AQA GCSE Maths: Higher

Your notes

Solving Inequalities

Contents

Your notes

Solving Linear Inequalities

What is an inequality?

- An **inequality** tells you that something is **greater than** (>) or **less than** (<) something else
  - $x > 5$ means x is greater than 5
    - x could be 6, 7, 8, 9, ...
- Inequalities may also include being **equal** (=)
  - $\geqslant$ means **greater than or equal to**
  - $\leqslant$ means **less than or equal to**
    - $x \leqslant 10$ means x is less than or equal to 10
      - x could be 10, 9, 8, 7, 6,....
- When they **cannot** be **equal**, they are called **strict** inequalities
  - > and < are **strict** inequalities
    - $x > 5$ **does not** include 5 (strict)
    - $x \geqslant 5$ **does** include 5 (not strict)

How do I find integers that satisfy inequalities?

- You may be given **two end points** and have to list the **integer** values of x that **satisfy** the inequality
  - Look at whether each end point is **included** or not
    - $3 \leqslant x \leqslant 6$
      - $x = 3, 4, 5, 6$
    - $3 \leqslant x < 6$
      - $x = 3, 4, 5$
    - $3 < x \leqslant 6$
      - $x = 4, 5, 6$
    - $3 < x < 6$

- $x = 4, 5$

- If only **one end point** is given, there are an infinite number of integers
  - $x > 2$
    - $x = 3, 4, 5, 6, ...$
  - $x \leqslant 2$
    - $x = 2, 1, 0, -1, -2, ...$
    - Remember **zero** and **negative** whole numbers are **integers**
    - If the question had said **positive** integers only then just list $x = 2, 1$
- You may be asked to find integers that satisfy **two** inequalities
  - $0 < x < 5$ and $x \geqslant 3$
    - List **separately**: $x = 1, 2, 3, 4$ and $x = 3, 4, 5, 6, ...$
    - Find the values that appear in **both** lists: $x = 3, 4$
- If the question does not say x is an integer, do **not assume** x is an integer!
  - $x > 3$ actually means **any value** greater than 3
    - 3.1 is possible
    - $\pi = 3.14159...$ is possible
- You may be asked to find the **smallest** or **largest** integer
  - The smallest integer that satisfies $x > 6.5$ is 7

Worked Example

List all the integer values of x that satisfy

$$-4 \leq x < 2$$

Integer values are whole numbers
$-4 \leq x$ shows that x includes -4, so this is the first integer

$$x = -4$$

$x < 2$ shows that x does not include 2
Therefore the last integer is $x = 1$

Your notes

$x = 1$

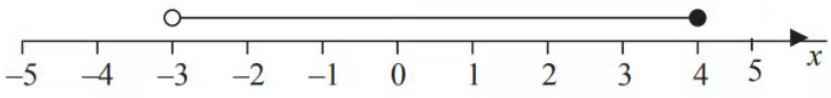
For the answer, list all the integers from -4 to 1
Remember integers can be zero and negative

$$x = -4, -3, -2, -1, 0, 1$$

Your notes

How do I represent an inequality on a number line?

- The inequality $-3 < x \leq 4$ is shown on a **number line** below



- Draw **circles** above the **end points** and connect them with a **horizontal line**
  - Leave an **open circle** for end points with **strict** inequalities, **< or >**
    - These end points **are not** included
  - Fill in a **solid circle** for end points with **≤ or ≥** inequalities
    - These end points **are** included

< or > use an open circle ○ (end points are excluded)
⩽ or ⩾ use a closed circle ● (end points are included)

- Use a **horizontal arrow** for inequalities with **one end point**
  - $x > 5$ is an open circle at 5 with a horizontal arrow pointing to the right

Worked Example

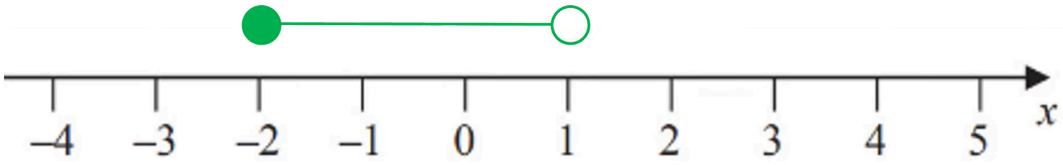
Represent the following inequalities on a number line.

(a) $-2 \leq x < 1$

-2 is included so use a closed circle

Your notes

1 is not included so use an open circle



(b) $t < 3$

3 is not included so use an open circle

There is no second end point
Any value less than three is accepted, so draw a horizontal arrow to the left

How do I solve a linear inequalities?

- Solving linear inequalities is just like **Solving Linear Equations**
  - Follow the same rules, but keep the **inequality sign** throughout
  - If you change the inequality sign to an equals sign you are changing the meaning of the problem
- When you **multiply** or **divide** both sides by a **negative** number, you must **flip the sign** of the inequality
  - E.g.

$$1 < 2$$
$$(\times -1) \qquad\qquad (\times -1)$$
$$-1 > -2$$

- **Never multiply** or **divide** by a **variable** (x) as this could be **positive or negative**
- The **safest way** to rearrange is simply to **add** and **subtrac**t to move all the terms onto **one side**

How do I solve double inequalities?

- Inequalities such as $a < 2x < b$ can be solved by doing the same thing to **all three parts** of the inequality
  - Use the same rules as solving linear inequalities

Your notes

How do I represent linear inequalities using set notation?

- We use curly brackets and a colon in set notation. $\{x: \ldots\}$ means "x is in the set ..."
- For example; if x is greater than 3, then in set notation, $\{x: x > 3\}$
- However, if x is between two values, then the two end values must be written in separate sets, using the **intersection** symbol, $\cap$
  - For example, if x is greater than 3 **and** less than or equal to 5, then in set notation, $\{x: x > 3\} \cap \{x: x \leq 5\}$
- Similarly, if x is less than one value or greater than another (disjoint), then the two end values must be written in separate sets using the **union** symbol, $\cup$
  - For example, if x is less than 3 **or** greater than or equal to 5, then in set notation, $\{x: x < 3\} \cup \{x: x \geq 5\}$

Examiner Tips and Tricks

- Do **not** change the inequality sign to an equals when solving linear inequalities.
  - In an exam you will lose marks for doing this.
- Remember to **reverse the direction** of the inequality sign when multiplying or dividing by a **negative** number!

Worked Example

Solve the inequality $2x - 5 \leq 21$.

Add 5 from both sides

$$2x \leq 26$$

Now divide both sides by 2

$$x \leq 13$$

$$\boldsymbol{x \leq 13}$$

Your notes

Worked Example

Solve the inequality $5 - 2x \leq 21$. Write your answer using set notation.

Subtract 5 from both sides, keeping the inequality sign the same

$$-2x \leq 16$$

Now divide both sides by -2
However because you are dividing by a negative number, you must flip the inequality sign

$$x \geq -8$$

Write the answer in set notation

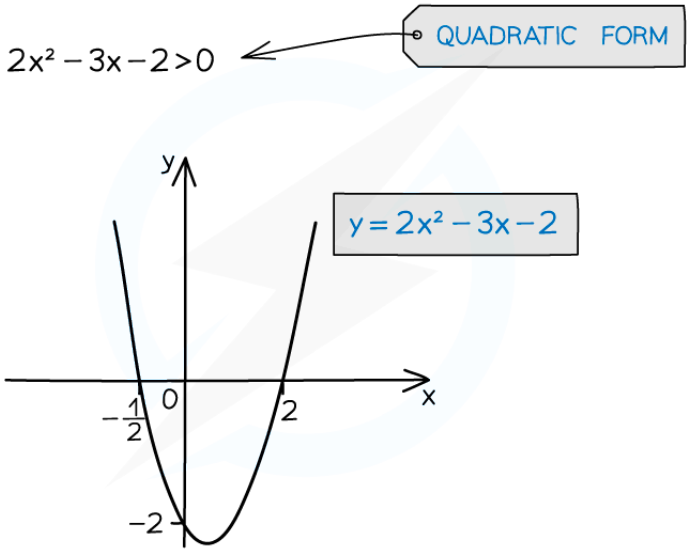
$$\boldsymbol{\{x : x \geq -8\}}$$

Your notes

Solving Quadratic Inequalities

What are quadratic inequalities?

- Similar to quadratic equations quadratic inequalities just mean there is a range of values that satisfy the solution
- Sketching a quadratic graph is essential



How do I solve quadratic inequalities?

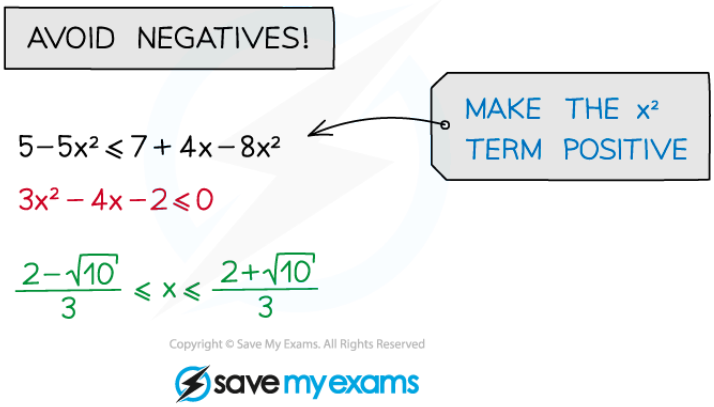
- **STEP 1**: **Rearrange** the inequality into quadratic form with a **positive squared term**
  - $ax^2 + bx + c > 0$ (>, <, ≤ or ≥)
- **STEP 2**: Find the **roots** of the quadratic equation

- **Solve** $ax^2 + bx + c = 0$ to get x_1 and x_2 where $x_1 < x_2$

- **STEP 3**: **Sketch** a graph of the quadratic and **label the roots**
  - As the squared term is positive it will be **"U" shaped**
- **STEP 4**: **Identify** the **region** that satisfies the inequality
  - For $ax^2 + bx + c > 0$ you want the region **above** the x-axis
    - The solution is $x < x_1$ **or** $x > x_2$
  - For $ax^2 + bx + c < 0$ you want the region below the x-axis
    - The solution is $x > x_1$ **and** $x < x_2$
    - This is more commonly written as $x_1 < x < x_2$
- **avoid** multiplying or dividing by a negative number

  if unavoidable, "**flip**" the inequality sign so $< \rightarrow >$, $\geq \rightarrow \leq$, etc
- **avoid** multiplying or dividing by a **variable** (**x**) that **could** be negative

  (multiplying or dividing by x^2 guarantees positivity (unless **x** could be 0) but this can create extra, invalid solutions)
- **do** rearrange to make the x^2 term positive. Be careful:



Your notes

Examiner Tips and Tricks

- Always start by rearranging to a quadratic with positive squared term
- Always sketch a graph of the quadratic before deciding the final answer

Your notes

Worked Example

Your notes

Find the set of values for which $3x^2 + 2x - 6 > x^2 + 4x - 2$ giving your answer in set notation.

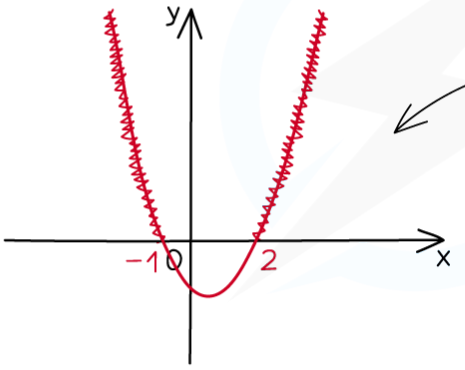
$3x^2 + 2x - 6 - x^2 - 4x + 2 > 0$

$2x^2 - 2x - 4 > 0$ ← REARRANGE TO QUADRATIC FORM

$2(x^2 - x - 2) > 0$

$x^2 - x - 2 > 0$

$(x-2)(x+1) > 0$



CRUCIAL: SKETCH THE GRAPH TO SEE WHERE THE SOLUTIONS ARE

$x < -1$ OR $x > 2$

$\{x : x < -1\} \cup \{x : x > 2\}$

FINAL ANSWER IN SET NOTATION