AQA GCSE Maths: Higher

Your notes

Ratios

Contents

Ratios

What is a ratio?

- A **ratio** is a way of comparing one **part** of a **whole** to another
    - **Ratios** are used to compare **one part** to **another part**

What do ratios look like?

- Ratios involve **two or three different numbers** separated using a **colon**

    - E.g. 2 : 5, 3 : 1, 4 : 2 : 3

- In all ratio questions, who or what is **mentioned first** in the question, will be associated with the **first part of the ratio**

    - E.g. The cake recipe with flour and butter in the ratio 2 : 1

        - 'Flour' is associated with '2' and 'butter' is associated with '1'

- The numbers in a ratio tell us, for **each quantity** involved, its **proportion of the whole**

    - In the ratio **4 : 3**

        - The **first quantity** comprises 4 parts (of the whole)

        - The **second quantity** comprises 3 parts (of the whole)

        - In total, the **whole** is made up of 4 + 3 = 7 parts

    - In the ratio **2 : 5 : 3**

        - The **first quantity** comprises 2 parts (of the whole)

        - The **second quantity** comprises 5 parts (of the whole)

        - The **third quantity** comprises 3 parts (of the whole)

        - In total, the **whole** is made up of 2 + 5 + 3 = 10 parts

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Worked Example

Your notes

A pot of money is shared between three friends, Dave, John and Mary.
Dave receives $450, John receives $200 and Mary receives $350.

(a) Find the total amount of money in the pot.

Add up the three separate amounts

$$450 + 200 + 350 = 1000$$

$1000

(b) Write down the ratio of money received by Dave, John and Mary.
(There is no need to simplify the ratio.)

Be careful with the order
Dave gets mentioned first, so 450 will be the first part of the ratio, then John and finally Mary

450 : 200 : 350

(c) Write down the fraction of the pot of money that Mary receives.
(There is no need to simplify the fraction.)

Fractions are compared to the whole, so this will be 'Mary's money' "out of" 'total money'

$$\frac{350}{1000}$$

What is an equivalent ratio?

- **Equivalent ratios** are two ratios that represent the **same proportion** of quantities within a whole

    - E.g. The ratio **5 : 10** is equivalent to **20 : 40**

- Equivalent ratios are frequently used when the values involved take on a **real-life meaning**

    - E.g. A cake recipe involves flour and butter being mixed in the ratio 3 : 2

        - 3 g of flour and 2 g of butter would not lead to a very big cake

        - An equivalent ratio of 300 : 200 gives a more realistic 300 g of flour and 200 g of butter

How do I find an equivalent ratio?

- You can find an **equivalent ratio** by **multiplying** (or **dividing**) **each part** of the ratio by the **same value**

    - E.g. Multiply each part of the ratio 2 : 3 : 7 by 4 to find an equivalent ratio of 8 : 12 : 28

    - Ratios can be scaled up or down to suit the context of a question

- The size of each part in the ratio, **relative** to the others, is still the same

- The **actual values** in the equivalent ratio may be **more meaningful** in the context of the situation
- Finding equivalent ratios is **similar** to finding **equivalent fractions**

  - However it is crucial to remember that 1 : 4 is **not** equivalent to $\dfrac{1}{4}$

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Examiner Tips and Tricks

Writing down what you are **doing** to each part of the ratio helps show your working and makes it easier to keep track of what you are doing.

E.g.

$$
\begin{array}{ccc}
A & : & B \\
3 & : & 4 \\
\end{array}
$$
$$\times 5 \downarrow \qquad\qquad \downarrow \times 5$$
$$
\begin{array}{ccc}
15 & : & 20
\end{array}
$$

✏️

Worked Example

The ratio of cabbage leaves eaten by two rabbits, Alfred and Bob, is 7 : 5.

(a) Write down an equivalent ratio that would involve a total of 48 cabbage leaves being eaten.

We have information about the whole so first add up the parts of the ratio to find how many parts make the whole
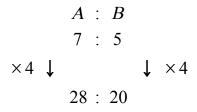
$$7 + 5 = 12$$

We require a total of 48 so divide this by 12 to find the multiplier

$$48 \div 12 = 4$$

Answer the question by multiplying each part of the ratio by this multiplier

$$A \; : \; B$$
$$7 \; : \; 5$$
$$\times 4 \downarrow \qquad\qquad \downarrow \times 4$$
$$28 \; : \; 20$$

28 : 20

(b) On another occasion Bob eats 35 leaves.

Find out how many leaves Alfred eats.

Use the information about Bob to find the multiplier using division

$$35 \div 5 = 7$$

Now multiply Alfred's part of the ratio by the multiplier to answer the question

$$A \; : \; B$$
$$7 \; : \; 5$$
$$\times 7 \downarrow \qquad\qquad \downarrow \times 7$$
$$49 \; : \; 35$$

Alfred eats 49 cabbage leaves

What is a simplified ratio?

- **Simplifying a ratio** involves finding an **equivalent ratio** where the numbers involved are **smaller**

  - E.g. The ratio **45 : 30** is equivalent to **9 : 6**

- A ratio is in its **simplest form** when

  - All of the values in the ratio are **integers**

  - There are no **common factors** between each of the values in the ratio

  - E.g. The simplest form of the ratio **45 : 30** is **3 : 2**

How do I simplify a ratio?

- **Divide each part** of the ratio by the **same value**

  - This value should be a **common factor** of all parts of the ratio

Your notes

Your notes

- Ideally, the **highest common factor (HCF)** should be used to get the ratio into its simplest form in one go

- If the HCF is not used, we can repeat the process of simplifying
- E.g. Divide all parts of the ratio **30 : 66 : 12** by 6 to find the ratio in its simplest form **5 : 11 : 2**

Worked Example

Amber and Naomi are sharing a large cake that has been cut into 48 pieces.
Amber receives 30 of these pieces, Naomi receives the rest.

Write down the ratio, in its simplest form, of the number of pieces of cake that Amber receives to the number of pieces of cake that Naomi receives.

First use subtraction to find the number of pieces that Naomi receives

$$48 - 30 = 18$$

Next write down the ratio without simplifying
Make sure you have the order correct ('Amber to Naomi')

$$30 : 18$$

Look for a common factor of both 30 and 18 to simplify the fraction
6 is the highest common factor
Divide both parts of the fraction by 6

$$
\begin{array}{ccc}
A & : & N \\
30 & : & 18 \\
\div 6 \downarrow & & \downarrow \div 6 \\
5 & : & 3
\end{array}
$$

5 : 3

Your notes

Sharing in a Ratio

Working with Ratios

How do I share an amount in a given ratio?

- **Add** together **all parts in the ratio** to find the **total number of parts** in the whole

  - $200 is to be shared between two people, A and B, in the ratio 5:3

  - There are 8 "parts" in total, as A receives 5 parts and B receives 3 parts

- **Divide** the **amount being shared** by the **total number of parts**

  - $200 must be split into 8 parts, so this means that 1 part must be worth $25

    - $200 \div 8 = 25$

- **Multiply** the **amount each part is worth** by the **number of parts** for each quantity in the ratio

  - Person A receives 5 parts, each worth $25

    - $5 \times 25 = \$125$ for person A

  - Person B receives 3 parts, each worth $25

    - $3 \times 25 = \$75$ for person B

- Check the **values** in the new ratio **add up to** the total **amount being shared**

  - $125 + $75 = $200

Examiner Tips and Tricks

- **Adding labels** to your ratios will help make your working clearer and help you remember which number represents which quantity, e.g.

$$\begin{array}{ccc} A & : & B \\ 3 & : & 4 \end{array}$$

Worked Example

Your notes

A particular shade of pink paint is made using three parts red paint, to two parts white paint. Mark needs 60 litres of pink paint in order to decorate a room in his house.

Calculate the volume of red and white paint that Mark needs to purchase in order to have enough paint to decorate the room.

The ratio of red to white is

$$3 : 2$$

Adding these together gives the total number of parts

$$3 + 2 = 5$$

The total amount of paint is 60 litres

$$\therefore 5 \text{ parts} = 60 \text{ litres}$$

Divide both sides by 5 to find out the number of litres in one part

$$
\begin{aligned}
5 \text{ parts} &= 60 \text{ litres} \\
\div 5 \qquad & \qquad \div 5 \\
1 \text{ part} &= 12 \text{ litres}
\end{aligned}
$$

The ratio was 3:2, so multiply both number of parts by 12

$$
\begin{array}{ccc}
R & : & W \\
3 & : & 2 \\
\times 12 \downarrow & & \downarrow \times 12 \\
36 & : & 24
\end{array}
$$

Answer in context, making sure you make it clear which value is associated with which colour paint

Mark will need to buy 36 litres of red paint and 24 litres of white paint

What type of ratio problems could I be asked to solve?

- **Simple ratio problems** are discussed earlier, including

  - Writing ratios

  - The link between ratios and fractions

  - Equivalent ratios

  - Simplifying ratios

Your notes

- ▪ Sharing an amount in a given ratio
- ▪ **Further problems** involving ratio include

  - ▪ Ratios where you are given the **difference between** the **two parts**

    - ▪ E.g. Kerry is given $30 more than Kacey who is given $50

  - ▪ Ratios where **one quantity** is **given** and you have to **find** the **other quantity**

    - ▪ E.g. Kerry and Kacey are sharing money in the ratio 8 : 5, Kacey gets $50

  - ▪ Situations where you are given **two separate (two-part) ratios** but can combine them in to **one (three-part) ratio**

    - ▪ E.g. Kerry and Kacey are sharing money in the ratio 8 : 5 whilst Kacey is also sharing money with Kylie in the ratio 1 : 2

How do I solve a ratio problem when given the difference between two parts?

- ▪ **Find** the **difference in the number of parts** between the two quantities in the ratio

- ▪ **Compare** the difference in the **number of parts** with the difference between the **actual numbers**

- ▪ **Simplify** to find out the **value of one part**

- ▪ **Multiply** the **value of one part** by the **number of parts** for each quantity in the ratio

- ▪ **Multiply** the **value of one part** by the **total number of parts** to find the **total amount**

Given one quantity of a ratio, how can I find the other quantity?

- ▪ **Compare** the **given quantity** with the relevant **number of parts** in the ratio

- ▪ **Simplify** to find the **value of one part**

- ▪ **Multiply** the **value of one part** by the **number of parts** in the remaining quantity in the ratio

- ▪ **Multiply** the **value of one part** by the **total number of parts** to find the **total amount**

Worked Example

(a) The ratio of cabbage leaves eaten by two rabbits, Alfred and Bob, is 8 : 4 respectively. It is known that Alfred eats 12 more cabbage leaves than Bob for a particular period of time.

Find the total number of cabbage leaves eaten by the rabbits and the number that each rabbit eats individually.

The difference in the number of parts is

$$8 - 4 = 4 \text{ parts}$$

This means that

$$4 \text{ parts} = 12 \text{ cabbage leaves}$$

Dividing both by 4

$$1 \text{ part} = 3 \text{ cabbage leaves}$$

Find the total number of parts

$$8 + 4 = 12 \text{ parts}$$

Find the total number of cabbage leaves

$$12 \times 3 = 36$$

36 cabbage leaves in total

Find the number eaten by Alfred

$$8 \times 3 = 24$$

24 cabbage leaves

Find the number eaten by Bob

$$4 \times 3 = 12$$

12 cabbage leaves

(b) A particular shade of pink paint is made using 3 parts red paint, to two parts white paint.

Mark already has 36 litres of red paint, but no white paint.

Calculate the volume of white paint that Mark needs to purchase in order to use all of his red paint, and calculate the total amount of pink paint this will produce.

The ratio of red to white is

$$3 : 2$$

Mark already has 36 litres of red, so

$$36 \text{ litres} = 3 \text{ parts}$$

Divide both sides by 3.

12 litres = 1 part

The ratio was 3 : 2
Find the volume of white paint, 2 parts

$$2 \times 12 = 24$$

24 litres of white paint

In total there are 5 parts, so the total volume of paint will be

$$5 \times 12 = 60$$

60 litres in total

Working with Proportion

Working with Proportion

What is direct proportion?

- **Direct** proportion

  - As one quantity **increases/decreases** by a certain rate (factor)

  - The other quantity will **increase/decrease** by the same rate

- The **ratio** of the two quantities is **constant**

  - E.g. 2 boxes of cereal is 800 g of cornflakes

  - **Doubling** the number of boxes of cereal (4 boxes) will **double** the amount of cornflakes (1600 g)

How do I solve direct proportion questions?

- **Read** through wordy **direct proportion** questions **carefully**

  - Ensure that you understand the **context** of the question

  - Some questions may tell you the relationship between the two values as a **ratio**

- **Identify** the two quantities involved

  - E.g. Hours worked and pay

- **Find** the **factor** that you will be **increasing/decreasing** by

  - This may be given to you in the question, e.g. 'the amount is tripled'

    - The quantity is multiplied by three

  - Alternatively, find the factor by dividing the 'new' quantity by the 'old' quantity

- **Multiply** the other quantity by this factor to find the required **quantity**

  - E.g. If three times as many hours are worked, the pay will be three times more in total

- Give your **final answer** in **context**

  - **Round** and **give units** where appropriate

Examiner Tips and Tricks

- You may have to round an answer to a whole number, but think carefully about the context of the question!

  - Rounding to the nearest whole number is often appropriate
  - Sometimes you need to round **up** to the next whole number even if it is not the nearest

    - E.g. If you need 1.3 tins of paint, round the number of tins required up to 2 to ensure that you have enough paint

What is the unitary method?

- The **unitary method** means finding **one of something** (1 **unit** of something)

  - This can be a useful strategy

- For example, find the weight of 7 boxes, if 8 boxes weigh 60 kg

  - Find the weight of **1 box** (1 **unit**) using division

    - 60 kg ÷ 8 boxes = 7.5 kg per box

  - Scale this unit up using multiplication

    - 7.5 kg per box × 7 boxes = 52.5 kg

Worked Example

The bonus received by an employee is directly proportional to the profit made by the company they work for.
Bonuses are paid at a rate of $250 per $3000 profit the company makes.

(i) Work out the bonus an employee receives if the company makes a profit of $18 000.

(ii) If the company makes less than $600 profit, no bonus is paid.

Find the lowest bonus an employee could receive.

(i) Identify the two quantities 'profit' and 'bonus'

Find the factor ('new' ÷ 'old') from the profit

$$\frac{18\,000}{3000} = 6$$

Multiply the bonus by the factor

$$250 \times 6 = 1500$$

Answer in context with units

An employee should receive a bonus of $1500

(ii) We are still working with profit and bonus

The lowest bonus will be when the company makes exactly $600 profit

Find the factor using 'new' ÷ 'old'

$$\frac{600}{3000} = \frac{1}{5}$$

Find the amount of bonus by multiplying by the factor

$$250 \times \frac{1}{5} = 50$$

Answer in context with units

The lowest amount of bonus an employee could receive is $50

What is inverse proportion?

- **Inverse** proportion
    - As one quantity **increases** by a certain rate (factor)
    - The other quantity will **decrease** by the same rate
- This relationship applies vice versa too, if one quantity **decreases** the other **increases**
- E.g. If 2 robots take 15 hours to build a car
    - **Tripling** the number of robots (6) would mean the time taken to build a car would be **divided by 3** (5 hours)

How do I solve inverse proportion questions?

- **Read** through wordy **inverse proportion** questions **carefully**
    - Ensure that you understand the **context** of the question
    - Some questions may tell you the relationship between the two values as a **ratio**
- **Identify** the two quantities involved
- **Find** the **factor** that you will be **increasing/decreasing** by

- This may be given to you in the question, e.g. 'the amount is tripled'

- Alternatively, find this by dividing the 'new' quantity by the 'old' quantity
- **Divide** the other quantity by this factor to find the required **quantity**

- Give your final answer in **context**

  - **Round** and **give units** where appropriate

Examiner Tips and Tricks

- Think about the **context** to determine if a question is **direct or inverse proportion**

  - As the number of robots goes up, the time to build a car comes down (**inverse proportion**)
  - If you buy more boxes of cereal, the amount of cereal also increases (**direct proportion**)

Worked Example

The time taken to fill a swimming pool is inversely proportional to the number of pumps used to pump the water in.
If 3 pumps are used it will take 12 hours to fill the pool.

(i) Work out the amount of time required to fill the pool if 9 pumps are used.

(ii) If only 2 pumps are available find out how much extra time will be needed to fill the pool.

(i) Identify the two quantities, 'number of pumps' and 'time (hours)'

Find the factor ('new' ÷ 'old') from the number of pumps

$$\frac{9}{3} = 3$$

Divide the time by the factor

$$12 \div 3 = 4$$

Answer in context with units

If 9 pumps are used it will take 4 hours to fill the swimming pool

(ii) We are still working with 'pumps' and 'time'

Find the factor using 'new' ÷ 'old'

$$\frac{2}{3} \;\left(= 0.666\,666\,\ldots\right)$$

Your notes

Avoid rounding, keep the exact value in your calculator (it will be stored under the ANS key)

Find the time taken by dividing by the factor

$$12 \div \frac{2}{3} = 18$$

Answer in context with units

If only 2 pumps are available then it will take 18 hours to fill the swimming pool