AQA GCSE Maths: Higher

Your notes

Simple Probability Diagrams

Contents

Two-Way Tables

Your notes

Two Way Tables

What are two-way tables?

- **Two-way tables** are tables that compare **two types** of **characteristics**
  - For example, a college of 55 students has two year groups (Year 12 and Year 13) and two language options (Spanish and German)
  - The two-way table is shown:

	Spanish	**German**
Year 12	15	10
Year 13	5	25

How do I find probabilities from a two-way table?

- Draw in the **totals** of each **row** and **column**
  - Include an **overall total** in the bottom-right corner
    - It should be the sum of the totals above, or to its left (both work)
  - For the example above:

	Spanish	**German**	***Total***
Year 12	15	10	*25*
Year 13	5	25	*30*
Total	*20*	*35*	*55*

- Use this to answer **probability** questions
  - If a random student is selected from the whole college, it will be out of 55

Your notes

- The probability a student selected from the college studies Spanish and is in Year 12 is $\frac{15}{55}$
- The probability a student selected from the college studies Spanish is $\frac{20}{55}$
- If a random student is selected from a **specific category**, the **denominator** will be that **category total**
    - The probability a student selected from Year 13 studies Spanish is $\frac{5}{30}$

Examiner Tips and Tricks

- Check your row and column totals add up to the overall total, otherwise all your probabilities will be wrong!

Worked Example

At an art group, children are allowed to choose between colouring, painting, clay modelling and sketching.

A total of 60 children attend and are split into two classes: class A and class B.
12 of class A chose the activity colouring and 13 of class B chose clay modelling.
A total of 20 children chose painting and a total of 15 chose clay modelling.
8 of the 30 children in class A chose sketching, as did 4 children in class B.

(a) Construct a two-way table to show this information.

Read through each sentence and fill in the numbers that are given

	Colouring	Painting	Clay modelling	Sketching	Total
Class A	12			8	30
Class B			13	4	
Total		20	15		60

Use the row and column totals to fill in any obvious missing numbers

	Colouring	Painting	Clay modelling	Sketching	Total
Class A	12		15 - 13 = 2	8	30
Class B			13	4	60 - 30 = 30
Total		20	15	8 + 4 = 12	60

Use the row and column totals again to find the last few numbers

	Colouring	Painting	Clay modelling	Sketching	Total
Class A	12	30 - 12 - 2 - 8 = 8	2	8	30
Class B	30 - 12 - 13 - 4 = 1	20 - 8 = 12	13	4	30
Total	12 + 1 = 13	20	15	12	60

Write out your final answer

	Colouring	**Painting**	**Clay modelling**	**Sketching**	**Total**
Class A	**12**	**8**	**2**	**8**	**30**
Class B	**1**	**12**	**13**	**4**	**30**
Total	**13**	**20**	**15**	**12**	**60**

(b) Find the probability that a randomly selected child

(i) chose colouring,

(ii) is in class A, who chose sketching.

(i) We are not interested in whether the child is in class A or B
A total of 13 children chose colouring, out of 60 children

P(colouring) = $\frac{13}{60}$

(ii) 8 children in class A chose sketching
There are 60 children to select from

P(class A and sketching) = $\frac{8}{60} = \frac{2}{15}$

(c) A child in class B is selected at random. Find the probability they chose painting.

Your notes

As we are only selecting from class B, this will be out of 30 (rather than the total of 60)
12 in class B chose painting

$$\textbf{P(painting, from class B only)} = \frac{12}{30} = \frac{2}{5}$$

Your notes

Frequency Trees

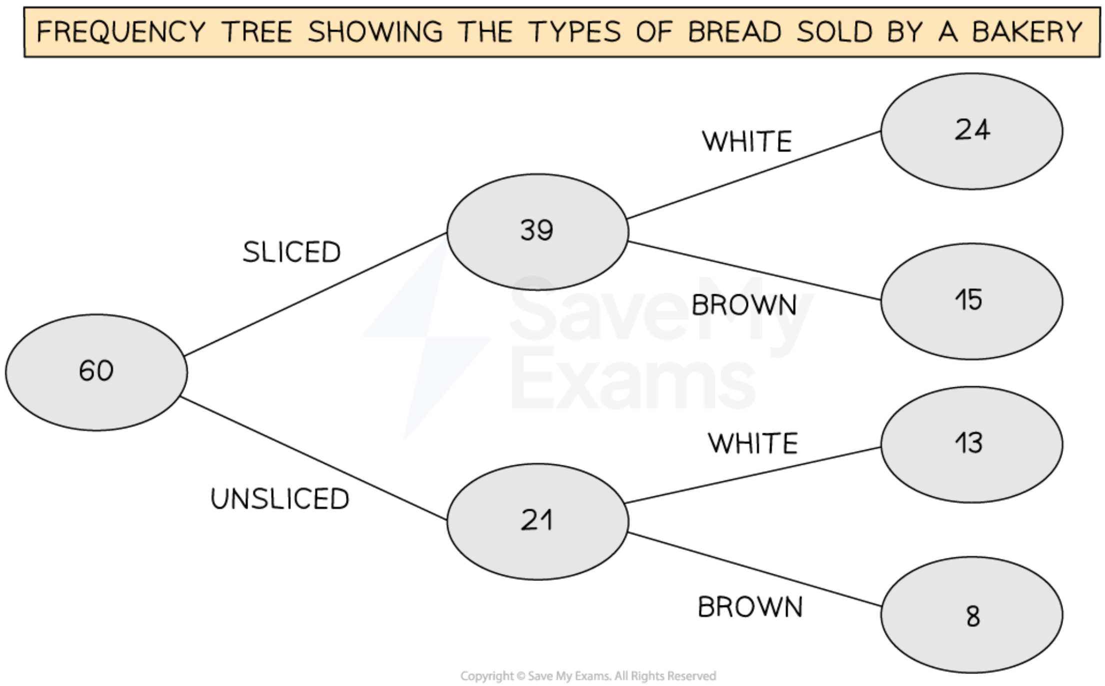
Your notes

Frequency Trees

What are frequency trees?

- **Frequency trees** show the frequencies associated with **two properties** of a set of data
- They are usually used when each property only has two possible outcomes
  - For example the types of bread sold by a bakery in a day
    - The first property could be if the bread is sliced or unsliced
    - The second property could be if the bread is white or brown
  - A frequency tree shows the **frequency** for **each combination**
    - e.g how many sliced, white loaves of bread were sold
- The **total frequency** appears in a 'bubble' at the **start** of a frequency tree
  - The first set of branches then break this down by the two outcomes for the first property
  - The second set of branches then further breaks down each of those frequencies
- It does not matter which set of branches shows which property
- It is possible to have three, or more, properties on a frequency tree by adding more sets of branches
  - However these would quickly become large and cumbersome
- For situations with more than two options for a property, **two-way tables** are more useful
  - For example if the bread in the bakery could brown, white, or seeded

Your notes



How do I draw a frequency tree?

- If **drawing** a frequency tree from scratch
  - **Identify** the two properties
  - Decide which property to put on the first set of branches and which to put on the second set of branches
  - Remember to include a 'bubble' at the **start** for the **total frequency** and a 'bubble' at the end of each branch
- Double check that the values at the ends of the branches, **sum** to the 'bubble' that they are connected to

How do I complete a frequency tree?

- Often in an exam there will be a **partially completed** frequency tree
- Check for any values **in the question** that you can use to fill in gaps
  - e.g. "A total of 100 people"

Your notes

- Remember that the **values at the ends of the branches, sum to the 'bubble' that they are connected to**
  - This should allow you to fill in any gaps that aren't revealed by the information in the question

How do I find probabilities from a frequency tree?

- Similar to finding probabilities from **two-way tables**, you need to select the appropriate numbers from the diagram
- It can help to rephrase the question to use **AND** & **OR** statements
  - e.g. The probability of selecting a loaf of sliced white bread is P("sliced AND white")
- Use the branches to help select the values you need to write down the probability
  - For "sliced AND white" this would be the along the branch saying 'sliced' on the first property and 'white' on the second
    - The value in the bubble at the end of the required branch(es) would be the numerator
  - The denominator will be the **total** of the group we are choosing from
    - This could be the **whole group** - the **total frequency** at the start of the diagram
    - Or if we are finding a probability from just sliced loaves, it would be the frequency in the bubble at the end of the 'sliced' branch
- You may need to **add together values**
  - e.g. To find the total number of white loaves of bread sold, sum together the sliced white loaves and the unsliced white loaves

Examiner Tips and Tricks

- Double check that the values at the ends of the tree add up to the starting value
- Some of the frequencies may be given as fractions or percentages of others
  - e.g. 65% of the loaves of bread sold were sliced

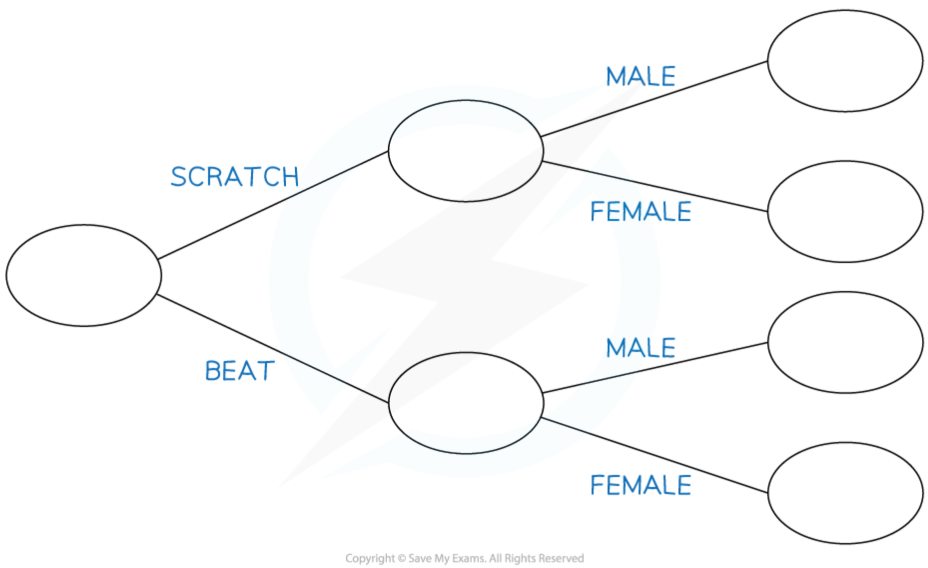
Worked Example

80 students are learning how to DJ. There are two courses; scratch mixing, and beat mixing.
60% of the students are studying scratch mixing, the rest are studying beat mixing.

Your notes

Of those studying scratch mixing, 15 are female.
Of those studying beat mixing, 12 are male.

(a) Use the information above to complete the frequency tree.



Start with the total frequency bubble at the start of the frequency tree - 80.
Work out 60% of 80 to find the frequency for scratch mixing.

$$10\% \text{ of } 80 = 80 \div 10 = 8$$
$$60\% \text{ of } 80 = 8 \times 6 = 48$$

Work your way through the rest of the tree.

beat: $80 - 48 = 32$

We are given that 15 of those studying scratch mixing are female and that 12 of those studying beat mixing are male.

scratch and female: $48 - 15 = 33$
beat and female: $32 - 12 = 20$

Now we have all the values, we can complete the frequency tree.

Check the bubble totals: 48 + 32 = 80, 33 + 15 + 12 + 20 = 80

(b)

Your notes

(i) A student is chosen at random. Find the probability that the student is a male studying beat mixing.

(ii) A student studying scratch mixing is chosen at random. Find the probability that the they are female.

(i)

Rephrasing this is P("male AND beat mixing").
The numerator will be the value in the bubble at the end of the branches "beat mixing" and "male" (12).

We are choosing from all of the students, so the denominator will be the total frequency (80).

$$\mathrm{P}(\text{male practising beat mixing}) = \frac{12}{80}\left(=\frac{3}{20}\right)$$

(ii)

Rephrasing this is P("female AND scratch mixing").
The numerator will be the value in the bubble at the end of the branches "scratch mixing" and "female" (15).
This time though we are **only** choosing from those studying scratch mixing, so the denominator will be at the end of the scratch mixing branch (48).

$$\mathrm{P}(\text{" female " GIVEN " scratch "}) = \frac{15}{48}\left(=\frac{5}{16}\right)$$

Your notes

Set Notation & Venn Diagrams

Set Notation

What is a set?

- A set is a **collection of elements**
  - Elements could be anything
    - Numbers, letters, coordinates, ...
- You could describe a set by writing its elements inside **curly brackets** {}
  - $\{1, 2, 3, 6\}$, is the set of factors of 6
- If the set of elements **follow a rule** then you can write this using a **colon inside the curly brackets** {... : ...}
  - The bit before the colon is the type of element
  - The bit after the colon is the rule
    - $\{x$ is a positive integer : $x^2 < 30\}$ is the set of positive integers which, when squared, are less than 30
    - This is equal to $\{1, 2, 3, 4, 5\}$
  - The colon is often read as '**such that**'
  - If **no type** is specified, x can take **any value** (fractions, decimals, irrationals, ...)
    - $\{x : x^2 < 30\}$ means any value whose square is less than 30
  - $\{(x, y) : y = mx + c\}$ would mean the coordinates (x, y) where $y = mx + c$
    - I.e. The set of all possible coordinates that lie on the line $y = mx + c$
  - A colon can also be replaced by a **vertical bar**
    - $\{x \mid x^2 < 30\}$

What do I need to know about set notation?

- $\mathscr{E}$ is the **universal** set (the set of **everything**)
  - For example, if we are only interested in factors of 24 then $\mathscr{E} = \{1, 2, 3, 4, 6, 8, 12, 24\}$
  - You may see alternative notations used for $\mathscr{E}$

Your notes

- U is a common alternative (different to $\cup$ for union!)
- S or the Greek letter ξ (xi) may also be seen
- We use **upper case** letters to represent **sets** (*A*, *B*, *C*, ...) and **lower case** letters to represent **elements** (*a*, *b*, c, ...)
- $\in$ means "**is an element of**"
  - E.g. $3 \in \{1,2,3\}$
- *A* $\cap$ *B* means the **intersection** of *A* and *B* (the **overlap** of *A* **and** *B*)
  - This is the set of elements that are in **both** set *A* and set *B*
    - E.g. If *A* = {1, 2, 3, 4, 5} and *B* = {-1, 1, 4, 7, 8}, then *A* $\cap$ *B* = {1, 4}
- *A* $\cup$ *B* means the **union** of *A* and *B* (**everything** in *A* **or** *B* **or** both)
  - This is the set of elements that are in **at least one** of the sets
  - This **includes** elements in both sets (in the intersection)
    - E.g. If *A* = {5, 6, 7, 8} and *B* = {3, 7, 11}, then *A* $\cup$ B = {3, 5, 6, 7, 8, 11}
- *A'* means the **complement of *A***
  - It is the **set** of all elements in the universal set $\mathscr{E}$ that are **not in *A***
    - E.g. If $\mathscr{E}$ = {1, 2, 3, 4, 5} and *A* = {1, 3}, *A'* = {2, 4, 5}

Venn Diagrams

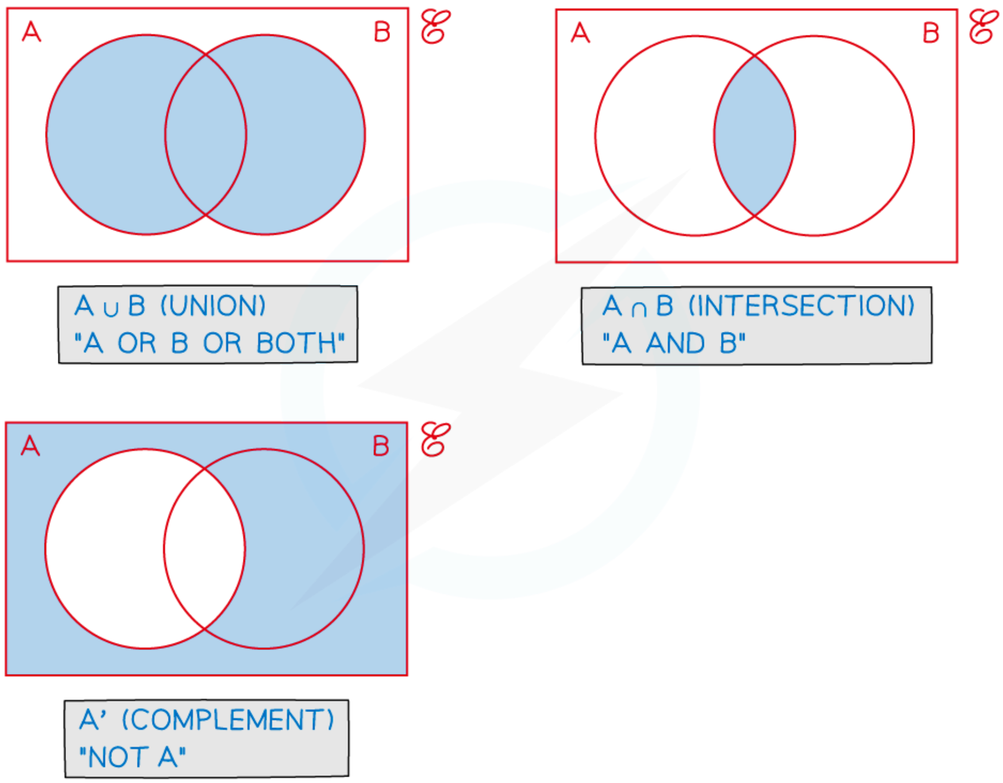
What is a Venn diagram?

- A Venn diagram is a way to illustrate **all the elements within sets** and any **intersections**
- A Venn diagram consists of
  - a **rectangle** representing the **universal set** (**$\mathscr{E}$**)
  - a **circle** for each **set**
    - Circles may or may not overlap depending on which **elements** are shared between **sets**

What do the different regions mean on a Venn diagram?

- $A \cap B$ is represented by the region where the *A* and *B* circles **overlap**
- $A \cup B$ is represented by the regions that **are in** *A* or *B* or both

Your notes

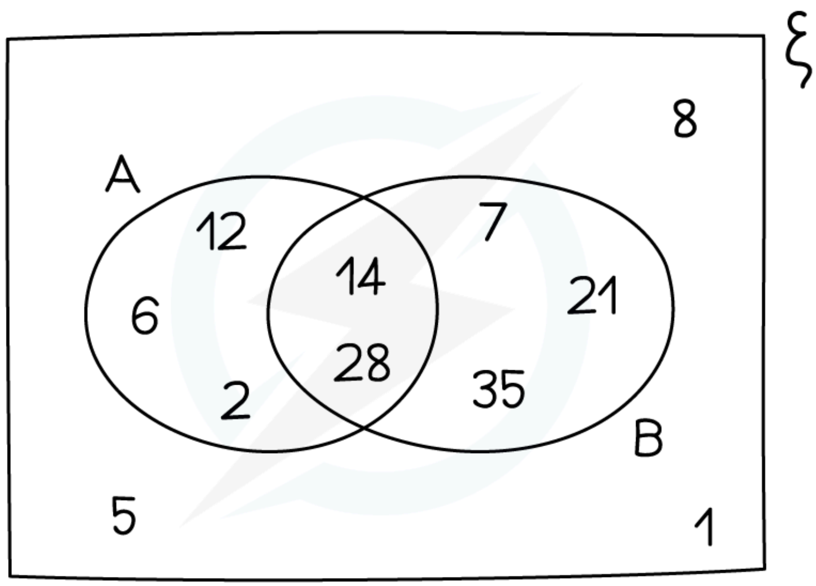


How do I find probabilities from Venn diagrams?

- Count the **number** of elements you want and **divide** by the **total number** of elements
- For the Venn diagram shown below,
  - The probability of being **in *A*** is $\frac{5}{11}$
    - There are 5 elements in A out of 11 in total
  - The probability of being in **both** A **and** B is $\frac{2}{11}$
    - There are 2 elements in A and B (the intersection)
  - The probability of being **in *A***, but **not *B***, is $\frac{3}{11}$

Your notes

    - 3 elements are in A but not B
- Some harder questions are **not** out of the **total** number, but out of a **restricted** number
  - The probability of being in B, **given that** you are already in A, is $\frac{2}{5}$
    - You are only interested in elements in A
    - There are 5 elements in A, out of which only 2 are also in B



Examiner Tips and Tricks

- Be careful when filling in numbers for a Venn diagram
  - Some of the given numbers may need to be split between two sections of the Venn diagram
- Suppose 10 people have a cat, 8 people have a dog and 6 people have both a cat and a dog
  - Out of the 10 people who have a cat

Your notes

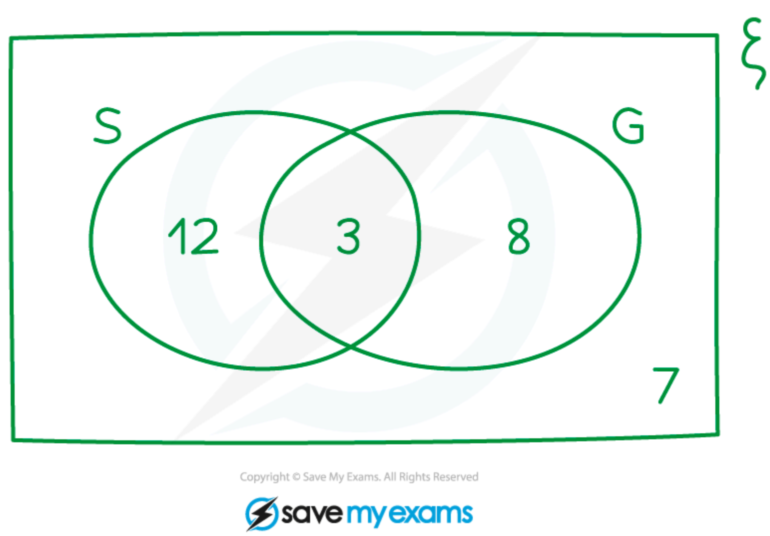
- 6 also have a dog
- 4 do not have a dog
- Out of the 8 people who have a dog
  - 6 also have a cat
  - 2 do not have a cat

Worked Example

In a class of 30 students, 15 students study Spanish and 3 of the Spanish students also study German.
7 students study neither Spanish nor German.

(a) Draw a Venn diagram to show this information.

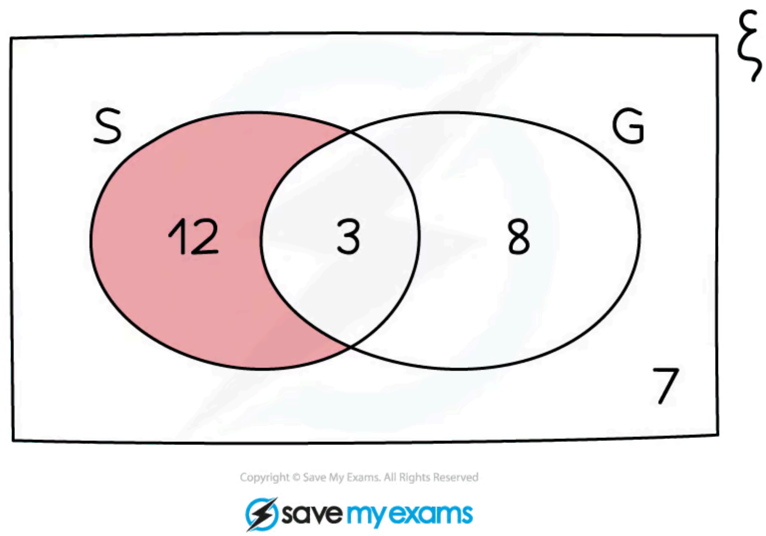
Draw the Venn diagram with its rectangular box and two (labelled) overlapping circles
3 students study both Spanish and German, so start here and work outwards
12 must study Spanish but not German (to get 15 in total for Spanish)
7 study neither, so this goes outside of the circles
To get 30 in total, 8 must study German but not Spanish



(b) Use your Venn diagram to find the probability that a student, selected at random from the class, studies Spanish but not German.

It helps to highlight Spanish but not German

Your notes



Divide the number of students studying Spanish but not German by the total number of students

Students studying Spanish but not German = 12
Total number of students = 30

P(Spanish but not German) $= \frac{12}{30} \left(= \frac{2}{5} \right)$