AQA GCSE Maths: Higher

Your notes

Rounding, Estimation & Bounds

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Your notes

Rounding & Estimation

How do I round a number to a given place value?

- Identify the **digit** in the **required place value**
- **Circle the number to the right** of the required place value
  - If the circled number is **5 or more** then you round to the **bigger number**
  - If the circled number is **less than 5** then you round to the **smaller number**
  - Put a **zero** in any **following place values before the decimal point**
    - E.g. 1567.45 to the nearest 100 would be 16**00**

How do I round a number to a given decimal place?

- Identify the **position** of the **decimal place** you are rounding to
- **Circle the number to the right** of the required decimal place
  - If the circled number is **5 or more** then you round to the **bigger number**
  - If the circled number is **less than 5** then you round to the **smaller number**
    - E.g. 2.435123 to the nearest 2 d.p. would be 2.44
- When rounding to decimal places make sure you leave your answer with the **required amount of decimal places**
  - Do not put any zeros **after** the position of the decimal place you are rounding to
    - E.g. 1267 to the nearest 100 is 13**00**
    - But 1.267 to two decimal places (nearest 100th) is 1.27 not 1.270
  - If asked for a **certain number of decimal places**, you must give an answer with that number of decimal places
    - E.g. 2.395 to two decimal places is 2.40 (do not write 2.4)

Worked Example

Your notes

Round the following numbers to 2 decimal places.

(i) 345.254

(ii) 0.295 631

(iii) 4.998

(i) Identify the second decimal place (5)
Circle the digit to the right of the second decimal place (4)

$$345.25\textcircled{4}$$

As this digit is less than 5 we will round the number down

345.25 (2 d.p.)

No zeros are required after the second decimal place

(ii) Identify the second decimal place (9)
Circle the digit to the right of the second decimal place (5)

$$0.29\textcircled{5}631$$

As this digit is greater than or equal to 5 we will round the number up

0.30 (2 d.p.)

The zero shows we have rounded to two decimal places

(iii) Identify the second decimal place (9)
Circle the digit to the right of the second decimal place (8)

$$4.99\textcircled{8}$$

As this digit is greater than or equal to 5 we will round the number up

5.00 (2 d.p.)

Two zeros show we have rounded to 2 decimal places

How do I round a number to a given significant figure?

- To find the **first significant figure** when reading from left to right, find the **biggest place value** that has a **non-zero digit**

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- The first significant figure of 3097 is 3
- The first significant figure of 0.0062070 is 6
    - The zeros **before** the 6 are **not** significant
    - The zero **after** the 2 but **before** the 7 **is** significant
    - The zero **after** the 7 is **not** significant
- Count along to the **right** from the first significant figure to **identify** the **position** of the **required significant figure**
    - **Do** count zeros that are between other non-zero digits
        - E.g. 0 **is** the **second** significant figure of 3097
        - 9 is the **third** significant figure of 3097
- Use the **normal rules for rounding**
- For **large numbers**, complete places **up to the decimal point** with **zeros**
    - E.g. 34 568 to 2 significant figures is 35 **000**
- For **decimals**, complete places **between** the **decimal point** and the **first significant figure** with **zeros**
    - E.g. 0.003 435 to 3 significant figures is **0.00**3 44

How do I know what degree of accuracy to give my answer to?

- If a question requires your answer to be an **exact value**
    - You can leave it as a **simplified fraction**
        - E.g. $\frac{5}{6}$
    - You can leave it in terms of π or a **square root**
        - E.g. 4π, or $\sqrt{3}$
    - If it is an **exact decimal** up to and including 5 s.f., you can write it out without rounding it
        - E.g. 0.9375, or 850.25
- If the answer is **not exact**, an exam question will often state the **required degree of accuracy** for an answer
    - E.g. Give your answer to 2 significant figures

Your notes

- If the degree of accuracy is not asked for, use **3 significant figures**
  - All **working** and the **final answer** should show values correct to at least 4 significant figures
  - The final answer should then be **rounded** to 3 significant figures
- In **money calculations**, unless the required degree of accuracy is stated in the question, you can look at the **context**
  - Round to **2 decimal places**
    - E.g. $64.749214 will round to $64.75
  - Or to the nearest whole number, if this seems sensible (for example, other values are whole numbers)
    - $246 029.8567 rounds to $246 030
- When calculating **angles**, all values should be given correctly to **1 decimal place**
  - An angle of 43.5789° will round to 43.6°
  - An angle of 135.211...° will round to 135.2°

Examiner Tips and Tricks

- In an exam question check that you have written your answer correctly by considering if the value you have ended up with makes sense
  - Remember the importance of zeros to indicate place value
  - E.g. Round 2 530 457 to 3 significant figures, 253 (without the zeros) and 2 530 000 are very different sizes!

Worked Example

Round the following numbers to 3 significant figures.

(i) 345 256

(ii) 0.002 956 314

(iii) 3.997

(i) The first (non-zero) significant digit is in the hundred thousands column (3)
The third significant figure is therefore the value in the thousands column (5)

Circle the digit on the right of the third significant figure (2)

$$345\,2\,56$$

This digit is less than 5 so round down

345 000 (3 s.f.)

(ii) The first significant digit is in the thousandths column (2)
The third significant figure is therefore in the hundred thousandths column (5)

Circle the digit to the right of the third significant figure (6)

$$0.002\ 95\,6\,314$$

6 is greater than 5 so we need to round up

0.002 96 (3 s.f.)

(iii) The first significant digit is in the units column (3)
The third significant figure is therefore in the hundredths column (9)

Circle the digit to the right of the third significant figure (7)

$$3.99\,7$$

This value is greater than 5 so it will round up

4.00 (3 s.f.)

The two zeros indicate that it has been rounded to 3 s.f.

Why do I need to estimate?

- **Estimation** can be used to find **approximations** for difficult calculations
- You can estimate a calculation to **check your answers**
  - You can identify if there is a mistake in your working out if your answer is much bigger or smaller than your estimated value

How do I estimate?

- **Round** each number in the question to a sensible degree, then perform the **calculation**

Your notes

- The exam question will usually tell you what to round each number to before carrying out any calculations
- The **general rule** is to round numbers to **1 significant figure**
  - 7.8 → 8
  - 18 → 20
  - $3.65 \times 10^{-4} \rightarrow 4 \times 10^{-4}$
  - 1080 → 1000
- In certain cases it may be more **sensible** (or easier) to round to something convenient
  - 16.2 → 15
  - 9.1 → 10
  - 1180 → 1200
- **Avoid** rounding values to **zero**

How do I know if I have underestimated or overestimated?

- For **addition** $a + b$ and **multiplication** $a \times b$

a (rounded **up**) and/or ***b*** (rounded **up**)	Overestimate
a (rounded **down**) and/or ***b*** (rounded **down**)	Underestimate

- For **subtraction** $a - b$ and **division** $a \div b$

a (rounded **up**) and/or ***b*** (rounded **down**)	Overestimate
a (rounded **down**) and/or ***b*** (rounded **up**)	Underestimate
a (rounded **up**) and ***b*** (rounded **up**)	Not easy to tell
a (rounded **down**) and ***b*** (rounded **down**)	Not easy to tell

Examiner Tips and Tricks

Your notes

- Estimation exam questions often involve small decimals
  - Avoid rounding to 0, especially if the small decimal is the denominator of a fraction, as dividing by 0 is undefined

Worked Example

Calculate an **estimate** for $\frac{17.3 \times 3.81}{11.5}$.

State, with a reason, whether the estimate is an overestimate or an underestimate.

Round each number to 1 significant figure

$$17.3 \rightarrow 20$$
$$3.81 \rightarrow 4$$
$$11.5 \rightarrow 10$$

Perform the calculation with the rounded numbers

$$\frac{20 \times 4}{10} = \frac{80}{10} = 8$$

An estimate is 8

This is an overestimate as the numerator was rounded up and the denominator was rounded down

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Upper & Lower Bounds

Bounds & Error Intervals

What are bounds?

- **Bounds** are the values that a **rounded number** can lie between
  - The **smallest value** that a number can take is the **lower bound (LB)**
  - The **largest value** that a number must be less than is the **upper bound (UB)**
- The bounds for a number, x, can be written as $\mathrm{LB} \le x < \mathrm{UB}$
  - Note that the **lower bound** is **included** in the range of values x but the **upper bound is not**

How do we find the upper and lower bounds for a rounded number?

- Identify the **degree of accuracy** to which the number has been **rounded**
  - E.g. 24 800 has been rounded correct to the nearest 100
- **Divide** the degree of accuracy by **2**
  - E.g. If an answer has been rounded to the nearest 100, half the value is 50
- **Add** this value to the number to find the **upper bound**
  - E.g. 24 800 + 50 = 24 850
- **Subtract** this value from the number to find the **lower bound**
  - E.g. 24 800 - 50 = 24 750
- The **error interval** is the range between the upper and lower bounds
  - Error interval: **LB ≤ *x* < UB**
  - E.g. 24 750 ≤ 24 800 < 24 850

Examiner Tips and Tricks

- Read the exam question carefully to correctly identify the **degree of accuracy**

- It may be given as a **place value**, e.g. rounded to 2 s.f.
- Or it may be given as a **measure**, e.g. nearest metre

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Worked Example

The length of a road, l, is given as $l = 3.6\ \text{km}$, correct to 1 decimal place.

Find the lower and upper bounds for l.

The degree of accuracy is 1 decimal place, or 0.1 km
Divide this value by 2

$$0.1 \div 2 = 0.05$$

The true value could be up to 0.05 km above or below the given value

Upper bound: $3.6 + 0.05 = 3.65$ km

Lower bound: $3.6 - 0.05 = 3.55$ km

Upper bound: 3.65 km
Lower bound: 3.55 km

This could also be written as f $3.55 \leq l < 3.65$

What is truncation?

- **Truncation** is essentially the same as rounding **down** to a given degree of accuracy
- It is usually used in situations where a **whole number of items** is needed
  - E.g. If the answer to a question about the maximum number of people that can fit in a taxi is 4.6
    - Truncate the answer to 4, rather than round to 5

How do we find the error interval for a truncated number?

- You may be given a question where the number has been **truncated**
  - E.g. The first 3 digits of an answer, a, to a calculation have been written down as 2.95
    - The **smallest value** that the answer could have been is 2.95
    - The **largest value** that the number could have been up to (but not equal to) is 2.96 before it was truncated to 2.95

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- The error interval for the size of the number is $2.95 \leq a < 2.96$
- The **truncated** value should be the same as the **smallest value** in the error interval

Worked Example

The mass of a dog, m kg, is given as $m = 14$, truncated to 2 significant figures.

Write down the error interval for m.

The degree of accuracy is 2 significant figures, which is 1 kg in this question
A mass of 13.999 kg would be truncated to 13 kg
The smallest possible mass would be 14 kg itself

Smallest possible mass = 14 kg

14.999 kg would be truncated to 14 kg
15 kg would be truncated to 15 kg
The largest possible mass is therefore 15 kg but it can not be equal to this value

Largest possible mass < 15 kg

Write down the error interval using inequality notation

$$14 \leq m < 15$$

Calculations using Bounds

How do I find the bounds of a calculation?

- To find the **upper bound** of a calculation, consider how the result can be made as **large as possible**
- To find the **lower bound** of a calculation, consider how the result can be made as **small as possible**
- E.g. For an addition, $a + b$
  - The **upper** bound will be when both a and b are at their upper bounds
  - The **lower** bound will be when both a and b are at their lower bounds
- Sometimes you need **different bounds** in the same question
  - The upper bound of $\frac{a}{b}$ is the upper bound of a subtract the lower bound of b

Your notes

- **Increasing** the **numerator** makes the fraction **bigger**, $\frac{2}{1} < \frac{3}{1} < \frac{4}{1} < \ldots$
- But **increasing** the **denominator** make the fraction **smaller**, $\frac{1}{2} > \frac{1}{3} > \frac{1}{4} > \ldots$
- How to find the upper and lower bound for each operation is summarised in the table below

	Upper Bound	Lower Bound
$a+b$	Upper + Upper	Lower + Lower
$a-b$	Upper - Lower	Lower - Upper
$a\times b$	Upper × Upper	Lower × Lower
$\frac{a}{b}$	Upper ÷ Lower	Lower ÷ Upper

How do I use upper and lower bounds in contexts?

- Questions often give real-life **contexts** and ask about bounds
  - For example
    - To see if two cars will fit on the back of a truck
    - Use the upper bounds of the lengths of the two cars
    - This is like finding the upper bound of $a+b$
  - For example
    - To find the minimum speed (speed = distance $\div$ time)
    - Divide the lower bound of the distance by the upper bound of the time
    - This is like finding the lower bound of $\frac{a}{b}$

How can bounds help with calculations?

- You can use bounds to determine the **level of accuracy** of a calculation

- E.g. If a value has a lower bound of 8.33217... and upper bound of s 8.33198...
  - The true value is between 8.33217... and 8.33198...
- Find the level of accuracy for which **both bounds** round to the **same number**
  - This happens at 4 sf (rounding to 8.332)
  - To 5 sf they are different (lower is 8.3322 and upper is 8.3320)
- Therefore you know the original value rounds to 8.332 to 4 significant figures

Your notes

Worked Example

A room measures 4 m by 7 m, where each measurement is made to the nearest metre.

Find the upper and lower bounds for the area of the room.

Find the bounds for each dimension, you could write these as error intervals, or just write down the upper and lower bounds

As they have been rounded to the nearest metre, the true values could be up to 0.5 m bigger or smaller

$$3.5 \leq 4 < 4.5$$
$$6.5 \leq 7 < 7.5$$

Calculate the lower bound of the area, using the two smallest measurements

$$3.5 \times 6.5$$

Lower Bound = 22.75 m^2

Calculate the upper bound of the area, using the two largest measurements

$$4.5 \times 7.5$$

Upper Bound = 33.75 m^2

Worked Example

David is trying to work out how many slabs he needs to buy in order to lay a garden path.

Slabs are 50 cm long, measured to the nearest 10 cm.

The length of the path is 6 m, measured to the nearest 10 cm.

Find the maximum number of slabs David will need to buy.

Find the bounds for each measurement

As they have been rounded to the nearest 10 cm, the true values could be up to 5 cm bigger or smaller

Change quantities into the same units

Length of the slabs: $45 \leq 50 < 55$ cm
or in metres: $0.45 \leq 0.5 < 0.55$ m

Length of the path: $5.95 \leq 6 < 6.05$ m

The maximum number of slabs needed will be when the path is as long as possible (6.05 m), and the slabs are as short as possible (0.45 m)

$$\text{Maximum number of slabs} = \frac{6.05}{0.45} = 13.444...$$

Assuming we can only purchase a whole number of slabs, round up to nearest integer

The maximum number of slabs to be bought is 14

Your notes