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AQA GCSE Maths: Higher



Factorising

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Factorising Out Terms

Your notes

Basic Factorising

What is factorisation?

- A factorised expression is one written as the product (multiplication) of two, or more, terms (factors)
 - = 3(x + 2) is factorised
 - It is $3 \times (x + 2)$
 - 3x + 6 is **not** factorised
 - 3xy is factorised
 - It is $3 \times x \times y$
 - Numbers can also be factorised
 - $12 = 2 \times 2 \times 3$
- In algebra, factorisation is the **reverse** of **expanding brackets**
 - It's putting it into brackets, rather than removing brackets

How do I factorise two terms?

- To factorise $12x^2 + 18x$
 - Find the highest common factor of the **number** parts
 - **-** 6
 - Find the highest common factor of the **algebra** parts
 - X
 - Multiply both to get the overall highest common factor
 - 6x
 - $12x^2 + 18x$ is the same as $6x \times 2x + 6x \times 3$
 - Using the highest common factor
 - Take out the highest common factor
 - Write it **outside** a set of **brackets**

- Put the remaining terms, 2x + 3, inside the brackets
- This gives the answer
 - -6x(2x+3)
- To factorise an expression containing multiple variables, e.g. $2a^3b 4a^2b^2$
 - Use the same approach as above
 - Find the highest common factor of the **number** parts
 - **2**
 - Find the highest common factor of the **algebra** parts
 - a and b appear in both terms
 - The highest common factor of a^3 and a^2 is a^2
 - The highest common factor of b and b² is b
 - a²b
 - Multiply both to get the overall highest common factor
 - 2a²b
 - $2a^3b 4a^2b^2$ is the same as $2a^2b \times a 2a^2b \times 2b$
 - Using the highest common factor
 - Take out the highest common factor
 - Write it outside a set of brackets
 - Put the remaining terms, a 2b, inside the brackets
 - This gives the answer
 - $a 2a^2b(a-2b)$



Examiner Tips and Tricks

- In the exam, check that your factorisation is correct by **expanding** the brackets!
- Factorise mean factorise fully.
 - x(6x + 10) is not fully factorised but 2x(3x + 5) is.



Your notes

Worked Example

(a) Factorise 5x + 15

Find the highest common factor of 5 and 15

5

There is no x in the second term, so no highest common factor in x is needed Think of each term as $5 \times$ something

$$5 \times x + 5 \times 3$$

Take out the 5 and put x + 3 in brackets

$$5(x + 3)$$

5(x + 3)

(b) Factorise fully $30x^2 - 24x$

Find the highest common factor of 30 and 24

6

Find the highest common factor of x^2 and x

Χ

Find the overall highest common factor by multiplying these together

6х

Think of each term as $6x \times$ something

$$6x \times 5x - 6x \times 4$$

Take out the 6x and put 5x - 4 in brackets

$$6x(5x - 4)$$

6x(5x - 4)



Factorising by Grouping

Your notes

Factorising by Grouping

How do I factorise expressions with a common bracket?

- Look at the expression 3x(t+4) + 2(t+4)
 - Both terms have a **common bracket**, (t + 4)
 - The whole bracket, (t + 4), can be "taken out" like a common factor:
 - (t+4)(3x+2)
- This is like factorising 3xy + 2y to get y(3x + 2)
 - y represents (t + 4) above

How do I factorise by grouping?

- Some questions may require you to form a common bracket yourself
 - For example xy + 3x + 5y + 15
 - The first two terms have a common factor of x
 - The second two terms have a common factor of 5
 - Factorising fully the **first pair** of terms, and the **last pair** of terms:
 - x(y+3)+5(y+3)
 - You can now spot a common bracket of (y + 3)
 - (y+3)(x+5)
- This is called factorising by grouping

Does it matter what order I group in?

- You can often rearrange terms to factorise in a different order
 - Rewriting the same example, xy + 3x + 5y + 15, but in a different order:
 - xy + 5y + 3x + 15
 - The first pair of terms have a common factor of y
 - The second pair of terms have a common factor of 3



(x+5)(y+3)

• This gives the **same result** as found previously

• Some rearrangements **cannot** be factorised as "first pair" then "second pair"

• For example, rewriting the above example as xy + 15 + 3x + 5y



Examiner Tips and Tricks

Once you have factorised something, expand it by hand to check your answer is correct.



Worked Example

Factorise ab + 3b + 2a + 6.

Method 1:

Notice that ab and 3b have a common factor of b

Notice that 2a and 6 have a common factor of 2

Factorise the first two terms, using b as a common factor

$$b(a+3)+2a+6$$

Factorise the second two terms, using 2 as a common factor

$$b(a+3)+2(a+3)$$

(a+3) is a common bracket

We can now factorise out the bracket (a + 3)

$$(a + 3)(b + 2)$$

Method 2:

Notice that ab and 2a have a common factor of a

Notice that 3b and 6 have a common factor of 3

Rewrite the expression, grouping these terms together

$$ab + 2a + 3b + 6$$

Factorise the first two terms, using a as a common factor





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a(b+2)+3b+6

Factorise the second two terms, using 3 as a common factor

a(b+2)+3(b+2)

(b+2) is a common bracket We can now factorise out the bracket (b+2)



(b + 2)(a + 3)

Factorising Simple Quadratics

Factorising Simple Quadratics

What is a quadratic expression?

- A quadratic expression is in the form:
 - $= ax^2 + bx + c$ (where $a \ne 0$)
- If there are any **higher powers** of x (like x^3 say) then it is **not** a quadratic

How do I factorise quadratics by inspection?

- $\qquad \text{This is shown most easily through an example: factorising } x^2-2x-8$
- We need a pair of numbers that for $x^2 + bx + c$
 - multiply to give c
 - which in this case is -8
 - and add to give b
 - which in this case is -2
 - +2 and -4 satisfy these conditions
 - $2 \times (-4) = -8$ and 2 + (-4) = -2
 - Write these numbers in a pair of brackets like this:
 - (x+2)(x-4)

How do I factorise quadratics by grouping?

- This is shown most easily through an example: factorising $x^2 2x 8$
- We need a pair of numbers that for $x^2 + bx + c$
 - multiply to give c
 - which in this case is -8
 - and add to give b
 - which in this case is -2

- +2 and -4 satisfy these conditions
 - $2 \times (-4) = -8$ and 2 + (-4) = -2
- Rewrite the middle term by using +2x and -4x

$$x^2 + 2x - 4x - 8$$

- Group and factorise the first two terms, using x as the common factor
- and group and factorise the last two terms, using -4 as the common factor

$$X(x+2)-4(x+2)$$

■ Note that these both now have a **common factor** of (x + 2) so this **whole bracket** can be factorised out

$$(x+2)(x-4)$$

How do I factorise quadratics using a grid?

- This is shown most easily through an example: factorising $x^2 2x 8$
- We need a **pair of numbers** that for $x^2 + bx + c$
 - multiply to give c
 - which in this case is -8
 - and **add** to give b
 - which in this case is -2
 - +2 and -4 satisfy these conditions
 - $2 \times (-4) = -8$ and 2 + (-4) = -2
 - Write the quadratic equation in a **grid** (as if you had used a grid to expand the brackets)
 - splitting the middle term as +2x and -4x
- The grid works by multiplying the row and column headings, to give a product in the boxes in the middle

x ²	-4x
+2x	-8

• Write a heading for the first row, using x as the highest common factor of x^2 and -4x

х	x ²	-4x
	+2x	-8



- You can then use this to find the headings for the columns
 - e.g. "What does x need to be multiplied by to give x^2 ?"
 - and "What does x need to be multiplied by to give -4x?"

	х	-4
х	x ²	-4x
	+2x	-8

- We can then fill in the remaining row heading using the same idea
 - e.g. "What does x need to be multiplied by to give +2x?"
 - or "What does -4 need to be multiplied by to give -8?"

	х	-4
х	x ²	-4x
+2	+2x	-8

- We can now read off the factors from the column and row headings
 - (x+2)(x-4)

Which method should I use for factorising simple quadratics?

- The first method, by **inspection**, is by far the **quickest**
 - So this is recommended in an exam for simple quadratics (where a = 1)

• However the other two methods (grouping, or using a grid) can be used for **harder quadratic equations** where $a \ne 1$



• So you should learn at least one of them too



Examiner Tips and Tricks

As a check, expand your answer and make sure you get the same expression as the one you were trying to factorise.



Worked Example

(a) Factorise $x^2 - 4x - 21$.

We will factorise by inspection

We need two numbers that multiply to give -21, and sum to give -4+3 and -7 satisfy this

$$3 \times (-7) = -21$$

$$3 + (-7) = -4$$

Write down the brackets

$$(x + 3)(x - 7)$$

(b) Factorise
$$x^2 - 5x + 6$$
.

We will factorise by splitting the middle term and grouping

We need two numbers that multiply to 6, and sum to -5 -3 and -2 satisfy this

$$(-3) \times (-2) = 6$$

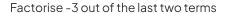
$$(-3) + (-2) = -5$$

Split the middle term

$$x^2 - 2x - 3x + 6$$

Factorise x out of the first two terms

$$x(x-2) - 3x + 6$$



$$x(x-2) - 3(x-2)$$

These have a common factor of (x - 2) which can be factored out

$$(x-2)(x-3)$$

(c) Factorise $x^2 - 2x - 24$.

We will factorise by using a grid

We need two numbers that multiply to -24, and sum to -2+4, and -6 satisfy this

$$4 \times (-6) = -24$$

$$4 + (-6) = -2$$

Use these to split the -2x term and write in a grid

x ²	+4x
-6x	-24

Write a heading using a common factor for the first row

x	x ²	+4x
	-6x	-24

Work out the headings for the rows

"What does x need to be multiplied by to make x^2 ?"

"What does x need to be multiplied by to make +4x?"

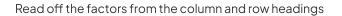
	х	+4
x	x ²	+4x
	-6x	-24



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Repeat for the heading for the remaining row "What does x need to be multiplied by to make -6x?" (Or "What does +4 need to be multiplied by to make -24?")

	х	+4
x	x ²	+4x
-6	-6x	-24









Factorising Harder Quadratics

Your notes

Factorising Harder Quadratics

How do I factorise a quadratic expression where $a \ne 1$ in $ax^2 + bx + c$?

Method 1: Factorising by grouping

- This is shown most easily through an example: factorising $4x^2 25x 21$
- We need a **pair of numbers** that, for $ax^2 + bx + c$
 - both **multiply** to give ac
 - ac in this case is $4 \times -21 = -84$
 - and both **add** to give b
 - b in this case is -25
 - -28 and +3 satisfy these conditions
 - **Rewrite** the middle term using -28x and +3x

$$4x^2 - 28x + 3x - 21$$

- **Group** and fully **factorise** the **first two terms**, using 4x as the common factor
- and **group** and fully **factorise** the **last two terms**, using 3 as the common factor

$$4x(x-7)+3(x-7)$$

- These terms now have a **common factor** of (x-7)
 - This whole bracket can be factorised out
 - This gives the answer (x-7)(4x+3)

Method 2: Factorising using a grid

- Use the same example: factorising $4x^2 25x 21$
- We need a **pair of numbers** that for $ax^2 + bx + c$
 - multiply to give ac



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- ac in this case is $4 \times -21 = -84$
- and add to give b
 - b in this case is -25
- -28 and +3 satisfy these conditions
- Write the quadratic equation in a grid
 - (as if you had used a grid to expand the brackets)
 - splitting the middle term up as -28x and +3x (either order)
- The grid works by multiplying the row and column headings, to give a product in the boxes in the middle

4x ²	-28x
+3x	-21

• Write a heading for the first row, using 4x as the highest common factor of $4x^2$ and -28x

4x	4x ²	-28x
	+3 <i>x</i>	-21

• You can then use this to find the headings for the columns, e.g. "What does 4x need to be multiplied by to give $4x^2$?"

	х	-7
4x	4x ²	-28x
	+3 <i>x</i>	-21

• We can then fill in the remaining row heading using the same idea, e.g. "What does x need to be multiplied by to give +3x?"



	х	-7
4x	4x ²	-28x
+3	+3x	-21



• We can now read off the brackets from the column and row headings:

$$(x-7)(4x+3)$$



Worked Example

(a) Factorise $6x^2 - 7x - 3$.

We will factorise by grouping

We need two numbers that:

multiply to
$$6 \times -3 = -18$$

and sum to -7

Split the middle term up using these values

$$6x^2 + 2x - 9x - 3$$

Factorise 2x out of the first two terms

$$2x(3x+1) - 9x - 3$$

Factorise - 3 of out the last two terms

$$2x(3x+1) - 3(3x+1)$$

These have a common factor of (3x + 1) which can be factorised out

$$(3x + 1)(2x - 3)$$

(b) Factorise $10x^2 + 9x - 7$.

We will factorise using a grid

We need two numbers that:



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multiply to $10 \times -7 = -70$ and sum to +9

-5, and +14

Use these values to split the 9x term and write in a grid

10x ²	-5x
+14x	-7

Write a heading using a common factor of 5x from the first row

5x	10 <i>x</i> ²	-5x
	+14x	-7

Work out the headings for the rows, e.g. "What does 5x need to be multiplied by to make $10x^2$?"

	2x	-1
5x	10 <i>x</i> ²	-5 <i>x</i>
	+14x	-7

Repeat for the heading for the remaining row, e.g. "What does 2x need to be multiplied by to make $\pm 14x$?"

	2x	-1
5x	10 <i>x</i> ²	-5x
+7	+14x	-7

Read off the brackets from the column and row headings

(2x-1)(5x+7)



Difference of Two Squares

Your notes

Difference of Two Squares

What is the difference of two squares?

- When a "squared" quantity is subtracted from another "squared" quantity, you get the difference of two squares
 - For example:
 - $a^2 b^2$
 - $9^2 5^2$
 - $(x+1)^2 (x-4)^2$
 - $= 4m^2 25n^2$, which is $(2m)^2 (5n)^2$

How do I factorise the difference of two squares?

- $a^2 b^2$ factorises to (a + b)(a b)
 - This can be shown by expanding the brackets

$$(a+b)(a-b) = a^2 - ab + ba - b^2 = a^2 - b^2$$

- The brackets can swap order
 - $= a^2 b^2 = (a + b)(a b) = (a b)(a + b)$
 - (but terms inside a bracket cannot swap order)
- For example, $x^2 9 = (x + 3)(x 3)$
 - This is the same as (x-3)(x+3)
 - But **not the same** as (3 + x)(3 x)
 - which expands to $9 x^2$

How can the difference of two squares be made harder?

- You may find it used with:
 - numbers
 - $7^2 3^2 = (7+3)(7-3) = (10)(4) = 40$

- A combination of square numbers and squared variables
 - $4m^2 9n^2 = (2m)^2 (3n)^2 = (2m + 3n)(2m 3n)$
- Any other powers which can be written as a difference of two squares

$$a^4 - b^4 = (a^2)^2 - (b^2)^2 = (a^2 + b^2)(a^2 - b^2)$$

$$r^8 - t^6 = (r^4)^2 - (t^3)^2 = (r^4 + t^3)(r^4 - t^3)$$

You may also need to take out a common factor first

$$2x^2 - 18 = 2(x^2 - 9) giving 2(x + 3)(x - 3)$$

■ The 2 comes out in front

Can I use the difference of two squares to expand?

- Using the difference of two squares to expand is quicker than expanding double brackets and collecting like terms
- Brackets of the form (a + b)(a b) expand to $a^2 b^2$
 - For example (2x+3)(2x-3) expands to $4x^2-9$



Examiner Tips and Tricks

- The difference between two squares is often the trick required to complete a harder algebraic question in the exam
 - Make sure you are able to spot it!



Worked Example

(a) Factorise $9x^2 - 16$.

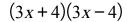
Recognise that $9x^2$ and 16 are both squared terms

Therefore you can factorise using the difference of two squares

Rewrite as a difference of two squared terms

$$9x^2 - 16 = (3x)^2 - (4)^2$$

Use the rule $a^2 - b^2 = (a + b)(a - b)$





(b) Factorise $4r^2 - t^4$.

Recognise that $4r^2$ and t^4 are both squared terms

Therefore you can factorise using the difference of two squares

Rewrite as a difference of two squared terms

$$4r^2 - t^4 = (2r)^2 - (t^2)^2$$

Use the rule $a^2 - b^2 = (a + b)(a - b)$

$$(2r+t^2)(2r-t^2)$$

(c) Factorise $2v^2 - 50$

This does not appear to be in the form $a^2 - b^2$

There is a common factor of 2, so take this factor out

$$2(y^2-25)$$

You can now see y^2-25 which has the form y^2-5^2

Use the rule $a^2 - b^2 = (a + b)(a - b)$

$$y^2 - 25 = (y+5)(y-5)$$

Now multiply this answer by 2 (leaving the 2 on the outside)

$$2(y+5)(y-5)$$

Deciding the Factorisation Method

Your notes

Quadratics Factorising Methods How do I know if an expression factorises?

- The easiest way to check if $ax^2 + bx + c$ factorises is to check if you can find a pair of integers which:
 - Multiply to give ac
 - Sum to give b
 - If you can find integers to satisfy this, the expression must factorise
- There are some alternate methods to check:
 - Method 1: Use a calculator to solve the quadratic expression equal to 0
 - Only some calculators have this functionality
 - If the solutions are integers or fractions (without square roots), then the quadratic expression will factorise
 - Method 2: Find the value under the square root in the quadratic formula
 - $b^2 4ac$
 - If this number is a **square number**, then the quadratic expression will factorise

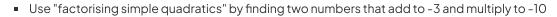
Which factorisation method should I use for a quadratic expression?

- Does it have 2 terms only?
 - Yes, like $x^2 7x$
 - Factorise out the highest common factor, x
 - X(x-7)
 - Yes, like $x^2 9$
 - Use the "difference of two squares" to factorise
 - (x+3)(x-3)

Does it have 3 terms?

Your notes

• Yes, starting with x^2 like $x^2 - 3x - 10$



- (x+2)(x-5)
- Yes, starting with ax^2 like $3x^2 + 15x + 18$
 - Check to see if the 3 in front of x^2 is a **common factor** for **all three terms** (which it is in this case), then **factorise** it **out** of all three terms
 - $3(x^2+5x+6)$
 - The quadratic expression inside the brackets is now $x^2 + ...$, which factorises more easily
 - 3(x+2)(x+3)
- Yes, starting with ax^2 like $3x^2 5x 2$
 - The 3 in front of x^2 is not a common factor for all three term
 - Use "factorising harder quadratics", for example factorising by grouping or factorising using a grid
 - (3x+1)(x-2)

What other expressions should I be able to factorise?

- You may have a **cubed term** like $x^3 3x^2 10x$
 - Check to see if x is a common factor for all three terms (which it is in this case), so factorise it
 out of all three terms
 - $x(x^2-3x-10)$
 - The remaining quadratic can then be factorised
 - X(x+2)(x-5)
- It can also be useful to spot a quadratic in the form $x^2 + 2ax + a^2$
 - This factorises to $(x + a)^2$
 - $E.g. x^2 + 6x + 9 = (x+3)^2$



Examiner Tips and Tricks

- A common **mistake** in the exam is to divide **expressions** by numbers, e.g. $2x^2 + 4x + 2$ becomes $x^2 + 2x + 1$ (which is incorrect)
 - This can only be done with equations
 - e.g. $2x^2 + 4x + 2 = 0$ becomes $x^2 + 2x + 1 = 0$ (dividing "both sides" by 2)



Worked Example

Factorise $-8x^2 + 100x - 48$.

Spot the common factor of -4 and factorise it out

$$-8x^2 + 100x - 48 = -4(2x^2 - 25x + 12)$$

Check to see if the quadratic in the bracket will factorise using $b^2 - 4ac$

$$(-25)^2 - (4 \times 2 \times 12)$$
= 625 - 96
= 529

529 is a square number (23^2) so the expression will factorise

Factorise $2x^2 - 25x + 12$

We require a pair of numbers which multiply to ac, and sum to b

$$a \times c = 2 \times 12 = 24$$

The only numbers which multiply to 24 and sum to -25 are

-24 and -1

Split the -25x term into -24x - x

$$2x^2 - 24x - x + 12$$

Group and factorise the first two terms, using 2x as the common factor Group and factorise the last two terms using -1 as the common factor

$$2x(x-12)-1(x-12)$$



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These factorised terms now have a common term of (x-12), so this can be factorised out

$$(2x-1)(x-12)$$



Recall that -4 was factorised out at the start

$$-8x^{2} + 100x - 48 = -4(2x^{2} - 25x + 12) = -4(2x - 1)(x - 12)$$
$$-4(2x - 1)(x - 12)$$