AQA GCSE Maths: Higher

Your notes

Angles in Polygons & Parallel Lines

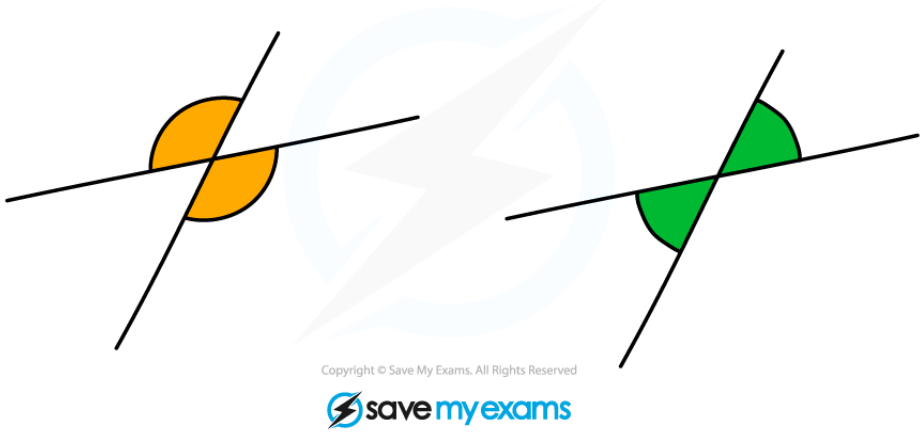
Contents

Your notes

Basic Angle Properties

What are the basic angle properties?

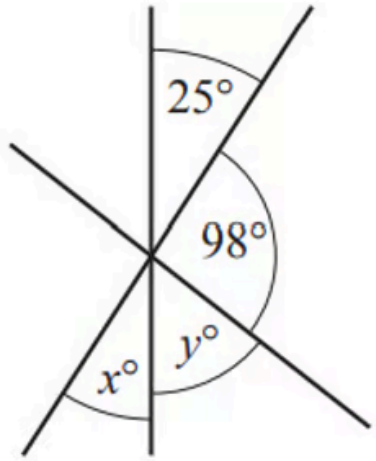
- Angles around a **point** add up to **360°**
- Angles that form a **straight line** add up to **180°**
- **Vertically opposite** angles are **equal**
  - Vertically opposite angles occur when **two lines intersect**, as in the diagram below



Worked Example

The diagram below shows three straight lines intersecting at a point.

Your notes



NOT TO SCALE

Find the values of x and y.

Vertically opposite angles between two intersecting lines are equal

$$x = 25$$

Angles that meet on a straight line add up to 180°

$$x + y + 98 = 180$$
$$25 + y + 98 = 180$$
$$123 + y = 180$$

Solve to find y

$$y = 180 - 123$$
$$y = 57$$

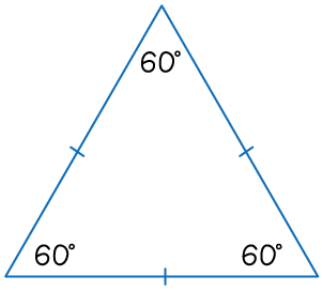
$$x = 25,\ y = 57$$

What are the angle properties of triangles?

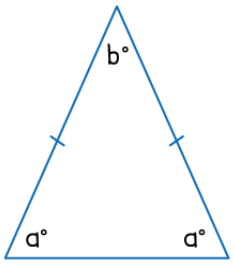
- The **three** interior angles inside any triangle **add up to 180°**
- If the triangle is **isosceles** then **two angles will be equal**
  - These will be the **two angles opposite** the **two sides of equal length**

- If the triangle is **equilateral** then all three angles will be equal
  - Each angle will equal **60°**
- A **right-angled triangle** has one 90° angle

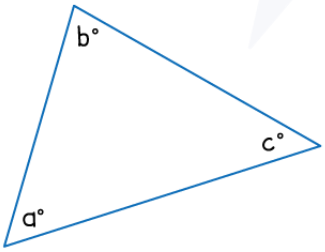
Your notes



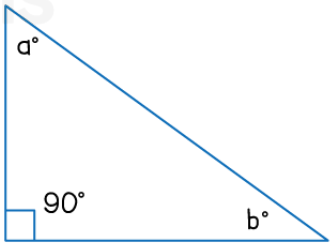
EQUILATERAL TRIANGLE
THREE EQUAL SIDES AND THREE EQUAL ANGLES
60 + 60 + 60 = 180



ISOSCELES TRIANGLE
TWO EQUAL SIDES AND TWO EQUAL ANGLES
a + a + b = 180



SCALENE TRIANGLE
NO EQUAL SIDES OR ANGLES
a + b + c = 180



RIGHT-ANGLED TRIANGLE
ONE ANGLE = 90°
a + b + 90 = 180
a + b = 90

Examiner Tips and Tricks

- Find all the missing angles that you can using the angles that are given to you in a question

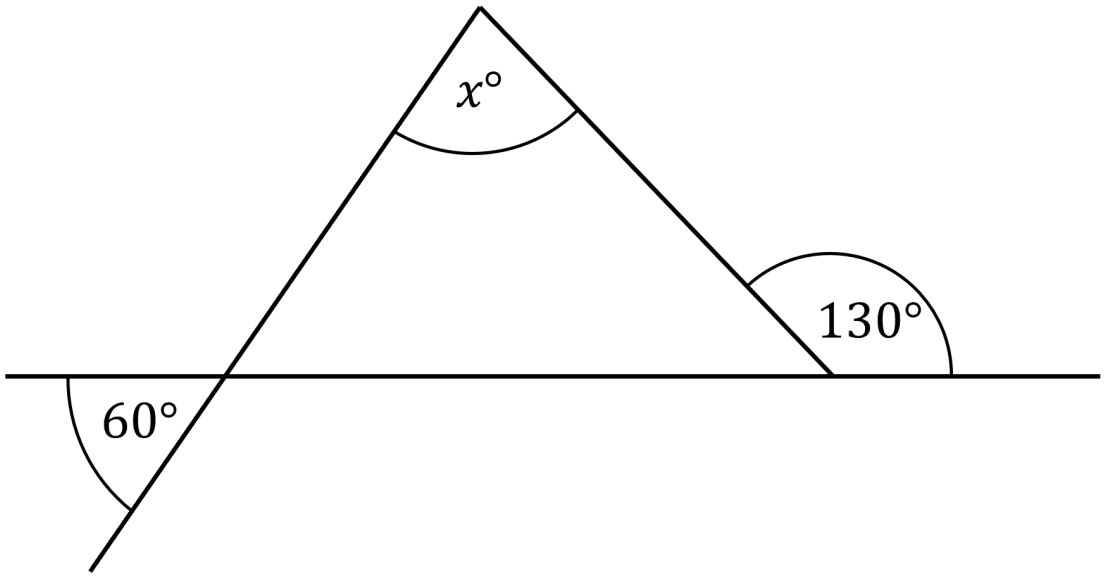
- They might not seem to help you straight away but having more angles will lead you to find the angle you need

Your notes

Worked Example

The diagram below is formed using three straight lines.

Find the value of x.



Label the other missing angles inside the triangle

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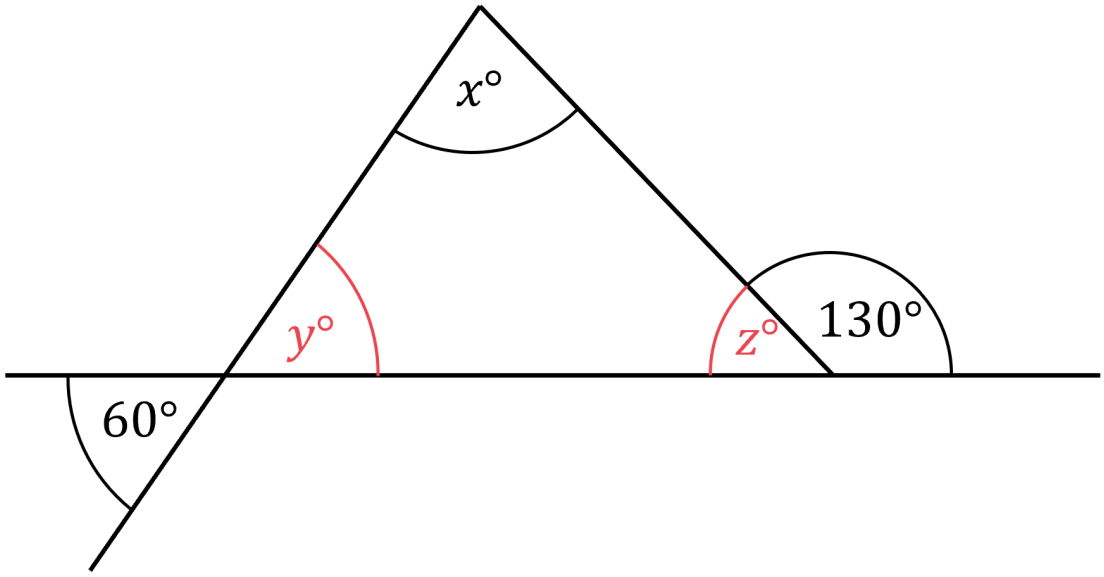


Vertically opposite angles between two intersecting lines are equal

$$y = 60$$

Angles that meet on a straight line add up to 180°

$$z + 130 = 180$$
$$z = 50$$

Interior angles in a triangle add up to 180°

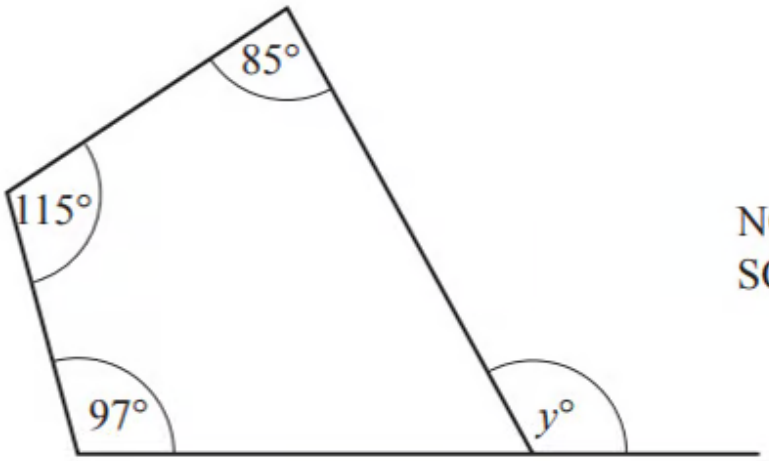
$$x + 60 + 50 = 180$$
$$x + 110 = 180$$

$$\boldsymbol{x = 70}$$

What are the angle properties of quadrilaterals?

- The **four** interior angles inside any quadrilateral **add up to 360°**
- If the quadrilateral is a **square** or a **rectangle** then all the angles are equal to **90°**
- You can use any **symmetries** of the quadrilateral to identify other equal angles
  - For a **parallelogram** or **rhombus**, **opposite angles** are **equal**
  - For a **kite**, **one pair** of **opposite angles** are **equal**

Your notes

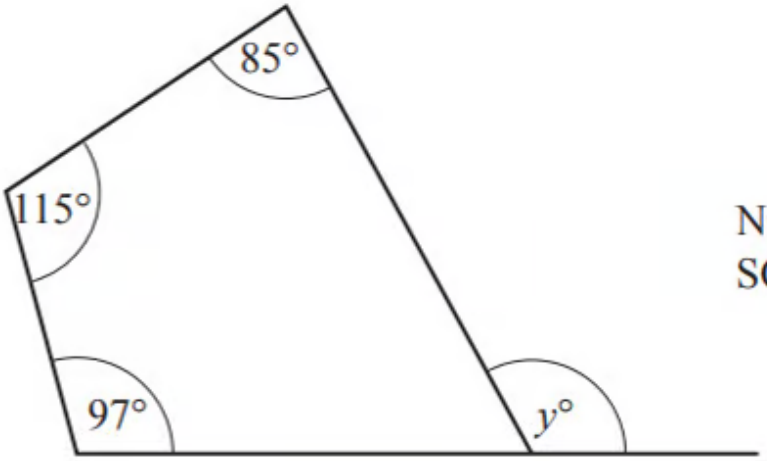


NOT TO SCALE

Worked Example

The diagram below shows an irregular quadrilateral.

Find the value of y.



NOT TO SCALE

Your notes

Find the missing angle inside the quadrilateral using the rule 'angles in a quadrilateral add up to 360°'

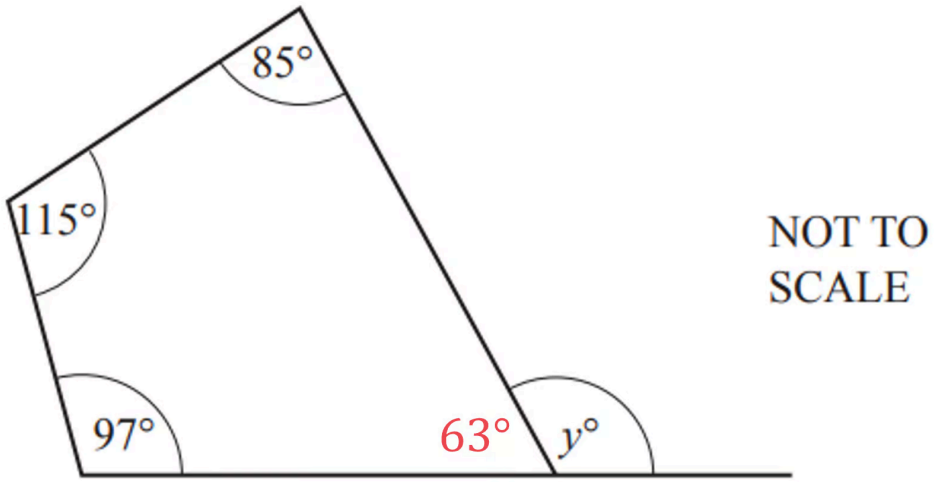
First, add together the three given angles

$$97 + 115 + 85 = 297$$

Subtract the answer from 360°

$$360 - 297 = 63$$

Add this to the diagram



Angles on a straight line add up to 180°, so subtract the answer from 180°

$$y + 63 = 180$$
$$y = 180 - 63$$
$$y = 117$$

$$\boldsymbol{y = 117}$$

Your notes

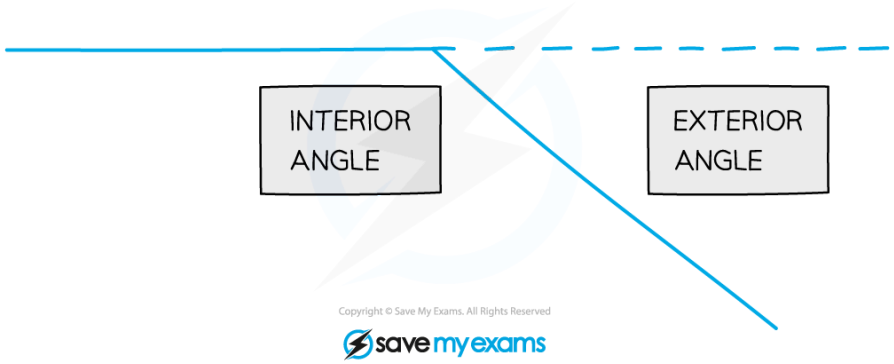
Angles in Polygons

What is a polygon?

- A **polygon** is a 2D shape with n straight sides
  - A **triangle** is a polygon with **3 sides**
  - A **quadrilateral** polygon with **4 sides**
  - A **pentagon** is a polygon with **5 sides**
- In a **regular** polygon **all the sides are the same length** and a**ll the angles are the same size**
  - A **regular polygon** with **3 sides** is an **equilateral triangle**
  - A **regular polygon** with **4 sides** is a **square**

What are the interior angles and the exterior angles of a polygon?

- **Interior angles** are the angles **inside** a polygon at the corners
- The **exterior angle** at a corner is the angle needed to **make a straight line with the interior angles**
  - It is **not** the angle that forms a **full turn** at the corner
- The **interior** angle and **exterior** angle **add up to 180°** at each corner



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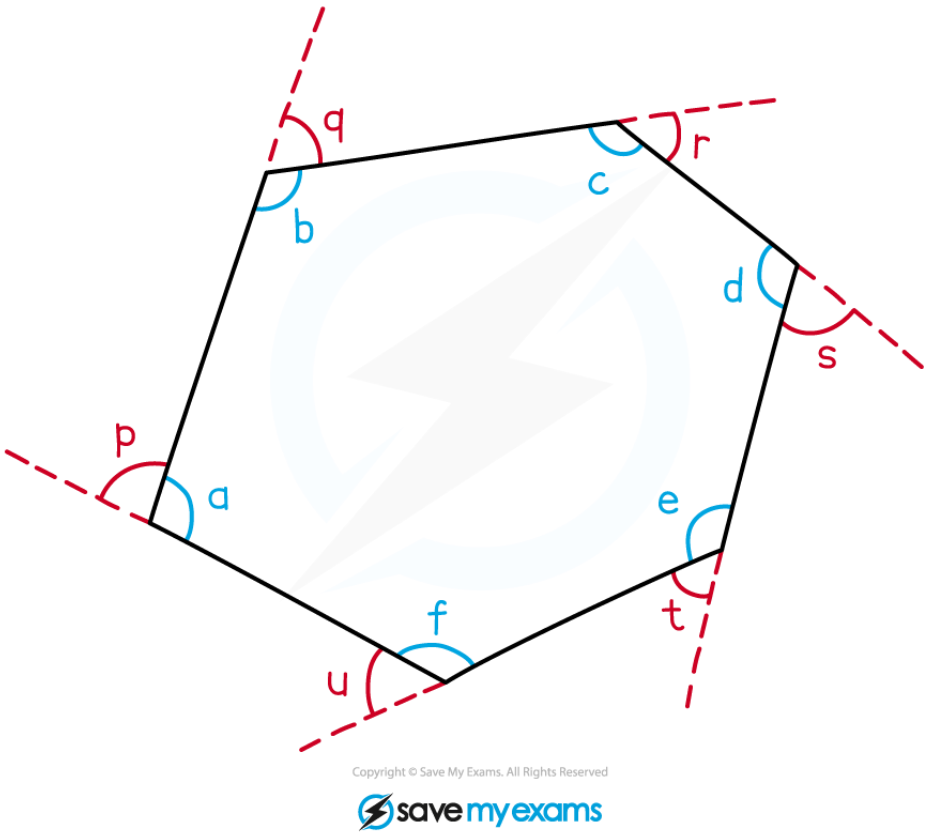


What is the sum of the interior angles in a polygon?

- To find the **sum of the interior angles** in a polygon of n sides, use the rule
  - Sum of interior angles = $180° \times (n - 2)$
    - This formula comes from the fact that n-sided polygons can be split into $n - 2$ triangles
- **Remember** the sums for these polygons
  - The interior angles of a **triangle** add up to **180°**
  - The interior angles of a **quadrilateral** add up to **360°**
  - The interior angles of a **pentagon** add up to **540°**

What is the sum of the exterior angles in a polygon?

- The **exterior angles** in **any** polygon always **sum to 360°**

How do I find the size of an interior or exterior angle in a regular polygon?


Your notes

- To find the **size of an interior angle** in a **regular polygon**:
  - Find the **sum** of the **interior angles**
    - For a pentagon: $180° \times (5-2) = 540°$
  - **Divide** by the **number of sides** (n)
    - For a pentagon: $540° \div 5 = 108°$
- To find the **size of an exterior angle** in a **regular polygon**:
  - Divide 360° by the number of sides (n)
    - For a pentagon: $360° \div 5 = 72°$
- The **interior** angle and **exterior** angle add to **180°**
  - **Subtract** the **exterior** angle from **180°** to find the **interior** angle
  - **Subtract** the **interior** angle from **180°** to find the **exterior** angle

Regular Polygon	Number of Sides	Sum of Interior Angles	Size of Interior Angle	Size of Exterior Angle
Equilateral Triangle	3	180°	60°	120°
Square	4	360°	90°	90°
Regular Pentagon	5	540°	108°	72°
Regular Hexagon	6	720°	120°	60°
Regular Octagon	8	1080°	135°	45°
Regular Decagon	10	1440°	144°	36°

How do I find a missing angle in a polygon?

- **STEP 1**
  Calculate the **sum** of the **interior angles** for the **polygon**
  - Use the formula $180° \times (n-2)$
- **STEP 2**
  Subtract the **other interior angles** in the polygon

Your notes

Examiner Tips and Tricks

- Make sure you identify whether you are dealing with a **regular** or **irregular** polygon before you start a question
- Finding the sum of the interior angles using $180 \times (n-2)$ can often be a good starting point for finding missing angles

Worked Example

The exterior angle of a regular polygon is 45°.

Write down the name of the polygon.

The formula for the exterior angle of a regular polygon is $\text{Exterior Angle} = \frac{360°}{n}$

Substitute the 45 for the exterior angle

$$45° = \frac{360°}{n}$$

Solve by rearranging

$$n = \frac{360}{45}$$

$$n = 8$$

Write down the name of a shape with 8 sides

Regular Octagon

Your notes

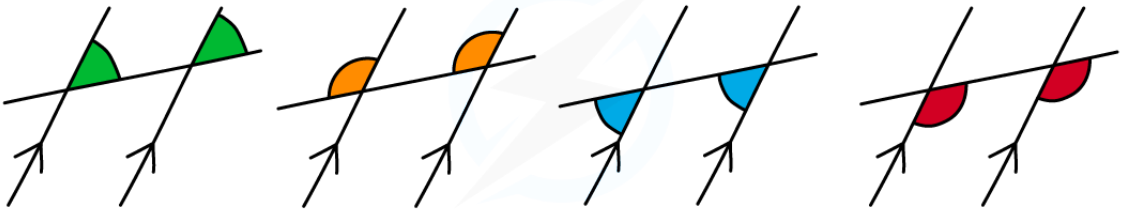
Angles in Parallel Lines

What are parallel lines?

- Parallel lines are lines that are always **equidistant** (the same distance apart)
  - No matter how far the lines are extended in either direction, they will **never meet**
- **Angles** are formed when a **straight line** cuts through **two parallel lines**

What are corresponding angles in parallel lines?

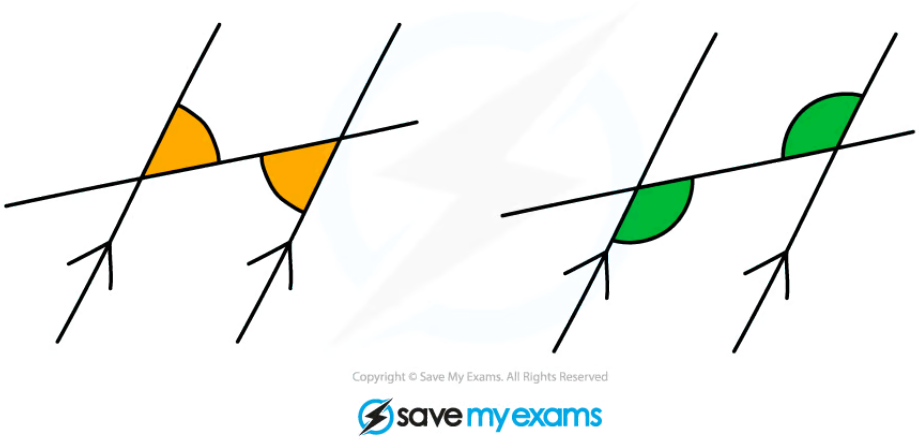
- Find **corresponding angles** by looking for an **F-shape**
- **Corresponding** angles are **equal**



What are alternate angles in parallel lines?

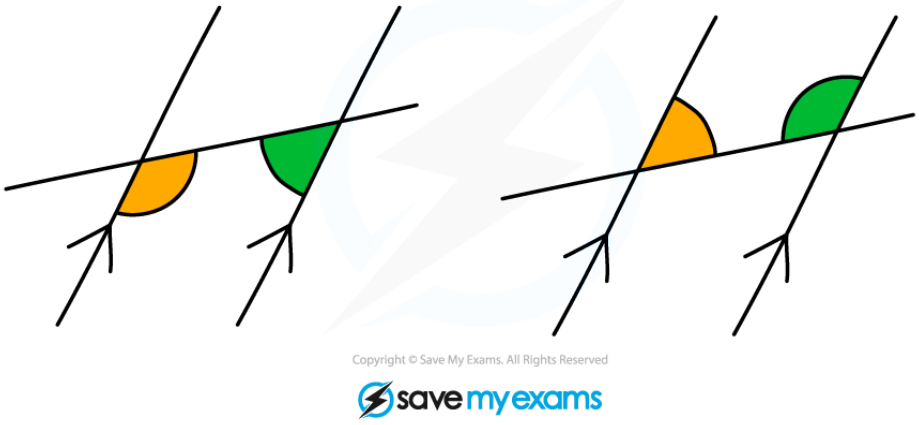
- Find **alternate angles** by looking for a **Z-shape**
- **Alternate** angles are **equal**

Your notes



What are co-interior (supplementary) angles in parallel lines?

- Find **co-interior angles** by looking for a **C-shape**
- **Co-interior** angles **add** up to **180°**

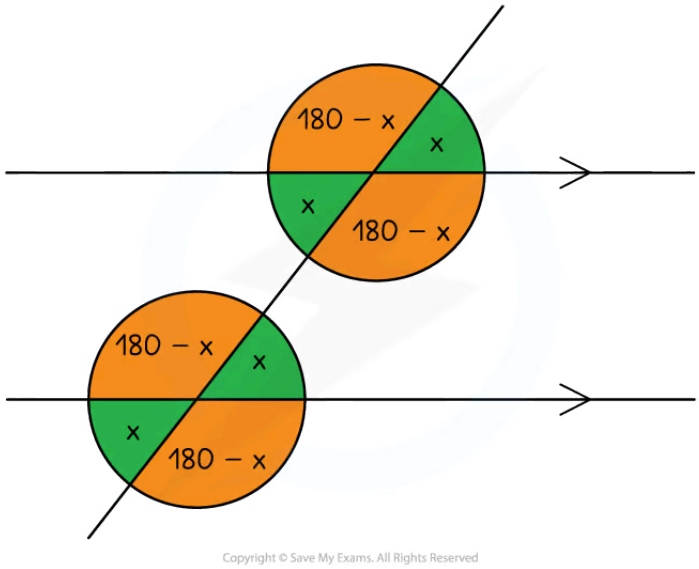


How do I find missing angles in parallel lines?

- Look for shapes that look like **F**, **Z**, or **C**
- **Vertically opposite angles** can also be used in problems involving parallel lines
  - The below diagram shows how identifying angle x, can lead to knowing information about several other angles

- The green angle opposite is also x, as it is vertically opposite
- The orange angle must be 180–x as angles on a straight line sum to 180°

- You should also be able to spot **corresponding** angles, **alternate** angles, and **co-interior** angles in this diagram



Your notes

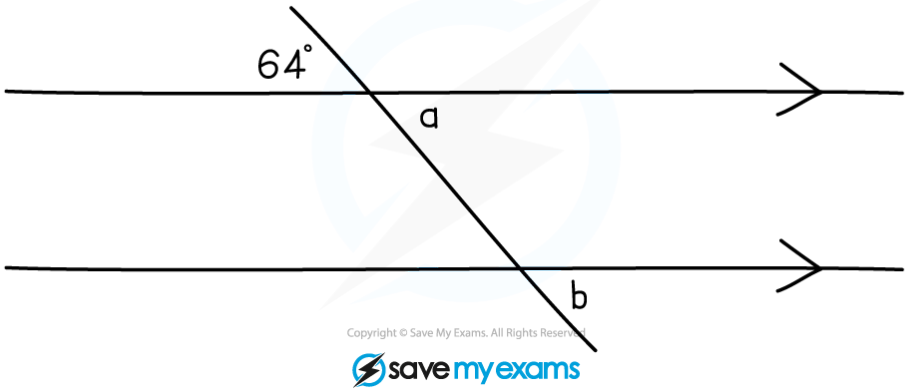
Examiner Tips and Tricks

- Do not forget to give **reasons** for each step of your working in an angles question
  - These are often needed to get **full marks**
  - You must use the **correct names** as listed above
    - **Do not** use F, Z and C angles otherwise you will **lose marks**!

Worked Example

Find the size of the angles *a* and *b* in the diagram below.

Give a reason for each step in your working.



Your notes

Vertically opposite angles are equal.
Corresponding angles on parallel lines are equal.

You **must** write down **both** of these reasons for full marks.

a = 64° (Vertically opposite angles are equal)
b = 64° (Corresponding angles on parallel lines are equal)