



AQA GCSE Maths: Higher



Your notes

Expanding Brackets

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Your notes

Expanding & Simplifying Single Brackets

Expanding One Bracket

How do I expand a bracket?

- The expression $3x(x + 2)$ means $3x$ **multiplied by** the bracket $(x + 2)$
 - $3x$ is the term **outside** the bracket
 - this is sometimes called a **factor**
 - and $x + 2$ are the terms **inside** the bracket
- Expanding the brackets means **multiplying** the **outside term** by **each term** on the **inside**
 - This will **remove** (get rid of) the **brackets**
 - $3x(x + 2)$ expands to $3x \times x + 3x \times 2$ which simplifies to $3x^2 + 6x$
- Beware of **minus signs**
 - Remember the rules
 - $- \times - = +$
 - $- \times + = -$
 - It helps to put **brackets** around **negative** terms



Worked Example

(a) Expand $4x(2x - 3)$.

Multiply the $4x$ term outside the brackets by both terms inside the brackets

$$4x \times 2x + 4x \times (-3)$$

Simplify

$$8x^2 - 12x$$

(b) Expand $-7x(4 - 5y)$.

Multiply the $-7x$ outside the brackets by both terms inside the brackets



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$$(-7x) \times 4 + (-7x) \times (-5y)$$

Simplify and remember that multiplying two negatives gives a positive

$$-28x + 35xy$$

Expand & Simplify

How do I simplify brackets that are added together?

- First **expand** both brackets **separately**
 - $4(x + 7) + 5x(3 - x)$
 - The **first** set of brackets expands to $4 \times x + 4 \times 7$ which **simplifies** to $4x + 28$
 - The **second** set of brackets expands to $5x \times 3 + 5x \times (-x)$ which **simplifies** to $15x - 5x^2$
 - So $4(x + 7) + 5x(3 - x) = 4x + 28 + 15x - 5x^2$
- Then **collect like terms**
 - $4x + 15x = 19x$
 - The other two terms are **not** like terms
 - So $4(x + 7) + 5x(3 - x) = 19x + 28 - 5x^2$



Worked Example

(a) Expand and simplify $2(x + 5) + 3x(x - 8)$.

Expand each set of brackets separately

You can keep negative terms inside brackets

$$2 \times x + 2 \times 5 + 3x \times x + 3x \times (-8)$$

Simplify each term

$$2x + 10 + 3x^2 - 24x$$

Collect like terms (the $2x$ and the $-24x$)



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$$-22x + 10 + 3x^2$$

(b) Expand and simplify $3x(x + 2) - 7(x - 6)$.

Expand each set of brackets separately

Be careful: the second set of brackets has a -7 in front, not +7

$$3x \times x + 3x \times 2 + (-7) \times x + (-7) \times (-6)$$

Simplify each term

Remember that multiplying two negatives gives a positive

$$3x^2 + 6x - 7x + 42$$

Collect like terms

$$3x^2 - x + 42$$



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Expanding Double Brackets

Expanding Two Brackets

How do I expand two brackets using FOIL?

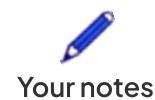
- **Every term** in the **first** bracket must be multiplied by **every term** in the **second** bracket
 - Expanding $(x + 1)(x + 3)$ requires 4 multiplications in total
- A good way to remember all the multiplications is **FOIL**
 - **F = First:** multiply together the **first** terms in each bracket
 - **O = Outside:** multiply the first term in the first bracket by the last term in the last bracket
 - Visually, these are the **outer** terms
 - **I = Inside:** multiply the last term in the first bracket by the first term in the last bracket
 - Visually, these are the **inner** terms
 - **L = Last:** multiply together the **last** terms in each bracket
- It helps to put **negative** terms in **brackets** when multiplying
- **Simplify** the final answer by **collecting like terms** (if there are any)

How do I expand two brackets using a grid?

- You may prefer a more **visual** method using a **grid**
- To expand $(x + 1)(x + 3)$, write out the brackets as **row** and **column headings** of a grid
 - They can be in either direction
 - Remember to write the appropriate **sign** in front of each term

	x	+1
x		
+3		

- For each cell in the grid, **multiply** the term in the **row** heading by the term in the **column** heading



	x	+1
x	x^2	x
+3	$3x$	3

- Add together all the terms **inside** the grid to get the answer

- $x^2 + x + 3x + 3$

- Collect **like** terms

- $x^2 + 4x + 3$

How do I expand when there are multiple variables?

- All the same rules and methods apply as when there is just one variable

- Remember to only simplify **like terms**

- For example: $(3x + 2y)(4x - 6y)$

- Expanding: $12x^2 - 18xy + 8xy - 12y^2$

- The **xy** terms can be combined

- $12x^2 - 10xy - 12y^2$



Worked Example

(a) Expand $(2x - 3)(x + 4)$.

Using FOIL, multiply together the first, outer, inner and last terms

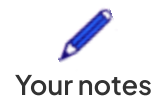
$$\begin{array}{cccc}
 \text{F} & & \text{O} & & \text{I} & & \text{L} \\
 (2x \times x) & + & (2x \times 4) & + & (-3 \times x) & + & (-3 \times 4)
 \end{array}$$

Simplify each term

$$2x^2 + 8x - 3x - 12$$

Collect like terms (the $8x$ and $-3x$)

$$2x^2 + 5x - 12$$



(b) Expand $(x - 3)(3x - 5)$.

Using FOIL, multiply together the first, outer, inner and last terms

$$\begin{array}{cccc} \text{F} & & \text{O} & & \text{I} & & \text{L} \\ \hline (x \times 3x) & + & (x \times (-5)) & + & (-3 \times 3x) & + & (-3 \times (-5)) \end{array}$$

Simplify each term

$$3x^2 - 5x - 9x + 15$$

Collect like terms (the $-5x$ and $-9x$)

$$3x^2 - 14x + 15$$



Worked Example

Expand $(3r + 2t)(5t - 8r)$.

Expand using your chosen method, here we will use a grid

	$3r$	$+2t$
$5t$		
$-8r$		

Work out the term in each place in the grid by multiplying

	$3r$	$+2t$
$5t$	$15rt$	$10t^2$
$-8r$	$-24r^2$	$-16rt$

So the expanded expression is

$$10t^2 + 15rt - 16rt - 24r^2$$



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The ***rt*** terms can be combined

$$10t^2 - rt - 24r^2$$

Expanding Squared Brackets

How do I expand a bracket squared?

- Remember that a **square** number is a number multiplied by itself
- Write $(x + 3)^2$ as $(x + 3)(x + 3)$ and use one of the methods above
 - With FOIL: $(x + 3)(x + 3) = x^2 + 3x + 3x + 9$
 - Then collect like terms: $x^2 + 6x + 9$
- Do **not** make the common **mistake** of saying $(x + 3)^2$ is $x^2 + 3^2$
 - This cannot be true, try substituting in $x = 1$
 - you would get $(1 + 3)^2 = 4^2 = 16$ on the left
 - but $1^2 + 3^2 = 1 + 9 = 10$ on the right



Worked Example

Expand $(2x + 3)^2$.

Remember that the answer is not $(2x)^2 + 3^2$

Rewrite the expression as two separate brackets multiplied together

$$(2x + 3)(2x + 3)$$

Using FOIL, multiply together the first, outer, inner and last terms

F	O	I	L
$(2x \times 2x)$	$+$	$(2x \times 3)$	$+$
$(3 \times 2x)$	$+$	(3×3)	

Simplify each term

$$4x^2 + 6x + 6x + 9$$

Collect like terms (the $6x$ and $6x$)

$$4x^2 + 12x + 9$$



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Expanding Triple Brackets

Expanding Three Brackets

How do I expand three brackets?

- Multiply out any **two of the brackets** using a standard method and **simplify**
- Then multiply the resulting expression by the third (unused) bracket
- This step often looks like $(x + a)(x^2 + bx + c)$
- Every term in the first bracket must be multiplied with every term in the second bracket
- A **grid** can help to keep track of all the terms
 - E.g. $(x + 2)(x^2 + 3x + 1)$

	x^2	$+3x$	$+1$
x	x^3	$3x^2$	x
$+2$	$2x^2$	$6x$	2

- Add all the terms inside the grid together
 - $x^3 + 2x^2 + 3x^2 + 6x + x + 2$
- **Simplify** by collecting any like terms
 - $x^3 + 5x^2 + 7x + 2$



Worked Example

Expand $(2x - 3)(x + 4)(3x - 1)$.

Expand and simplify the first two brackets, for example using the FOIL method



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$$\begin{aligned}
 &(2x - 3)(x + 4) \\
 &= 2x \times x + 2x \times 4 + (-3) \times x + (-3) \times 4 \\
 &= 2x^2 + 8x - 3x - 12 \\
 &= 2x^2 + 5x - 12
 \end{aligned}$$

Rewrite the original expression with the first two brackets expanded

$$(2x^2 + 5x - 12)(3x - 1)$$

Multiply all of the terms in the first set of brackets by all of the terms in the second set of brackets

A grid can help when there are many terms to multiply together (e.g. write $2x^2 + 5x - 12$ in the vertical column and $3x - 1$ in the horizontal column, then multiply corresponding terms)

	$3x$	-1
$2x^2$	$6x^3$	$-2x^2$
$5x$	$15x^2$	$-5x$
-12	$-36x$	12

Write out the multiplied terms

$$6x^3 - 2x^2 + 15x^2 - 5x - 36x + 12$$

Collect the like terms to simplify

$$6x^3 + 13x^2 - 41x + 12$$