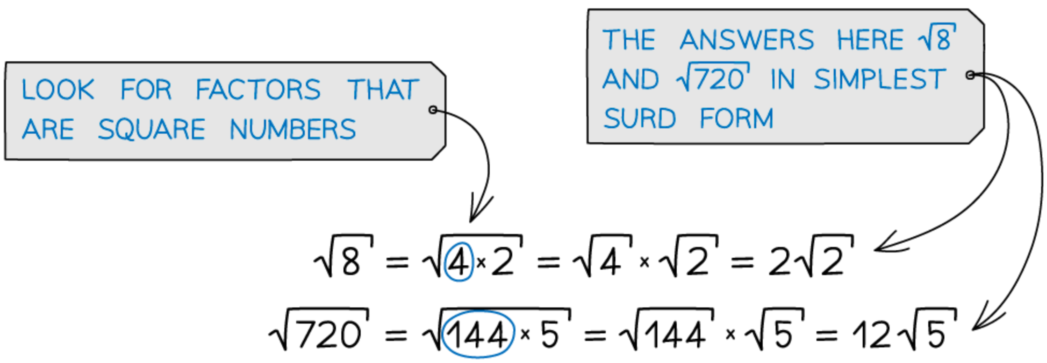
Simplifying Surds

How do I simplify surds?

- To **simplify a surd**, factorise the number using a square number, if possible
  - If multiple square numbers are a factor, use the largest
- Use the fact that $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ and then work out any square roots of square numbers

Your notes

- E.g. $\sqrt{48} = \sqrt{16 \times 3} = \sqrt{16} \times \sqrt{3} = 4 \times \sqrt{3} = 4\sqrt{3}$



- When simplifying multiple surds, simplify **each separately**
  - This may produce surds which can then be collected together
    - E.g. $\sqrt{32} + \sqrt{8}$ can be rewritten as $\sqrt{16}\sqrt{2} + \sqrt{4}\sqrt{2}$
    - This simplifies to $4\sqrt{2} + 2\sqrt{2}$
    - These surds can then be collected together
    - $6\sqrt{2}$
- You may have to expand double brackets containing surds
  - This can be done in the same way as multiplying out double brackets algebraically, and then simplifying
  - The property $\left(\sqrt{a}\right)^2 = a$ can be used to simplify the expression, once expanded
  - E.g. $\left(\sqrt{6} - 2\right)\left(\sqrt{6} + 4\right)$ expands to $\left(\sqrt{6}\right)^2 + 4\sqrt{6} - 2\sqrt{6} - 8$
    - This simplifies to $6 + 2\sqrt{6} - 8$ which gives $-2 + 2\sqrt{6}$

Worked Example

Write $\sqrt{54} - \sqrt{24}$ in the form $\sqrt{q}$ where q is a positive integer.

Simplify both surds separately by finding the highest square number that is a factor of each of them

9 is a factor of 54, so $\sqrt{54} = \sqrt{9 \times 6} = 3\sqrt{6}$

4 is a factor of 24, so $\sqrt{24} = \sqrt{4 \times 6} = 2\sqrt{6}$

Simplify the whole expression by collecting the like terms

$$\sqrt{54} - \sqrt{24} = 3\sqrt{6} - 2\sqrt{6} = \sqrt{6}$$

$\sqrt{6}$

Your notes

Rationalising Denominators

Your notes

Rationalising Denominators

What does rationalising the denominator mean?

- If a fraction has a denominator containing a surd then it has an **irrational** denominator
  - E.g. $\frac{4}{\sqrt{5}}$ or $\sqrt{\frac{2}{3}} = \frac{\sqrt{2}}{\sqrt{3}}$
- The fraction can be rewritten as an equivalent fraction, but with a **rational** denominator
  - E.g. $\frac{4\sqrt{5}}{5}$ or $\frac{\sqrt{6}}{3}$
- The numerator may contain a surd, but the denominator is **rationalised**

How do I rationalise denominators?

- If the **denominator** is a **surd**:
  - Multiply **the top and bottom of the fraction** by the **surd on the denominator**
    - $\frac{a}{\sqrt{\mathrm{b}}} = \frac{a}{\sqrt{\mathrm{b}}} \times \frac{\sqrt{\mathrm{b}}}{\sqrt{\mathrm{b}}}$
    - This is equivalent to multiplying by 1, so does not change the value of the fraction
    - $\sqrt{b} \times \sqrt{b} = b$ so the denominator is no longer a surd
  - Multiply the fractions as you would usually, and simplify if needed
    - $\frac{a\sqrt{b}}{b}$

Worked Example

Your notes

Write $\frac{4}{\sqrt{6}}$ in the form $q\sqrt{r}$ where q is a fraction in its simplest form and r has no square factors.

There is a surd on the denominator, so the fraction will need to be multiplied by a fraction with this surd on both the numerator and denominator

$$\frac{4}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$$

Multiply the fractions together by multiplying across the numerator and the denominator.

$$\frac{4 \times \sqrt{6}}{\sqrt{6} \times \sqrt{6}}$$

By multiplying out the denominator, you will notice that the surds are removed

$$\frac{4 \times \sqrt{6}}{\sqrt{6} \times \sqrt{6}} = \frac{4\sqrt{6}}{6}$$

Rewriting in the form $q\sqrt{r}$ and simplifying the fraction

$$\frac{4\sqrt{6}}{6} = \frac{4}{6} \times \sqrt{6} = \frac{2}{3}\sqrt{6}$$

$$\mathbf{\frac{2}{3}\sqrt{6}}$$

$$\boldsymbol{q = \frac{2}{3}}$$

$$\boldsymbol{r = 6}$$