AQA GCSE Maths: Higher

Your notes

Number Operations

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Mathematical Symbols

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Mathematical Symbols

What mathematical symbols do I need to know?

- Starting with the basic four operations
  - $+$ **Addition,** plus, sum, total
  - $-$ **Subtraction,** minus, difference, take away
  - $\times$ **Multiplication,** product, times
  - $\div$ **Division,** quotient, divide, share
- **Equals** signs
  - $=$ Equal to
    - e.g. $3x + 7 = 19$
  - $\neq$ Not equal to
    - e.g. $2 - 5 \neq 5 - 2$
  - $\approx$ Approximately equal to
    - e.g. $\pi \approx 3.14$
  - $\equiv$ Identical (equivalent) to
    - e.g. $12x + 6 \equiv 3(4x + 2)$
- **Inequality** signs
  - $>$ **Greater** than
    - e.g. $5 > -2$
  - $<$ **Less** than
    - e.g. $\frac{1}{2} < \frac{2}{3}$
  - $\geq$ Greater than or equal to

Your notes

  - $\leq$ Less than or equal to
- Other helpful symbols
  - $(\ )$ **Brackets**
    - used to group symbols
    - e.g. $(2x + 4) - 3(x + 7)$
  - 3^x **Powers** (also called indices, orders, or exponents)
    - repeated multiplication
    - e.g. $3^4 = 3 \times 3 \times 3 \times 3 = 81$
  - $\sqrt{\ }$ **Square root**
    - opposite of squaring
    - e.g. $\sqrt{81} = 9$ is the opposite to $9^2 = 81$
  - $\pm$ **Plus Minus**
    - used to allow for both a positive and negative answer (two distinct answers)
    - $a \pm b$ is the same as $a + b$ and $a - b$
    - e.g. $x = 3 \pm 2.5$ is the same as $x = 3 + 2.5 = 5.5$ and $x = 3 = 2.5 = 0.5$
  - π **Pi**
    - the ratio of the circumference of a circle to its diameter

Your notes

Order of Operations (BIDMAS/BODMAS)

Order of Operations (BIDMAS/BODMAS)

What is the order of operations (BODMAS/BIDMAS)?

- If there is more than one **operation** in a calculation then they should be done in the following order
  - **B**rackets: ( )
    - Perform any calculation(s) **inside** brackets first
  - p**O**wers or **I**ndices (sometimes **O**rder): ^{2}, ^{3}, √ and similar
    - These include **powers, roots, reciprocals**
  - **D**ivisions or **M**ultiplications: × or ÷
    - If there are more than one of these then work them out left to right
    - Division includes **fractions**
  - **A**dditions or **S**ubtractions: + or -
    - If there are more than one of these then work them out left to right
- The acronym **BODMAS** or **BIDMAS** can help you remember the order of operations

In what order should fractions and roots be dealt with?

- **Fractions** mean **division** in calculations (BO**D**MAS/BI**D**MAS)
- There may be "invisible brackets" around the numerator and around the denominator
  - e.g. $\frac{2+5}{7-2}$ means $(2+5) \div (7-2)$
  - Instead of brackets we extend the fraction line to show exactly what is on the top and what is on the bottom
- **Roots** are p**O**wers (B**O**DMAS/B**I**DMAS)
- There may be "invisible brackets" under the root
  - e.g. $\sqrt{9+16}$ means $\sqrt{(9+16)}$
  - Instead of brackets we extend the top line on the root symbol to show everything that is to be rooted

How do I use a calculator for BIDMAS/BODMAS questions?

- Ensure that you enter a long or complicated calculation carefully
  - For most modern calculators, the calculation can be typed in exactly as it is written on paper
  - You may need to use additional brackets on older calculators

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Examiner Tips and Tricks

- Make sure that you always check that your answer seems sensible!

Worked Example

Work out $(5-3)+2\times 7^2$.

Use **BODMAS**

First calculate anything inside "**B**"rackets, $5-3=2$, so the question becomes

$$2+2\times 7^2$$

Then any p"**O**"wers, $7^2=49$

$$2+2\times 49$$

followed by any "**M**"ultiplications and "**D**"ivisions, $2\times 49=98$

$$2+98$$

and finally any "**A**"dditions and "**S**"ubtractions, $2+98=100$

$$\mathbf{(5-3)+2\times 7^2=100}$$

Your notes

Irrational Numbers

Irrational Numbers

What is a rational number?

- A **rational** number is a number that can be written as a fraction in its simplest form
  - It must be possible to write in the form $\frac{a}{b}$, where a and b are both **integers**
    - b cannot be zero
  - This includes all terminating and recurring decimals
    - E.g. 0.15 (which is $\frac{15}{100}$) and 0.151515151515... (which is $\frac{15}{99}$)
  - This also includes all integers
    - 5 can be written as $\frac{5}{1}$
    - -3 can be written as $\frac{-3}{1}$ or $\frac{3}{-1}$
    - 0 can be written as $\frac{0}{1}$ (it is ok to have 0 in the numerator)

What is an irrational number?

- An **irrational** number is a number that **cannot** be written in the form $\frac{a}{b}$, where a and b are **integers** and $\frac{a}{b}$ is in its simplest form
  - A decimal which is non-terminating and non-recurring is an irrational number
  - The number $\sqrt{n}$, where n is not a square number, is an irrational number
    - This is also known as a **surd**

What irrational numbers should I know?

Your notes

- You may be asked to identify an irrational number from a list
- Irrational numbers that you should recognise are π, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$,
  - Any multiple of these is also irrational
    - For example 2π, $3\sqrt{2}$, $3\sqrt{5}$ are all irrational
  - Be careful: $\sqrt{2} \times \sqrt{2}$ is not irrational as it equals 2
- Most calculators will show irrational numbers in their exact form rather than as a decimal

Examiner Tips and Tricks

If you're not sure if a number is rational or irrational, type it into your calculator and see if it can be displayed as a fraction.

Your notes

Negative Numbers

What are the rules for calculations with negative numbers?

- When **multiplying** and **dividing** with negative numbers
  - Two numbers with the **same sign** make a **positive**
    - $(-12) \div (-4) = 3$
    - $(-6) \times (-4) = 24$
  - Two numbers with **different signs** make a **negative**
    - $(-12) \div 4 = -3$
    - $6 \times (-4) = -24$
- When **adding** and **subtracting** with negative numbers
  - **Subtracting a negative** number is the same as **adding** the positive
    - e.g. $5 - (-3) = 5 + 3 = 8$
  - **Adding a negative** number is the same as **subtracting** the positive
    - e.g. $7 + (-3) = 7 - 3 = 4$

Where are negative numbers used in real-life?

- **Temperature** is a common context for negative numbers
  - If the temperature is 3°C, and it cools by 5°C, the new temperature will be -2°C
    - This is equivalent to 3 - 5 = - 2
  - If the temperature is -4°C, and it warms up by 6°C, the new temperature will be 2°C
    - This is equivalent to (-4) + 6 = 2
  - To explain why (-5) - (-6) = 1, you could think of it as follows:
    - A room is -5°C, then -6°C of cold air is 'removed'
    - The room now warms to 1°C

Your notes

- **Money** and **debt** is another common context for negative numbers
  - A negative sign means you owe money
  - If someone has a debt of \$200, and they borrow another \$400, their total debt is now \$600
    - This is equivalent to (-200) + (-400) = -600
  - If someone is in debt by \$300, but then pays off \$200 of their debt, they are now only \$100 in debt
    - This is equivalent to (-300) + 200 = -100

Examiner Tips and Tricks

- Your calculator isn't always as clever as you may think!
- Using brackets around negative numbers will always make sure the calculator is doing what you want
  - e.g. The square of negative three is $(-3) \times (-3) = 9$
    On many calculators, $-3^2 = -9$ but $(-3)^2 = 9$
    The second one is correct

Worked Example

Complete the following table.

Calculation	Answer
3 + (-4)	
(-5) + (-8)	
7 - (-10)	
(-8) - (-6)	
(-3) × 6	
(-9) × (-2)	
(-9) ÷ (-3)	
(-10) ÷ 5	

Calculation	Working	Answer
$3 + (-4)$	$3 - 4$	**-1**
$(-5) + (-8)$	$(-5) - 8$	**-13**
$7 - (-10)$	$7 + 10$	**17**
$(-8) - (-6)$	$(-8) + 6$	**-2**
$(-3) \times 6$	$3 \times 6 = 18$ one is negative	**-18**
$(-9) \times (-2)$	$9 \times 2 = 18$ both are negative	**18**
$(-9) \div (-3)$	$9 \div 3 = 3$ both are negative	**3**
$(-10) \div 5$	$10 \div 5 = 2$ one is negative	**-2**

Your notes

Your notes

Money Calculations

What currencies can be used?

- Many different currencies are used
- The **most commonly** used **currencies** are
  - US Dollars ($ or USD)
  - Great British Pounds (£ or GBP)
  - Euros (€ or Euros)
  - It is possible to see other currencies used, with or without their symbols

What might I be asked to do in money calculations?

- Read exam questions carefully to identify **key words**
  - **Total** or **sum** will mean to **add up**
  - **Difference** or **increase/decrease** in costs will involve **subtracting**
  - Changing from one currency to another (**exchange rates**) will involve **multiplying or dividing**
- Some questions may involve a **combination** of these
  - E.g., Working out the total cost of an energy bill
- Questions that involve money may also involve **other topics** in the course
  - E.g., Fractions, percentages, simple and compound interest etc.

How should I round values in a money calculation?

- Many currencies will be rounded to **two decimal places**
  - Dollars, pounds and euro should all be used to two decimal places
  - Always write **down both decimal places**, even if the second is zero
    - This is particularly important when using a **calculator**
    - 1.4 dollars on a calculator should be written as $1.40
- For a question involving **large numbers** such as the cost of a car

Your notes

- Rounding to the nearest dollar, 10 dollars or 100 dollars may be **more appropriate**
    - Use the **information in the question** to make a judgement
- **Some currencies** have large numbers due to exchange rates
    - These are usually rounded to the **nearest whole number**
    - E.g., $10 is 816.38 rupees making $100 the same as 8163.80 rupees
        - Rounding the exchange rate to $100 is 8160 rupees would be appropriate

What should I do when money calculations involve more than two decimal places?

- In some contexts money facts may be given to **more than two decimal places**
    - E.g. One litre of petrol in the UK costs an average price of £1.579
- Use **all of the decimal places** given in your **working**
    - Round (to two decimal places or whatever is appropriate) for your **final answer only**

Examiner Tips and Tricks

- Use the **information given in the question** to decide how to **round** your final answer
- Check that your answer **matches** the currency in the question

Worked Example

In his favourite UK fashion store, Thomas buys 4 t-shirts costing £8.50 each and 2 pairs of shorts costing £7.20 each. On his way home Thomas fills his car up with 45 litres of petrol at a price of £1.579 per litre.

Find out how much Thomas spent in total on clothes and petrol.

Find the total cost of the t-shirts, shorts and petrol separately
Use the figures as they are given, do not round any at this stage

$$4 \times 8.50 = 34$$
$$2 \times 7.20 = 14.4$$
$$45 \times 1.579 = 71.055$$

Total (add) these amounts

$$34 + 14.4 + 71.055 = 119.455$$

The currency is Great British Pounds (£) and values are relatively small so it makes sense to round the final answer to two decimal places

Thomas spends a total of £119.46 on clothes and petrol

Your notes

Your notes

Addition & Subtraction

Addition & Subtraction

How do I add large numbers without a calculator?

- There are a variety of written methods that can be used to add large numbers
  - The order in which numbers are added is **not** important
- The **column method** is the most commonly used hand written method
  - The numbers are written one number above the other,
    - Line up the digits using **place value columns**
  - Add each pair of corresponding digits from the top and bottom rows (work **right to left**)
  - If the result is a single digit
    - write the result in the relevant place value column below the line
  - If the result is a 2 digit number
    - the ones are written in the relevant place value column below the line
    - the tens are carried to the top of the next column
  - If the addition of the final pair of digits results in a 2 digit number
    - write both digits below the line
- For example, the addition 9789 + 563 = 10 352

$$\begin{array}{r} 1\,1\,1 \\ 9\,7\,8\,9 \\ +\quad 5\,6\,3 \\ \hline 1\,0\,3\,5\,2 \end{array}$$

How do I subtract large numbers?

- A variety of written methods exist, but you only need to know one
  - The order in which two numbers are subtracted **is** important so ensure the calculation is the right way round
- The **column method** is the most commonly used hand written method
  - The numbers are written one number above the other

Your notes

    - Line up the digits using **place value columns**
    - The number being subtracted should be below the original amount
  - Subtract each digit in the bottom value from the corresponding digit in the top value (work **right to left**)
  - If the digit being subtracted is bigger than the one it is subtracted from
    - **"borrow ten"** from the next column to the left
- For example, 392 - 28 = 364

$$\begin{array}{r} 3\overset{8}{\cancel{9}}{}^{1}2 \\ -\;\;28 \\ \hline 364 \end{array}$$

What words are used for addition and subtraction?

- **Addition** may be phrased using the words: **plus, total** or **sum**
- **Subtraction** may be phrased using the words: **difference** or **take away**

Examiner Tips and Tricks

- A good way to check your answer without a calculator is to estimate it
  - e.g. if you work out 32 870 ÷ 865 to be 295, check by doing 30 000 ÷ 1 000 in your head which is 30, so your answer is probably wrong (the actual answer is 38)

Worked Example

(a) Find the sum of 3985 and 1273.

Notice that the word sum is used but this means add
Quickly estimate the answer

$$4000 + 1000 = 5000$$

Write the numbers in two rows and columns aligned

Your notes

$$\begin{array}{r} 3985 \\ +1273 \\ \hline \end{array}$$

Start with the ones (units) column, writing the answer below the line but in the same column

$$\begin{array}{r} 3985 \\ +1273 \\ \hline 8 \end{array}$$

Move on to the tens (next on the left) column
The sum is 15 so the 5 (ones) is written below the line and the 1 (tens) 'carries over' to the next (hundreds) column

$$\begin{array}{r} 1 \\ 3985 \\ +1273 \\ \hline 58 \end{array}$$

Next is the hundreds column which again results in a two-digit answer

$$\begin{array}{r} 11 \\ 3985 \\ +1273 \\ \hline 258 \end{array}$$

Finally add the digits in the thousands column

$$\begin{array}{r} 11 \\ 3985 \\ +1273 \\ \hline 5258 \end{array}$$

Check the final answer is similar to your estimate; 5000 and 5258 are reasonably close

3985 + 1273 = **5258**

(b) Find the difference between 506 and 28.

Notice that the word difference is used; this means subtract
Quickly estimate the answer

500 - 30 = 470

Your notes

Write the numbers in two rows, column aligned, ensuring the top number is the number being subtracted *from*

$$\begin{array}{r} 506 \\ -\ \ 28 \\ \hline \end{array}$$

In the ones (units) column, 6 is smaller than 8, so borrow from the next column (tens)
The tens column is 0, so borrow from the column to the left of that (hundreds)

$$\begin{array}{rrr} 4 & & \\ \cancel{5} & {}^{1}0 & 6 \\ - & 2 & 8 \\ \hline \end{array}$$

This turns the 0 (in the tens column) into a 10 which we can then borrow from (for the ones column)

$$\begin{array}{rrr} 4 & 9 & \\ \cancel{5} & \cancel{1}\cancel{0} & {}^{1}6 \\ - & 2 & 8 \\ \hline \end{array}$$

16 - 8 can be now be calculated in the ones column

$$\begin{array}{rrr} 4 & 9 & \\ \cancel{5} & \cancel{1}\cancel{0} & {}^{1}6 \\ - & 2 & 8 \\ \hline & & 8 \end{array}$$

Move onto the tens column which is 9 - 2 = 7
There is nothing to subtract in the last (hundreds) column (4 - 0 = 4)

$$\begin{array}{rrr} 4 & 9 & \\ \cancel{5} & \cancel{1}\cancel{0} & {}^{1}6 \\ - & 2 & 8 \\ \hline 4 & 7 & 8 \end{array}$$

Check the final answer is similar to the estimate; 470 and 478 are reasonably close

506 - 28 = **478**

Multiplication

How do I multiply two numbers without a calculator?

- There are a variety of written methods that can be used to add large numbers
  - You only need to know **one** method, but be able to use it confidently
  - Four common methods are described below, but there are many other valid methods

How do I use the column method?

- This is an efficient method if you are confident with multiplication
- To use the **column method**:
  - Write one number above the other lining up the digits using **place value columns**
  - Multiply the first digit (on the right) from the bottom value by each digit in the top value
    - Write the result under the line with the digits in the correct place value columns
  - Multiply the next digit in the bottom value by each digit in the top value
    - Always work from **right to left**
    - Use 0s as place holders when multiplying digits in columns other than the ones column
  - For example, 87 × 426 = 37 062

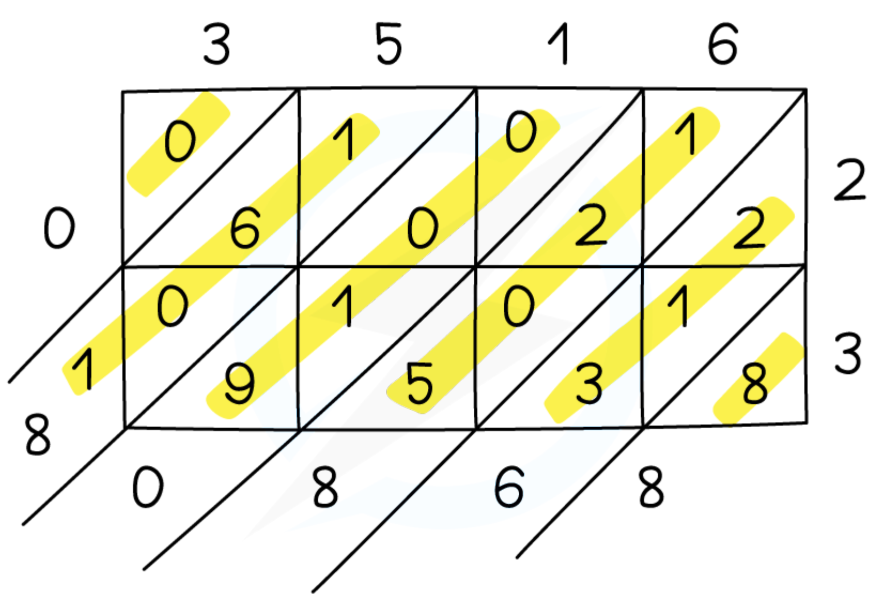
$$
\begin{array}{r}
8\ 7 \\
\times\ 4\ 2\ 6 \\
\hline
5\ 2\ 2 \\
1\ 7\ 4\ 0 \\
3\ 4\ 8\ 0\ 0 \\
\hline
3\ 7\ 0\ 6\ 2
\end{array}
$$

How do I use the lattice method?

- The lattice method is good for numbers with two or more digits
  - This method allows you to work with individual digits

Your notes

- To use the **lattice method**:
  - Draw a grid
    - The **number of rows** should be the same as the number of digits in one number
    - The **number of columns** should be the same as the number of digits in the other number
    - Draw diagonal lines through the boxes
  - Multiply each pair of digits, writing the result in the relevant box
    - Ones should be written in the bottom half of the box and tens in the top half of the box
  - Add the digits along the diagonals and write the result in the diagonal outside the grid
    - Carry the tens of any 2 digit result into the next diagonal
- For example, 3516 × 23 = 80 868



How do I use the grid method?

- This method keeps the value of the larger number intact
  - It may take longer with two larger numbers
  - Be careful lining up numbers with lots of zeros!

Your notes

- To use the **grid method**
  - Draw a grid
    - The **number of rows** should be the same as the number of digits in one number
    - The **number of columns** should be the same as the number of digits in the other number
  - Label the rows and columns with the values of each digit
    - E.g. For 3516 you would use 3000, 500, 10 and 6
  - Multiply together the relevant values and put the results in the boxes
  - Add up all of the cells in the boxes
- For example, 3516 × 7 = 24 612

	3000	500	10	6
7	21 000	3500	70	42

$$\begin{array}{r} 21\,000 \\ 3500 \\ 70 \\ +\quad 42 \\ \hline 24612 \end{array}$$

How do I use the repeated addition method?

- This is best for smaller, simpler cases

Your notes

- You may have seen this called 'chunking'
- To use the **repeated addition method**
  - Build up to the answer using simple multiplication facts that can be worked out easily
  - To find 13×23 :
    $1 \times 23 = 23$

    $2 \times 23 = 46$

    $4 \times 23 = 92$

    $8 \times 23 = 184$
  - So, $13 \times 23 = 1 \times 23 + 4 \times 23 + 8 \times 23 = 23 + 92 + 184 = 299$

What words are used for multiplication and division?

- **Multiplication** may be phrased using the words **lots of, times** or **product**
- **Division** may be phrased using the words **quotient, share** and **per**

Examiner Tips and Tricks

- A good way to check your answer without a calculator is to estimate it (e.g. by rounding everything to 1 significant figure)

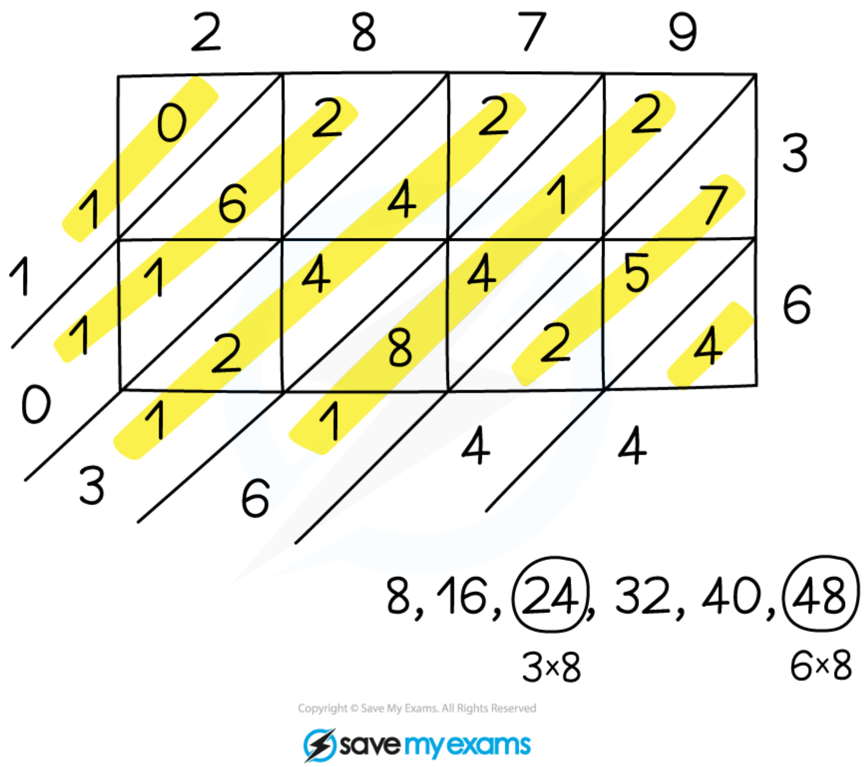
Worked Example

Multiply 2879 by 36.

As you have a 4-digit number multiplied by a 2-digit number then the lattice method is a good choice

Start with a 4×2 grid....

Your notes



Notice the use of listing the 8 times table underneath to help with some of the multiplication within the lattice

Use an estimate to check your answer; 3000×40 is equal to 120 000

2879 × 36 = **103 644**

Division

How do I divide a number by another without a calculator?

- The most common written method for division is short division (or "the bus stop method")
  - There are other methods such as long division, but short division is generally the most efficient
- While short division is best when dividing by a single digit, for bigger numbers you may need a different approach
- You can use other number skills to help

Your notes

- eg. cancelling fractions, "shortcuts" for dividing by 2 and 10, and the repeated addition ("chunking") method covered in Multiplication

Short division (bus stop method)

- Unless you can use simple shortcuts such as dividing by 2 or by 10, this method is best used when dividing by a single digit
- To find 174 ÷ 3
  - Starting from the left; 3 fits into 1, 0 times, with a remainder of 1
    - Carry the remainder of 1 over to the next digit, which forms 17
    - $$3\overline{)1\ {}^{1}7\ \ 4}^{\displaystyle 0}$$
  - 3 fits into 17, 5 times, with a remainder of 2
    - Carry the remainder of 2 over to the next digit, which forms 24
    - $$3\overline{)1\ {}^{1}7\ {}^{2}4}^{\displaystyle 0\ 5}$$
  - 3 fits into 24, 8 times, with no remainder
    - $$3\overline{)1\ {}^{1}7\ {}^{2}4}^{\displaystyle 0\ 5\ \ 8}$$
  - So, 174 ÷ 3 = 58

Factoring & cancelling

- This involves treating division as you would if you were asked to simplify fractions
  - For example, 1008 ÷ 28 can be written as $\frac{1008}{28}$
  - This can then be simplified
    - $$\frac{1008}{28}=\frac{504}{14}=\frac{252}{7}$$
    - 252 ÷ 7 can then be calculated using short division; the answer is 36

Dividing by 10, 100, 1000, ... (Powers of 10)

- This is a case of shifting digits along the place value columns

  For example

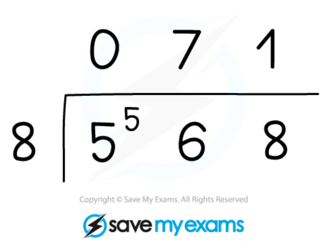
  - 380 ÷ 10 = 38.0 (shifts by 1 column)
  - 45 ÷ 100 = 0.45 (shifts by 2 columns)
  - 28 ÷ 1000 = 0.028 (shifts by 3 columns)
    - For cases like this, it can help to add leading zeros
    - 0028 ÷ 1000 may be easier to visualise

Worked Example

Divide 568 by 8.

This is division by a single digit so short division would be an appropriate method
If you spot it though, 8 is also a power of 2 so you could just halve three times

Using short division, the bus stop method:



8 fits into 5, 0 times, with a remainder of 5
8 fits into 56, 7 times exactly
8 fits into 8, 1 time exactly

Use an estimate to check your answer; 600 ÷ 10 is equal to 60

568 ÷ 8 = **71**

Your notes

Related Calculations

Your notes

Related Calculations

What are related calculations?

- If you know the result of one calculation, you can use it to find the result of a similar, **related** calculation
- For example, because $3 \times 2 = 6$
  - We also know the following:
    - $2 \times 3 = 6$
    - $6 \div 2 = 3$
    - $30 \times 2 = 60$

What are inverse operations?

- An **inverse operation** is an operation which undoes another
  - **Adding** and **subtracting** are **inverses** of one another
  - **Multiplying** and **dividing** are **inverses** of one another
- Inverse operations can be used to help find related calculations
- For example,
  - If we know that $3 \times 5 = 15$, then we also know that $15 \div 3 = 5$ and $15 \div 5 = 3$
  - If we know that $3^2 = 9$, then we also know that $\sqrt{9} = 3$

What types of related calculations are there?

- For example, consider the calculation **12 × 13 = 156**
- Using the fact that the **order** of multiplication does not matter (**commutativity**)
  - $13 \times 12 = 156$
  - The order of **multiplication** and **addition does not** matter
  - The order of **division** and **subtraction does** matter
- Using **inverse** operations
  - $156 \div 13 = 12$

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    - $156 \div 12 = 13$
- Using **multiples of ten**
    - Ten times larger
        - $120 \times 13 = 1560$
    - Ten times smaller
        - $1.2 \times 13 = 15.6$
    - One value ten times larger, one value 1000 times smaller
        - Answer will therefore be 100 times smaller
        - $0.013 \times 120 = 1.56$
- Using a **combination of multiples of ten and inverse operations**
    - $15\,600 \div 12$
    - $= 100 \times 156 \div 12$
    - $= 100 \times 13$
    - $= 130$
- If you are **dividing by a decimal**, use a multiple of ten to **change it to an integer** first
    - Writing the calculation as a fraction can help
    - Consider $1560 \div 1.2$
    - $\frac{1560}{1.2} = \frac{15600}{12} = \frac{156 \times 100}{12} = 13 \times 100 = 1300$

Examiner Tips and Tricks

- Use **estimation** to check your answer is sensible
- Rounding numbers to one significant figure can help you estimate the correct order of magnitude
- E.g. If the answer should be closer to 20 or 2 or 200

Worked Example

Your notes

Given that 43 × 16 = 688, find the answer to

(i) 688 ÷ 16

Division is the inverse operation to multiplication

$$\text{If } 16 \times 43 = 688 \text{ then}$$

$$688 \div 16 = 43$$

43

(ii) 1.6 × 4300

1.6 is 16 divided by 10

4300 is 43 multiplied by 100

$$(43 \times 100) \times (16 \div 10)$$

$$43 \times 16 \times 100 \div 10$$

$$43 \times 16 \times 10$$

We know that 43 × 16 = 688

$$688 \times 10$$

6880

(iii) 68.8 ÷ 4.3

Begin by writing as a fraction and changing the denominator to an integer

$$\frac{68.8}{4.3} = \frac{688}{43}$$

Division is the inverse operation to multiplication

$$\text{If } 43 \times 16 = 688 \text{ then } 688 \div 43 = 16$$

16

(iv) Explain how you can use estimation to check your answer for part (iii).

Estimate 68.8 ÷ 4.3 by rounding each number to one significant figure

$$70 \div 4 = 17.5$$

This shows that 16 is likely to be correct, if we had an answer of 1.6 or 160 then we would know we are wrong

We can estimate 68.8 ÷ 4.3 by carrying out the calculation 70 ÷ 4 = 17.5 and comparing our answer

Your notes

Your notes

Systematic Lists

Systematic Lists

What are systematic listing strategies?

- **Systematic listing strategies** are a way of writing out **all possible combinations or arrangements** of items in an organised way, without missing any
- A strategy can be used to make sure that **no options are missed out** by choosing one item as the first possible option, then arranging the rest using a similar technique
- For example, to list all the ways arrange the letters A, B and C
  - Begin by fixing the letter A and arranging the others
    A B C or A C B
  - Then fix the letter B and arrange the others
    B A C or B C A
  - Then fix the letter C and arrange the others
    C A B or C B A
  - So the six arrangements are ABC, ACB, BAC, BCA, CAB and CBA
  - This idea can be applied to arrangements of more items and longer lists

Examiner Tips and Tricks

- Using **systematic listing techniques** can be particularly helpful in **probability.**
  - Practice finding methods that will ensure you do not miss out any options.

Worked Example

List all the possible combinations of one filling, one side and one sauce from the sandwich menu below.

Filling	Side	Sauce

Steak	Lettuce	Ketchup
Chicken	Tomato	Mayo
Veggie Patty	Bacon	Apple

Simplify the problem by assigning a letter to each of the items.

Filling	**Side**	**Sauce**
Steak S	Lettuce L	Ketchup K
Chicken C	Tomato T	Mayo Y (M is taken, use a different letter)
Veggie Patty V	Mushroom M	Apple A

'Fix' the letter S and the letter L and arrange the other possibilities.

S L K
S L Y
S L A

'Fix' the letter S and the letter T and arrange the other possibilities.

S T K
S T Y
S T A

'Fix' the letter S and the letter M and arrange the other possibilities.

S M K
S M Y
S M A

Repeat the process by fixing the letter C in the first position.

C L K	C T K	C M K
C L Y	C T Y	C M Y
C L A	C T A	C M A

Repeat the process again by fixing the letter V in the first position.

V L K	V T K	V M K
V L Y	V T Y	V M Y

Your notes

VLA	VTA	VMA

So there are 27 different ways of choosing one item from each of the options on the sandwich menu, and they can be listed as follows:

SLK, SLY, SLA	**STK, STY, STA**	**SMK, SMY, SMA**
CLK, CLY, CLA	**CTK, CTY, CTA**	**CMK, CMY, CMA**
VLK, VLY, VLA	**VTK, VTY, VTA**	**VMK, VMY, VMA**

Your notes

Product Rule for Counting

Your notes

Product Rule for Counting

What is meant by counting principles?

- Counting principles state that if there are m ways to do one thing **and** n ways to do another there are $m \times n$ ways to do **both** things
- Applying counting principles allows us to ...
  - ... see patterns in real world situations
  - ... find the number of **combinations** or **arrangements** of a number of items
  - ... find the number of ways of choosing some items from a list of items
- When you have a question like "How many ways...?" you should always look for the words "AND" and "OR"
  - "AND means ×"
    "OR means +"

How do I choose an item from a list of items AND another item from a different list of items?

- If a question requires you to choose an item from one list **AND** an item from another list you should **multiply** the number of options in each list
  - In general if you see the word 'AND' you will most likely need to 'MULTIPLY'
- For example if you are choosing a pen and a pencil from 4 pens and 5 pencils:
  - You can choose 1 item from 4 pens AND 1 item from 5 pencils
  - You will have 4×5 different options to choose from

How do I choose an item from a list of items OR another item from a different list of items?

- If a question requires you to choose an item from one list **OR** an item from another list you should **add** the number of options in each list
  - In general if you see the word 'OR' you will most likely need to 'ADD'
- For example if you are choosing a pen or a pencil from 4 pens and 5 pencils:

- You can choose 1 item from 4 pens OR 1 item from 5 pencils
- You will have $4 + 5$ different options to choose from

Your notes

Examiner Tips and Tricks

Always read a question carefully and identify where it requires you to add or multiply before beginning the problem

Worked Example

Harry is going to a formal event and is choosing what accessories to add to his outfit. He has seven different ties, four different bow ties and five different pairs of cufflinks. How many different ways can Harry get ready if he chooses:

i) Either a tie, a bow tie or a pair of cufflinks?

Harry has $7 + 4 + 5$ different items to choose from.
He wants a tie OR a bow tie OR a pair of cufflinks.
OR means add them.

$$7 + 4 + 5 = 16$$

16 different ways to choose either a tie, a bow tie or a pair of cufflinks

ii) A pair of cufflinks and either a tie or a bow tie?

Harry wants a tie AND a pair of cufflinks OR a bow tie AND a pair of cufflinks.
AND means multiply them.

A tie AND cufflinks: $7 \times 5 = 35$
A bow tie AND cufflinks: $4 \times 5 = 20$

OR means add them.

$$35 + 20 = 55 \text{ ways}$$

55 different ways to choose pair of cufflinks and either a tie or a bow tie