AQA GCSE Maths: Higher

Your notes

Powers, Roots & Standard Form

Contents

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Your notes

Powers & Roots

What are powers (indices)?

- **Powers (or indices)** are the small 'floating' values that are used when a number is multiplied by itself repeatedly
  - 6^1 means 6
  - 6^2 means 6 × 6
  - 6^3 means 6 × 6 × 6
- The big number at the **bottom** is called the **base**
- The small number that is **raised** is called the **index, power,** or **exponent**
- Any non-zero number to the **power of 0** is **equal to 1**
  - $3^0 = 1$
- Any number to the **power of 1** is **equal to itself**
  - $3^1=3$

What are square roots?

- **Roots** are the **reverse of powers**
- A **square root** of 25 is a number that when squared equals 25
  - The **two square roots** of 25 are 5 and -5
    - $5^2 = 25$ and $(-5)^2 = 25$
- Every **positive number** has **two square roots**
  - One is **positive** and one is **negative**
  - Negative numbers do not have a square root
- The notation $\sqrt{}$ refers to the **positive square root** of a number
  - $\sqrt{25} = 5$
  - You can show both roots at once using the **plus or minus symbol** ±

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- Square roots of 25 are $\pm\sqrt{25} = \pm 5$

What are cube roots?

- A **cube root** of 125 is a number that when **cubed** equals 125
  - The cube root of 125 is 5
    - $5^3 = 125$
  - Unlike square roots, each number only has **one cube root**
  - Every **positive** and **negative number** has **a cube root**
  - The notation $\sqrt[3]{}$ refers to the **cube root** of a number
    - $\sqrt[3]{125} = 5$

What are nth roots?

- An ***nth* root** of a number is a value that when raised to the power *n* equals the original number
  - 3^5=243 therefore 3 is a 5th root of 243
- If *n* is **even**, there will be a **positive and negative** *n*th root
  - The 6th roots of 64 are 2 and -2
    - $2^6 = 64$ and $(-2)^6 = 64$
  - The notation $\sqrt[n]{}$ refers to the **positive *nth* root** of a number
    - $\sqrt[6]{64} = 2$
  - Negative numbers do not have an *n*th root if *n* is even
- If *n* is **odd** then there will only be **one** *n*th root
  - The 5th root of -32 is -2
    - $(-2)^5 = -32$
  - Every **positive** and **negative** number will have an *n*th root

How do I estimate a root?

- You can **estimate roots** by finding the **closest integer** roots
  - To estimate $\sqrt{20}$

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- We know that $\sqrt{16} = 4$ and $\sqrt{25} = 5$
- So $\sqrt{20}$ must be between 4 and 5

What are reciprocals?

- The **reciprocal** of a number is the number that you multiply it by to get 1
  - The reciprocal of 2 is $\frac{1}{2}$
  - The reciprocal of $\frac{1}{4}$ is 4
  - The reciprocal of $\frac{3}{2}$ is $\frac{2}{3}$
- The **reciprocal** of a number can be written as an **index of -1**
  - 5^{-1} is the reciprocal of 5, so $\frac{1}{5}$
- This can be extended to other **negative indices**
  - 5^{-2} means the reciprocal of 5^2, so $\frac{1}{5^2}$ or $\frac{1}{25}$

Examiner Tips and Tricks

- If your calculator shows "Math Error" or similar when finding a square root, this is probably because you have accidentally entered a negative number!

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Laws of Indices

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Laws of Indices

What are the laws of indices?

- **Index laws** are **rules** you can use when doing **operations** with **powers**
  - They work with both **numbers** and **algebra**

Law	Description	How it works
$a^1 = a$	Anything to the power of 1 is itself	$6^1 = 6$
$a^0 = 1$	Anything to the power of 0 is 1	$8^0 = 1$
$a^m \times a^n = a^{m+n}$	To multiply indices with the same base, add their powers	$4^3 \times 4^2$ $= (4 \times 4 \times 4) \times (4 \times 4)$ $= 4^5$
$a^m \div a^n = \dfrac{a^m}{a^n} = a^{m-n}$	To divide indices with the same base, subtract their powers	$7^5 \div 7^2$ $= \dfrac{7 \times 7 \times 7 \times \cancel{7} \times \cancel{7}}{\cancel{7} \times \cancel{7}}$ $= 7^3$
$(a^m)^n = a^{mn}$	To raise indices to a new power, multiply their powers	$(14^3)^2$ $= (14 \times 14 \times 14) \times (14 \times 14 \times 14)$ $= 14^6$
$(ab)^n = a^n b^n$	To raise a product to a power, apply the power to both numbers, and multiply	$(3 \times 4)^2 = 3^2 \times 4^2$

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$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	To raise a fraction to a power, apply the power to both the numerator and denominator	$\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2} = \frac{9}{16}$
$a^{-1} = \frac{1}{a}$ $a^{-n} = \frac{1}{a^n}$	A negative power is the reciprocal	$6^{-1} = \frac{1}{6}$ $11^{-3} = \frac{1}{11^3}$
$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$	A fraction to a negative power, is the reciprocal of the fraction, to the positive power	$\left(\frac{2}{5}\right)^{-3} = \left(\frac{5}{2}\right)^3 = \frac{5^3}{2^3} = \frac{125}{8}$
$a^{\frac{1}{n}} = \sqrt[n]{a}$	The fractional power $\frac{1}{n}$ is the n^{th} root ($\sqrt[n]{\ }$)	$25^{\frac{1}{2}} = \sqrt[2]{25} = 5$ $27^{\frac{1}{3}} = \sqrt[3]{27} = 3$
$a^{-\frac{1}{n}} = \left(a^{\frac{1}{n}}\right)^{-1}$ $= \left(\sqrt[n]{a}\right)^{-1} = \frac{1}{\sqrt[n]{a}}$	A negative, fractional power is one over a root	$64^{-\frac{1}{2}} = \frac{1}{\sqrt[2]{64}} = \frac{1}{8}$ $125^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{125}} = \frac{1}{5}$
$a^{\frac{m}{n}} = a^{\frac{1}{n} \times m}$ $= \left(a^{\frac{1}{n}}\right)^m = (a^m)^{\frac{1}{n}}$	The fractional power $\frac{m}{n}$ is the n^{th} root all to the power m, $\left(\sqrt[n]{\ }\right)^m$, or the n^{th} root of the power m, $\sqrt[n]{(\ )^m}$ (both are the same)	$8^{\frac{2}{3}} = \left(8^{\frac{1}{3}}\right)^2 = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$ $8^{\frac{2}{3}} = (8^2)^{\frac{1}{3}} = \sqrt[3]{64} = 4$

How do I deal with different bases?

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- Index laws only work with terms that have the **same base**
  - $2^3 \times 5^2$ cannot be simplified using index laws
- Sometimes expressions involve different base values, but **one is related to the other by a power**
  - e.g. $2^5 \times 4^3$
- You can use powers to **rewrite one of the bases**
  - $2^5 \times \mathbf{4}^3 = 2^5 \times (\mathbf{2^2})^3$
  - This can then be simplified more easily, as the two bases are now the same
  - $2^5 \times (2^2)^3 = 2^5 \times 2^6 = 2^{11}$

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Worked Example

(a) Find the value of x when $6^{10} \times 6^x = 6^2$

Using the law of indices $a^m \times a^n = a^{m+n}$ we can rewrite the left hand side

$$6^{10} \times 6^x = 6^{10+x}$$

So the equation is now

$$6^{10+x} = 6^2$$

Comparing both sides, the bases are the same, so we can say that

$$10 + x = 2$$

Subtract 10 from both sides

$$\boldsymbol{x = -8}$$

(b) Find the value of n when $5^n \div 5^4 = 5^6$

Using the law of indices $a^m \div a^n = a^{m-n}$ we can rewrite the left hand side

$$5^n \div 5^4 = 5^{n-4}$$

So the equation is now

$$5^{n-4} = 5^6$$

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Comparing both sides, the bases are the same, so we can say that

$$n - 4 = 6$$

Add 4 to both sides

$$\boldsymbol{n = 10}$$

(c) Without using a calculator, find the value of 2^{-4}

Using the law of indices $a^{-n} = \frac{1}{a^n}$ we can rewrite the expression

$$2^{-4} = \frac{1}{2^4}$$

$2^4 = 2 \times 2 \times 2 \times 2 = 16$ so we can rewrite the expression

$$\frac{1}{2^4} = \frac{1}{16}$$

$$\boldsymbol{\frac{1}{16}}$$

(d) Without using a calculator, find the value of $8^{-\frac{1}{3}}$

Using the law of indices $a^{-n} = \frac{1}{a^n}$ we can rewrite the expression

$$8^{-\frac{1}{3}} = \frac{1}{8^{\frac{1}{3}}}$$

Using the law of indices $a^{\frac{1}{n}} = \sqrt[n]{a}$ we can rewrite the expression

$$\frac{1}{8^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{8}}$$

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The cube root of 8 is 2

$$\frac{1}{2}$$

(e) Without using a calculator, find the value of $81^{\frac{3}{4}}$.

Use the law of indices $(a^m)^n = a^{mn}$ we can rewrite the expression in two ways

$$81^{\frac{3}{4}} = (81^3)^{\frac{1}{4}} \text{ or } \left(81^{\frac{1}{4}}\right)^3$$

Both forms are equivalent, but $(81^3)^{\frac{1}{4}}$ would require calculating 81 cubed, so use the second form instead

Using the law of indices $a^{\frac{1}{n}} = \sqrt[n]{a}$ we can rewrite the expression

$$\left(81^{\frac{1}{4}}\right)^3 = \left(\sqrt[4]{81}\right)^3$$

The 4th root of 81 is 3 as $3\times3\times3\times3=3^4=81$

$$(3)^3$$

Lastly, calculate or recall 3 cubed

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Converting to & from Standard Form

What is standard form and why is it used?

- **Standard form** is a way of writing **very large** and **very small numbers** using **powers of 10**
- This allows us to:
  - Write them more concisely
  - Compare them more easily
  - Perform calculations with them more easily

How do I write a number in standard form?

- Numbers written in standard form are always written as:

$$a \times 10^n$$

- Where:
  - $1 \leq a < 10$ (a is **between 1 and 10**)
  - $n > 0$ (n is **positive**) for **large** numbers
  - $n < 0$ (n is **negative**) for **small** numbers

How do I write a large number in standard form?

- To write a **large number** such as 3 240 000 in standard form
  - Identify the value of a
    - 3.24
  - Find **how many times** you must **multiply** 3.24 **by 10**, to make 3 240 000
    - Count how many places you need to move the decimal point
    - We need to multiply by 10 six times
  - 3 240 000 = 3.24 × 10 × 10 × 10 × 10 × 10 × 10 = **3.24 × 10^6**

How do I write a small number in standard form?

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- To write a **small number** such as 0.000567 in standard form
  - Identify the value of a
    - 5.67
  - Find **how many times** you must **divide** 5.67 **by 10**, to make 0.000567
    - Count how many places you need to move the decimal point
    - We need to divide by 10 four times
    - We are dividing rather than multiplying so the **power will be negative**
  - 0.000567 = 5.67 ÷ 10 ÷ 10 ÷ 10 ÷ 10 = $\mathbf{5.67 \times 10^{-4}}$

Examiner Tips and Tricks

- On some calculators, typing in a very large or very small number and pressing $\boxed{=}$ will convert it to standard form

Worked Example

(a) Without a calculator, write 0.007052 in standard form.

Standard form will be written as $a \times 10^n$ where a is between 1 and 10
Find the value for a

$$a = 7.052$$

The original number is smaller than 1 so n will be negative
Count how many times you need to divide a by 10 to get the original number

$$0.007052 = 7.052 \div 10 \div 10 \div 10 \quad \text{(3 times)}$$

Therefore $n = -3$.

0.007052 = $\mathbf{7.052 \times 10^{-3}}$

(b) Without a calculator, write 324 500 000 in standard form.

Standard form will be written as $a \times 10^n$ where a is between 1 and 10
Find the value for a

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$$a = 3.245$$

The original number is larger than 1 so n will be positive
Count how many times you need to multiply a by 10 to get the original number

$$324\,500\,000 = 3.245 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \quad \text{(8 times)}$$

Therefore $n = 8$

$$324\,500\,000 = \mathbf{3.245 \times 10^8}$$

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Operations with Standard Form

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Operations with Standard Form

How do I perform calculations in standard form using a calculator?

- Make **use of brackets** around each number, and use the $\boxed{\times 10^x}$ button to enter numbers in standard form
  - e.g. $(3 \times 10^8) \times (2 \times 10^{-3})$
  - You can instead use the standard multiplication and index buttons
- If your calculator answer is **not in standard form**, but the question requires it:
  - Either **rewrite** it using the standard process
    - e.g. 3 820 000 = 3.82×10^6
  - Or rewrite numbers in standard form, then apply the **laws of indices**
    - e.g. $243 \times 10^{20} = (2.43 \times 10^2) \times 10^{20} = 2.43 \times 10^{22}$

How do I perform calculations with numbers in standard form without a calculator?

Multiplication and division

- Consider the "number parts" **separately** to the powers of 10
  - E.g. $(3 \times 10^2) \times (4 \times 10^5)$
    - Can be written as $(3 \times 4) \times (10^2 \times 10^5)$
  - Then **calculate each part separately**
    - Use **laws of indices** when combining the powers of 10
    - 12×10^7
  - This can then be rewritten in standard form
    - $1.2 \times 10 \times 10^7 = 1.2 \times 10^8$
- This process is the same for a division

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- E.g. $(8 \times 10^{-5}) \div (2 \times 10^{-3})$
  - Can be written as $\frac{8 \times 10^{-5}}{2 \times 10^{-3}} = \frac{8}{2} \times \frac{10^{-5}}{10^{-3}}$
- Then calculate each part separately
  - Use laws of indices when combining the powers of 10
  - Be careful with negative powers -5 -(-3) is -5 + 3
  - 4×10^{-2}

Addition and subtraction

- One strategy is to **write both numbers in full**, rather than standard form, and then add or subtract them
  - E.g. $(3.2 \times 10^3) + (2.1 \times 10^2)$
  - Can be written as $3200 + 210 = 3410$
  - Then this can be rewritten in standard form if needed, 3.41×10^3
- However this method is **not efficient for very large or very small powers**
- For very large or very small powers:
  - Write the values with **the same, highest, power of 10**
  - And then calculate the addition or subtraction, keeping the power of 10 the same
  - Consider $(4 \times 10^{50}) + (2 \times 10^{48})$
    - Rewrite both with the highest power of 10, i.e. 50
    - Changing 10^{48} to 10^{50} has made it 10^2 times larger, so make the 2 smaller by a factor of 10^2 to **compensate**
    - $(4 \times 10^{50}) + (0.02 \times 10^{50})$
    - These can now be added
    - 4.02×10^{50}
  - Consider $(8 \times 10^{-20}) - (5 \times 10^{-21})$
    - Rewrite both with the higher power of 10, i.e. -20

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- Changing 10^{-21} to 10^{-20} has made it 10^1 times larger, so make the five 10^1 times smaller to compensate
- $(8 \times 10^{-20}) - (0.5 \times 10^{-20})$
- These can now be subtracted
- 7.5×10^{-20}

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Worked Example

Without using a calculator, find $(45 \times 10^{-3}) \div (0.9 \times 10^5)$.

Write your answer in the form $A \times 10^n$, where $1 \leq A < 10$ and n is an integer.

Rewrite the division as a fraction, then separate out the powers of 10

$$\frac{45 \times 10^{-3}}{0.9 \times 10^5} = \frac{45}{0.9} \times \frac{10^{-3}}{10^5}$$

Work out $\frac{45}{0.9}$

$$\frac{45}{0.9} = \frac{450}{9} = 50$$

Work out $\frac{10^{-3}}{10^5}$ using laws of indices

$$\frac{10^{-3}}{10^5} = 10^{-3-5} = 10^{-8}$$

Combine back together

$$(45 \times 10^{-3}) \div (0.9 \times 10^5) = 50 \times 10^{-8}$$

Rewrite in standard form, where a is between 1 and 10

$$50 \times 10^{-8} = 5 \times 10 \times 10^{-8} = 5 \times 10^{-7}$$

5×10^{-7}

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Worked Example

Without using a calculator, find $(2.8 \times 10^{-6}) + (9.7 \times 10^{-8})$.

Write your answer in the form $A \times 10^n$, where $1 \leq A < 10$ and n is an integer.

Rewrite both numbers with the highest power of ten, which is -6

Changing 10^{-8} to 10^{-6} has made it 10^2 times larger, so make the 9.7 a factor of 10^2 times smaller to compensate

$$(2.8 \times 10^{-6}) + (0.097 \times 10^{-6})$$

The numbers can now be added together, keeping the power of 10 the same

$$2.897 \times 10^{-6}$$

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