Problem 8.3
The main idea is to use the equivalent version of convergence in distribution.
Suppose that $X_n \xrightarrow{\mathbb{P}} X$ and define $Y_n = |X_n - X|$. We need to show that $\mathbb{P}(Y_n > \varepsilon) \to 0$ holds for any $\varepsilon > 0$. First recall that $X_n \xrightarrow{\mathbb{P}} X$ is defined as weak convergence of Y_n to the constant zero random variable. By Lemma 8.2 this is equivalent to

$$\lim_{n\to\infty} \mathbb{P}(Y_n \leq t) = \mathbb{P}(0 \leq t),$$

for all continuity points of the function $\omega \mapsto 0$. We now note that any $\varepsilon > 0$ is a continuity point of this function. Hence, we get

$$\lim_{n\to\infty} \mathbb{P}(Y_n > \varepsilon) = 1 - \lim_{n\to\infty} \mathbb{P}(Y_n \leq \varepsilon) = 1 - \mathbb{P}(0 \leq \varepsilon) = 0$$

Now we prove the other implication. So suppose that $\mathbb{P}(Y_n > \varepsilon) \to 0$ holds for any $\varepsilon > 0$. We then have to prove that $(Y_n)_{\#}\mathbb{P} \Rightarrow 0_{\#}\mathbb{P}$. Due to Lemma 8.2 it is enough to show that

$$\lim_{n\to\infty} \mathbb{P}(Y_n \leq t) = \mathbb{P}(0 \leq t) = \mathbb{1}_{t\geq 0},$$

holds for all continuity points t of the function $\omega \mapsto 0$. Notice that the only non-continuity point is 0. Moreover, for all $t < 0$ we have that $\mathbb{P}(Y_n \leq t) = 0$ since $Y_n \geq 0$ almost every-where. Finally, for all $t > 0$ we have

$$\lim_{n\to\infty} \mathbb{P}(Y_n \leq t) = 1 - \lim_{n\to\infty} \mathbb{P}(Y_n > t) = 1 = \mathbb{P}(0 \leq t).$$

Problem 8.5 Suppose that $X_n \xrightarrow{\text{a.s.}} X$. Then by Lemma 5.2.16 this is equivalent to $\mathbb{P}(\|X_n - X\| > \varepsilon \text{ i.o.}) = 0$ for all $\varepsilon > 0$.
For now fix an $\varepsilon > 0$ and write $A_n := \{\|X_n - X\| > \varepsilon\}$. Recall that

$$\{A_n \text{ i.o.}\} = \bigcap_{k=1}^{\infty} \bigcup_{k\geq n} A_n$$

and note two things:

(a) The sets $B_k := \bigcup_{n\geq k} A_n$ are non-increasing, i.e. $B_k \supset B_{k+1}$, and

(b) $\mathbb{P}(A_k) \leq \mathbb{P}(\bigcup_{n\geq k} A_n) = \mathbb{P}(B_k)$.

We then have that:

$$\begin{aligned}
0 &= \mathbb{P}(\{A_n \text{ i.o.}\}) && \text{by assumption} \\
&= \mathbb{P}(\bigcap_{k=1}^{\infty} B_k) && \text{by Lemma 5.2.16} \\
&= \lim_{k\to\infty} \mathbb{P}(B_k) && \text{by continuity form above (Proposition 2.2.15)} \\
&\geq \lim_{k\to\infty} \mathbb{P}(A_k) && \text{by (b).}
\end{aligned}$$