AQA GCSE Maths: Higher

Area & Perimeter

Contents

Perimeter

Perimeter

What is perimeter?

- Perimeter is the **total distance** around the **outside** of a 2D shape
  - The perimeter of a **circle** is called the **circumference**
- Perimeter is a length in **one dimension**
  - **Units of measure** include mm, cm, m etc

How do I find the perimeter of a 2D shape?

- **Add together** the **lengths** of all of the sides of the shape
- For any **regular** 2D shape, the perimeter will be the **number of sides**, multiplied by the **length of one side**
  - For example, the perimeter of a square of side length x cm will be $4x$ cm
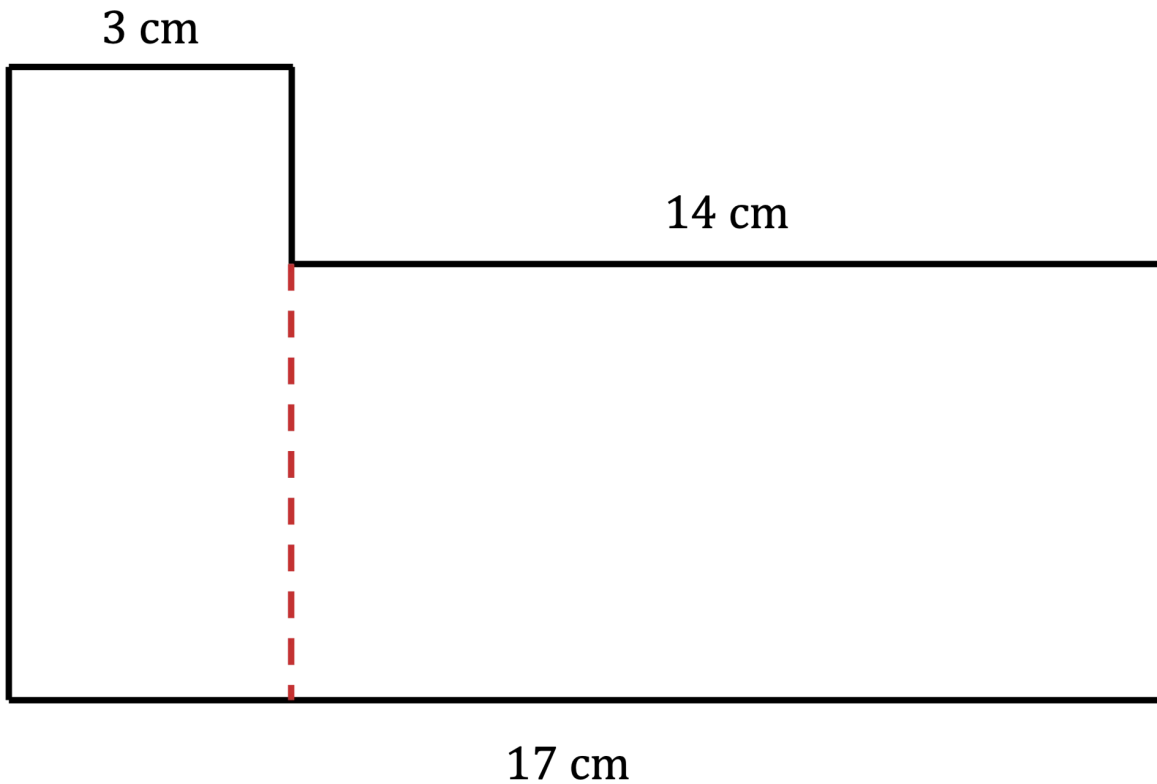
How do I find the perimeter of a compound shape?

- Shapes may be made up of two or more 2D shapes, these are called **compound shapes**
  - Compound shapes can usually be split into **rectangles**, **triangles** and **parts of circles**
  - You will need to be confident with the **properties of 2D shapes**
    - For example, the distance between the **centre point** of a **circle** and a point on the **circumference** is its **radius**
  - Look out for **sides** that are **equal**, for example in a rectangle, parallelogram, or isosceles triangle
    - **Dashes** may be used to mark the **equal sides**, or the question may tell you which sides are equal
  - You may need to use certain **formulas** to calculate lengths
    - For example, you may need to use **Pythagoras' theorem** to calculate a length on a **right-angled triangle**
  - You may need to use the **given information** to find the lengths of some of the sides
    - For example, the L-shape below can be split into two rectangles

Your notes

- The **sum of the lengths of two shorter sides** will be the same as the length of the **longer side opposite**

3 cm

14 cm

17 cm

Examiner Tips and Tricks

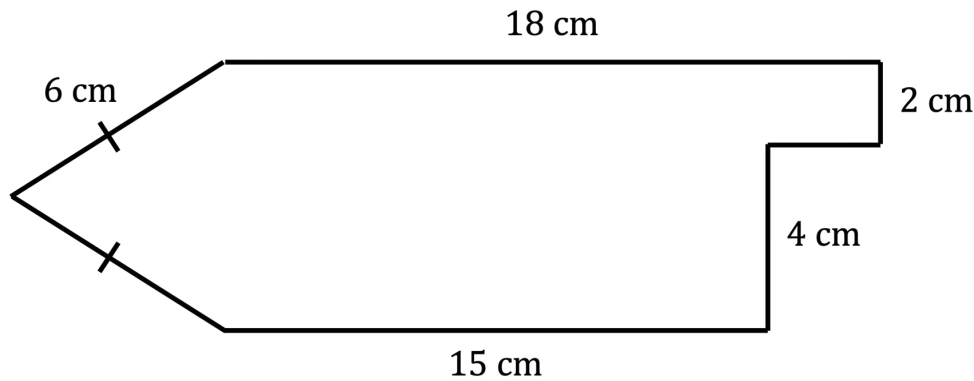- Understanding properties of different 2D shapes can be essential for missing lengths.

Worked Example

Find the perimeter of the compound shape.

18 cm

6 cm

2 cm

4 cm

15 cm

The shape can be split up into a triangle and two rectangles

The dashes on the triangle mean that the lengths are equal
It is an isosceles triangle and the missing length is 6 cm

The horizontal lengths of the two rectangles add up to the length of the longest horizontal line
15 cm and the missing length, are equal to the 18 cm length at the top
The missing horizontal length is therefore 3 cm

18 cm

6 cm

2 cm

3 cm

4 cm

6 cm

15 cm
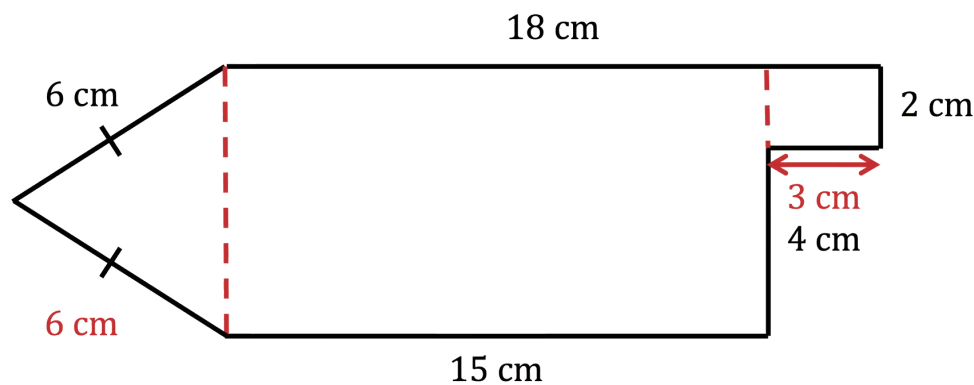
You can now find the sum of all the sides to find the total perimeter

$$6 + 6 + 18 + 2 + 3 + 4 + 15$$

54 cm

Your notes

Area Formulae

What is area?

- **Area** is the **amount of space** within the perimeter of a 2D shape

  - For example, the size of a sports field

- Area is calculated using lengths in **two dimensions**

  - **Units of measure** include mm^2, cm^2, m^2 etc

How do I find the area of a shape on a square grid?

- Count the **total number** of **whole squares** inside the shape

  - You can **shade** or **mark** the squares you have counted so far

- Parts of the shape **may not contain whole squares**

  - Pair up **half squares**, or **parts** of squares, to form whole ones

- There will be a **scale** telling you how much area **one square** represents

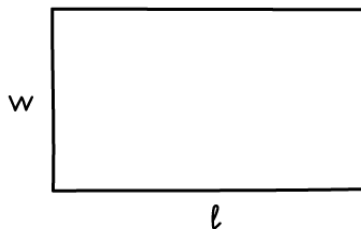 - **Multiply** the number of squares you have counted by this value to find the **total area** of the shape
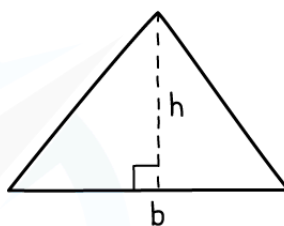
Which area formulae do I need to know?

RECTANGLE

AREA = $l \times w$

w

l

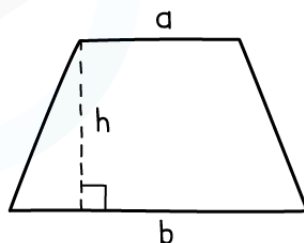TRIANGLE

AREA = $\frac{1}{2}bh$

h

b

TRAPEZIUM

AREA = $\frac{1}{2}(a+b)h$

a

h

b

PARALLELOGRAM

AREA = $b \times h$

h

b
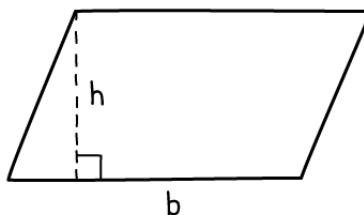
How do I find the area of a rectangle?

- The area, A, of a **rectangle** of length, l, and width, w, using the formula

  - $A = l \times w$

    - Multiply together the length and the width

How do I find the area of a triangle?

- The area, *A*, of a **triangle** with base, *b*, and length, *l*, can be found using the formula

  - $A = \dfrac{1}{2}bh$

    - **Multiply** the length of the **base** (*b*) by the **perpendicular height** (*h*)

    - **Halve** the answer

- The **perpendicular height** may not be the length of one of the sides of the triangle

How do I find the area of a trapezium?

- The area, *A*, of a **trapezium** with parallel lengths, *a* and *b*, and perpendicular height, *h*, can be found using the formula

  - $A = \dfrac{1}{2}(a + b)h$

    - **Add** together the lengths of the **parallel sides**

    - **Multiply** the result by the distance between the parallel sides

    - **Halve** the answer

- You **may** be able to work out the area of a trapezium by **splitting the shape** into a rectangle and triangles if you can't remember the formula

How do I find the area of a parallelogram?

- You can find the area, *A*, of a **parallelogram** of length, *l*, and perpendicular height, *h*, by using the formula

  - $A = bh$

    - **Multiply** the length of the **base** by the **perpendicular height**

- The **perpendicular height** is **not a length** of the parallelogram

- It is the **distance between** the **base** and its **opposite side**

- You can work the area of a parallelogram out by **splitting the shape** into a rectangle and triangles if you can't remember the formula
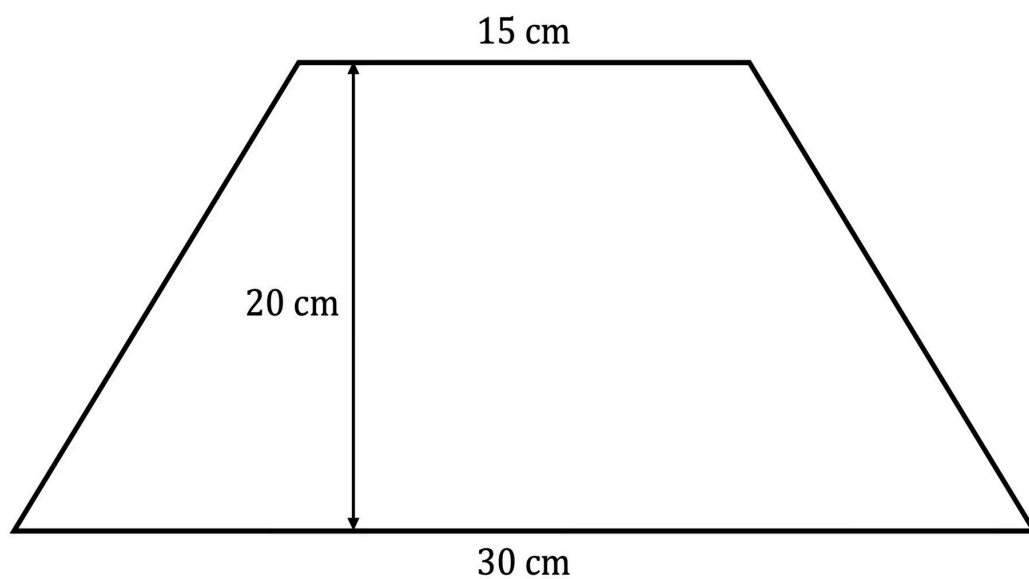
Examiner Tips and Tricks

- You may have to do some work to find **missing lengths** first

  - For example, you may need to use **Pythagoras' Theorem** to find a missing length on a triangle

Worked Example

Calculate the area of the following shapes.

(a)

15 cm

20 cm

30 cm

Find the area of the trapezium using $A = \dfrac{1}{2}(a + b)h$

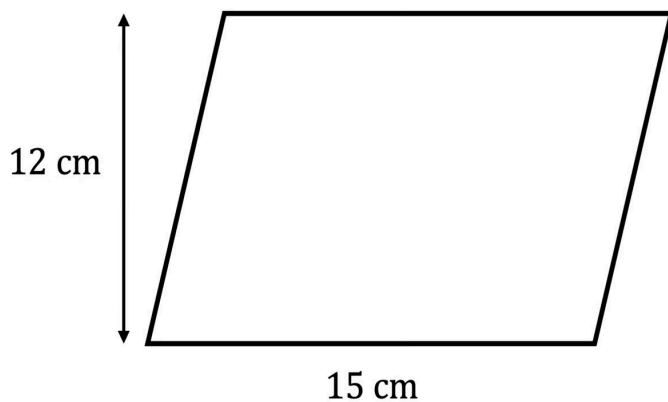Remember that a and b are the two parallel sides and h is the perpendicular height

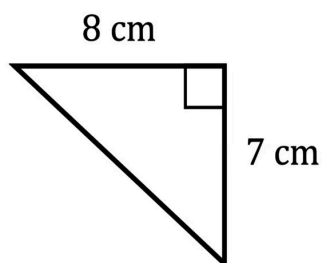$$A = \dfrac{1}{2}(30 + 15) \times 20$$

450 cm^2

(b)

Find the area of the parallelogram using $A = b \times h$

Remember that b is the base and h is the perpendicular height

$$A = 15 \times 12$$

180 cm^2

(c)

Find the area of the right-angled triangle using $A = \dfrac{1}{2} bh$

Remember that b is the base and h is the perpendicular height

$$A = \frac{1}{2} \times 8 \times 7$$

28 cm^2

Adding & Subtracting Areas

Adding & Subtracting Areas

What is a compound shape?

- Sometimes you will have a shape that is **not a standard shape** such as a rectangle, triangle, trapezium etc.

  - These are often called **compound shapes**

  - We can **split** the non-standard shape into **standard shapes**

How do I find the area of a compound shape?

- **Split** the **compound** shape into **standard shapes**

- Find the **areas** of the **standard shapes**

- **Add** these areas together to find the area of the **compound shape**

- Occasionally it may be easier to **add an extra shape** to the diagram and **subtract the area** of the **extra shape** from the **new bigger shape**

  - For the shape below you might complete the rectangle by putting a triangle in the top right corner

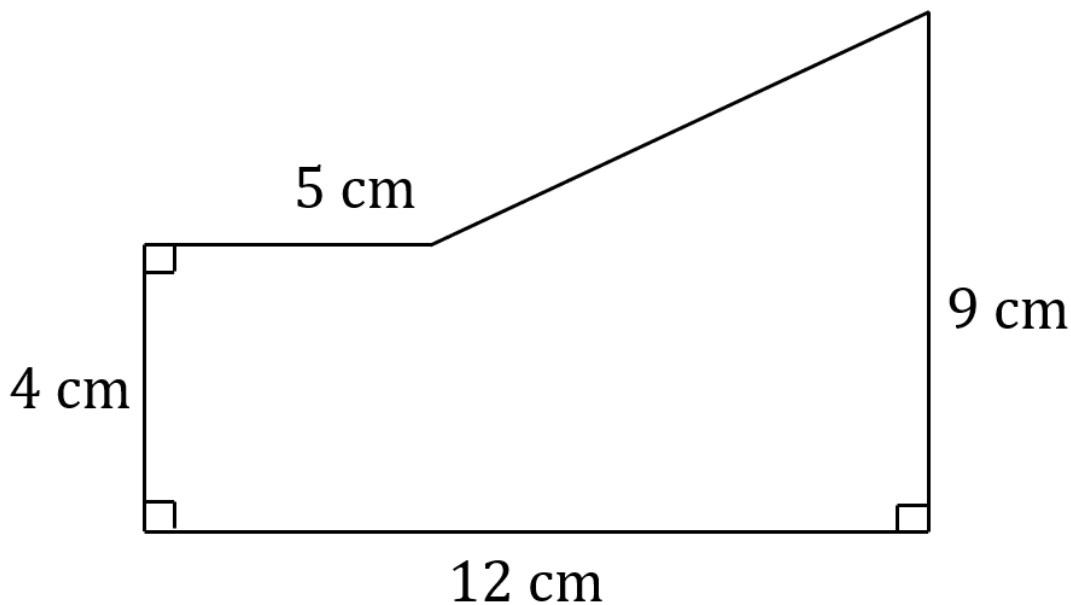 - The area of the **original shape** is the rectangle minus the triangle

"SQUARE – OFF"
THE RECTANGLE.
AREA IS THEN
"RECTANGLE – TRIANGLE"

💡
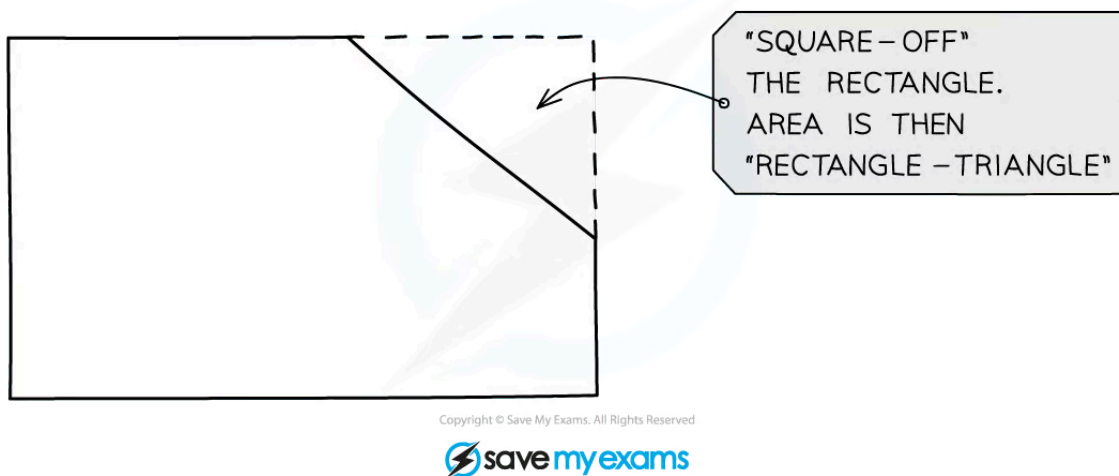
Examiner Tips and Tricks

Take a moment to think about how to split up the shape into the easiest shapes possible – there will probably be more than one way to do it!

✏️

Worked Example

Find the area of the pentagon shown in the diagram below.

Separate the diagram into two shapes that you are familiar with and know the area formulae for
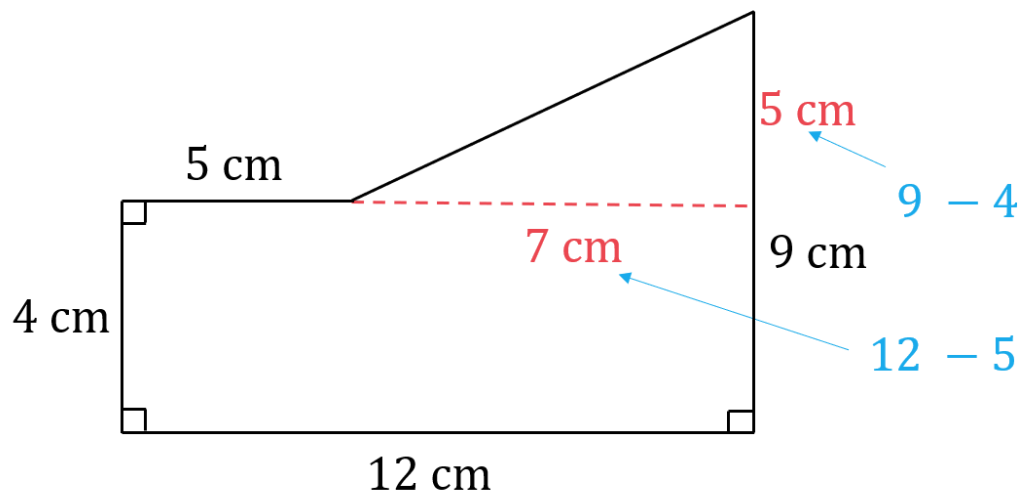This pentagon can easily be split into a rectangle and a triangle

Use the values given to find the length of the base and the height of the triangle and add these to the diagram.

Your notes

The total area will be the area of the rectangle + the area of the triangle.

$$\text{Total area} = (12 \times 4) + \frac{1}{2}(7 \times 5)$$

$$= 48 + \frac{1}{2}(35)$$

Area = 65.5 cm^2

Problem Solving with Areas

Problem Solving with Areas

What is problem solving?

- **Problem solving**, usually has two key features:
  - A question is given as a **real-life scenario**
    - eg. Mary is painting a bedroom in her house...
  - There is usually **more than one topic of maths** you will need in order to answer the question
    - eg. Area and percentages

What are common problems that involve area?

- **Area** is a commonly used topic of **'real-world'** maths
  - For example, laying a carpet, painting a house or designing a sports field all involve area
- Typically, the 'real-world' scenarios also have a **cost**
  - A lot of area problems also involve **calculations with money**

How do I solve problems that involve area?

- There is often a **lot of text** in a problem solving question, which can make it seem harder than it is
  - Avoid focusing **only** on what the question asks you, think about what you can do with the **information given**
    - This may lead you to think of **something else** you can do
    - Eventually you may be able to see your way to answering the **original question**
  - Think about the context of the question, **what makes sense?**
- Look out for **key information** in the text:
  - **Real-life context**
    - A question involving the **size** of a field, will mean be talking about its **area**
  - **Key words**
    - Types of **measure**: area, length, cost, ...

Your notes

- **Conditions**: minimum, maximum, greatest, ...
- **Units**
  - You may see **compound units**, e.g. $/m^2$, these may help you to **identify calculations** that you need to do
- **Annotate diagrams** with information that you can work out
  - Remember to do this in pencil in case you need to erase it!
- Problem solving questions could appear on either a **non-calculator paper** or a **calculator paper**

Examiner Tips and Tricks

- Even if you never get to a final answer always try to do some maths with the information from the question
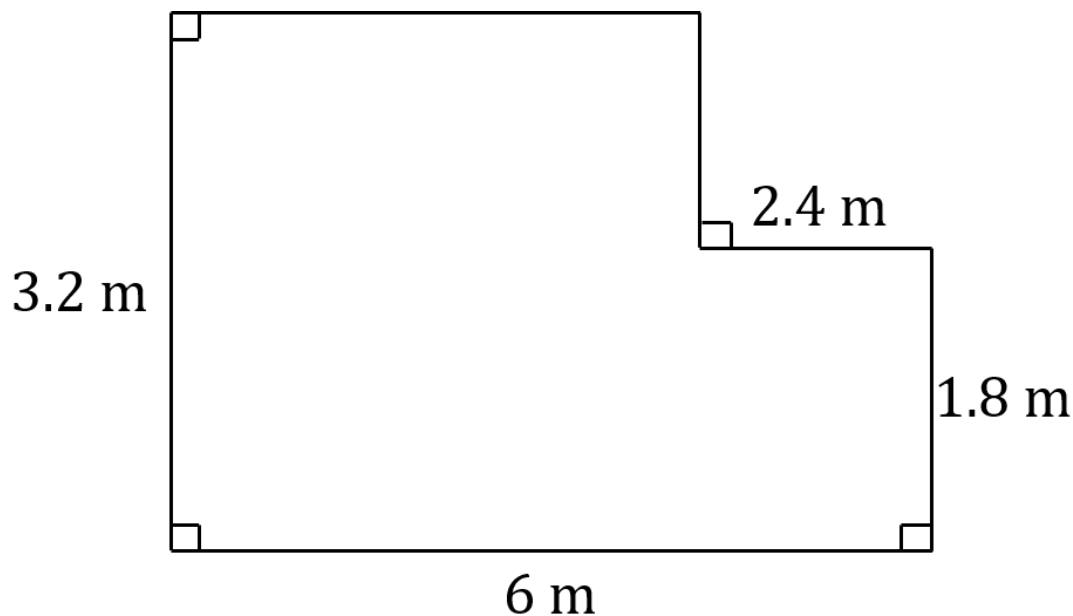 - You are likely to score some extra marks from your working!

Worked Example

John wants a new carpet for the lounge in his house.

A sketch of his lounge is given below.

He gets quote from two local companies, Company A and Company B.

The amount they charge for laying a carpet is given below.

- Company A: Fixed price of $5.50 per square metre
- Company B: $6 per square metre for the first ten square metres, then $4 per square metre for anything over that.

Which company should John choose in order to keep the cost of laying the carpet to a minimum?

Although this question doesn't specifically tell you you need to find the area, it is implied as the costs both use 'square metre'
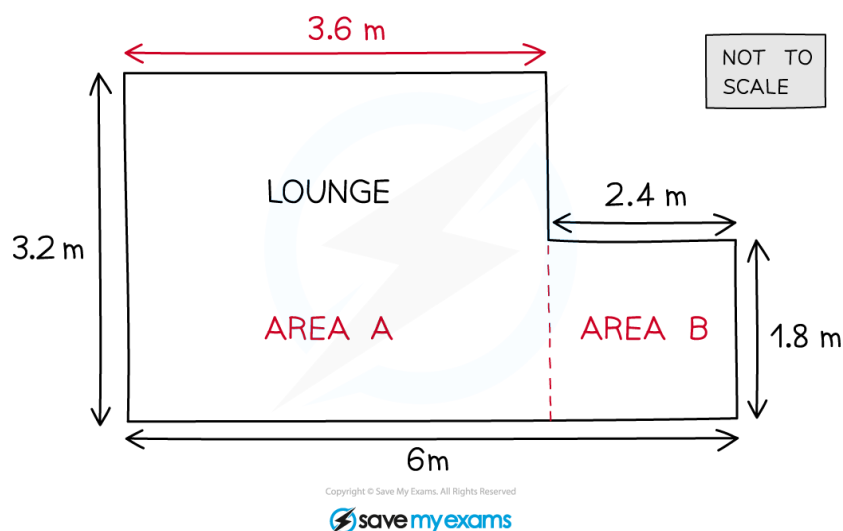
The shape of the lounge is a compound shape consisting of two rectangles
Split the area into these two rectangles and find the missing distances by subtracting the smaller length (2.4 m) from the longer one (6 m)

$$6 - 2.4 = 3.6$$

Your notes

Find the area of the lounge by adding the two areas together

$$\text{Total Area} = \text{Area A} + \text{Area B}$$
$$= (3.2 \times 3.6) + (2.4 \times 1.8)$$
$$= 11.52 + 4.32$$
$$= 15.84 \text{ m}^2$$

Find the cost for each of the two companies separately

Company A:

$$\text{Total Cost} = 15.84 \times 5.50$$
$$= \$87.12$$

Company B:

$$\text{Total Cost} = \$6 \times \text{first } 10 \text{ m}^2 + \$4 \times \text{remaining}$$
$$= (6 \times 10) + 4 \times (15.84 - 10)$$
$$= 60 + 23.36$$
$$= \$83.36$$

John should choose Company B as it will cost \$3.76 less than Company A