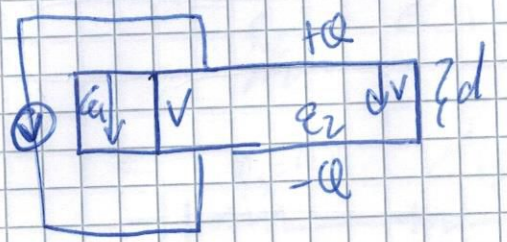
PNS - 23/11/18

5.1)



$\varepsilon_1 \Rightarrow \frac{1}{3}$ Área

$\varepsilon_2 \Rightarrow \frac{2}{3}$ Área

a) Capacidade

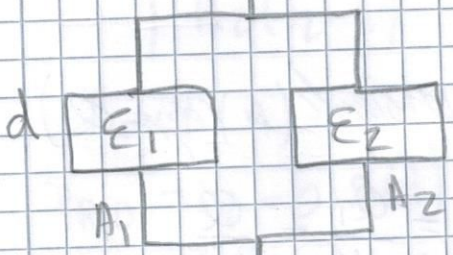
$$C = \frac{Q}{V}$$

Considerando placas paralelas

$$C = \frac{A\varepsilon}{d}$$

$$A_1 = \frac{1}{3}A$$

$$A_2 = \frac{2}{3}A$$



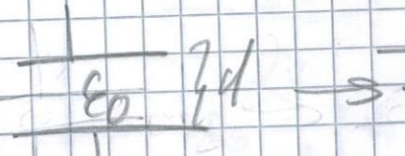
$$C_1 = \frac{A_1 \cdot \varepsilon_1}{d} = \frac{A\varepsilon_1}{3d}$$

$$C_2 = \frac{A_2\varepsilon_2}{d} = \frac{2A\varepsilon_2}{3d}$$

As somas de condensadores em paralelo

$$C_{eq} = \sum_i C_i$$

$$C_{eq} = C_1 + C_2 = \frac{A}{3d}(\varepsilon_1 + 2\varepsilon_2)$$

b) 

$$E = \frac{1}{2}CV^2$$

Uma vez

$$C_i = \frac{A\varepsilon_0}{d_i} = \frac{A\varepsilon_0}{d}$$

$$C_f = \frac{A\varepsilon_0}{d_f} = \frac{A\varepsilon_0}{d/2} = \frac{2A\varepsilon_0}{d}$$

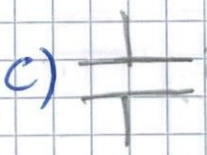
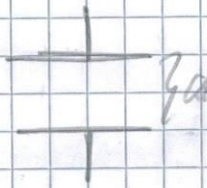
$$E_i = \frac{1}{2}\left(\frac{A\varepsilon_0}{d}\right)V^2$$

$$E_f = \frac{1}{2}\left(\frac{2A\varepsilon_0}{d}\right)V^2$$

$$\Big\} E_F = 2E_i$$

$$\begin{cases} C_i = \dfrac{Q_i}{V_i} = \dfrac{Q_i}{V} \\ C_F = \dfrac{Q_F}{V_F} = \dfrac{Q_F}{V} \end{cases}$$

$$\Rightarrow C_F = 2C_i \Leftrightarrow Q_F = 2Q_i$$

c)  $d_i = d$  $d_f = 2d$

desligar a fonte

$\hookrightarrow V_i \neq V_F$ (só se mantiver constante a carga Q)

$V_i \neq V_f$

$Q_i = Q_F = Q$

$$C = \frac{Q}{V} \begin{cases} C_i = \dfrac{Q_i}{V_i} = \dfrac{Q}{V_i} \\ C_F = \dfrac{Q_F}{V_F} = \dfrac{Q}{V_F} \end{cases}$$

$$C_i = \frac{A\varepsilon_0}{d_i} = \frac{A\varepsilon_0}{d}$$

$$C_F = \frac{A\varepsilon_0}{d_f} = \frac{A\varepsilon_0}{2d}$$

$$V_F = 2V_i$$

$$C_f = \frac{1}{2}C_i$$

d) Aproxima as placas com fonte ligada

$$\begin{cases} C_F = 2C_i \\ Q_F = 2Q_i \\ E_F = 2E_i \end{cases}$$

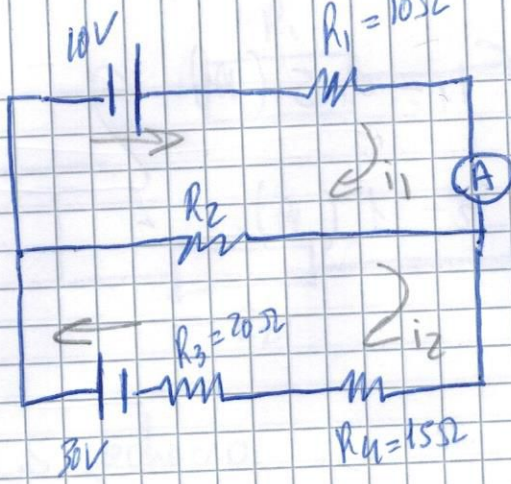
Afastar as placas sem fonte

$$E_i = \frac{1}{2}C_i V_i^2 = \frac{1}{2}CV^2$$

$$E_f = \frac{1}{2}\left(\frac{1}{2}C\right)(2V)^2$$

$$\begin{cases} C_f = \frac{1}{2}C_i \\ V_0 = 2V_i \\ E_F = 2E_i \end{cases}$$

5.2



Lei das Malhas

$$\begin{cases} 1: -10 + i_1 \cdot 10 + R_2(i_1 - i_2) = 0 \\ 2: R_2(i_2 - i_1) + 35 i_2 - 30 = 0 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} -0{,}5 \cdot 10 + R_2(i_1 - i_2) = 10 \\ R_2(i_2 - i_1) + 35 i_2 = 30 \end{cases} \Rightarrow \begin{cases} R_2(i_1 - i_2) = 10 - 5 = 5 \\ -5 + 35 i_2 = 30 \end{cases}$$

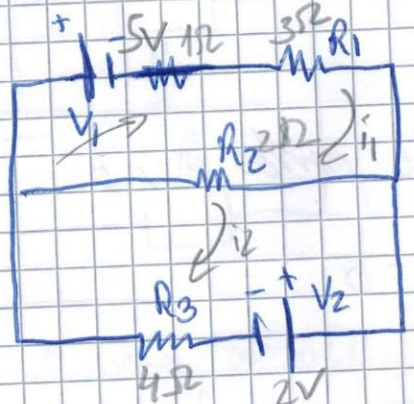
$i_2 = 1\,(A)$

$i_{R10} = i_1 = 0{,}5\,(A)$

$i_{R_2} = (i_1 - i_2) = 0{,}5\,(A)$

$i_{R20} = i_{R15} = i_2 = 1\,(A)$

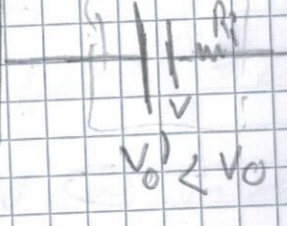
5.3



R_i (fonte) = 1 Ω

fonte ideal

fonte real



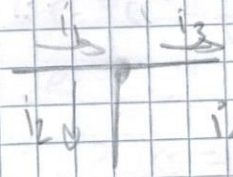
a) Lei das Malhas

$$\begin{cases} 1: 5 + i_1 \cdot 1 + i_1 \cdot 3 + 2(I_1 - I_2) \\ 2: (i_2 - i_1)2 + i_2 \cdot 1 + 2 + i_2 \cdot 4 = 0 \end{cases}$$

$$\begin{cases} 6i_1 - 2i_2 = 5 \\ -2i_1 + 7i_2 = -2 \end{cases} \Rightarrow \begin{cases} i_1 = 0{,}5\ (A) \\ i_2 = 1\ (A) \end{cases}$$

b) Leis de Kirchoff

Lei dos Nós $\Rightarrow \sum_i I_i = 0$



Princípio da conservação da carga elétrica

Lei das Malhas $\sum_i V_i = 0$

$V = R \cdot I$

$$\Delta V = -\int_a^b E \, dl$$

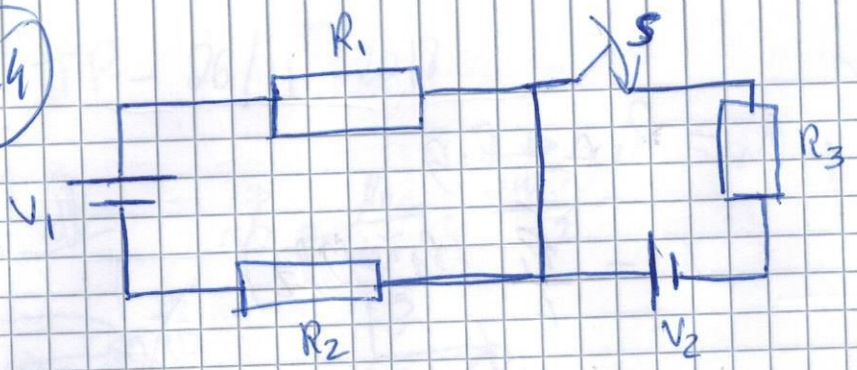
Campo E é conservativo

↳ V só depende dos pontos a e b

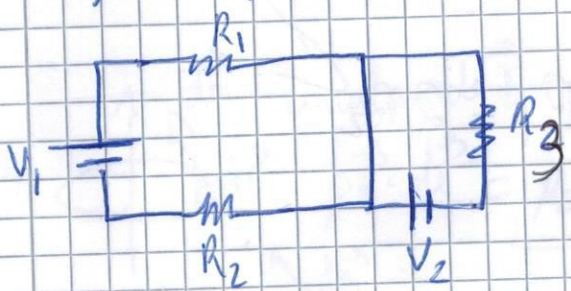
$a = b \Rightarrow \Delta V = 0$

$\rightarrow V_{R_1} + V_{R_2} = V$

5.4



a) S fechado



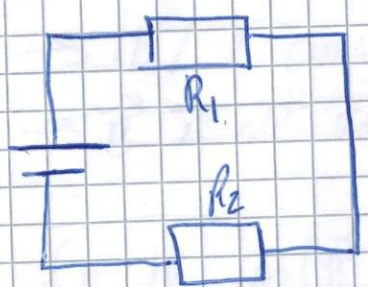
$$\begin{cases} 1: -V_1 + i_1 R_1 + i_1 R_2 = 0 \\ 2: i_2 R_3 - V_2 = 0 \end{cases}$$

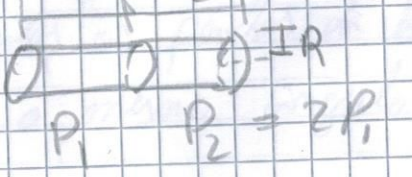
$$\begin{cases} i_1 = \dfrac{V_1}{R_1 + R_2} \\ i_2 = \dfrac{V_2}{R_3} \end{cases}$$

$$V_{R_1} = \frac{R_1 V_1}{R_1 + R_2} \quad , \quad V_{R_2} = \frac{R_2 V_1}{R_1 + R_2}$$

$$V_{R_3} = V_2$$

b)





$\frac{l_1}{l_2}$? de forma a que $V_{P_2} = 2V$

R_1 é uma associação de duas resistências em Sé

$$R_{1A} = P_1 \frac{l_1}{A_1} \quad ; \quad A_1 = A_2 = \pi r^2$$

$$R_{1B} = 2P_1 \frac{l_1}{A}$$

$$R_{1A} = \rho_1 \frac{l_1}{A}$$

$$R_{1B} = 2\rho_1 \frac{l_1}{A}$$

$$R_1 = R_{1A} + R_{1B}$$

$$R_1 = \frac{\rho_1}{A}(l_1 + 2l_2)$$

$$V_{R_1} = 2V_{R_2}$$

$$V = R \cdot I$$

$$R_1 \cdot S_1 = 2 \cdot R_2 \cdot S_2$$

$$\boxed{S_1 = S_2}$$

$$\Rightarrow R_1 = 2R_2$$

Substituting

$$\frac{\rho_1}{A}(l_1 + 2l_2) = 2R_2$$

$$l_1 = \frac{2R_1 \cdot A - R_2 l_2}{\rho_1}$$