

Extend Computer Science - Week 1

1.

Write a program to find the smallest prime factor of a positive integer.

Use your program to find the smallest prime factor of:

- (a) 72
- (b) 377
- (c) 1147
- (d) 2011

2.

Write a number to find the largest prime factor of a positive integer.

Use your program to find the largest prime factor of:

- (a) 72
- (b) 377
- (c) 1147
- (d) 2011

3.

- (i) Write a program to find all the positive integer solutions to $ax + by = c$ ($x, y \in \mathbb{Z}$)
- (ii) Use your program to find the positive integer solutions to
 - (a) $3x + 5y = 60$ ($x, y \in \mathbb{Z}$)
 - (b) $2x + 4y = 77$ ($x, y \in \mathbb{Z}$)
 - (c) $12x + 18y = 100$ ($x, y \in \mathbb{Z}$)
 - (d) $12x + 18y = 120$ ($x, y \in \mathbb{Z}$)
- (iii) Show that if c is not a multiple of the highest common factor of a and b then $ax + by = c$ ($x, y \in \mathbb{Z}$) has no solutions.

4. Watching Clocks!

Two *clocks*, which show the time in hours and minutes using the 24 hour clock, are running at different speeds. Each clock is an exact number of minutes per hour fast. Both clocks start showing the same time (00:00) and are checked regularly every hour (starting after one hour) according to an accurate timekeeper. What time will the two clocks show on the first occasion when they are checked and show the same time?

NB: For this question we *only* care about the clocks matching when they are checked.

For example, suppose the first clock runs 1 minute fast (per hour) and the second clock runs 31 minutes fast (per hour).

- When the clocks are first checked after one hour, the first clock will show 01:01 and the second clock will show 01:31;
- When the clocks are checked after two hours, they will show 02:02 and 03:02;
- After 48 hours the clocks will both show 00:48.

(a) [25 marks]

Write a program which reads in a two integers, each between 0 and 50,000 inclusive, indicating the number of minutes fast (per hour) of the first and second clock respectively.

You should output the time shown on the clocks when they first match. Both the hour and the minutes should be displayed with two digits.

Sample run 1

```
1 31
00:48
```

(b) [3 marks]

Suppose the first clock is accurate and the clocks do *not* show 00:00 when they first match. The second clock is less than 20 minutes fast (per hour). How many minutes fast is the second clock? Write out all the possible answers.

(c) [4 marks]

Suppose the two clocks can be *any* number of full minutes fast per hour. What is the largest number of hours that can pass before the clocks first match?

5. Write a program that returns true if an input string is a palindrome. A palindrome is a word that reads the same backwards as it does forwards *e.g* ABBA.
6. i) Write a program that converts all characters of an input string to upper case characters.
ii) Write a program that capitalises the first character of each new word.

7.

(a) [24 marks]

Write a program to determine a sequence of button presses to *unlock* the system.

Your program should read in a string of between 1 and 12 *distinct* letters (inclusive) in alphabetical order, representing the current lighting of the security system.

You should output a string (in alphabetical order) that indicates a sequence of buttons which can be pressed to unlock the system, or output IMPOSSIBLE if it cannot be unlocked. A button can be pressed at most twice; a lowercase letter in your string will indicate that the corresponding button should be pressed once and an uppercase letter indicating that it should be pressed twice.

If there are multiple solutions you are only required to print out a single solution.

Sample run

mnoqRTwxy
RST

Alternative answer

mnoqRTwxy
aCeFhJprSUwY

(b) [2 marks]

What is the lighting of the system if the system is unlocked and then B is pressed, followed by I and O? (Each button being pressed once.)

(c) [4 marks]

Starting from an unlocked system, in how many ways can three different buttons be pressed once each, in alphabetical order, so that no light becomes bright?

(d) [5 marks]

Suppose that the system has a configuration of lighting and that a particular sequence of button presses will unlock the system. Is it possible for the same sequence of button presses to unlock the system when it is in a different configuration? Justify your answer.