AQA GCSE Maths: Higher

Your notes

Area & Volume of Similar Shapes

Contents

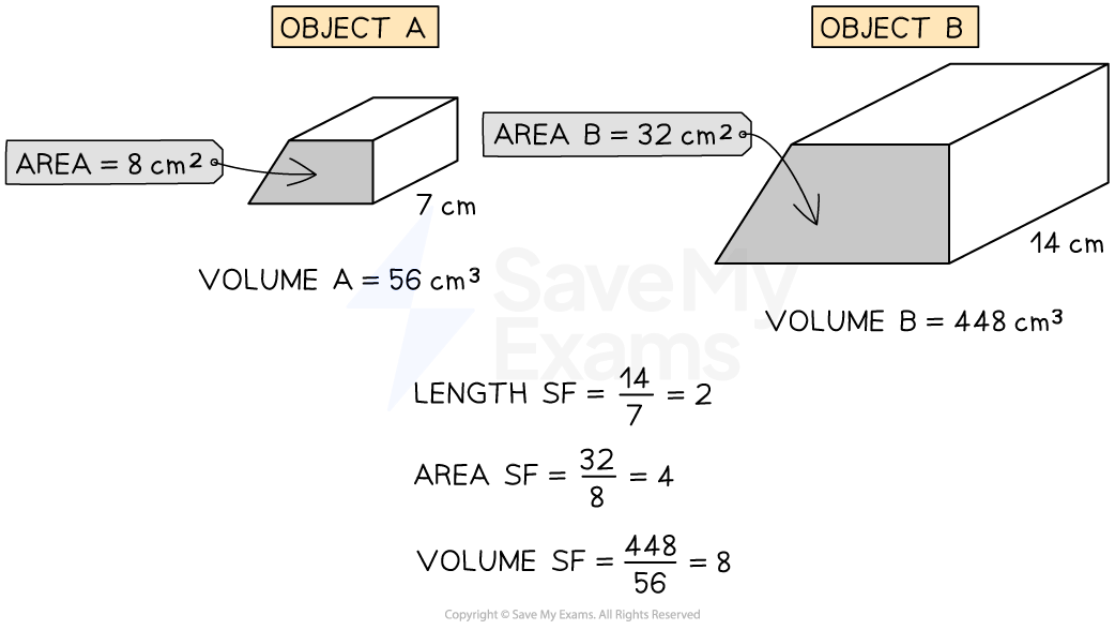
Your notes

Similar Areas & Volumes

What are similar shapes?

- Two shapes are **mathematically similar** if one is an **enlargement** of the other
- If the **lengths** of two similar shapes are linked by the **scale factor,** k
  - Equivalent **areas** are linked by an **area factor,** k^2
  - Equivalent **volumes** are linked by a **volume factor,** k^3
- The **scale factor** (SF) for a given quantity (length, area or volume), can be found using the formula:

$$\text{scale factor} = \frac{\text{second quantity}}{\text{first quantity}}$$



- An object could be made either **bigger or smaller** by a scale factor
  - When $k > 1$, the object is getting bigger
    - This is also true for $k^2 > 1$ and $k^3 > 1$
  - When $0 < k < 1$, the object is getting smaller

- This is also true for $0 < k^2 < 1$ and $0 < k^3 < 1$

How do I work with similar shapes involving area or volume?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- **STEP 1**
  Identify the **equivalent** known quantities
  - Recognise if the quantities are lengths, areas or volumes
- **STEP 2**
  Find the **scale factor** from two known **lengths, areas** or **volumes**
  - $\text{scale factor} = \dfrac{\text{second quantity}}{\text{first quantity}}$
  - For two lengths, k = length SF
  - For two areas, k^2 = area SF
  - For two volumes, k^3 = volume SF
- **STEP 3**
  Check the scale factor
  - SF > 1 if getting bigger
  - 0 < SF < 1 if getting smaller
- **STEP 4**
  If necessary, use the scale factor you have found to find **other scale factors**
  - If you have the **length** scale factor
    - $\text{Area scale factor} = (\text{Length scale factor})^2$
    - $\text{Volume scale factor} = (\text{Length scale factor})^3$
  - If you have the **area** scale factor
    - $\text{Length scale factor} = \sqrt{(\text{Area scale factor})}$
    - Find the volume scale factor by finding the length scale factor first
  - If you have the **volume** scale factor
    - $\text{Length scale factor} = \sqrt[3]{(\text{Volume scale factor})}$
    - Find the area scale factor by finding the length scale factor first

- **STEP 5**
  Multiply or divide by relevant scale factor to find a **new quantity**

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Examiner Tips and Tricks

- Take extra care not to mix up **which shape is which** when you have started carrying out the calculations
  - It can help to **label the shapes** and write an equation

Worked Example

Solid *A* and solid *B* are mathematically similar.

The volume of solid *A* is 32 cm^3.
The volume of solid *B* is 108 cm^3.
The height of solid *A* is 10 cm.

Find the height of solid *B*.

Calculate k^3, the scale factor of enlargement for the volumes, using:

$$\text{volume } B = k^3(\text{volume } A)$$

Or $k^3 = \dfrac{\text{larger volume}}{\text{smaller volume}}$

$$108 = 32k^3$$

$$k^3 = \frac{108}{32} = \frac{27}{8}$$

Find the length scale factor k by taking the cube root of the volume scale factor k^3

$$k = \sqrt[3]{\frac{27}{8}} = \frac{3}{2}$$

Substitute the value for k into formula for the heights of the similar shapes:

$$\text{Height } B = k(\text{height } A)$$

$h = 10k$

$h = 10\left(\frac{3}{2}\right) = \frac{30}{2} = 15$

Height of *B* = 15 cm

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