AQA GCSE Maths: Higher

Your notes

Fractions

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Basic Fractions

What is a fraction?

- A **fraction** is **part** of a whole
- A fraction is written as $\frac{a}{b}$ where *a* and *b* are **integers**
  - The number on the top, *a*, is called the **numerator**
  - The number on the bottom, *b*, is called the **denominator**
- The **values** of the numerator and denominator tell us about the **size** of the fraction
  - The denominator is how many **parts** the **whole is split into**
  - The numerator is how many **parts** you **have**
    - For example, $\frac{2}{3}$ means that the whole is divided into 3 parts and you have 2 of these parts

How do I find a fraction of an amount?

- **Method 1**
  Divide by the **denominator** and **multiply** by the **numerator**
  - To find $\frac{2}{5}$ of 60, divide it by 5 then multiply it by 2
    - $60 \div 5 = 12$, then $12 \times 2 = 24$
- **Method 2**
  Change the fraction into a **decimal**, then **multiply**
  - To find $\frac{9}{10}$ of 15, turn $\frac{9}{10}$ into 0.9 then multiply
    - $0.9 \times 15 = 13.5$
- **Method 3**
  Write both numbers as fractions and **multiply the two fractions** together

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- To find $\frac{5}{6}$ of 8, turn 8 into $\frac{8}{1}$
  - Work out $\frac{5}{6} \times \frac{8}{1} = \frac{40}{6} = \frac{20}{3}$

What are equivalent fractions?

- **Equivalent fractions** are two fractions that represent the **same amount**
  - They are different ways of writing the same thing
  - For example, $\frac{1}{2}$ and $\frac{6}{12}$ are equivalent fractions

How do I find an equivalent fraction?

- To find an equivalent fraction, **multiply the top and bottom** of a fraction by the same number
  - $\frac{5}{6}$ is equivalent to $\frac{5 \times 2}{6 \times 2} = \frac{10}{12}$ and $\frac{5 \times 3}{6 \times 3} = \frac{15}{18}$ and $\frac{5 \times 4}{6 \times 4} = \frac{20}{24}$ etc.
  - For each fraction, there are an **infinite number** of equivalent fractions

What is a simplified fraction?

- A **simplified fraction** is a fraction that is written using the **smallest** possible **integer values**
  - Simplifying turns a fraction into an 'easier' **equivalent fraction**
- If both the numerator and denominator can be divided by the **same number** (called a **common factor**), the fraction can be **simplified**
- Simplifying is also called **cancelling** a fraction

How do I simplify fractions?

- To simplify the fraction, **divide the top and bottom** by the common factor
- $\frac{12}{18} = \frac{12 \div 6}{18 \div 6} = \frac{2}{3}$
- $\frac{25}{45} = \frac{25 \div 5}{45 \div 5} = \frac{5}{9}$

Examiner Tips and Tricks

- Make use of your calculator in your exam
  - Type any fraction into it and pressing equals will automatically simplify the fraction for you
  - If the question asks you to show your working, you must show which number you divided by

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Mixed Numbers & Top Heavy Fractions

What are mixed numbers & top heavy fractions?

- A **mixed number** has an **integer** part and a **fraction** part
  - $3\frac{3}{4}$ has the whole number 3 and the fraction $\frac{3}{4}$, meaning "three and three quarters"
- A **top heavy fraction** is also known as an **improper fraction**
- An **improper fraction** is a fraction where the **numerator** is **bigger** than the **denominator**
  - $\frac{15}{4}$ means "fifteen quarters"
- Any fraction that is **greater than 1** can be written as either a **mixed number** or an equivalent **improper fraction**

How do I convert a mixed number into an improper fraction?

- **Multiply** the **integer part** by the **denominator** of the **fraction part**
  - For example, convert $4\frac{6}{7}$ into a top heavy fraction
  - $4 \times 7 = 28$
- **Add** the result to the **numerator** of the fraction part
  - $28 + 6 = 34$
- **Write** the **'new' numerator** over the **same denominator** and ignore the integer part
  - $4\frac{6}{7} = \frac{34}{7}$

How do I convert an improper fraction into a mixed number?

- **Divide** the numerator by the bottom

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- For example, convert $\frac{22}{3}$ into a mixed number
- $22 \div 3 = 7$ remainder 1
- The **integer part** of the mixed number is the **whole number**
- The **fraction part** is the **remainder** over the denominator
- $\frac{22}{3} = 7\frac{1}{3}$

Examiner Tips and Tricks

- The term '**improper fraction**' is frequently used in exam papers
  - Make sure you remember that improper fractions are the same as top heavy fractions

Worked Example

(a) Write $5\frac{3}{4}$ as an improper fraction.

Multiply the whole number by the denominator, and add to the numerator

Keep the denominator the same

$$\frac{(5 \times 4) + 3}{4}$$

Simplify

$$\frac{23}{4}$$

(b) Write $\frac{17}{5}$ as a mixed number.

Divide the top by the bottom

17 ÷ 5 = 3 remainder 2

The final answer is 3, with 2 parts still left over

Leave the 3 as the whole number part and put the 2 over the original denominator of 5

$$3\frac{2}{5}$$

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Adding & Subtracting Fractions

How do I add or subtract two fractions?

- **Addition** and **subtraction** are treated in exactly the same way when dealing with fractions
  - Find the **lowest common denominator**
  - Write each fraction as an **equivalent fraction** over this denominator
  - **Add** (or **subtract**) the **numerators** and write this over a **single** denominator
    - Do **not** add the denominators
  - **Simplify** the fraction by cancelling common factors

What do I do if any of the fractions are mixed numbers?

- **Convert any** mixed numbers into **improper fractions**
  - Add or subtract the fractions as usual
  - Remember to **convert the answer** back to a mixed number if required

Worked Example

(a) Find $\frac{2}{3}+\frac{1}{5}$.

Find the lowest common denominator of 3 and 5
15 is the smallest number that can be divided by both 3 and 5

The lowest common denominator is 15

Write both fractions as equivalent fractions with a common denominator of 15
Multiply the numerator and denominator of the first fraction by 5
Multiply the numerator and denominator of the second fraction by 3

$$\frac{2\times5}{3\times5}+\frac{1\times3}{5\times3}=\frac{10}{15}+\frac{3}{15}$$

Add the numerators and write over a single denominator

$$\frac{10+3}{15}=\frac{13}{15}$$

There are no common factors that will cancel

$$\frac{13}{15}$$

(b) Find $3\frac{3}{4}-\frac{5}{8}$ giving your answer as a mixed number.

Change the mixed number into an improper fraction

Multiply the denominator by the whole part and add to the numerator
Write this value over the denominator

$$\frac{4\times3+3}{4}=\frac{15}{4}$$

Find the lowest common denominator of 4 and 8
8 is the smallest number that by divided by both 4 and 8
The lowest common denominator is 8

Write both fractions as equivalent fractions over 8
Multiply the numerator and denominator of the first fraction by 2

$$\frac{15\times2}{4\times2}-\frac{5}{8}=\frac{30}{8}-\frac{5}{8}$$

Subtract the numerators and write over a single denominator

$$\frac{30-5}{8}=\frac{25}{8}$$

Change into a mixed number
Divide 25 by 8 to get 3 remainder 1

$$3\frac{1}{8}$$

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Multiplying & Dividing Fractions

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Multiplying Fractions

How do I multiply two fractions?

- **Cancel** any factors that are common to both the **numerator** and the **denominator**
  - The common factors could be in either of the fractions
  - $\frac{3}{25} \times \frac{10}{11} = \frac{3}{\cancel{25}^{5}} \times \frac{\cancel{10}^{2}}{11} = \frac{3}{5} \times \frac{2}{11}$
- **Multiply**
  - Multiply the **numerators**
  - Multiply the **denominators**
  - $\frac{3}{5} \times \frac{2}{11} = \frac{3 \times 2}{5 \times 11} = \frac{6}{55}$
- **Cancel** any common factors again if possible

How do I multiply two fractions if one is a mixed number?

- Always convert **mixed numbers** into **improper fractions** before multiplying
  - **Convert** improper fractions **back** into mixed numbers at the end if required

Worked Example

Find $\frac{4}{15} \times \frac{25}{11}$.

The 15 and 25 can be cancelled before multiplying

$$\frac{4}{3 \times 5} \times \frac{5 \times 5}{11} = \frac{4}{3 \times \cancel{5}} \times \frac{\cancel{5} \times 5}{11} = \frac{4}{3} \times \frac{5}{11}$$

Multiply the numerators together and the denominators together

$$\frac{4 \times 5}{3 \times 11} = \frac{20}{33}$$

There is no further cancelling that can be done
Write down the fraction

$$\frac{20}{33}$$

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Dividing Fractions

How do I divide two fractions?

- **Flip** the second fraction and **change** ÷ into × "
  - $\frac{1}{3} \div \frac{4}{5}$ becomes $\frac{1}{3} \times \frac{5}{4}$
  - The 'flipped' fraction is called a **reciprocal fraction**
- **Cancel** any factors that are common to both the **numerator** and the **denominator**
- **Multiply** the fractions
  - Multiply the **numerators**
  - Multiply the **denominators**
- **Cancel** any common factors again if possible

How do I divide two fractions when one of them is a mixed number?

- Always convert **mixed numbers** into **improper fractions** before dividing
  - **Convert** improper fractions **back** into mixed numbers at the end if required

Examiner Tips and Tricks

- Remember "**flip'n'times**"
  - When **dividing** fractions you are **multiplying** by the **reciprocal**

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Worked Example

Divide $3\frac{1}{4}$ by $\frac{3}{8}$, giving your answer as a mixed number.

Rewrite $3\frac{1}{4}$ as an improper fraction

$$3\frac{1}{4} = \frac{3\times 4+1}{4} = \frac{13}{4}$$

Turn the division into a multiplication
'Flip' the second fraction and turn ÷ into ×

$$\frac{13}{4} \div \frac{3}{8} = \frac{13}{4} \times \frac{8}{3}$$

Cancel a factor of 4 from the numerators and denominators

$$\frac{13}{4} \times \frac{8}{3} = \frac{13}{\cancel{4}} \times \frac{\cancel{4}\times 2}{3} = \frac{13}{1} \times \frac{2}{3}$$

Multiply the fractions

$$\frac{13\times 2}{1\times 3} = \frac{26}{3}$$

Rewrite as a mixed number

$$\frac{26}{3} = \frac{24}{3} + \frac{2}{3} = 8\frac{2}{3}$$

$$8\frac{2}{3}$$