



AQA GCSE Maths: Higher



Your notes

Fractions, Decimals & Percentages

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Your notes

Converting Fractions, Decimals & Percentages

FDP Conversions

How do I convert from a percentage to a decimal?

- **Divide by 100** (move digits two places to the right)
 - 6% as a decimal is $6 \div 100 = 0.06$
 - 40% as a decimal is $40 \div 100 = 0.4$
 - 350% as a decimal is $350 \div 100 = 3.5$
 - 0.2% as a decimal is $0.2 \div 100 = 0.002$

How do I convert from a decimal to a percentage?

- **Multiply by 100** (move digits two places to the left and add a % sign)
 - 0.35 as a percentage is $0.35 \times 100 = 35\%$
 - 1.32 as a percentage is $1.32 \times 100 = 132\%$
 - 0.004 as a percentage is $0.004 \times 100 = 0.4\%$

How do I convert from a decimal to a fraction?

- If it has **one decimal place**, write the digits over **10**
 - 0.3 is $\frac{3}{10}$
 - 1.1 is $\frac{11}{10}$
- If it has **two decimal places**, write the digits over **100**
 - 0.07 is $\frac{7}{100}$
 - 0.13 is $\frac{13}{100}$



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- 30.01 is $\frac{3001}{100}$
- If it has n **decimal places**, write the digits over 10^n
 - 0.513 is $\frac{513}{1000}$
 - 0.0007 is $\frac{7}{10\,000}$
- Learn **simple recurring decimals** as fractions
 - $0.33333\dots = 0.\dot{3}$ is $\frac{1}{3}$
 - $0.66666\dots = 0.\dot{6}$ is $\frac{2}{3}$
- **Whole numbers** can be written as fractions by writing them over 1
 - 5 is $\frac{5}{1}$

How do I convert from a percentage to a fraction?

- Write the **percentage over 100**
 - 37% is $\frac{37}{100}$

How do I convert from a fraction to a decimal?



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CHANGE TO DECIMALS

FRACTIONS TO DECIMALS

◦ MANY SHOULD BE FAMILIAR:

$$\frac{1}{2} = 0.5$$

$$\frac{1}{5} = 0.2$$

$$\frac{1}{10} = 0.1$$

$$\frac{1}{4} = 0.25$$

$$\frac{3}{4} = 0.75$$

$$\frac{1}{3} = 0.\dot{3}$$

$$\frac{2}{3} = 0.\dot{6}$$

◦ IF UNFAMILIAR, DIVIDE NUMERATOR BY DENOMINATOR:

$$\frac{5}{8} \quad \begin{array}{r} 0.625 \\ 8 \overline{) 5.000} \end{array}$$

ADD ZEROES AS NEEDED
USE SHORT DIVISION

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▪ Fractions written **over powers of 10** are quicker

▪ $\frac{3}{5} = \frac{6}{10}$ which is 0.6

▪ $\frac{7}{20} = \frac{35}{100}$ which is 0.35

▪ $\frac{1}{500} = \frac{2}{1000}$ which is 0.002

How do I convert from a fraction to a percentage?

- Change fractions into **decimals** then **multiply by 100**

- $\frac{4}{5} = \frac{8}{10}$ which is 0.8 as a decimal, which is $0.8 \times 100 = 80\%$



Examiner Tips and Tricks

- A calculator can be used to check conversions between fractions and decimals (even if the question says to show working without a calculator)



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Recurring Decimals

Recurring Decimals

What are recurring decimals?

- When writing a **rational number** as a decimal, it will either be:
 - A decimal that stops, called a "**terminating**" decimal
 - $\frac{1}{4} = 0.25$
 - Or a decimal that repeats with a pattern, called a "**recurring**" decimal
 - $\frac{32}{99} = 0.32323232\dots$
- The recurring part can be written with a **dot above the digit that repeats**
- If multiple digits repeat, dots are used on the **first and last digits that repeat**
 - $0.3333\dots = 0.\dot{3}$
 - $0.121212\dots = 0.\dot{1}\dot{2}$
 - $0.325632563256\dots = 0.\dot{3}25\dot{6}$

How do I write recurring decimals as fractions?

Write out the first few decimal places to show the recurring pattern and then:

- STEP 1**
Write the recurring decimal as $x = \dots$
 - $x = 0.35353535\dots$
- STEP 2**
Multiply both sides by 10 repeatedly until two lines have the same recurring decimal part
 - $x = 0.35353535\dots$
 - $10x = 3.5353535\dots$
 - $100x = 35.353535\dots$



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- Note that x and $100x$ have 35 repeating after the decimal point, the repeating pattern after $10x$ is 53 repeating

STEP 3

Subtract the two lines which have matching recurring decimal parts

$$100x - x = 35.353535... - 0.35353535...$$

$$99x = 35$$

STEP 4

Divide both sides to get $x = \dots$

Cancel if necessary to get fraction in its lowest terms

$$x = \frac{35}{99}$$



Worked Example

Write $0.\dot{3}0\dot{7}$ as a fraction in its lowest terms.

Write as $x = \dots$ to show the pattern

$$x = 0.307307307307...$$

Multiply both sides by 10 repeatedly until two lines have the same recurring decimal part

$$10x = 3.07307307307...$$

$$100x = 30.730730730...$$

$$1000x = 307.307307307...$$

Notice that x and $1000x$ have matching recurring decimal parts

Subtract one from the other

$$1000x - x = 307.307307307... - 0.307307307...$$

$$999x = 307$$

Divide both sides by 999

$$x = \frac{307}{999}$$

This cannot be simplified, so this is the final answer

$$\frac{307}{999}$$



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Ordering Fractions, Decimals & Percentages

Ordering FDP

How do I put fractions in order of size?

- When comparing **only fractions**, write them over a **lowest common denominator**

- For $\frac{3}{5}, \frac{1}{2}, \frac{13}{20}, \frac{7}{12}$, the lowest common denominator is **60**

- So change them to $\frac{36}{60}, \frac{30}{60}, \frac{39}{60}, \frac{35}{60}$ and then **order them by their numerators**

- From smallest to largest: $\frac{30}{60}, \frac{35}{60}, \frac{36}{60}, \frac{39}{60}$

- Rewrite in their original form: $\frac{1}{2}, \frac{7}{12}, \frac{3}{5}, \frac{13}{20}$

How do I put fractions, decimals and percentages in order of size?

- When comparing a mixture of fractions, decimals and percentages, **change everything into decimals**



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e.g. WRITE THE FOLLOWING IN ORDER OF SIZE,
STARTING WITH THE SMALLEST.
(DO NOT USE A CALCULATOR)

$$0.31 \quad \frac{2}{3} \quad \sqrt{0.09} \quad 32\% \quad 3^{-1}$$

STEP 1:

CONVERT TO DECIMALS...

$$0.31$$

ALREADY A DECIMAL

$$\frac{2}{3} = 0.666\ 666\ \dots \quad (0.\dot{6})$$

RECOGNISE

$$\sqrt{0.09} = 0.3$$

RECOGNISE FROM $\sqrt{9} = 3$

$$32\% = 0.32$$

$$32 \div 100$$

$$3^{-1} = \frac{1}{3} = 0.333\ 333\ \dots \quad (0.\dot{3})$$

NEGATIVE POWER
MEANS "1 OVER"

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STEP 2:

WRITE AS A NEAT COLUMN
WITH DIGITS LINED UP

0.31

0.66...

0.3

0.32

0.33...

CAN ALWAYS ADD MORE
DIGITS LATER IF NEEDED

STEP 3:

REVEAL ONE DIGIT AT A TIME
AND ORDER THEM WHEN POSSIBLE

⑤ 0.3
0.6
0.3
0.3
0.3

FIRST DIGITS ALL ZERO...
BUT LAST PLACE (5th)
IS CLEAR TO SEE

② 0.31
⑤ 0.66
① 0.30
③ 0.32
④ 0.33

WITH SECOND DIGITS OF
0,1,2 AND 3 IT IS EASY
TO LABEL ALL PLACES

FILL BLANK PLACES IN
WITH ZEROS IF IT HELPS

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STEP 4:

WRITE DECIMALS IN ORDER

① ② ③ ④ ⑤
0.3, 0.31, 0.32, 0.33..., 0.66...

STEP 5:

WRITE FINAL ANSWER USING NUMBERS
IN THEIR ORIGINAL FORMAT

$\sqrt{0.09}$ 0.31 32% 3^{-1} $\frac{2}{3}$

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Which symbols can I use?

- Rather than just listing values in order, symbols can be used to compare them
 - For example, $\frac{1}{4} < \frac{1}{3} < \frac{1}{2}$
- Recall that $>$ means **greater than** and \geq means greater than **or equal to**
 - Similarly, $<$ means **less than** and \leq means less than or equal to
- You may also see $=$ and \neq (which means "**not** equal to")



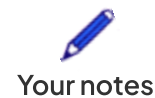
Examiner Tips and Tricks

A calculator can be used to quickly convert any quantities into decimals.

E.g. A fraction can be entered as a division.



Worked Example



Without use of a calculator, write these numbers in order, starting with the smallest.

$$\frac{1}{3} \quad \frac{2}{5} \quad \frac{9}{25} \quad \frac{4}{15}$$

As they are all fractions, write them with a common denominator

The lowest common denominator is 75

Rewrite each fraction with a denominator of 75

$$\frac{1}{3} = \frac{25}{75} \quad \frac{2}{5} = \frac{30}{75} \quad \frac{9}{25} = \frac{27}{75} \quad \frac{4}{15} = \frac{20}{75}$$

Compare and write in order from smallest to largest

$$\frac{20}{75}, \frac{25}{75}, \frac{27}{75}, \frac{30}{75}$$

Rewrite in their original form

$$\frac{4}{15}, \frac{1}{3}, \frac{9}{25}, \frac{2}{5}$$



Worked Example

Without use of a calculator, write these numbers in the spaces below.

$$\frac{7}{8} \quad \frac{5}{6} \quad 0.8 \quad 78\%$$

..... < < <

As there is a mixture of fractions, decimals, and percentages, rewrite each as a decimal

0.8 is already a decimal

Convert 78% to a decimal by dividing by 100

$$78\% = 0.78$$

Convert $\frac{7}{8}$ to a decimal by either using short division, or halving three times



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$$\begin{aligned}7 \div 2 &= 3.5 \\3.5 \div 2 &= 1.75 \\1.75 \div 2 &= 0.875 \\ \text{so } \frac{7}{8} &= 0.875\end{aligned}$$

Convert $\frac{5}{6}$ to a decimal by using short division

$$\begin{array}{r}0.833... \\ 6 \overline{) 5.22} \\ \underline{5.000} \\ \end{array}$$
$$\text{so } \frac{5}{6} = 0.8333... = 0.8\dot{3}$$

Write them all with 3 decimal places to determine the order

0.875
0.833
0.800
0.780

Rewrite the decimals in order

0.78, 0.8, 0.8 $\dot{3}$, 0.875

Rewrite in their original form, and recall that < means "less than", so the smallest value will be first

$$78\% < 0.8 < \frac{5}{6} < \frac{7}{8}$$

If a calculator was allowed for this question, it could be used to find $7 \div 8$ and $5 \div 6$ more easily