Microelectronic Circuits

Filters and Tuned Amplifiers

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Outline

- Filter Transmission, Types, and Specification
- The Filter Transfer Function
- Butterworth Filters and Chebyshev Filters
- First-Order and Second-Order Filter Functions
- The Second-Order LCR Resonator
- Second-Order Active Filters Based on Inductor Replacement
- Second-Order Active Filters Based on the two-Integrator-Loop Topology
- Single-Amplifier Biquadratic Active Filters
- Sensitivity
- Switch-Capacitor Filters
- Tuned Amplifiers

Filters

- Electronic filters are an important building block of communication and instrumentation systems.

- Filter design is one of the very few areas of engineering for which a complete design theory exists, starting from specification and ending with a circuit realization.

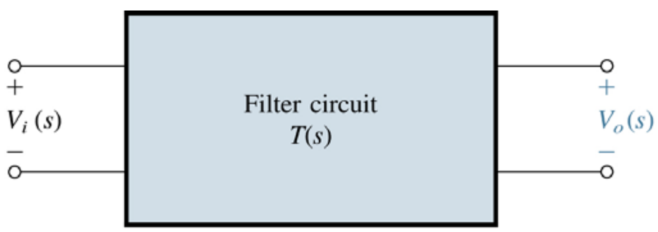
Filter Types (Implementation)

- Passive *LC* filters
  - Oldest technology
  - Work well at high frequency
  - For low frequency application (DC ~ 100kHz), *L*'s are large and impossible to fabricate in ICs.
- Inductorless filter
  - Active-*RC* filters
  - Switched-capacitor filters
  - Transconductance-*C* filters
  - …

Filter Transmission, Types, and Specification

Two-Port Network

The filters studied in this chapter are linear circuits represented by the general two-port network.



Transfer function: $T(s) \equiv \dfrac{V_o(s)}{V_i(s)} \quad \Rightarrow \quad T(j\omega) = \underbrace{|T(j\omega)|}_{\text{magnitude}}\, e^{\overbrace{j\phi(\omega)}^{\text{phase}}}$

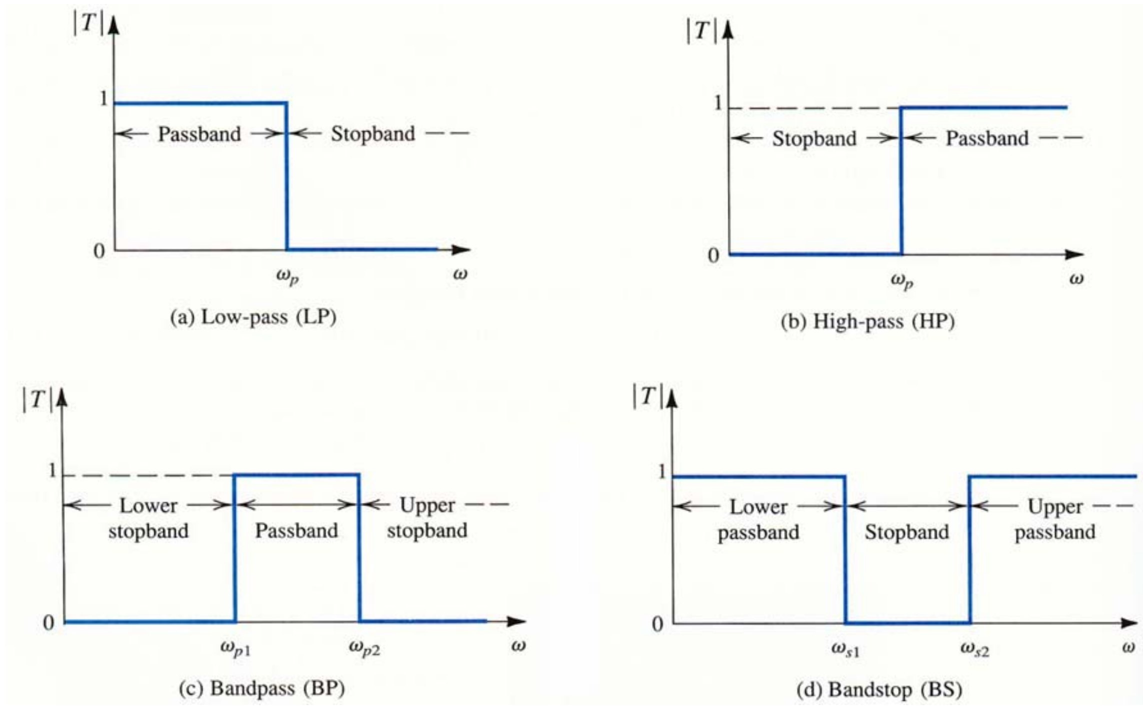
Gain function: $G(\omega) = 20\log|T(j\omega)|$ dB

Attenuation function: $A(\omega) = -20\log|T(j\omega)|$ dB

Output spectrum: $|V_o(j\omega)| = |T(j\omega)||V_i(j\omega)|$

Filter Specification

Ideal transmission characteristics of the four major filter types:



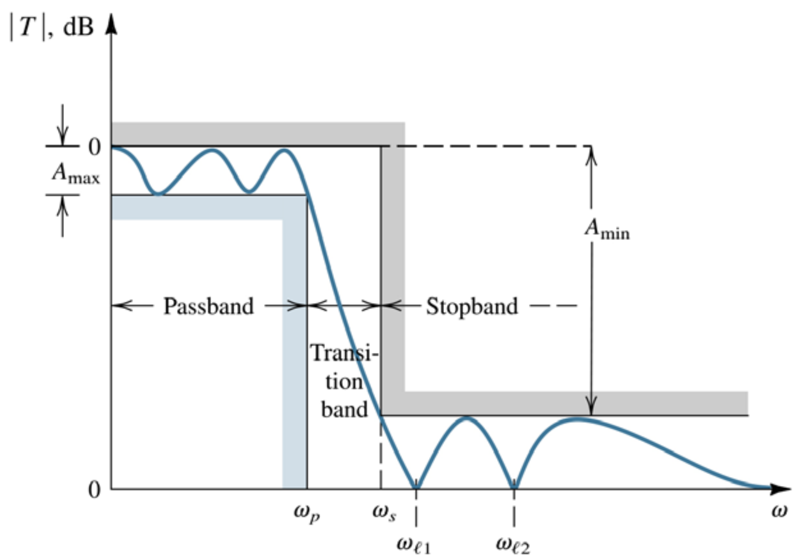
(a) Low-pass (LP)

(b) High-pass (HP)

(c) Bandpass (BP)

(d) Bandstop (BS)

Specification of the Transmission Characteristics of a Low-Pass Filter



ω_p : the pass band edge

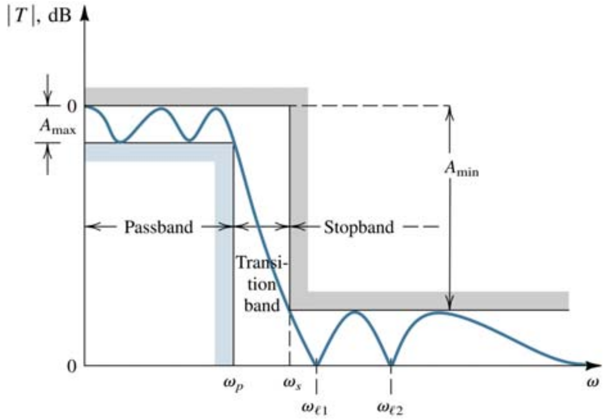
A_{max} : the max. allowed variation in passband transmission

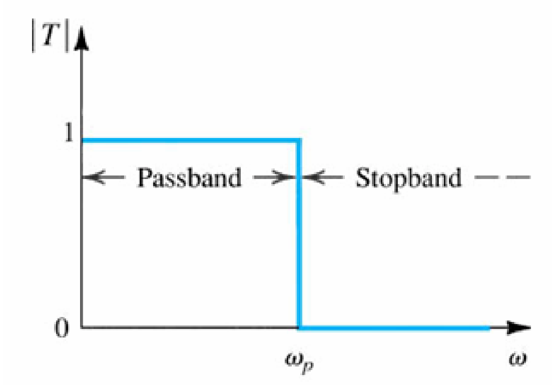
ω_s : the stop band edge

A_{min} : the min. required stopband attenuation

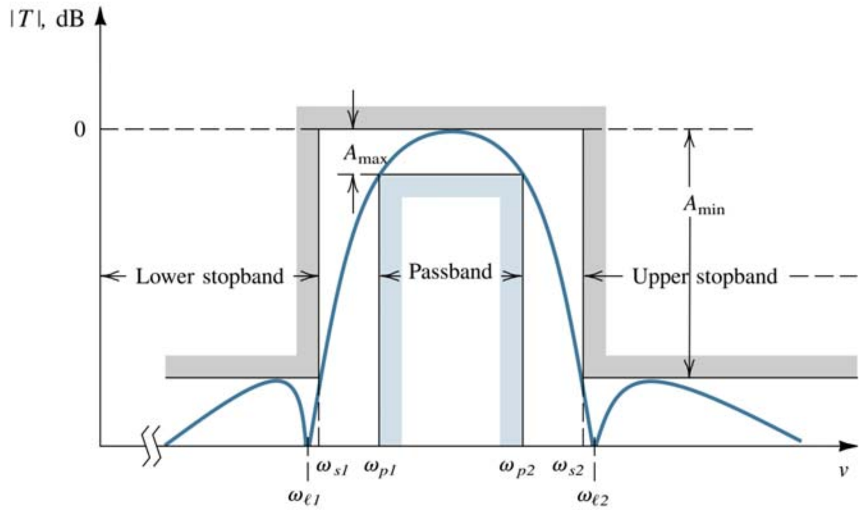
Ideal LP:

- Lower $A_{\max}$
- Higher $A_{\min}$
- Selectively ratio $\dfrac{\omega_s}{\omega_p} \to 1$





Transmission Specification for a Bandpass Filter



Note that this bandpass filter has a monotonically decreasing transmission in the passband on both sides of the peak frequency.

The Filter Transfer Function

Filter Transfer Function

- Filter transfer function

$$T(s) = \frac{a_M s^M + a_{M-1}s^{M-1} + \cdots + a_0}{s^N + b_{N-1}s^{N-1} + \cdots + b_0}$$

N: the filter order (the degree of the denominator)

$$T(s) = \frac{a_M(s-z_1)(s-z_2)\cdots(s-z_M)}{(s-p_1)(s-p_2)\cdots(s-p_N)}$$

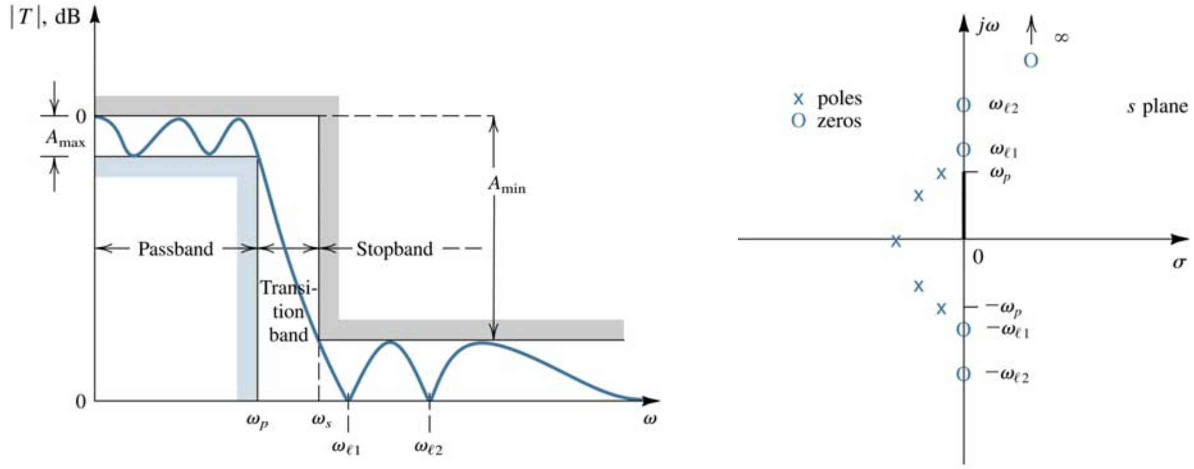
$z_1, z_2, \cdots, z_M$: transfer function zeros
$p_1, p_2, \cdots, p_N$: transfer function poles

- Poles and zeros must be real or complex conjugate pairs.
- To obtain zero or small stopband transmission, zeros are usually placed on the $j\omega$-axis at stopband frequencies.

- Pole-zero pattern

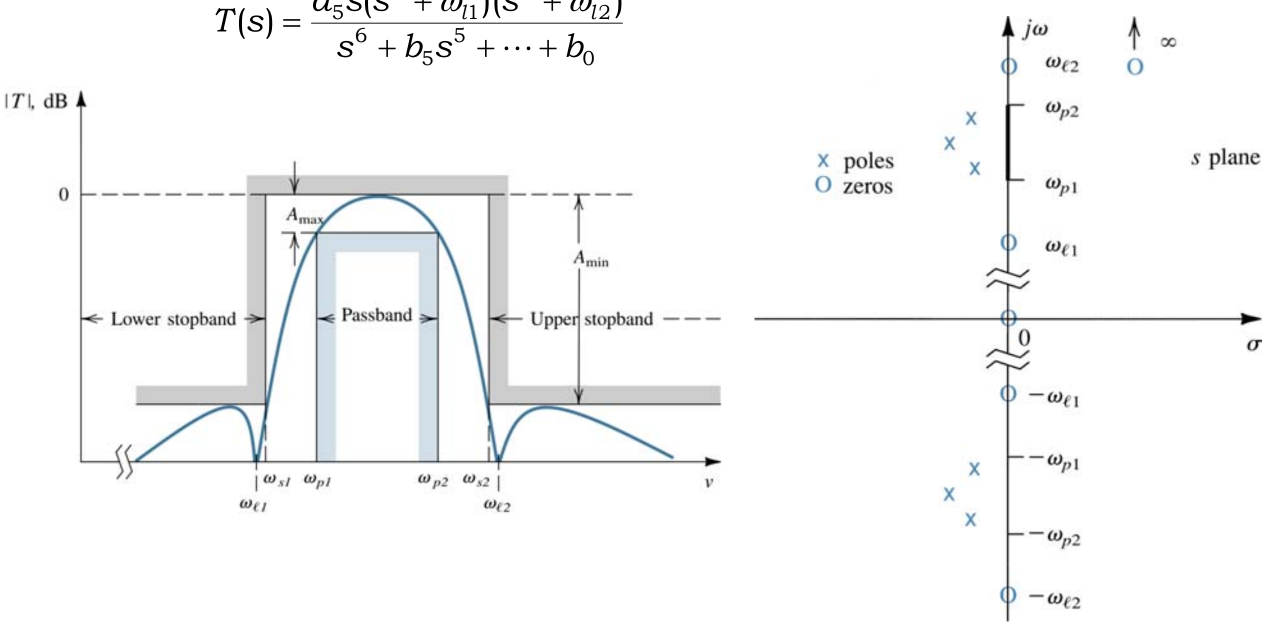
$$T(s) = \frac{a_4(s^2+\omega_{l1}^2)(s^2+\omega_{l2}^2)}{s^5+b_4s^4+b_3s^3+b_2s^2+b_1s+b_0}$$

N = 5, fifth-order LPF



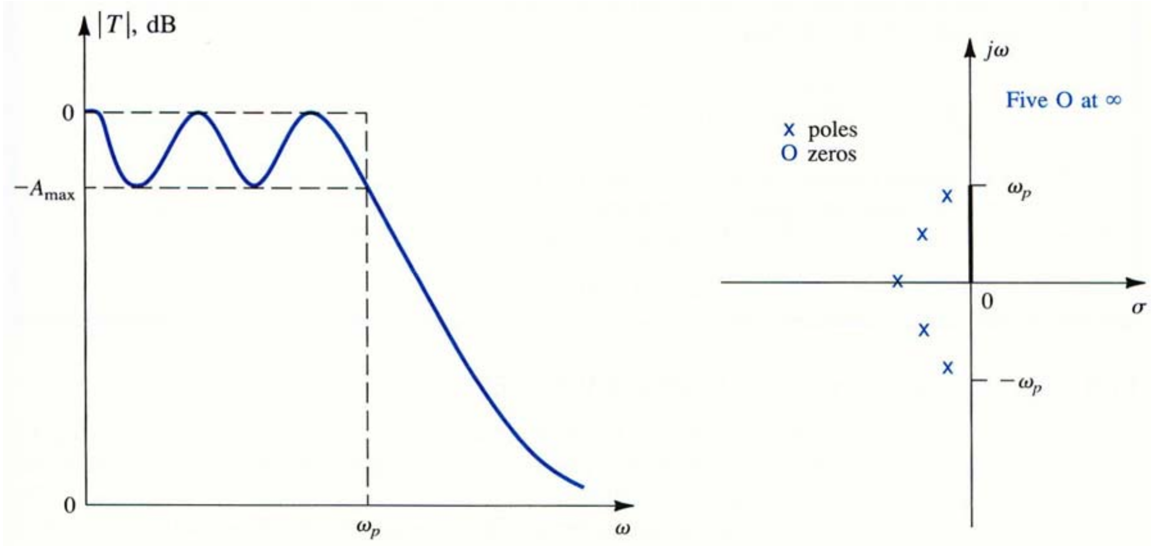
Pole-zero pattern for the bandpass filter (N = 6)

$$T(s) = \frac{a_5 s(s^2+\omega_{l1}^2)(s^2+\omega_{l2}^2)}{s^6+b_5s^5+\cdots+b_0}$$



Pole-zero pattern for the low-pass filter (N = 5)

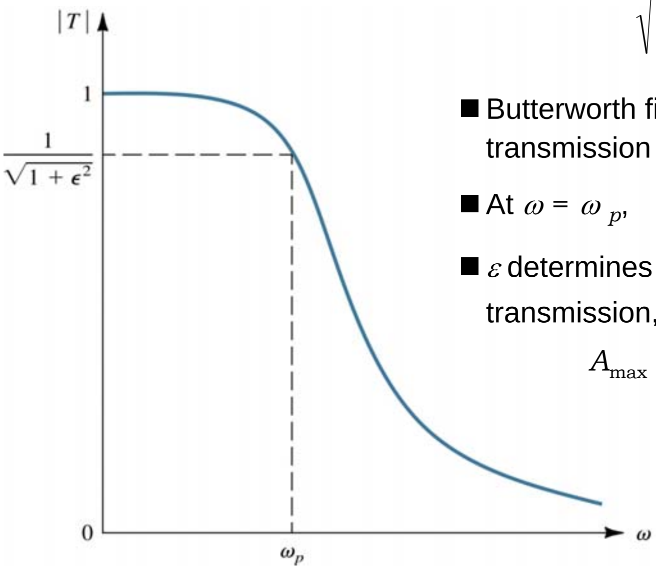
$$T(s) = \frac{a_0}{s^5 + b_4 s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0}$$



Butterworth Filters

Butterworth Filter

Magnitude response:



N-th order Butterworth filter:

$$|T(j\omega)| = \frac{1}{\sqrt{1+\varepsilon^2\left(\frac{\omega}{\omega_p}\right)^{2N}}}$$

ω_p: passband edge

- Butterworth filter exhibits a monotonically decreasing transmission with all the transmission zeros at $\omega = \infty$.
- At $\omega = \omega_p$, $|T(j\omega_p)| = \frac{1}{\sqrt{1+\varepsilon^2}}$
- ε determines the maximum variation in passband transmission,

$$A_{\max} = 20\log\sqrt{1+\varepsilon^2} \quad \Rightarrow \quad \varepsilon = \sqrt{10^{A_{\max}/10}-1}$$

- At the edge of the stopband, $\omega = \omega_s$, the attenuation of the Butterworth filter is

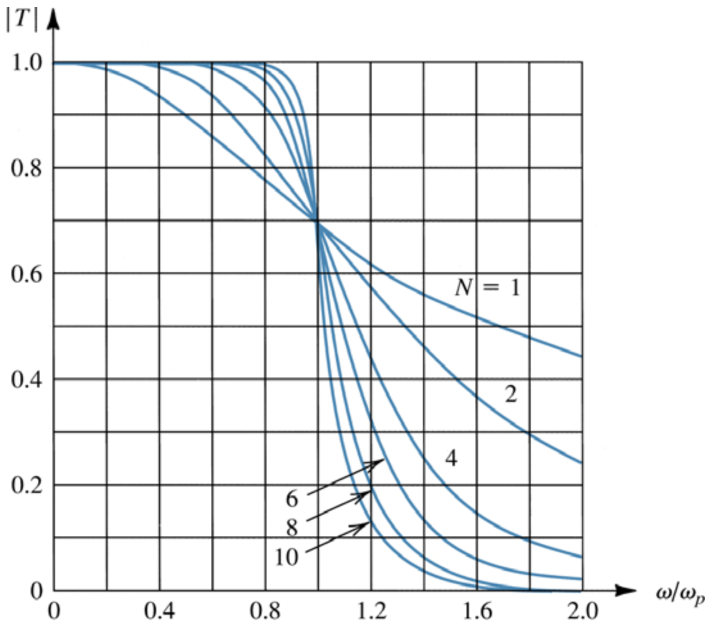
$$A(\omega_s) = -20\log\left[1\Big/\sqrt{1+\varepsilon^2(\omega_s/\omega_p)^{2N}}\right]$$
$$= 10\log\left[1+\varepsilon^2(\omega_s/\omega_p)^{2N}\right] \quad \geq \quad A_{\min}$$

- Maximally flat response
  - The first 2N–1 derivative of |T | relative to ω are zero at ω = 0.
    - ➡ Response is very flat near ω = 0.
  - Order N ↑ ➡ passband flatness ↑
- Attenuation at stopband edge $\omega = \omega_s$

$$A(\omega_s) = -20\log\frac{1}{\sqrt{1+\varepsilon^2(\omega_s/\omega_p)^{2N}}}$$
$$= 10\log\left[1+\varepsilon^2(\omega_s/\omega_p)^{2N}\right] \quad \geq \quad A_{\min}$$

- Filter order can usually be obtained from $A(\omega_s)$.

- Magnitude response of Butterworth filters of various order with $\varepsilon = 1$



Normalized Butterworth Polynomials

$$T(s) \xrightarrow{s=j\omega} T(j\omega)$$

$$T(j\omega)T(-j\omega) = |T(j\omega)|^2 = \frac{1}{1+\varepsilon^2\left(\frac{\omega}{\omega_p}\right)^{2N}} = \frac{1}{1+(-1)^N\varepsilon^2\left(\frac{\omega}{\omega_p}\right)^{2N}}$$

➡ $$|T(s)|^2 = \frac{1}{1+(-1)^N\varepsilon^2\left(\frac{s}{\omega_p}\right)^{2N}}$$

Find poles: $\left(\frac{s}{\omega_p}\right)^{2N}\varepsilon^2 = -(-1)^N$ ➡ $\left(\frac{s}{\omega_p}\right)^{2N} = -\frac{(-1)^N}{\varepsilon^2}$

$$\text{poles} = \begin{cases} -\dfrac{1}{\varepsilon^2} = \dfrac{e^{j(\pi+l\cdot 2\pi)}}{\varepsilon^2} & ;\ N \text{ is even.} \\ \dfrac{1}{\varepsilon^2} = \dfrac{e^{j(l\cdot 2\pi)}}{\varepsilon^2} & ;\ N \text{ is odd.} \end{cases}$$

where $l = 0,1,2,\cdots,(2N-1)$

- E.g. $N = 2$, $l = 1$

$$\left(\frac{s}{\omega_p}\right)^4 = -1 = e^{j(\pi + l\cdot 2\pi)}$$

$$\Rightarrow \quad s = \omega_p e^{j\left(\frac{\pi}{4}+\frac{l}{4}\cdot 2\pi\right)} \quad ; \; l = 0,1,2,3$$

$$\text{poles} = \begin{cases} s_1 = \omega_p e^{-j\frac{\pi}{4}} = \omega_p\left(\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right) \\ s_2 = \omega_p e^{-j\left(\frac{\pi}{4}+\frac{\pi}{2}\right)} = \omega_p\left(-\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right) \\ s_3 = \omega_p e^{-j\left(\frac{\pi}{4}+\pi\right)} = \omega_p\left(-\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right) \\ s_4 = \omega_p e^{-j\left(\frac{\pi}{4}+\frac{3\pi}{2}\right)} = \omega_p\left(\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right) \end{cases}$$

- Normalized polynomials for $N = 2$
  1. Take left plane poles
  2. Let $\omega_p = 1$ → frequency normalization

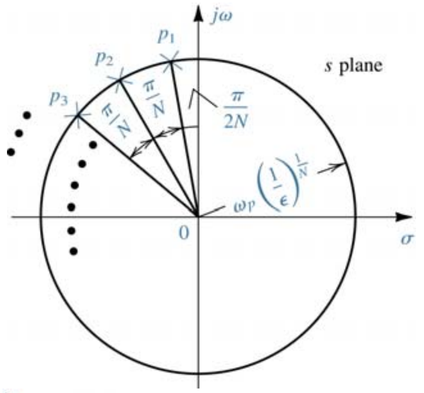
$$\Rightarrow \quad T(s) = \frac{1}{\left(1+\frac{s}{s_2}\right)\left(1+\frac{s}{s_3}\right)} = \frac{1}{1+\sqrt{2}s+s^2} \quad \text{for } N = 2.$$

$$B_2(s) = 1+\sqrt{2}s+s^2$$

- Normalized Butterworth polynomials

N	Factors of polynomial $B_n(s)$
1	$(s + 1)$
2	$(s^2 + 1.414s + 1)$
3	$(s + 1)(s^2 + s + 1)$
4	$(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)$
5	$(s + 1)(s^2 + 0.618s + 1)(s^2 + 1.618s + 1)$
6	$(s^2 + 0.518s + 1)(s^2 + 1.414s + 1)(s^2 + 1.932s + 1)$
7	$(s + 1)(s^2 + 0.445s + 1)(s^2 + 1.247s + 1)(s^2 + 1.802s + 1)$
8	$(s^2 + 0.390s + 1)(s^2 + 1.111s + 1)(s^2 + 1.663s + 1)(s^2 + 1.962s + 1)$

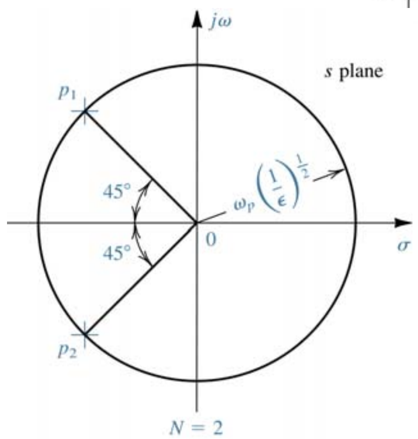
Graphical Construction of Butterworth Filters

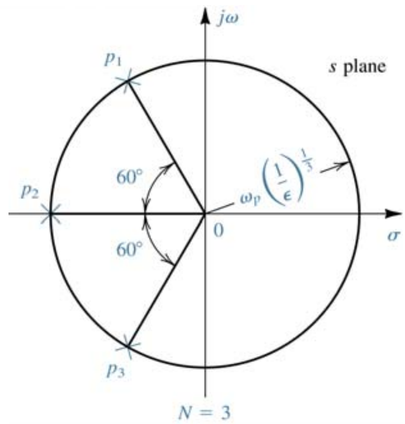


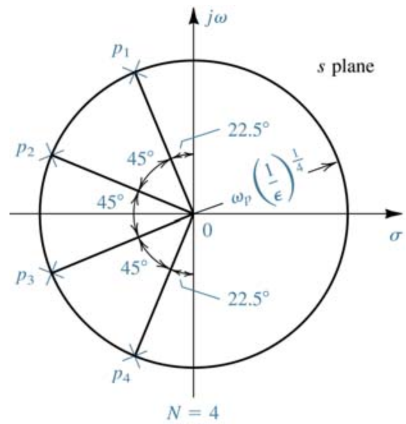
Transfer function:

$$T(s) = \frac{K\omega_0^N}{(s-p_1)(s-p_2)\cdots(s-p_N)}$$

where K is a constant equal to the required dc gain of the filter.

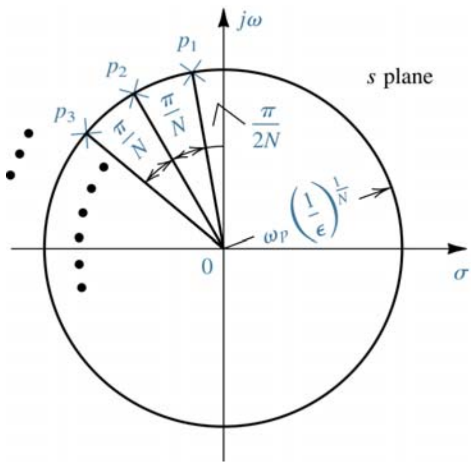






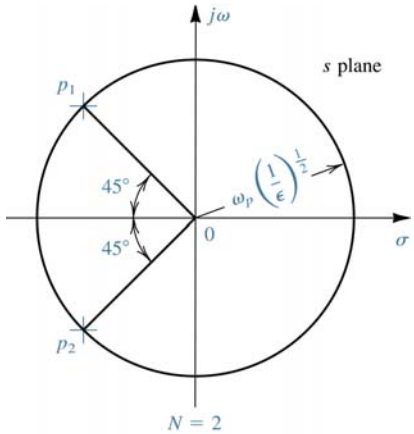
Graphical Construction of Butterworth Filters

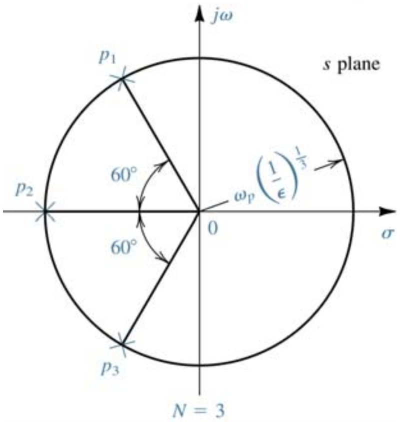
General case: Graphical construction for determining Butterworth filter of order N.

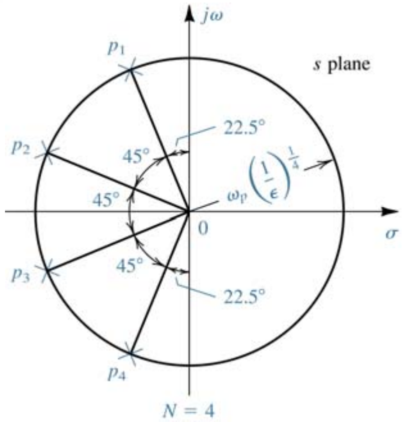


- All the poles lie in the left half of the s-plane on a circle of radius $\omega_0 = \omega_p(1/\varepsilon)^{1/N}$, where ε is the passband deviation parameter. ($\varepsilon = \sqrt{10^{A_{\max}/10} - 1}$)
- Transfer function: $T(s) = \dfrac{K\omega_0^N}{(s-p_1)(s-p_2)\cdots(s-p_N)}$

  where K is a constant equal to the required dc gain of the filter.







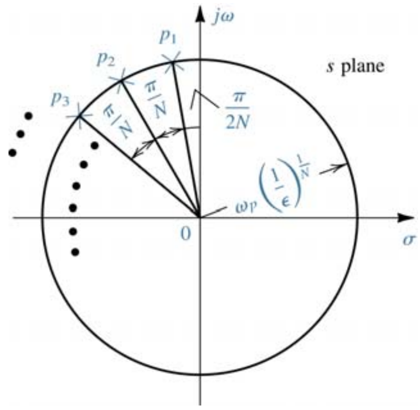
How to design a Butterworth filter

❶ Determine ε. $A_{\max} = 20\log\sqrt{1+\varepsilon^2} \quad \Rightarrow \quad \varepsilon = \sqrt{10^{A_{\max}/10}-1}$

❷ Determine the required filter order N.

$$A(\omega_s) = 10\log\left[1+\varepsilon^2(\omega_s/\omega_p)^{2N}\right] \quad \geq \quad A_{\min}$$

❸ Determine the N natural modes.



❹ Determine $T(s)$.

$$T(s) = \frac{K\omega_0^N}{(s-p_1)(s-p_2)\cdots(s-p_N)}$$

Example: Find the Butterworth Transfer Function

Low-pass filter specifications: f_p = 10 kHz, A_{max} = 1 dB, f_s = 15 kHz, A_{min} = 25 dB, dc gain = 1.

❶ Determine ε: $\varepsilon = \sqrt{10^{A_{max}/10} - 1} = \sqrt{10^{1/10} - 1} = 0.5088$

❷ Determine N: $A(\omega_s) = 10\log\left[1 + \varepsilon^2(\omega_s/\omega_p)^{2N}\right] \geq A_{min}$

$$\Rightarrow \quad N \geq \frac{1}{2}\frac{\log\left(\dfrac{10^{A_{min}/10} - 1}{\varepsilon^2}\right)}{\log\left(\dfrac{\omega_s}{\omega_p}\right)} \qquad \therefore \quad N \geq 8.76$$

Select N = 9

❸ Determine the N natural modes:

The poles all have the same frequency:

$$\omega_0 = \omega_p(1/\varepsilon)^{1/N} = 2\pi \times 10 \times 10^3 (1/0.5088)^{1/9} = 6.773 \times 10^4 \text{ rad/s}$$

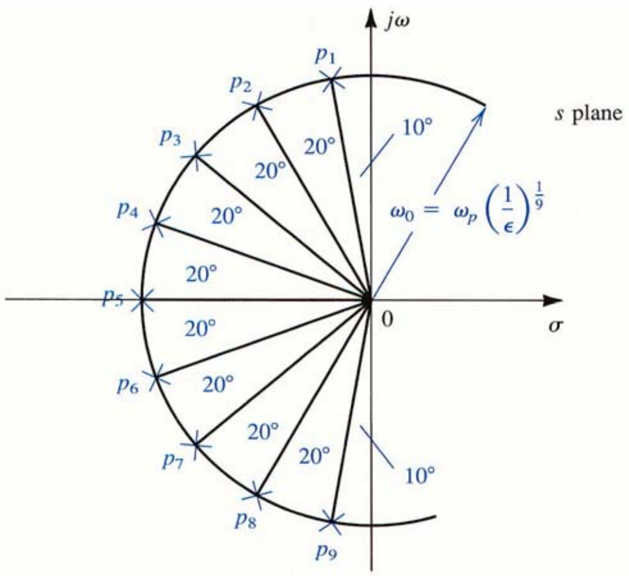
The first pole: $p_1 = \omega_0(-\cos 80^\circ + j\sin 80^\circ) = \omega_0(-0.1736 + j0.9848)$

Combining p_1 with its complex conjugate p_9 yields the factor ($s^2 + s0.3472\omega_0 + \omega_0^2$) in the denominator of the transfer function. The same can be done for the other complex poles.

❹ Determine $T(s)$:

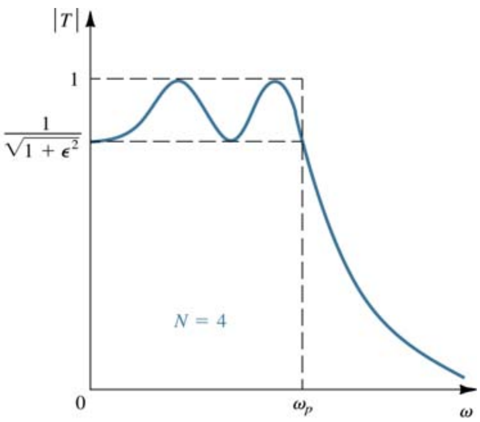
$$T(s) = \frac{\omega_0^9}{(s+\omega_0)(s^2 + s1.8794\omega_0 + \omega_0^2)(s^2 + s1.5321\omega_0 + \omega_0^2)(s^2 + s\omega_0 + \omega_0^2)(s^2 + s0.3472\omega_0 + \omega_0^2)}$$

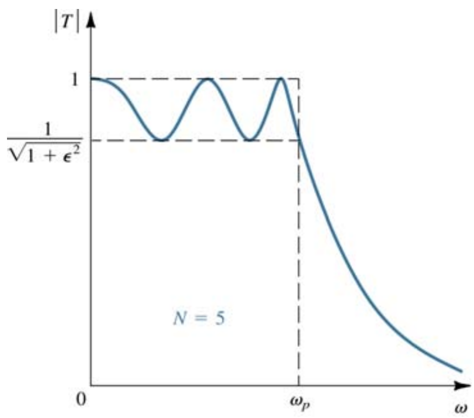
- Poles of the ninth-order Butterworth filter



Chebyshev Filters

- Equiripple in the passband
- Monotonically decreasing in the stopband
- Odd-order filter exhibits $|T(0)| = 1$
  Even-order filter exhibits $|T(0)| = |T(\omega_p)| = A_{\max}$
- The total no. of passband maxima and minima equals the order of the filter, N. All the transmission zeros at $\omega = \infty$, making it an all-pole filter.





- N-th order Chebyshev filter ω_p: passband edge

$$|T(j\omega)| = \frac{1}{\sqrt{1+\varepsilon^2\cos^2[N\cos^{-1}(\omega/\omega_p)]}} \quad \text{for } \omega \le \omega_p$$

$$|T(j\omega)| = \frac{1}{\sqrt{1+\varepsilon^2\cosh^2[N\cosh^{-1}(\omega/\omega_p)]}} \quad \text{for } \omega \ge \omega_p$$

- At $\omega = \omega_p$, $|T(j\omega_p)| = \frac{1}{\sqrt{1+\varepsilon^2}}$
- ε determines the passband ripple,

$$A_{\max} = 20\log\sqrt{1+\varepsilon^2} \quad \Rightarrow \quad \varepsilon = \sqrt{10^{A_{\max}/10} - 1}$$

- The attenuation achieved by the Chebyshev filter at the stopband edge ($\omega = \omega_s$) is

$$A(\omega_s) = 10\log[1+\varepsilon^2\cosh^2(N\cosh^{-1}(\omega_s/\omega_p))] \quad \geq \quad A_{\min}$$

- The poles of the Chebyshev filter:

$$p_k = -\omega_p \sin\left(\frac{2k-1}{N}\frac{\pi}{2}\right)\sinh\left(\frac{1}{N}\sinh^{-1}\frac{1}{\varepsilon}\right) + j\omega_p\cos\left(\frac{2k-1}{N}\frac{\pi}{2}\right)\cosh\left(\frac{1}{N}\sinh^{-1}\frac{1}{\varepsilon}\right)$$
$$k = 1, 2, \cdots, N$$

- The transfer function of the Chebyshev filter:

$$T(s) = \frac{K\omega_p^N}{\varepsilon 2^{N-1}(s-p_1)(s-p_2)\cdots(s-p_N)}$$

where K is a constant equal to the required dc gain of the filter.

- How to design a Chebyshev filter
  1. Determine ε from A_{max}.
  2. Determine N, the number of the order, from given $A(\omega_s)$.
  3. Determine the poles, p_k, using Chebyshev equation.

$$p_k = -\omega_p \sin\left(\frac{2k-1}{N}\frac{\pi}{2}\right)\sinh\left(\frac{1}{N}\sinh^{-1}\frac{1}{\varepsilon}\right) + j\omega_p\cos\left(\frac{2k-1}{N}\frac{\pi}{2}\right)\cosh\left(\frac{1}{N}\sinh^{-1}\frac{1}{\varepsilon}\right)$$
$$k = 1, 2, \cdots, N$$

  4. Determine the transfer function, $T(s)$.

$$T(s) = \frac{K\omega_p^N}{\varepsilon 2^{N-1}(s-p_1)(s-p_2)\cdots(s-p_N)}$$

where K is the dc gain.

Example: Find the Chebyshev Transfer Function

Low-pass filter specifications: f_p = 10 kHz, A_{max} = 1 dB, f_s = 15 kHz, A_{min} = 25 dB, dc gain = 1.

❶ Determine ε: $\varepsilon = \sqrt{10^{A_{max}/10} - 1} = \sqrt{10^{1/10} - 1} = 0.5088$

❷ Determine N: $A(\omega_s) = 10\log[1 + \varepsilon^2 \cosh^2(N \cosh^{-1}(\omega_s / \omega_p))] \geq A_{min}$

$$N = 4 \Rightarrow A(\omega_s) = 21.6 \text{ dB}$$

$$N = 5 \Rightarrow A(\omega_s) = 29.9 \text{ dB}$$ Select N = 5

To meet identical specifications, one requires a lower order for the Chebyshev than the Butterworth filter. Alternatively, for the same order and the same A_{max}, the Chebyshev filter provides greater stopband attenuation than the Butterworth filter.

❸ Determine the N natural modes:

$$p_1, p_5 = \omega_p(-0.0895 \pm j0.9901)$$

$$p_2, p_4 = \omega_p(-0.2342 \pm j0.6119) \qquad p_3 = \omega_p(-0.2895)$$

❹ Determine $T(s)$:

$$T(s) = \frac{\omega_p^5}{8.1408(s + 0.2895\omega_p)(s^2 + s0.4684\omega_p + \omega_p^2)(s^2 + s0.1789\omega_p + 0.9883\omega_p^2)}$$

where $\omega_p = 2\pi \times 10^4$ rad/s

First-Order and Second-Order Filter Functions

First-Order filters

Transfer function:

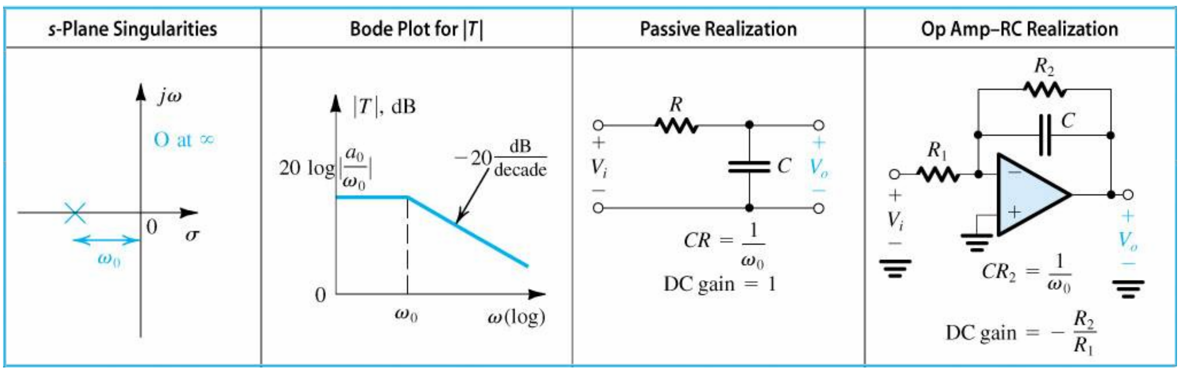
$$T(s)=\frac{a_1 s+a_0}{s+\omega_0}$$

The bilinear transfer function characterizes:

- a pole (natural mode) at $s = -\omega_0$,
- a transmission zero at $s = -a_0 / a_1$,
- a high-frequency gain that approaches a_1, i.e., $T(j\infty) = a_1$.
- The numerator coefficients, a_0 and a_1, determine the type of filter (e.g., low-pass, high-pass, etc.)

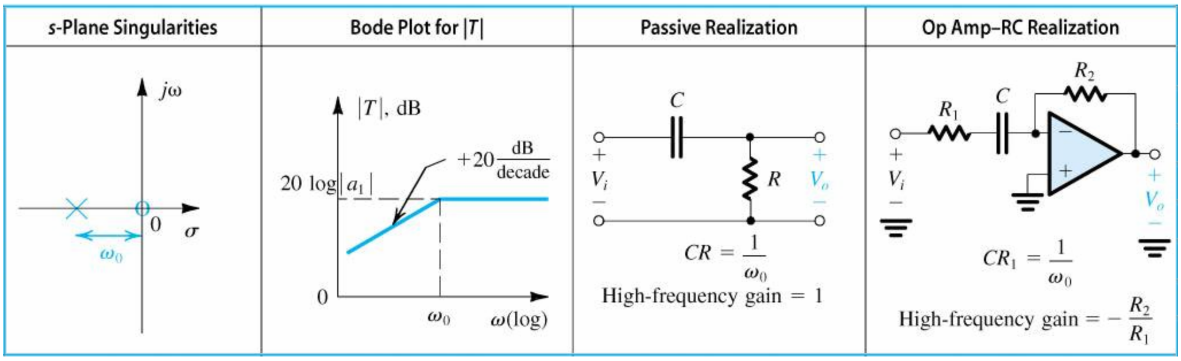
- Low-pass (LP) filters

$$T(s)=\frac{a_0}{s+\omega_0}$$



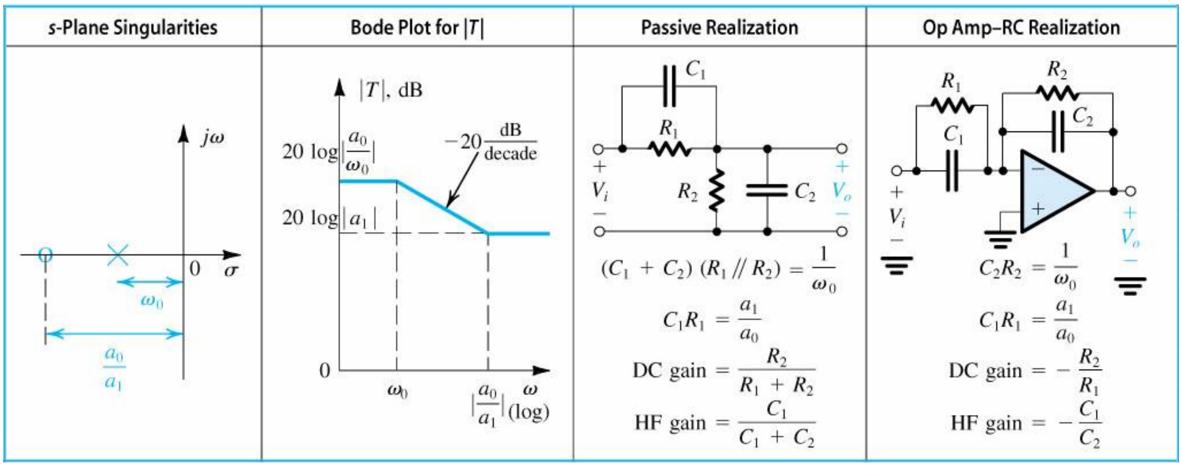
- High-pass (HP) filters

$$T(s) = \frac{a_1 s}{s + \omega_0}$$



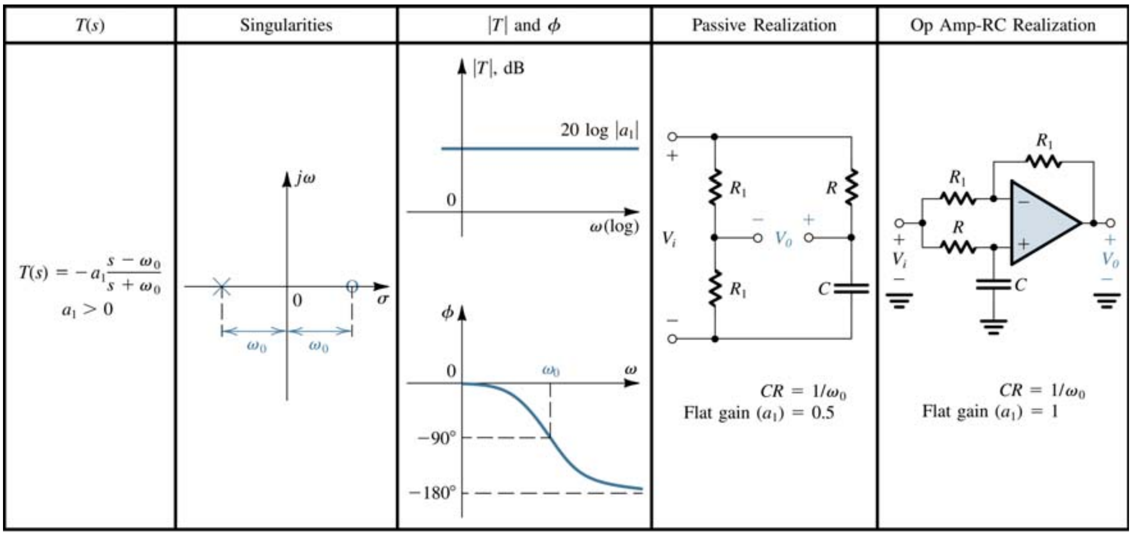
- General filters

$$T(s) = \frac{a_1 s + a_0}{s + \omega_0}$$



First-Order All-Pass Filters

- Special case of first-order filters
- Transmission zero and pole are symmetrically located relative to $j\omega$-axis
- Transmission is constant at all frequencies
- Phase is not constant at all frequencies
- Most often used in the design of delay equations



Second-Order Filters (Biquadratic Filters)

Transfer function:

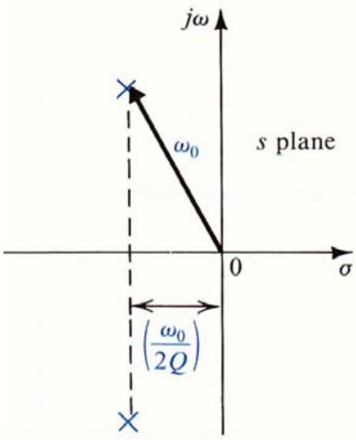
$$T(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + \left(\frac{\omega_0}{Q}\right)s + \omega_0^2}$$

where ω_0 and Q determine the natural modes (poles) according to

$$p_1, p_2 = -\frac{\omega_0}{2Q} \pm j\omega_0\sqrt{1 - \frac{1}{4Q^2}}$$

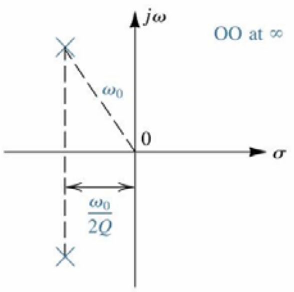
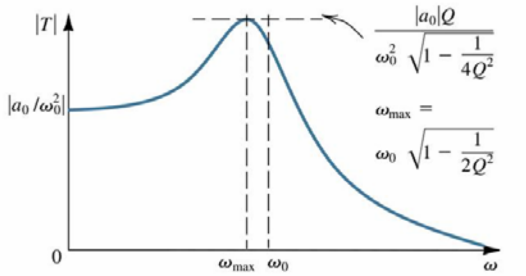
- We are usually interested in the case of complex-conjugate natural modes, obtained for $Q > 0.5$.
- The numerator coefficients (a_0, a_1, a_2) determine the type of filter function (i.e., LP, HP, etc.).

Definition of the parameter ω_0 and Q of a pair of complex conjugate poles:



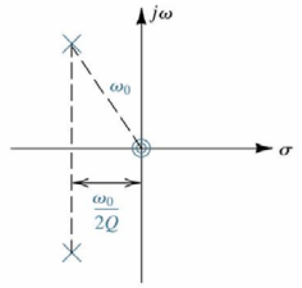
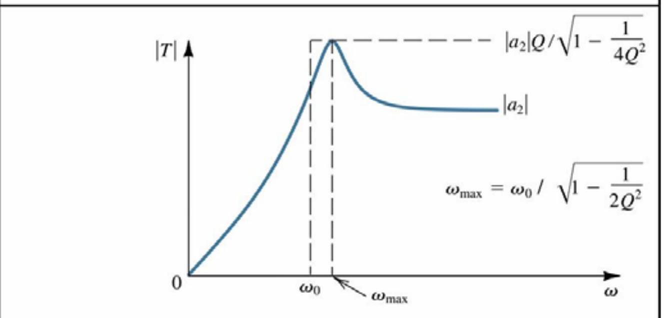
- Pole frequency ω_0 : distance of pole (from origin).
- Pole quality factor Q : distance of the poles from the $j\omega$-axis
  - Higher Q ➡ higher selectivity (pole is closer to $j\omega$-axis)
  - $Q = \infty$ ➡ poles are on the $j\omega$-axis
    ➡ can yield sustained oscillation
  - Q is negative ➡ poles are in the right half of s-plane
    ➡ unstable

2^nd-order Low-pass Filters

<table>
<tr><th>Filter Type and $T(s)$</th><th>s-Plane Singularities</th><th>$|T|$</th></tr>
<tr><td>(a) Low-Pass (LP)
$T(s) = \dfrac{a_0}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$
dc gain $= \dfrac{a_0}{\omega_0^2}$</td><td>
</td><td>
</td></tr>
</table>

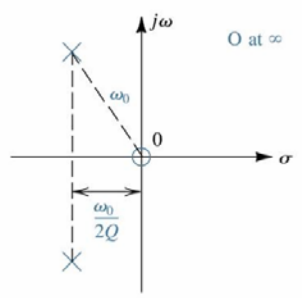
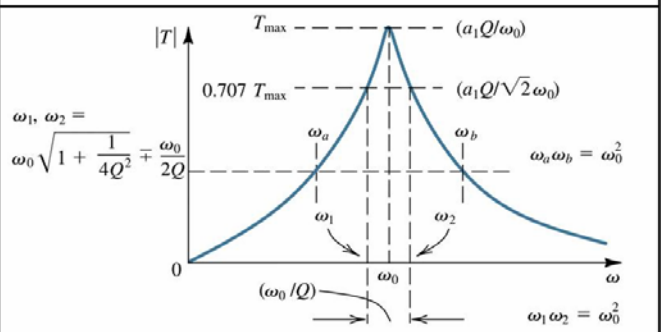
- Two transmission zeros are at $s = \infty$.
- Magnitude response
  - ☐ The peak occurs only for $Q > 1/\sqrt{2}$.
  - ☐ The response obtained for $Q = 1/\sqrt{2}$ is the Butterworth, or maximally flat, response.

2nd-order High-pass Filters

Filter Type and $T(s)$	s-Plane Singularities	$\lvert T\rvert$
(b) High-Pass (HP) $T(s) = \dfrac{a_2 s^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ High-frequency gain $= a_2$		

- Two transmission zeros are at $s = 0$ (dc).
- Magnitude response shows a peak for $Q > 1/\sqrt{2}$.
- Duality between the LP and HP response.

2nd-order Bandpass Filters

Filter Type and $T(s)$	s-Plane Singularities	$\lvert T\rvert$
(c) Bandpass (BP) $T(s) = \dfrac{a_1 s}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ Center-frequency gain $= \dfrac{a_1 Q}{\omega_0}$		

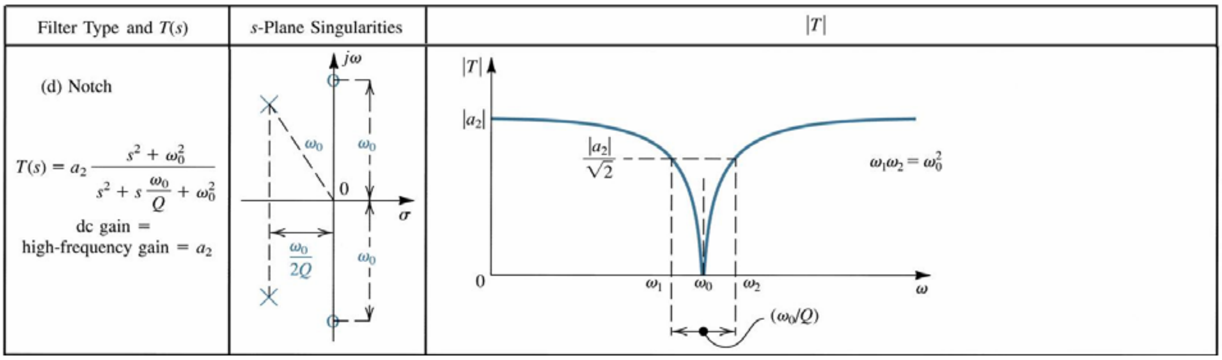
- One transmission zero is at $s = 0$ (dc), and the other is at $s = \infty$.
- Magnitude response peaks at $\omega = \omega_0$. The center frequency is equal to the pole frequency ω_0.
- The selectivity of 2nd-order bandpass filter is measured by 3-dB bandwidth.

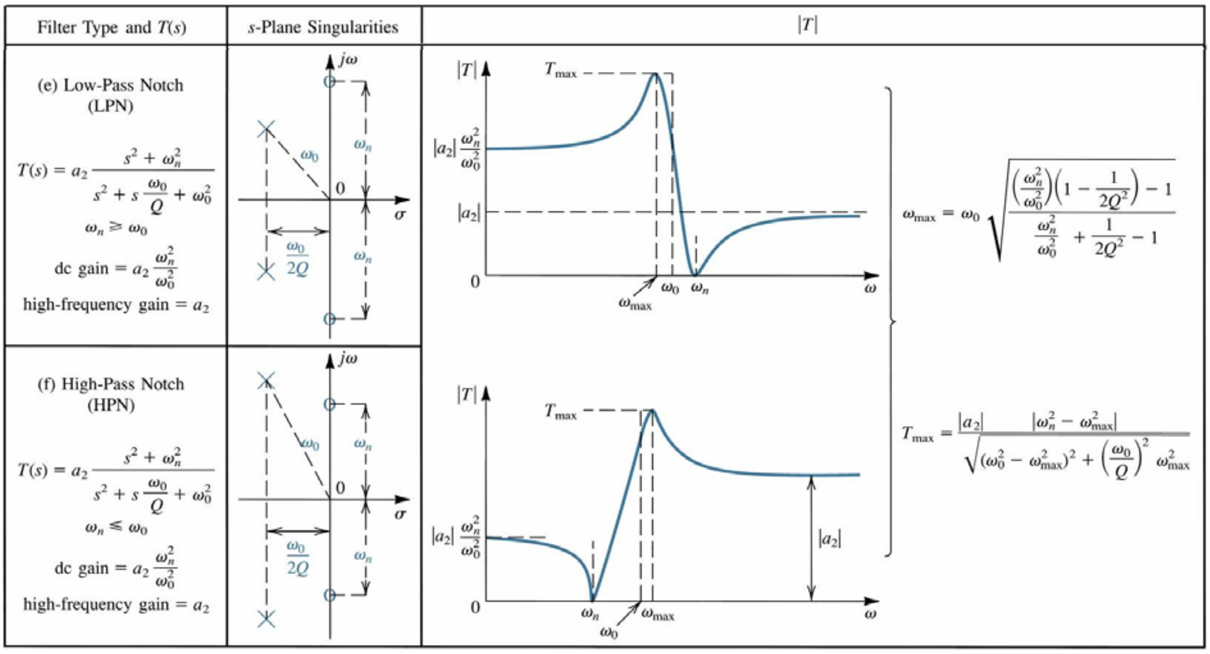
$$\omega_1, \omega_2 = \omega_0\sqrt{1+\frac{1}{4Q^2}} \pm \frac{\omega_0}{2Q} \quad \Rightarrow \quad BW \equiv \omega_2 - \omega_1 = \frac{\omega_0}{Q}$$

As Q increases, BW decreases and the bandpass filters become more selective.

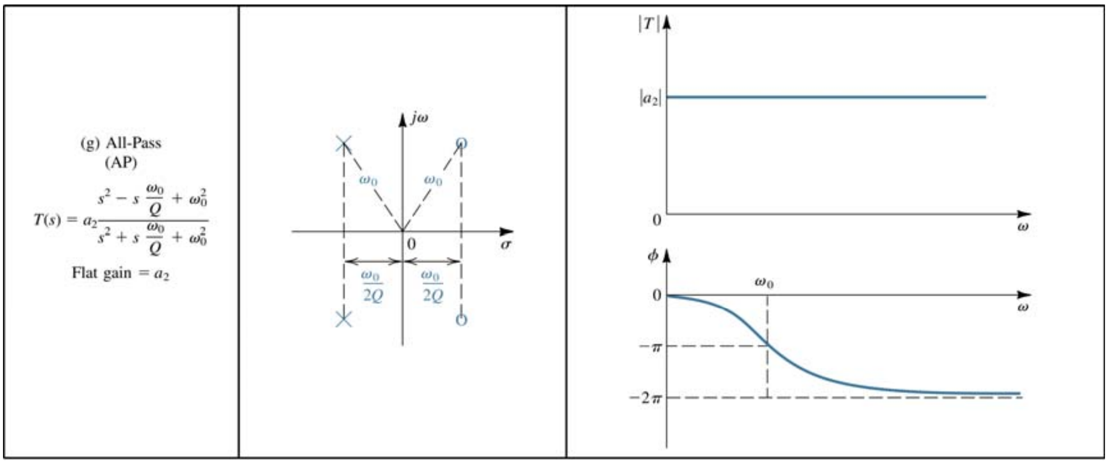
Notch Filters

- Transmission zeros are located on the $j\omega$-axis. ($\omega_z = \pm j\omega_n$)
  ➡ A notch in the magnitude response occurs at $\omega = \omega_n$. (ω_n : notch frequency)
- 3 cases of 2nd-order notch filters:
  ❶ Regular notch ($\omega_n = \omega_0$)
  ❷ Low-pass notch ($\omega_n > \omega_0$)
  ❸ High-pass notch ($\omega_n < \omega_0$)
- The transmission at dc and at $s = \infty$ is finite.
  (Because there are no transmission zeros at either $s = 0$ or $s = \infty$.)





2^{nd}-order All-pass Filters

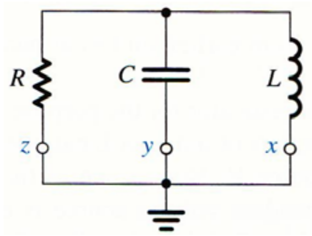


- Two transmission zeros are in the right half of the s-plane, at the mirror-image locations of the poles.
- Magnitude response is constant over all frequencies; the flat gain is equal to $|a_2|$.
- The frequency selectivity of the all-pass function is its phase response.

The Second-Order LCR Resonator

Second-Order LCR Resonator

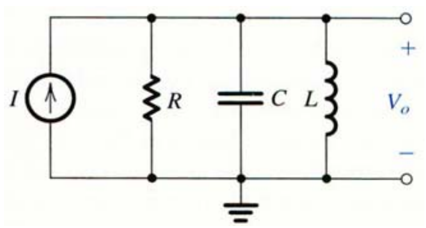
- Use LCR resonator to derive various second-order filter functions.
- Replacing inductor L by a simulated inductance obtained using an OPAMP-RC circuit results in an OPAMP-RC resonator.
- The second-order parallel LCR resonator:



(without changing poles)

Two ways for exciting the resonator without change its natural structure:

❶ The resonator is excited with a current source I connected in parallel.



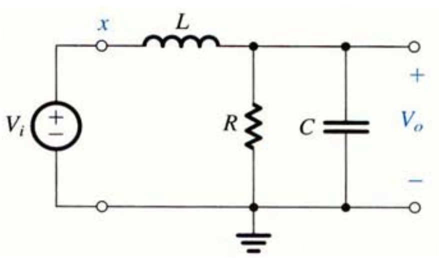
The resonator poles are poles of V_o / I.

$$\frac{V_o}{I} = \frac{1}{Y} = \frac{1}{\frac{1}{sL} + sC + \frac{1}{R}} = \frac{\frac{s}{C}}{s^2 + s\frac{1}{CR} + \frac{1}{LC}}$$

$$\Rightarrow \quad \omega_o^2 = \frac{1}{LC} \quad \frac{\omega_0}{Q} = \frac{1}{CR}$$

$$\Rightarrow \quad \omega_0 = \frac{1}{\sqrt{LC}} \quad Q = \omega_0 CR$$

❷ The resonator is excited with a voltage source V_i.



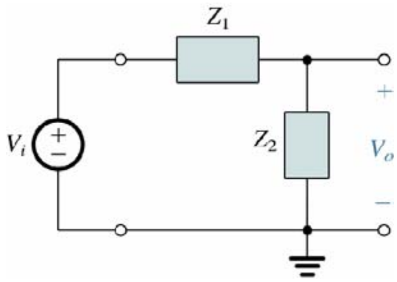
The resonator poles are poles of V_o / V_i.

$$\frac{V_o}{V_i} = \frac{R \| \frac{1}{sC}}{sL + \left(R \| \frac{1}{sC}\right)} = \frac{\frac{1}{LC}}{s^2 + s\frac{1}{CR} + \frac{1}{LC}}$$

$$\Rightarrow \quad \omega_o^2 = \frac{1}{LC} \quad \frac{\omega_0}{Q} = \frac{1}{CR}$$

$$\Rightarrow \quad \omega_0 = \frac{1}{\sqrt{LC}} \quad Q = \omega_0 CR$$

Various Second-Order Function Using Resonator

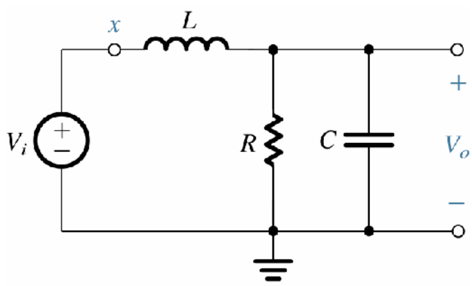


$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)}$$

Transmission zeros:

- The values of s are at which $Z_2(s)$ is zero, provided that $Z_1(s)$ is not simultaneously zero.
- The values of s are at which $Z_1(s)$ is infinite, provided that $Z_2(s)$ is not simultaneously infinite.

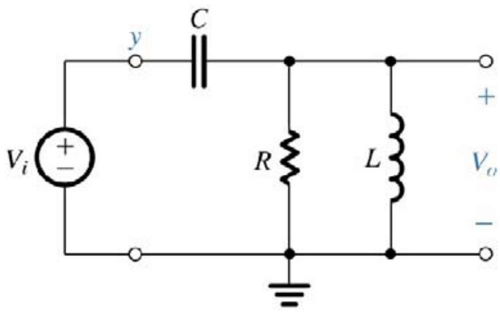
Realization of the Low-Pass Function



$$T(s) \equiv \frac{V_o}{V_i} = \frac{Z_2}{Z_1 + Z_2} = \frac{Y_1}{Y_1 + Y_2} = \frac{\boxed{\frac{1}{sL}}}{\boxed{\frac{1}{sL}} + \boxed{sC + \frac{1}{R}}}$$

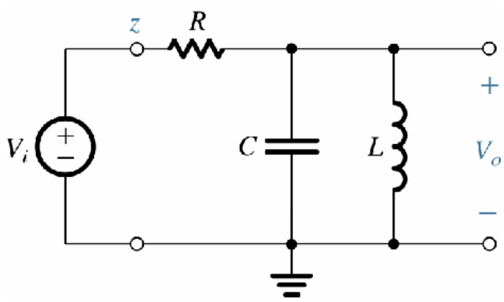
$$= \frac{\frac{1}{LC}}{s^2 + s\frac{1}{CR} + \frac{1}{LC}}$$

Realization of the High-Pass Function



$$T(s) \equiv \frac{V_o}{V_i} = \frac{a_2 s^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$$

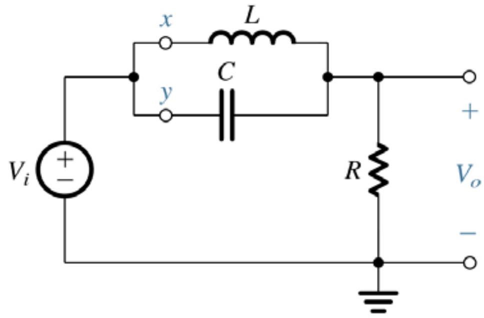
Realization of the Bandpass Function



$$T(s) = \frac{Y_R}{Y_R + Y_L + Y_C} = \frac{\dfrac{1}{R}}{\dfrac{1}{R} + \dfrac{1}{sL} + sC} = \frac{s\dfrac{1}{CR}}{s^2 + s\dfrac{1}{CR} + \dfrac{1}{LC}}$$

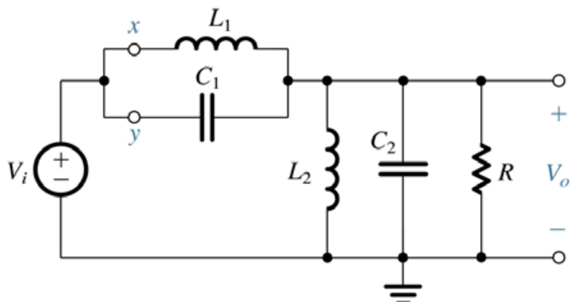
Realization of the Notch Functions

$$T(s) \equiv \frac{V_o}{V_i} = a_2 \frac{s^2 + \omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$



(e) Notch at ω_0

General notch function:



(f) General notch

$$L_1C_1 = \frac{1}{\omega_n^2}$$

$$C_1 + C_2 = C \qquad L_1 \| L_2 = L$$

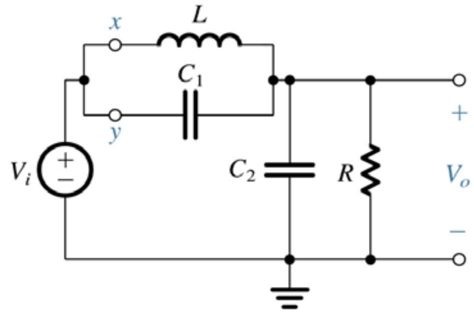
The L_1C_1 tank circuit introduces a pair of transmission zeros at $\pm j\omega_n$.

Realization of the low-pass notch (LPN) function $\omega_n > \omega_0$

$$L_1C_1 < (L_1\|L_2)(C_1 + C_2)$$

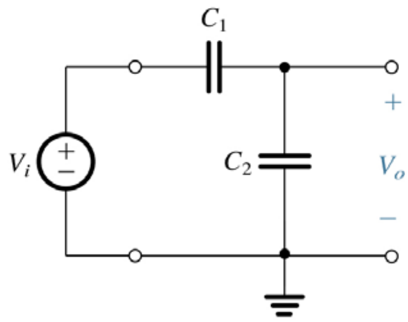
⇨ Satisfied with $L_2 = \infty$, $L_1 = L$.

$$T(s) \equiv \frac{V_o}{V_i} = a_2 \frac{s^2 + \omega_n^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$



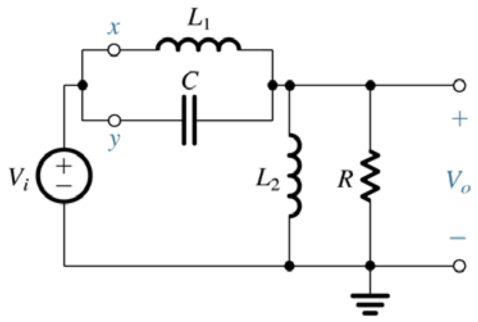
(g) LPN ($\omega_n > \omega_0$)

where $\omega_n^2 = 1/LC_1$, $\omega_0^2 = 1/L(C_1 + C_2)$, $\omega_0/Q = 1/CR$,
and *the high frequency* $a_2 = C_1/(C_1 + C_2)$.



(h) LPN as $s \to \infty$

Realization of the High-Pass Notch (HPN) Function $\omega_n < \omega_0$



(i) HPN ($\omega_n < \omega_0$)

$$L_1C_1 > (L_1 \| L_2)(C_1 + C_2)$$

⇨ Satisfied with $C_2 = 0$, $C_1 = C$.

$$T(s) \equiv \frac{V_o}{V_i} = \frac{s^2 + \dfrac{1}{L_1C}}{s^2 + s\dfrac{1}{CR} + \dfrac{1}{(L_1 \| L_2)C}}$$

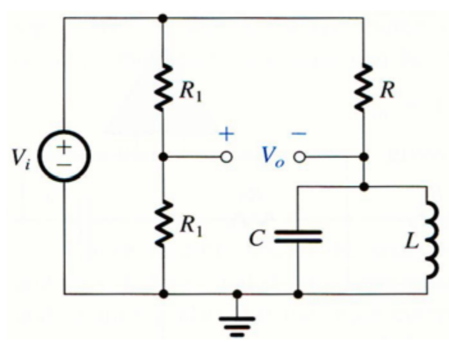
Realization of the All-Pass Function

$$T(s) \equiv \frac{V_o}{V_i} = \frac{s^2 - s\dfrac{\omega_0}{Q} + \omega_0^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2} \quad \Rightarrow \quad T(s) = 1 - \boxed{\frac{s\dfrac{2\omega_0}{Q}}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}}$$

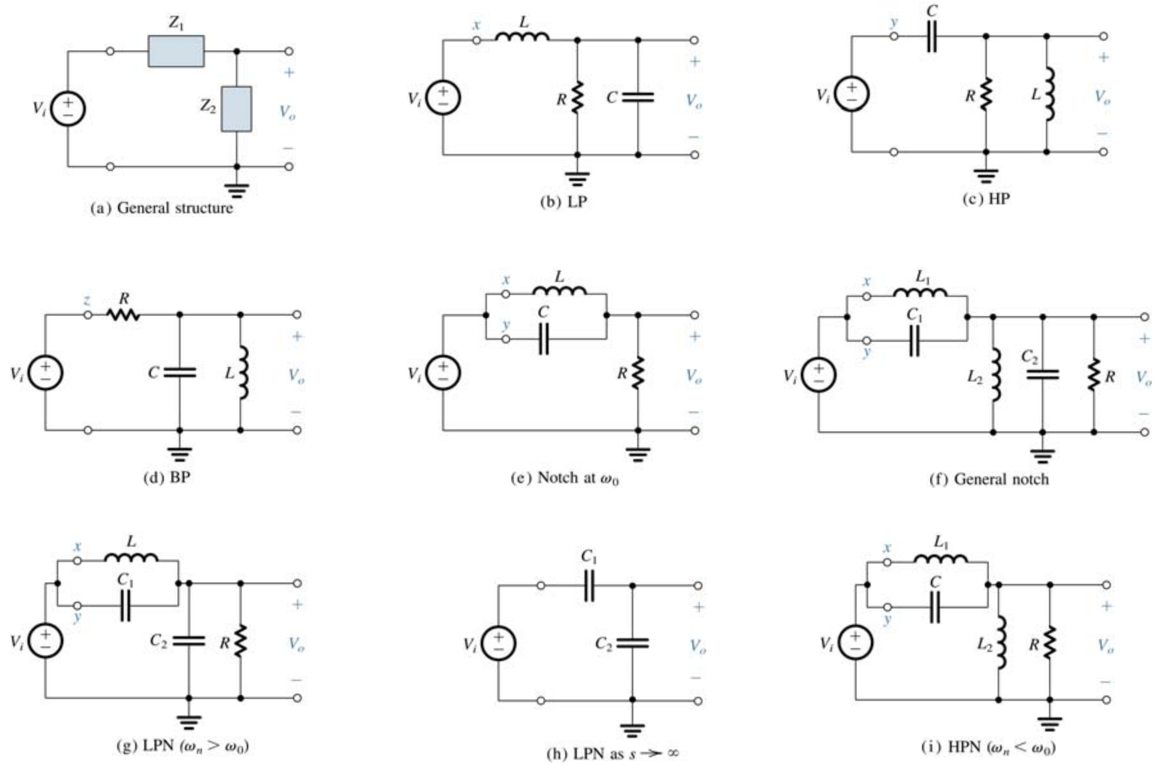
Bandpass filter with a center-frequency gain of 2.

Allpass filter with a flat gain of 0.5:

$$T(s) = 0.5 - \frac{s\dfrac{\omega_0}{Q}}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$$



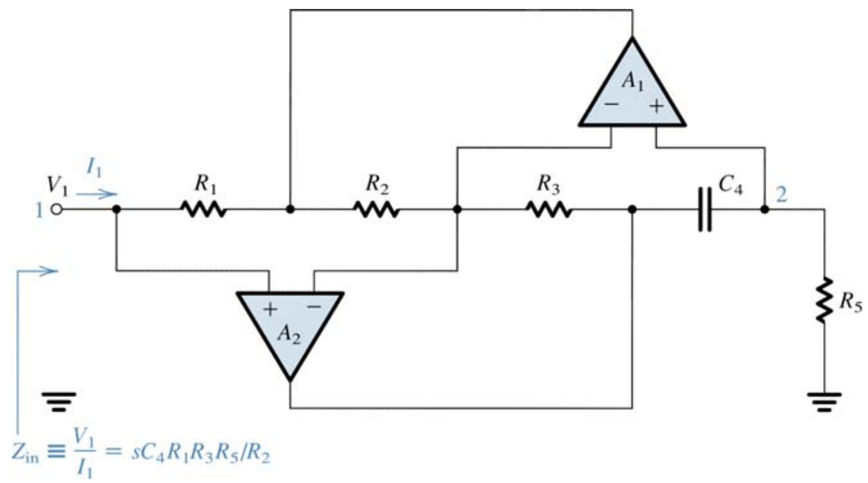
Realization of Various Second-Order Filter Functions Using LCR Resonator



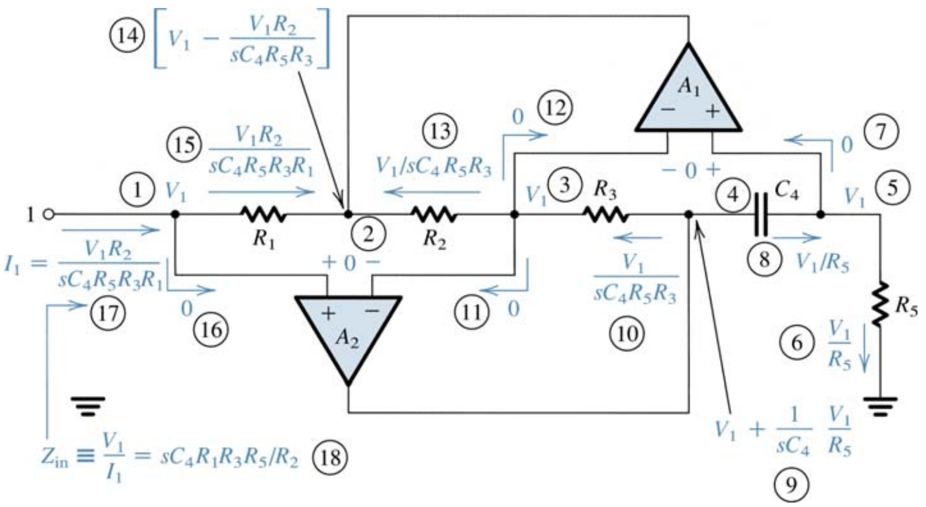
(a) General structure (b) LP (c) HP (d) BP (e) Notch at ω_0 (f) General notch (g) LPN ($\omega_n > \omega_0$) (h) LPN as $s \to \infty$ (i) HPN ($\omega_n < \omega_0$)

Second-Order Active Filters Based on Inductor Replacement

The Antoniou Inductance-Simulation Circuit (1969)



- Many inductor replacement circuit exists – Antoniou inductance simulation circuit is the best, i.e., it is very tolerant to the nonideal properties of the OPAMP, gain and bandwidth.



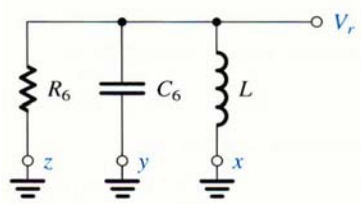
The input impedance is $Z_{in} \equiv \dfrac{V_1}{I_1} = s\dfrac{C_4R_1R_3R_5}{R_2}$

which is that of an inductance L given by $L = \dfrac{C_4R_1R_3R_5}{R_2}$.

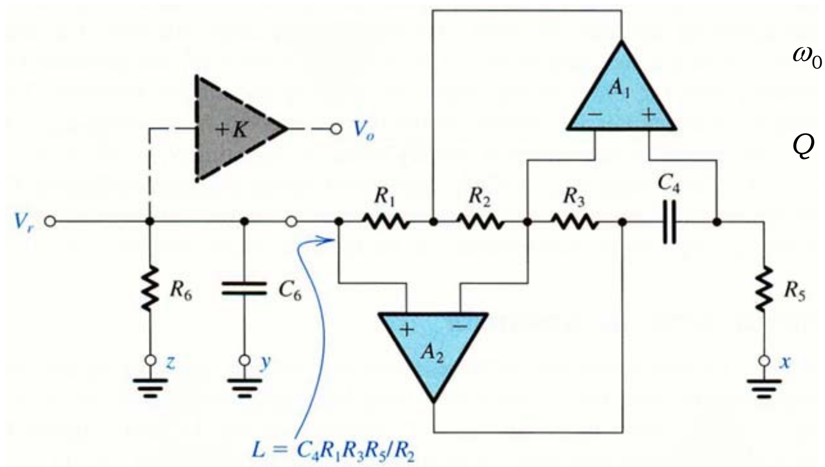
The design is usually based on selecting $R_1 = R_2 = R_3 = R_5 = R$ and $C_4 = C$, which leads to $L = CR^2$.

The Op Amp-RC Resonator

An LCR resonator



An op amp-RC resonator obtained by replacing the inductor L in the LCR resonator with a simulated inductance realized by the Antoiou circuit.



$$\omega_0 = \frac{1}{\sqrt{LC_6}} = \frac{1}{\sqrt{C_4C_6R_1R_3R_5/R_2}}$$

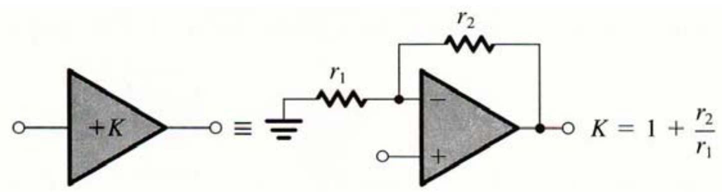
$$Q = \omega_0 C_6 R_6 = R_6\sqrt{\frac{C_6}{C_4}\frac{R_2}{R_1R_3R_5}}$$

Selecting $R_1 = R_2 = R_3 = R_5 = R$ and $C_4 = C_6 = C$, then

$$\omega_0 = \frac{1}{CR}$$

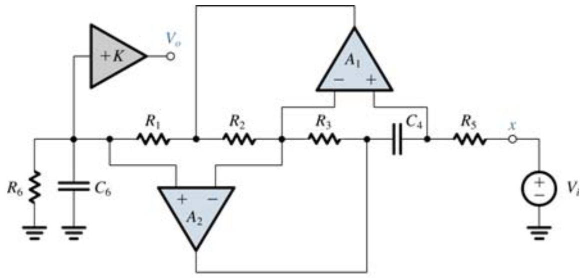
$$Q = \frac{R_6}{R}$$

Implementation of the buffer amplifier K

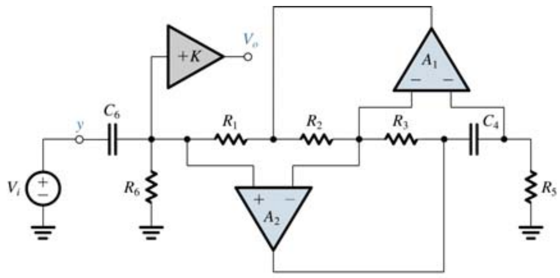


Note that not only does the amplifier K buffer the output of the filter, but it also allows the designer to set the filter gain to any desired value by appropriately selecting the value of K.

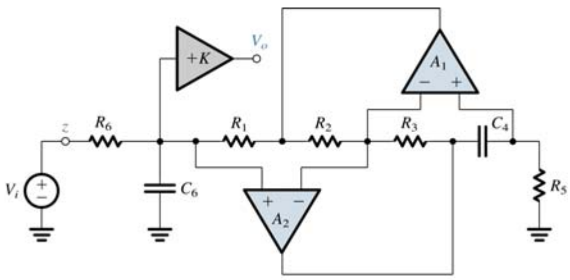
Realizations for the Various Second-Order Filter Functions Using the Op-Amp-RC Resonator



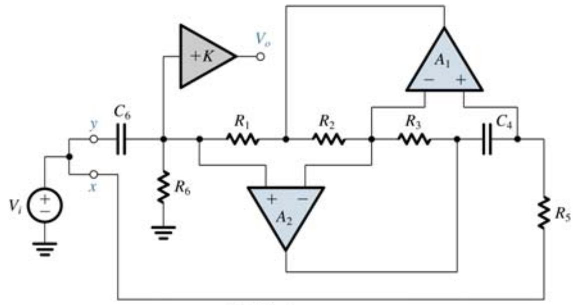
(a) LP



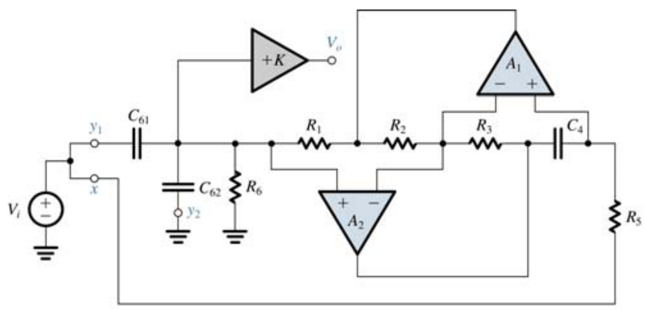
(b) HP



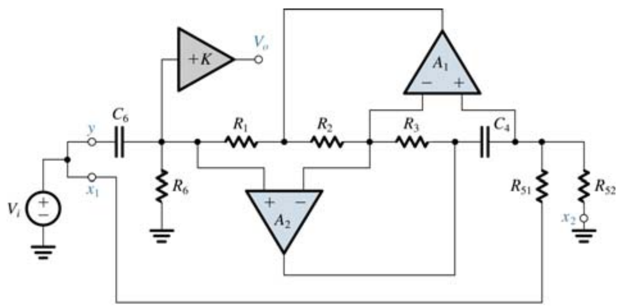
(c) BP



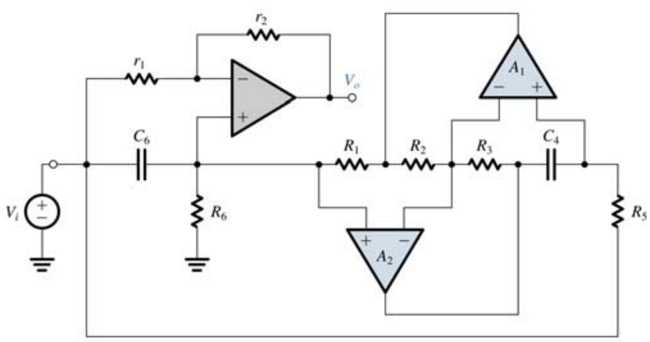
(d) Notch at ω_0



(e) LPN, $\omega_n \geq \omega_0$



(f) HPN, $\omega_n \leq \omega_0$



(g) All-pass

Table 12.1 DESIGN DATA FOR THE CIRCUITS OF FIG. 12.22

Circuit	Transfer Function and Other Parameters	Design Equations
Resonator Fig. 12.21(b)	$\omega_0 = 1/\sqrt{C_4 C_6 R_1 R_3 R_5 / R_2}$ $Q = R_6 \sqrt{\frac{C_6}{C_4 R_1 R_3 R_5} \frac{R_2}{}}$	$C_4 = C_6 = C$ (practical value) $R_1 = R_2 = R_3 = R_5 = 1/\omega_0 C$ $R_6 = Q/\omega_0 C$
Low-pass (LP) Fig. 12.22(a)	$T(s) = \dfrac{K R_2 / C_4 C_6 R_1 R_3 R_5}{s^2 + s\dfrac{1}{C_6 R_6} + \dfrac{R_2}{C_4 C_6 R_1 R_3 R_5}}$	K = dc gain
High-pass (HP) Fig. 12.22(b)	$T(s) = \dfrac{K s^2}{s^2 + s\dfrac{1}{C_6 R_6} + \dfrac{R_2}{C_4 C_6 R_1 R_3 R_5}}$	K = High-frequency gain
Bandpass (BP) Fig. 12.22(c)	$T(s) = \dfrac{K s / C_6 R_6}{s^2 + s\dfrac{1}{C_6 R_6} + \dfrac{R_2}{C_4 C_6 R_1 R_3 R_5}}$	K = Center-frequency gain
Regular notch (N) Fig. 12.22(d)	$T(s) = \dfrac{K[s^2 + (R_2 / C_4 C_6 R_1 R_3 R_5)]}{s^2 + s\dfrac{1}{C_6 R_6} + \dfrac{R_2}{C_4 C_6 R_1 R_3 R_5}}$	K = Low- and high-frequency gain

Low-pass notch (LPN) Fig. 12.22(e)	$T(s) = K\dfrac{C_{61}}{C_{61} + C_{62}}$ $\times \dfrac{s^2 + (R_2 / C_4 C_{61} R_1 R_3 R_5)}{s^2 + s\dfrac{1}{(C_{61} + C_{62}) R_6} + \dfrac{R_2}{C_4 (C_{61} + C_{62}) R_1 R_3 R_5}}$ $\omega_n = 1/\sqrt{C_4 C_{61} R_1 R_3 R_5 / R_2}$ $\omega_0 = 1/\sqrt{C_4 (C_{61} + C_{62}) R_1 R_3 R_5 / R_2}$ $Q = R_6 \sqrt{\dfrac{C_{61} + C_{62}}{C_4} \dfrac{R_2}{R_1 R_3 R_5}}$	K = dc gain $C_{61} + C_{62} = C_6 = C$ $C_{61} = C(\omega_0 / \omega_n)^2$ $C_{62} = C - C_{61}$
High-pass notch (HPN) Fig. 12.22(f)	$T(s) = K\dfrac{s^2 + (R_2 / C_4 C_6 R_1 R_3 R_{51})}{s^2 + s\dfrac{1}{C_6 R_6} + \dfrac{R_2}{C_4 C_6 R_1 R_3}\left(\dfrac{1}{R_{51}} + \dfrac{1}{R_{52}}\right)}$ $\omega_n = 1/\sqrt{C_4 C_6 R_1 R_3 R_{51} / R_2}$ $\omega_0 = \sqrt{\dfrac{R_2}{C_4 C_6 R_1 R_3}\left(\dfrac{1}{R_{51}} + \dfrac{1}{R_{52}}\right)}$ $Q = R_6 \sqrt{\dfrac{C_6}{C_4} \dfrac{R_2}{R_1 R_3}\left(\dfrac{1}{R_{51}} + \dfrac{1}{R_{52}}\right)}$	K = High-frequency gain $\dfrac{1}{R_{51}} + \dfrac{1}{R_{52}} = \dfrac{1}{R_5} = \omega_0 C$ $R_{51} = R_5 (\omega_0 / \omega_n)^2$ $R_{52} = R_5 / [1 - (\omega_n / \omega_0)^2]$
All-pass (AP) Fig. 12.22(g)	$T(s) = \dfrac{s^2 - s\dfrac{1}{C_6 R_6}\dfrac{r_2}{r_1} + \dfrac{R_2}{C_4 C_6 R_1 R_3 R_5}}{s^2 + s\dfrac{1}{C_6 R_6} + \dfrac{R_2}{C_4 C_6 R_1 R_3 R_5}}$ $\omega_z = \omega_0 \quad Q_z = Q(r_1 / r_2) \quad$ Flat gain = 1	$r_1 = r_2 = r$ (arbitrary) Adjust r_2 to make $Q_z = Q$.

Homework

- HW1
  Problems 8, 12, 19, 35, 41

Second-Order Active Filters Based on the two-Integrator-Loop Topology

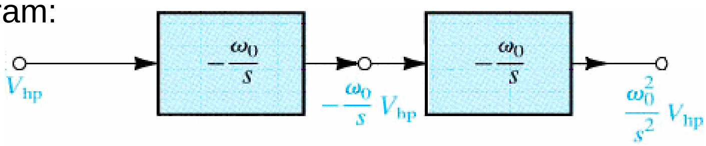
Two-Integrator-Loop Biquad

- Derivation
  - High-pass: $\dfrac{V_{hp}}{V_i} = \dfrac{Ks^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$

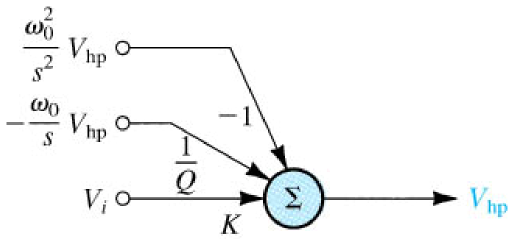
$$\Rightarrow \quad V_{hp} + \frac{1}{Q}\left(\frac{\omega_0}{s}V_{hp}\right) + \left(\frac{\omega_0^2}{s^2}V_{hp}\right) = KV_i$$

$$\Rightarrow \quad V_{hp} = KV_i - \frac{1}{Q}\left(\frac{\omega_0}{s}V_{hp}\right) - \left(\frac{\omega_0^2}{s^2}V_{hp}\right)$$

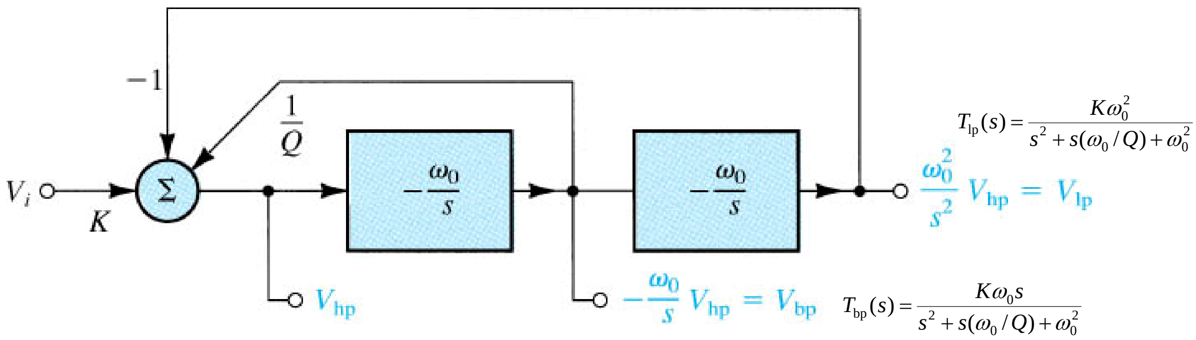
  - Bandpass: $\dfrac{V_{bp}}{V_i} = \dfrac{K\omega_0 s}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2} = \dfrac{(-\omega_0/s)V_{hp}}{V_i}$
  - Low-pass: $\dfrac{V_{lp}}{V_i} = \dfrac{K\omega_0^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2} = \dfrac{(\omega_0^2/s^2)V_{hp}}{V_i}$
- Block diagram:



- Universal active filter realizes LP, BP, and HP, simultaneously.
  - Versatile
  - Easy to design
  - Very popular



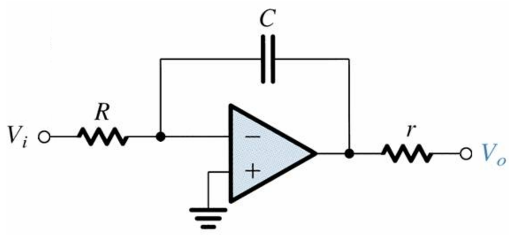
- Block-diagram realization:



Performance limited by the finite BW of OPAMP.

Integrator

- Miller integrator
  - Ideal OPAMP



$$T(s) = \frac{V_o(s)}{V_i(s)} = -\frac{1}{sRC}$$

  - Actual OPAMP (one-pole example)

$$A(s) = \frac{A_0}{1 + s/s_1}$$ OPAMP transfer function

$$R_i \to \infty, \quad R_o = 0, \;\; A_0 >> 1, \;\; A_o RC >> 1/|s_1|$$

(i.e., unity-gain BW $\omega_u = |s_1| >> 1/RC$)

$$T(s) = \frac{-A_0}{\left(1 + \dfrac{s}{A_0|s_1|}\right)(1 + sRCA_0)}$$

$$T(s) = -\frac{1}{sRC + \dfrac{1 + sRC}{A(s)}}$$

$$= -\frac{1}{\dfrac{RC}{\omega_u}\left[s^2 + s\left(\omega_u + s_1 + \dfrac{1}{RC}\right)\right]\dfrac{s_1}{RC}}$$

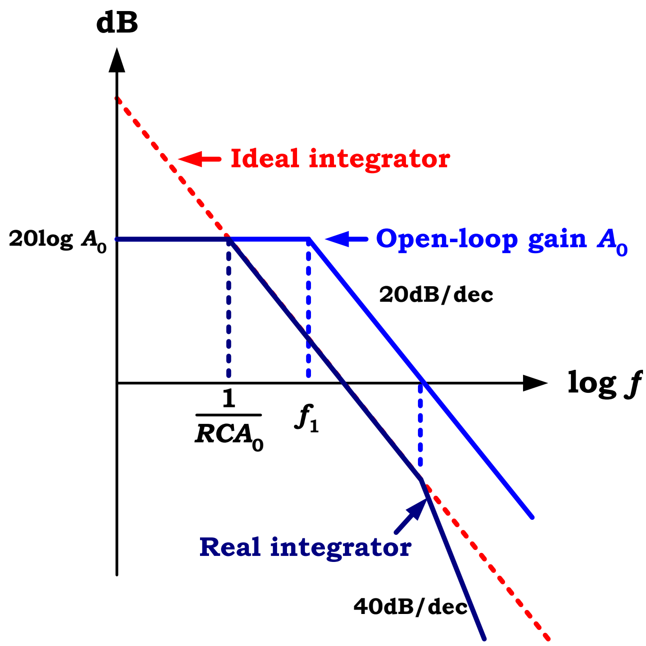
Since $\omega_u = A_0 s_1 >> s_1$

$$>> \frac{1}{RC}$$

$$>> \frac{1}{A_0 RC}$$

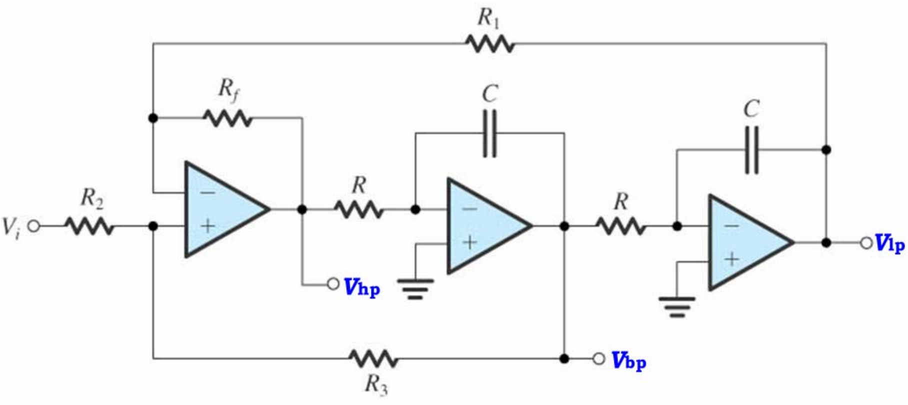
➡ $\omega_u + s_1 + \dfrac{1}{RC} \approx \omega_u \approx \omega_u + \dfrac{1}{A_0 RC}$

➡ $T(s) = -\dfrac{1}{\dfrac{RC}{\omega_u}\left(s + \dfrac{1}{A_0 RC}\right)(s + \omega_u)}$



Circuit Implementation of Universal Filter

Kerwin-Huelsman-Newcomb (KHN) biquad



$$\begin{cases} \dfrac{R_f}{R_1} = 1 \\ \dfrac{R_2}{R_3} = 2Q - 1 \end{cases}$$

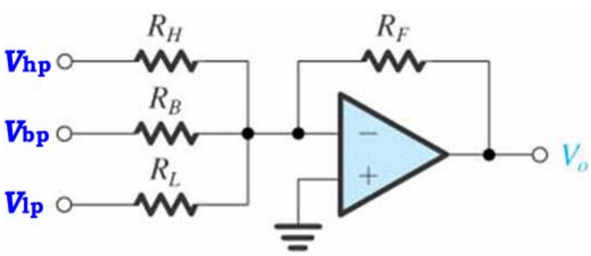
- Arbitrarily and practically choose R_1, R_f, R_2, and R_3 to meet the above relationship.

- To obtain notch and all-pass functions, the three outputs are assumed with appropriate weight.

$$V_o = -\left(\frac{R_F}{R_H}V_{\text{hp}} + \frac{R_F}{R_B}V_{\text{bp}} + \frac{R_F}{R_L}V_{\text{lp}}\right) = -V_i\left(\frac{R_F}{R_H}T_{\text{hp}} + \frac{R_F}{R_B}T_{\text{bp}} + \frac{R_F}{R_L}T_{\text{lp}}\right)$$

$$\frac{V_o}{V_i} = -K\frac{(R_F/R_H)s^2 - s(R_F/R_B)\omega_0 + (R_F/R_L)\omega_0^2}{s^2 + s(\omega_0/Q) + \omega_0^2}$$

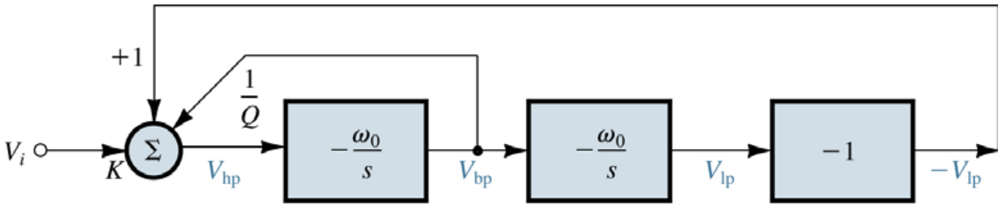
E.g. Notch filter: choose $R_B = \infty$, $\dfrac{R_H}{R_L} = \left(\dfrac{\omega_n}{\omega_0}\right)^2$

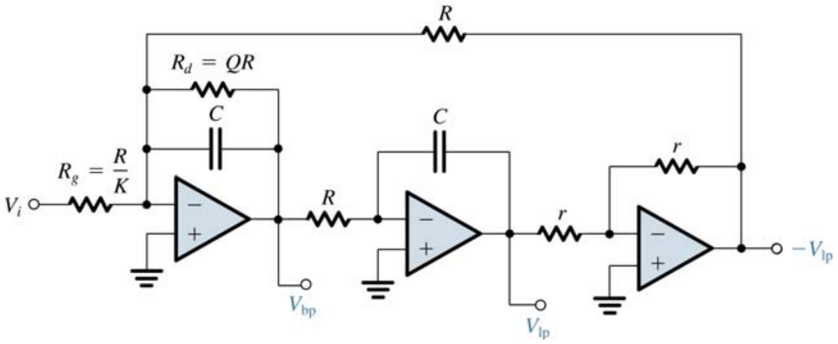


Alternative Two-Integrator-Loop Biquad

Two-Thomas biquad (Single-ended fashion)

- Universal filter

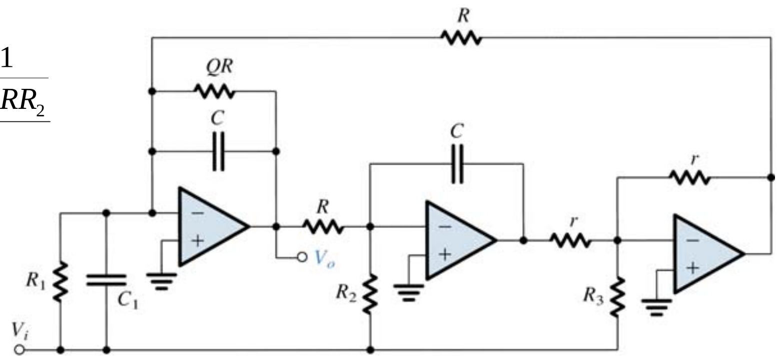




Two-Thomas biquad with input feedward paths

- Realize all special second-order function

$$\frac{V_o}{V_i} = -\frac{s^2\left(\frac{C_1}{C}\right) + s\frac{1}{C}\left(\frac{1}{R_1} - \frac{r}{RR_3}\right) + \frac{1}{C^2RR_2}}{s^2 + s\frac{1}{QCR} + \frac{1}{C^2R^2}}$$



- Design table

All cases	C = arbitrary, $R = 1/\omega_0 C$, r = arbitrary
LP	$C_1 = 0$, $R_1 = \infty$, $R_2 = R$/dc gain, $R_3 = \infty$
Positive BP	$C_1 = 0$, $R_1 = \infty$, $R_2 = \infty$, $R_3 = Qr$/center-frequency gain
Negative BP	$C_1 = 0$, $R_1 = QR$/center-frequency gain, $R_2 = \infty$, $R_3 = \infty$
HP	$C_1 = C \times$ high-frequency gain, $R_1 = \infty$, $R_2 = \infty$, $R_3 = \infty$
Notch (all types)	$C_1 = C \times$ high-frequency gain, $R_1 = \infty$, $R_2 = R(\omega_0/\omega_n)^2$/high-frequency gain, $R_3 = \infty$
AP	$C_1 = C \times$ flat gain, $R_1 = \infty$, $R_2 = R$/gain, $R_3 = Qr$/gain

Single-Amplifier Biquadratic Active Filters

Single-OPAMP Biquad (SAB) – compared with two-integrator biquad

- Economic

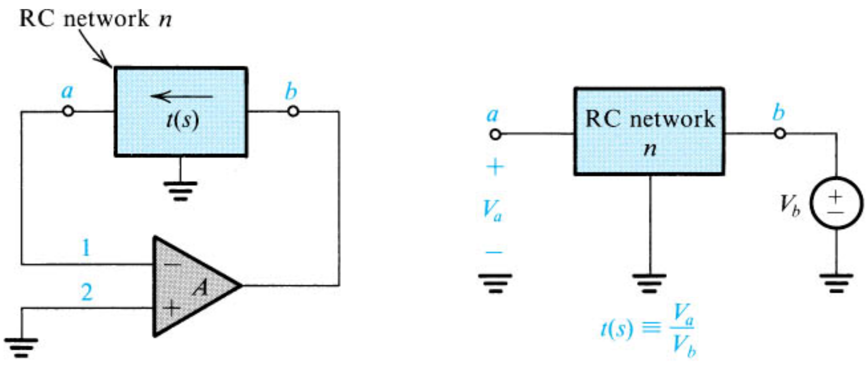
  require 1 OPAMP instead of 3 or 4.
- More sensitive to R and C variations
- Greater dependence on limited gain and bandwidth

➡ Single-amplifier biquads (SABs) are therefore limited to the less strigent filter spec. E.g., pole Q < 10.

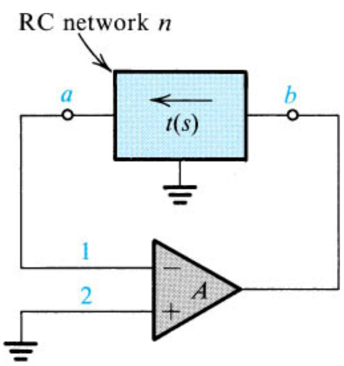
- Biquads can be cascaded to construct high-order filters.

SAB Synthesis

- Two-step procedure
  - Synthesis of a feedback loop that realizes a pair of complex conjugate poles characterized by ω_0 and Q.
  - Injecting the input signal in a way that realizes the desired transmission zeros.
- Synthesis of the feedback loop



(a) (b)



The produce of the op amp gain A and $t(s)$:

$$L(s) = At(s) = \frac{AN(s)}{D(s)}$$

$N(s)$: zeros of the RC network
$D(s)$: poles of the RC network

The characteristic equation: $1 + L(s) = 0$

- The poles s_P of the close-loop circuit obtain as solutions to the equation

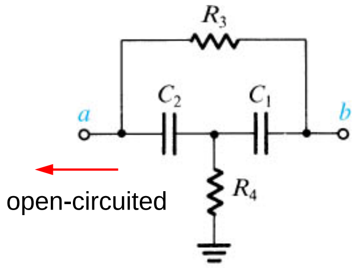
$$t(s_P) = -\frac{1}{A}$$

  In the ideal case, $A = \infty$ and the pole are obtained from

$$N(s_P) = 0$$

- The circuit poles are identical to the zeros of the RC network.

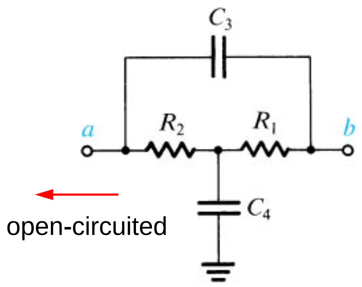
Bridged-T RC Networks



$$t(s)=\frac{s^2+s\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\frac{1}{R_3}+\frac{1}{C_1C_2R_3R_4}}{s^2+s\left(\frac{1}{C_1R_3}+\frac{1}{C_2R_3}+\frac{1}{C_1R_4}\right)+\frac{1}{C_1C_2R_3R_4}}$$

$$t(s)=\frac{V_a}{V_b}$$

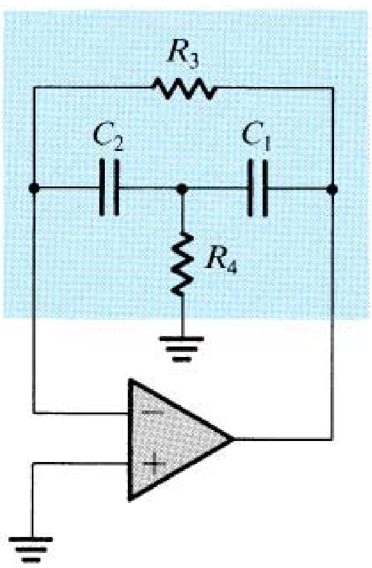
(a)



$$t(s)=\frac{s^2+s\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\frac{1}{C_4}+\frac{1}{C_3C_4R_1R_2}}{s^2+s\left(\frac{1}{C_4R_1}+\frac{1}{C_4R_2}+\frac{1}{C_3R_2}\right)+\frac{1}{C_3C_4R_1R_2}}$$

(b)

- The pole polynomial of the active-filter circuit will equal to the numerator polynomial of the Bridged-T network.



$$s^2+s\frac{\omega_0}{Q}+\omega_0^2=s^2+s\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\frac{1}{R_3}+\frac{1}{C_1C_2R_3R_4}$$

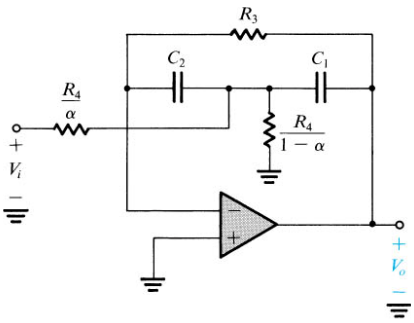
$$\omega_0=\frac{1}{\sqrt{C_1C_2R_3R_4}}$$

$$Q=\left[\frac{\sqrt{C_1C_2R_3R_4}}{R_3}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\right]^{-1}$$

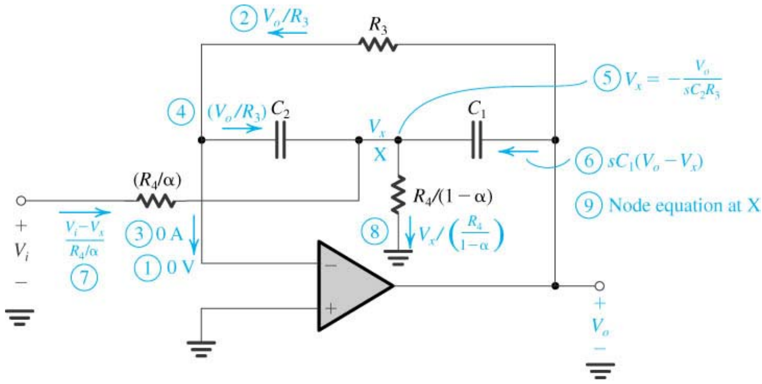
Let $\begin{cases} C_1=C_2=C \quad \Rightarrow \quad m=4Q^2 \ \text{ and } \ RC=\dfrac{2Q}{\omega_0} \\ R_3=R \\ R_4=\dfrac{R}{m} \end{cases}$

Q and ω_0 can be used to determine the component values.

- Injecting the input signal
  - To obtain transmission zero
  - Example: A bandpass filter



(a)



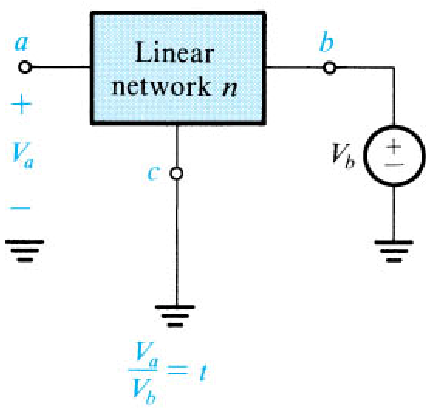
(b)

$$\frac{V_o}{V_i} = \frac{-s\dfrac{\alpha}{C_1R_4}}{\boxed{s^2 + s\left(\dfrac{1}{C_1} + \dfrac{1}{C_2}\right)\dfrac{1}{R_3} + \dfrac{1}{C_1C_2R_3R_4}}}$$

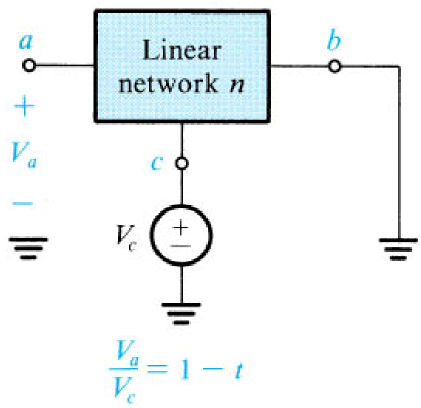
The denominator polynomial is identical to the numerator polynomial of $t(s)$.

Complementary Transformation

- Complement of transfer function – interchanging input and ground



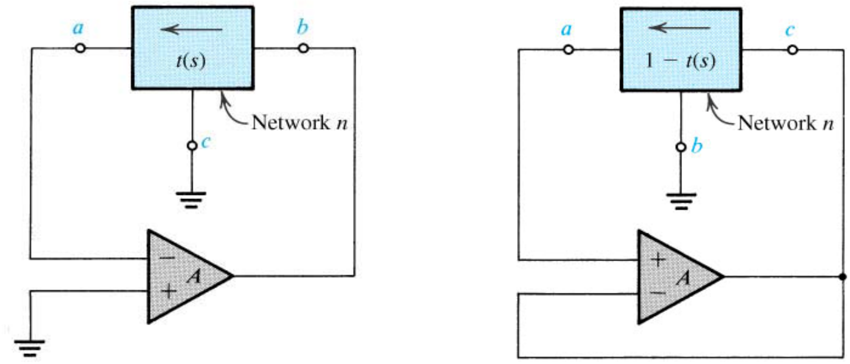
(a)



(b)

Note $\dfrac{V_{ac}}{V_{bc}} = \dfrac{V_a - V_c}{V_b - V_c} = t$

- Two-step procedure
  - ❶ Nodes of the feedback network and any of the op amp inputs that are connected to ground should be disconnected from ground and connected to the op amp output. Conversely, those nodes that were connected to the op amp output should be now connected to ground. That is, we simply interchange the op amp output terminal with ground.
  - ❷ The two input terminals of the op amp should be interchanged.
- Example



Characteristic equation:

$$1 + At = 0$$

$$1 - \frac{A}{A+1}(1-t) = 0$$

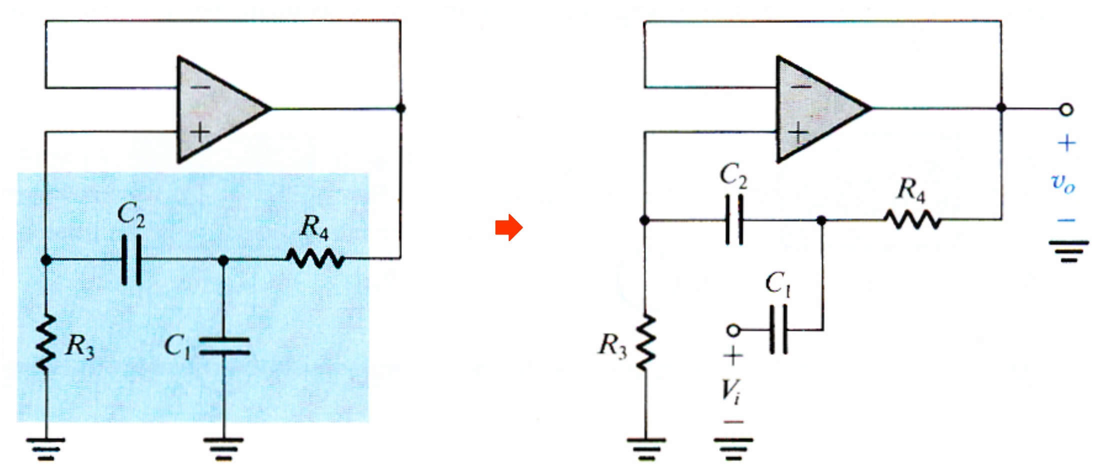
same poles

$$\Rightarrow 1 + At = 0$$

Ex. Appling the complementary transformation to the feedback loop

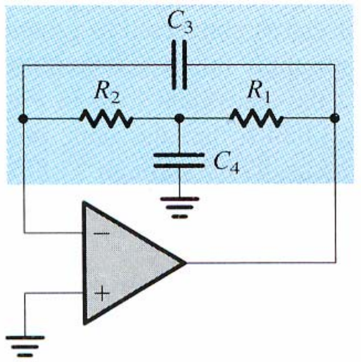
- Sallen-and-Key SAB circuit: (HP function)

  $C_1 = C_2 = C$, $R_3 = R$, $R_4 = R/4Q^2$, $CR = 2Q/\omega_0$, and the value of C is arbitrarily chosen to be practically convenient.



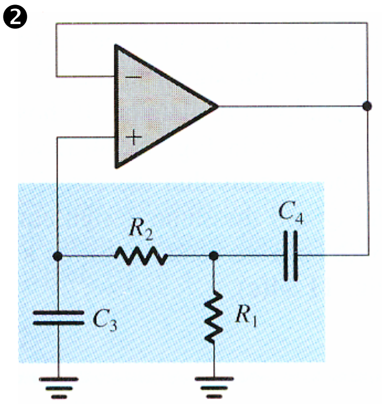
- Another example: LP filter obtained by injecting

❶

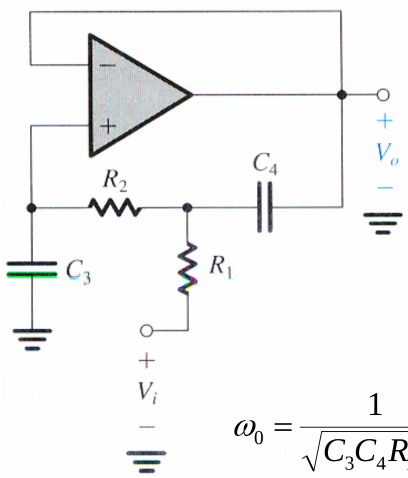


Complementary Trans.

❷



❸ Injecting V_i through R_1:



$$\omega_0 = \frac{1}{\sqrt{C_3 C_4 R_1 R_2}}$$

$$Q = \left[\frac{\sqrt{C_3 C_4 R_1 R_2}}{C_4}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\right]^{-1}$$

Selecting $R_1 = R_2 = R$, $C_4 = C$, $C_3 = C/m$,

➡ $m = 4Q^2 \quad CR = 2Q/\omega_0$

Sensitivity

Sensitivity

- Real components deviate from their designed values
  - initially inaccurate due to fabrication tolerances.
  - drift due to environmental effects such as temperature and humidity.
  - chemical changes which occurs as the circuit ages.
  - inaccuracies in modeling the passive and active devices, e.g., nonideal OPAMP and parasitics.
- All coefficients, and therefore poles and zeros of H(s), depend on circuit element.
- The size of H(s) error depends on how large the component tolerances are and how sensitive the circuit's performance is to these tolerances.

- Sensitivity calculation, allow the designer
  - to select the better circuit from those in the literature.
  - to determine whether a chosen filter circuit satisfies and will keep satisfying the given specifications.
- Component χ
- Performance criterion $p(\chi)$, such as
  - Quality factor
  - Pole frequency
  - Zero frequency

  or $p(s, \chi)$, if p is also a function of frequency and stands for

  ❶ H(s),

  ❷ magnitude of H(s),

  ❸ phase of H(s).

- Sensitivity S_χ^p

Taylor series $P(s,\chi) = P(s,\chi_0) + \left.\frac{\partial P(s,\chi)}{\partial \chi}\right|_{\chi_0} d\chi + \frac{1}{2}\left.\frac{\partial^2 P(s,\chi)}{\partial \chi^2}\right|_{\chi_0} (d\chi)^2 + \cdots$

if $\frac{d\chi}{\chi_0} << 1$ and $\left.\frac{dP}{d\chi}\right|_{\chi=\chi_0}$ is small

$$\begin{cases} \Delta P(s,\chi_0) = P(s,\chi_0 + d\chi) - P(s,\chi_0) \approx \left.\frac{\partial P(s,\chi)}{\partial \chi}\right|_{\chi_0} d\chi \\ \frac{\Delta P(s,\chi_0)}{P(s,\chi_0)} \approx \frac{\chi_0}{P(s,\chi_0)} \left.\frac{\partial P(s,\chi)}{\partial \chi}\right|_{\chi_0} \frac{d\chi}{\chi_0} \end{cases}$$

$$S_\chi^P = \frac{\chi_0}{P(s,\chi_0)} \left.\frac{\partial P(s,\chi)}{\partial \chi}\right|_{\chi_0} = \left.\frac{\partial P/P}{\partial \chi/\chi}\right|_{\chi_0} = \left.\frac{d(\ln P)}{d(\ln \chi)}\right|_{\chi_0}$$

if $\frac{\Delta\chi}{\chi_0} << 1$, then $S_\chi^P \approx \frac{\partial P/P}{\partial \chi/\chi}$

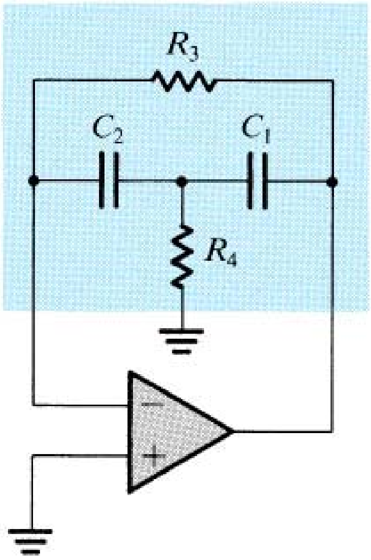
- Sensitivity is to predict the deviations from
  - the tolerances in component values
  - the finite op-amp gain
- Definition

$$S_x^y \equiv \lim_{\Delta x \to 0} \frac{\Delta y / y}{\Delta x / x} = \frac{\partial y}{\partial x}\frac{x}{y}$$

x denotes the value of a component (a resistor, a capacitor, or an amplifier gain) and y denotes a circuit parameter of interest (ω_0 or Q).

For small changes $S_x^y \approx \frac{\Delta y / y}{\Delta x / x}$

Example: **find the sensitivities of ω_0 and Q relative to all passive components**



$$s^2 + s\frac{\omega_0}{Q} + \omega_0^2 = s^2 + s\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\frac{1}{R_3} + \frac{1}{C_1C_2R_3R_4}$$

$$\Rightarrow \quad \omega_0 = \frac{1}{\sqrt{C_1C_2R_3R_4}}$$

$$Q = \left[\frac{\sqrt{C_1C_2R_3R_4}}{R_3}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\right]^{-1}$$

$$S_x^y \equiv \lim_{\Delta x \to 0}\frac{\Delta y / y}{\Delta x / x} = \frac{\partial y}{\partial x}\frac{x}{y}$$

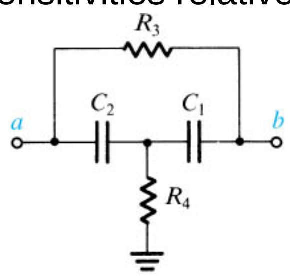
$$S_{C_1}^Q = \frac{1}{2}\left(\sqrt{\frac{C_2}{C_1}} - \sqrt{\frac{C_1}{C_2}}\right)\left(\sqrt{\frac{C_2}{C_1}} + \sqrt{\frac{C_1}{C_2}}\right)^{-1}$$

Assume $C_1 = C_2$,

$$S_{C_1}^{\omega_0} = S_{C_2}^{\omega_0} = S_{R_3}^{\omega_0} = S_{R_4}^{\omega_0} = -\frac{1}{2} \qquad S_{C_1}^Q = S_{C_2}^Q = 0, \quad S_{R_3}^Q = \frac{1}{2}, \quad S_{R_4}^Q = -\frac{1}{2}$$

Note that: A 10% increase in R_3 results in a 5% decrease in the value of ω_0 and a 5% increase in the value of Q.

- The sensitivities relative to the amplifier gain



$$t(s) = \frac{s^2 + s\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\frac{1}{R_3} + \frac{1}{C_1C_2R_3R_4}}{s^2 + s\left(\frac{1}{C_1R_3}+\frac{1}{C_2R_3}+\frac{1}{C_1R_4}\right) + \frac{1}{C_1C_2R_3R_4}}$$

Assume the op amp have a finite gain A, the characteristic equation for the loop

$$1 + At(s) = 0$$

- Using the obtained design, $C_1 = C_2 = C$, $R_3 = R$, $R_4 = R/4Q^2$, and $CR = 2Q/\omega_0$,

$$t(s) = \frac{s^2 + s(\omega_0/Q) + \omega_0^2}{s^2 + s(\omega_0/Q)(2Q^2+1) + \omega_0^2}$$

- $1 + At(s) = 0 \quad \Rightarrow \quad s^2 + s(\omega_0/Q)(2Q^2+1) + \omega_0^2 + A\left(s^2 + s(\omega_0/Q) + \omega_0^2\right) = 0$

$$\therefore \; s^2 + s\frac{\omega_0}{Q}\left(1 + \frac{2Q^2}{A+1}\right) + \omega_0^2 = 0 \quad \Rightarrow \quad \begin{cases} \omega_{0a} = \omega_0 \\ Q_a = \dfrac{Q}{1 + 2Q^2/(A+1)} \end{cases}$$

$$\Rightarrow \quad S_A^{\omega_{0a}} = 0 \quad S_A^{Q_a} = \frac{A}{A+1}\cdot\frac{2Q^2/(A+1)}{1+2Q^2/(A+1)} \qquad \text{Note: } S_A^{Q_a} \approx \frac{2Q^2}{A} \text{ for } A \gg 2Q^2 \text{ and } A \gg 1.$$

Switch-Capacitor Filters

Switched-Capacitor Filters

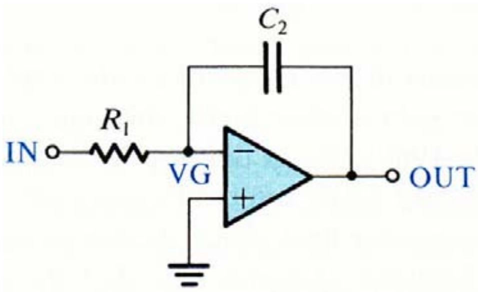
- Switched capacitor performing as a simulated resistor

  large area resistor ➡ smaller area capacitor
- VLSI technology requirements
  - Good switch
  - Well-defined capacitor
  - OPAMP
- First discussion on the equivalent resistance of a periodically switched capacitor methods in 1946.
- Rapid evolution of practical SC is due to the implementation in MOS technology.

  Followed by a rapid development and implementation of analog signal processing techniques.
- SC circuits are sampled-data analog MOS integrated circuits.

RC Time Constant

- Active-RC integrator

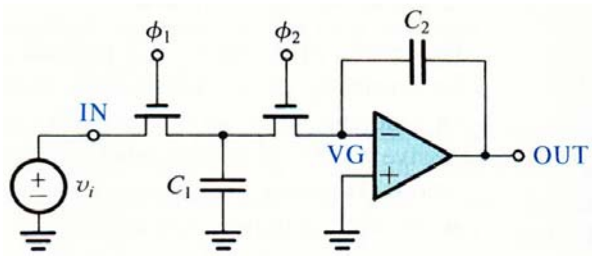


If $\tau = R_1 C_2$, then $\frac{d\tau}{\tau} = \frac{dR_1}{R_1} + \frac{dC_2}{C_2}$

The absolute accuracies of R and C is very poor.

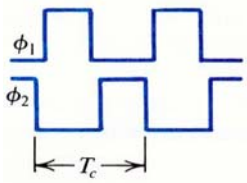
Basic Principle of the Switched-Capacitor Filter Technique

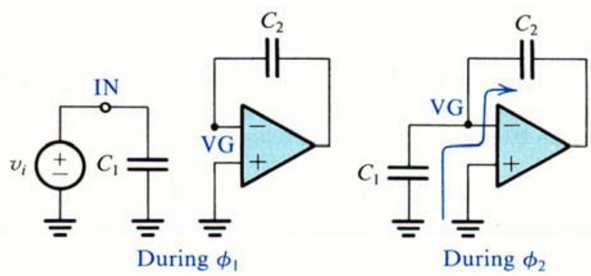
- Switched-capacitor integrator



- Operation

Nonoverlapping two-phase clock





$q_{C_1} = C_1 v_i$ $i_{av} = \frac{q_{C_1}}{T_c}$ $R_{eq} \equiv \frac{v_i}{i_{av}} = \frac{T_c}{C_1}$

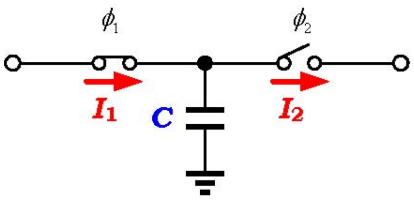
An equivalent time-constant for the integrator:

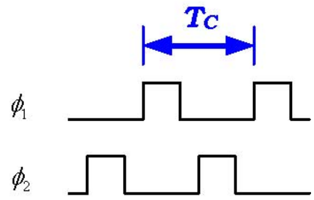
$$\tau = C_2 R_{eq} = T_c \frac{C_2}{C_1}$$

determined by ❶ T_c ❷ C_2/C_1 ⇨ accuracy

Accurate RC time constant

- SC as a resistor





$$\overline{I_1} = \overline{I_2} = \frac{Q}{T_C} = \frac{CV}{T_C} = \frac{V}{R} \quad \text{Equivalent } R = \frac{T_C}{C} = \frac{1}{f_C C}$$

- If R_1 is replaced by a switched capacitor resistance realization with a C_1, then

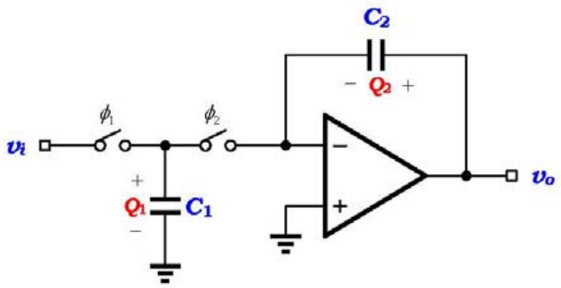
$$\tau = \frac{1}{f_C}\frac{C_2}{C_1} = T_C \frac{C_2}{C_1} \qquad \frac{d\tau}{\tau} = \frac{dT_C}{T_C} + \frac{dC_2}{C_2} - \frac{dC_1}{C_1}$$

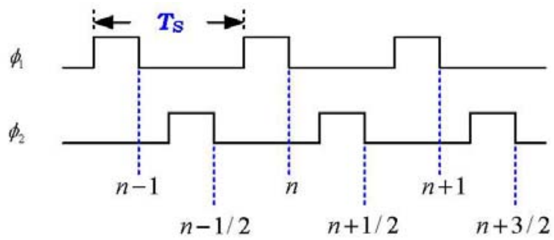
If clock frequency is assumed to be constant, then

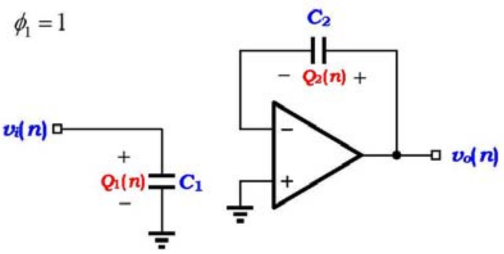
$$\frac{d\tau}{\tau} = \frac{dC_2}{C_2} - \frac{dC_1}{C_1} \quad \to 0$$

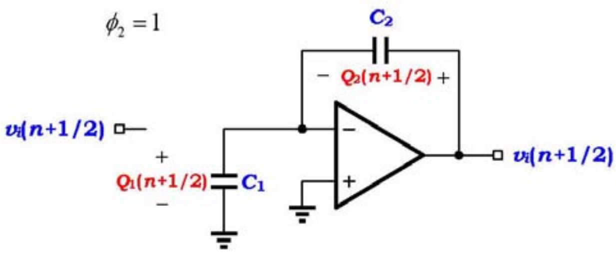
Relative accuracy is good for two capacitors fabricated in the same integrated circuit.

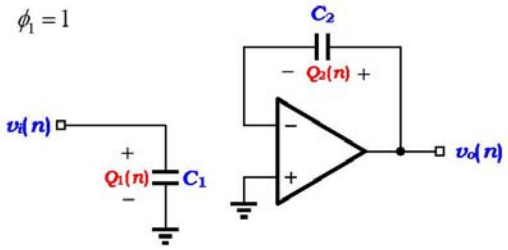
Z-Transform

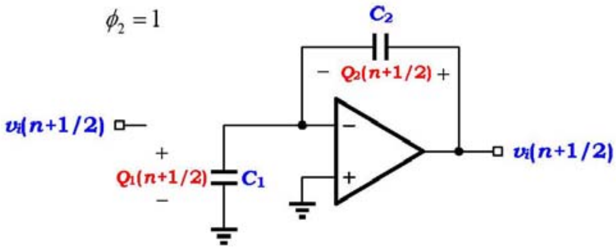












At cycle n, i.e., $t = nT_s$, we have $Q_1(n) = C_1 v_i(n)$ and $Q_2(n) = C_2 v_o(n)$

At cycle $n + 1/2$, i.e., $t = (n + 1/2)T_s$,

$Q_1(n + 1/2) = 0 \qquad Q_2(n + 1/2) = Q_2(n) - Q_1(n) = C_2 v_o(n) - C_1 v_i(n)$

At cycle $n + 1$, i.e., $t = (n + 1)T_s$,

$Q_1(n + 1) = C_1 v_i(n + 1)$

$Q_2(n + 1) = C_2 v_o(n + 1) = Q_2(n + 1/2) = C_2 v_o(n) - C_1 v_i(n)$

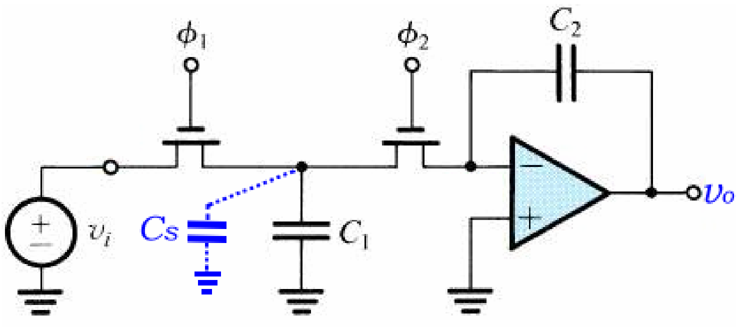
Thus, the time-domain difference equation is

$C_2 v_o(n + 1) = C_2 v_o(n) - C_1 v_i(n)$

In the z-domain

$$zC_2 v_o(z) = C_2 v_o(z) - C_1 v_i(z) \quad \Rightarrow \quad \frac{v_o(z)}{v_i(z)} = -\frac{C_1}{C_2} \times \frac{1}{z-1} = -\frac{C_1}{C_2} \times \frac{z^{-1}}{1-z^{-1}}$$

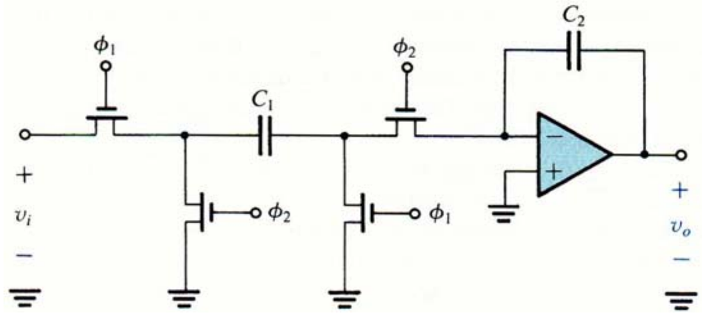
Stray Capacitance in SC Integrators



$$\frac{v_o(z)}{v_i(z)} = -\left(\frac{C_1 + C_S}{C_2}\right) \times \frac{z^{-1}}{1 - z^{-1}}$$

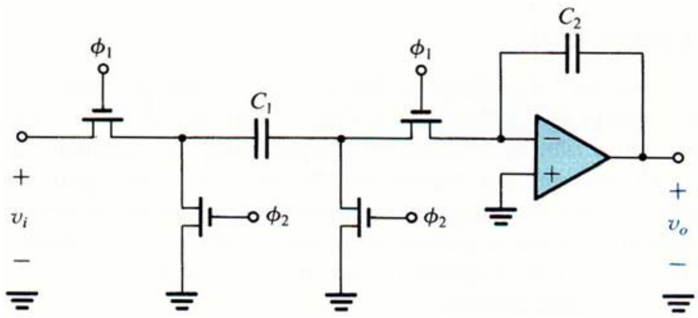
Practical Circuits of the Switched-Capacitor integrators

- *Noninverting* switched-capacitor integrator



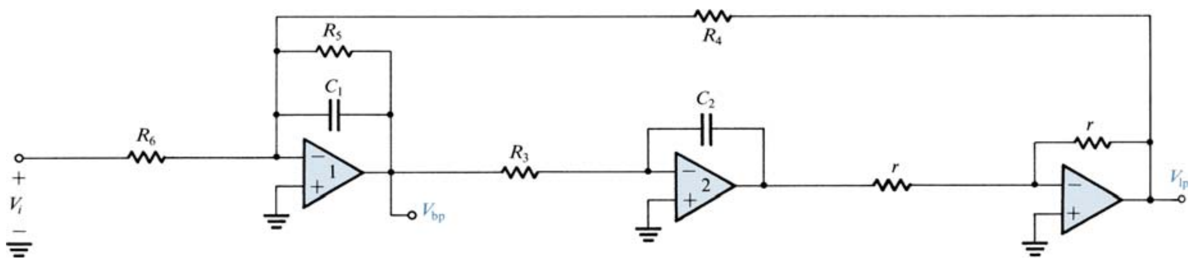
$$H(z)=\frac{V_o(z)}{V_i(z)}=\frac{C_1}{C_2}\frac{z^{-1}}{1-z^{-1}}$$

- *Inverting* switched-capacitor integrator

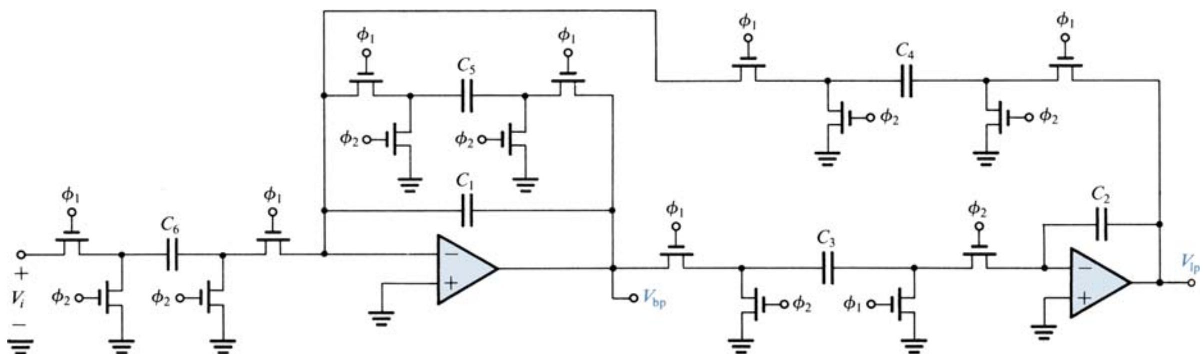


$$H(z)=\frac{V_o(z)}{V_i(z)}=-\frac{C_1}{C_2}\frac{1}{1-z^{-1}}$$

Practical Circuits of the Switched-Capacitor Filters



$$\omega_0=\frac{1}{\sqrt{C_1C_2R_3R_4}}$$



- Substituting for R_2 and R_4 by their SC equivalent values, that is

$$R_3 = \frac{T_c}{C_3} \quad \text{and} \quad R_4 = \frac{T_c}{C_4}$$

Give ω_0 of the SC biquad as

$$\omega_0 = \frac{1}{\sqrt{C_1 C_2 R_3 R_4}} \;\Rightarrow\; \omega_0 = \frac{1}{T_c}\sqrt{\frac{C_3 C_4}{C_2 C_1}} \qquad (12.93)$$

- If $\frac{T_c}{C_3}C_2 = \frac{T_c}{C_4}C_1$ and $C_1 = C_2 = C$, then $C_3 = C_4 = KC$, where $K = \omega_0 T_c$ (from Eq. 12.93).
- Determine C_5 :

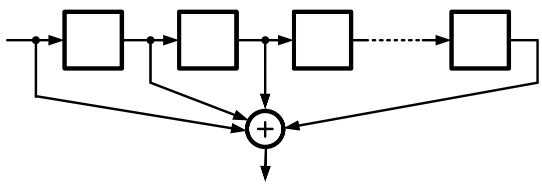
$$Q = \frac{T_c / C_5}{T_c / C_4} \qquad \Rightarrow \qquad C_5 = \frac{C_4}{Q} = \frac{KC}{Q} = \omega_0 T_c \frac{C}{Q}$$

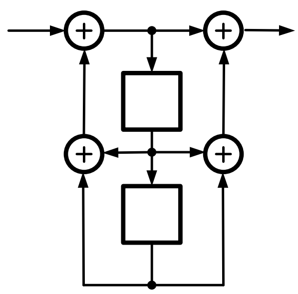
- The center-frequency gain of the bandpass function:

$$\text{Center - frequency gain} = \frac{C_6}{C_5} = Q\frac{C_6}{\omega_0 T_c C}$$

Summary of Filters

- Continuous-time filter
  - RLC passive
  - RC active
- Sampled-Data filter
  - Switched-Capacitor Filter
- Digital Filter



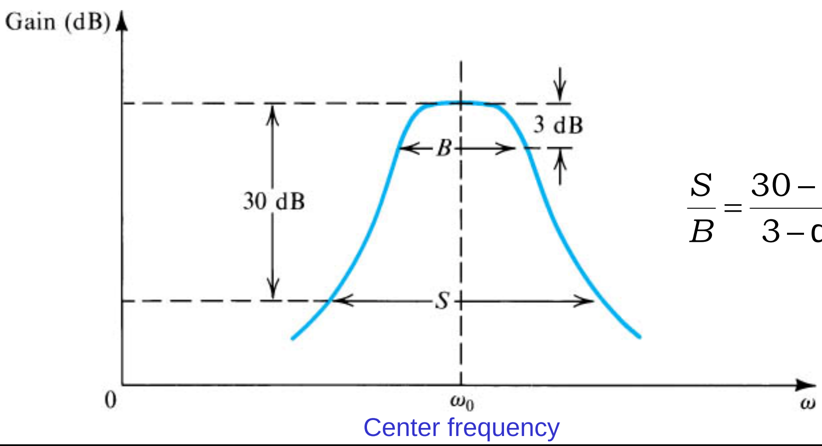


Operations
- ☐ Multiply
- ☐ Delay
- ☐ Add

Tuned Amplifiers

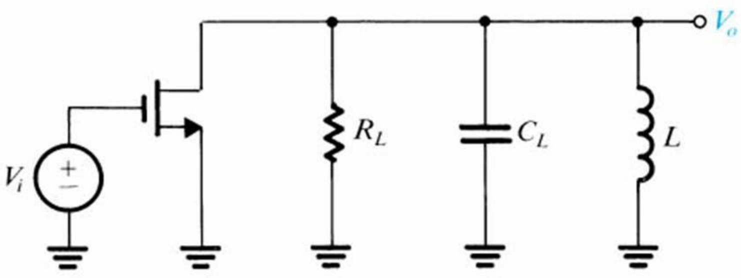
Tuned Amplifiers

- A special kind of frequency-selective network
- RF and IF application
- Response is similar to that of bandpass filters
- Center frequency ranges from a few hundred kHz to a few hundred MHz
- Skirt selectivity $\left(\frac{S}{B}\right)$



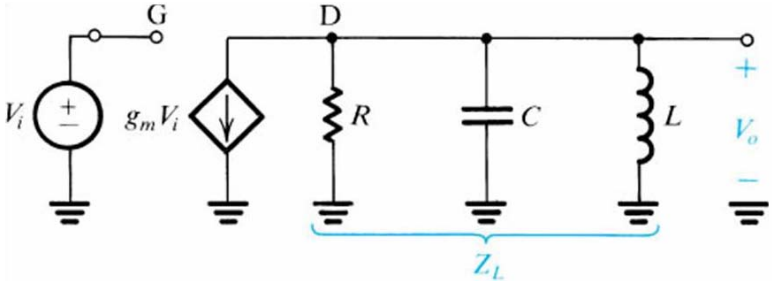
$$\frac{S}{B} = \frac{30-\text{dB bandwidth}}{3-\text{dB bandwidth}}$$

Single-Tuned amplifier



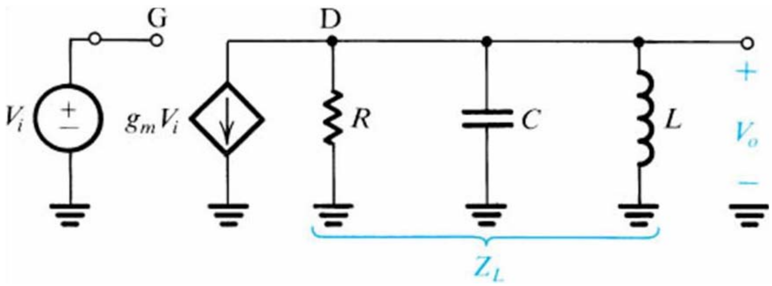
(Transistor operates in class A mode)

Equivalent circuit



$$R = R_L \| r_{o(FET)}$$

$$C = C_L + C_{(FET)}$$



$$V_o = \frac{-g_m V_i}{Y_L} = \frac{-g_m V_i}{sC + \frac{1}{R} + \frac{1}{sL}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad B = \frac{1}{CR}$$

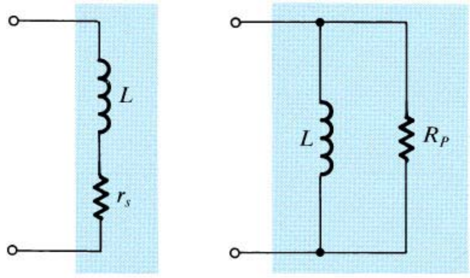
$$\Rightarrow \frac{V_o}{V_i} = -\frac{g_m}{C}\frac{s}{s^2 + s(1/CR) + 1/LC} \quad \Rightarrow \quad Q \equiv \frac{\omega_0}{B} = \omega_0 CR$$

2^nd-order bandpass function

Center frequency gain $\left.\frac{V_o}{V_i}\right|_{\omega=\omega_0} = -g_m R$

Inductor Losses

- Inductor equivalent circuits



(a) (b)

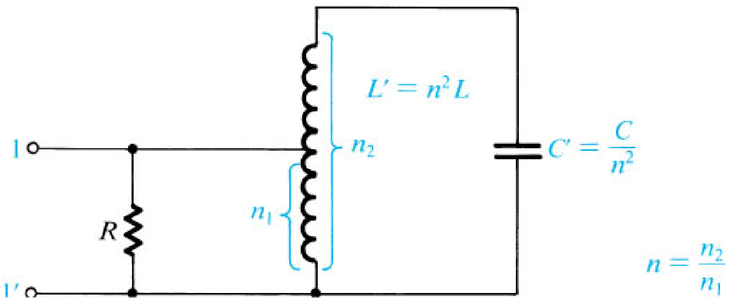
- The inductor Q factor: $Q_0 \equiv \dfrac{\omega_0 L}{r_s}$ (typically 50 ~ 200)
- Relationship between Q and R_p: $Y(j\omega_0) = \dfrac{1}{r_s + j\omega_0 L} = \dfrac{1}{j\omega_0 L}\dfrac{1 + j(1/Q_0)}{1 + (1/Q_0^2)}$

For Q >> 1, $Y(j\omega_0) \approx \dfrac{1}{j\omega_0 L}\left(1 + j\dfrac{1}{Q_0}\right) = \dfrac{1}{j\omega_0 L} + \dfrac{1}{\omega_0 L Q_0} = \dfrac{1}{j\omega_0 L} + \dfrac{1}{R_p}$

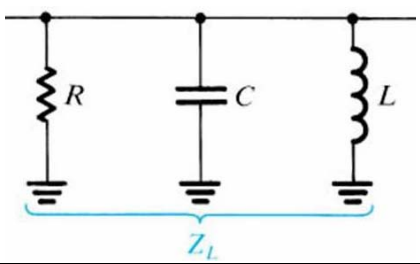
➡ $L_p = L\left(1 + \dfrac{1}{Q_0^2}\right) \approx L \quad R_p = \omega_0 L Q_0$ Fig.(a) Fig.(b)

Use of Transformers

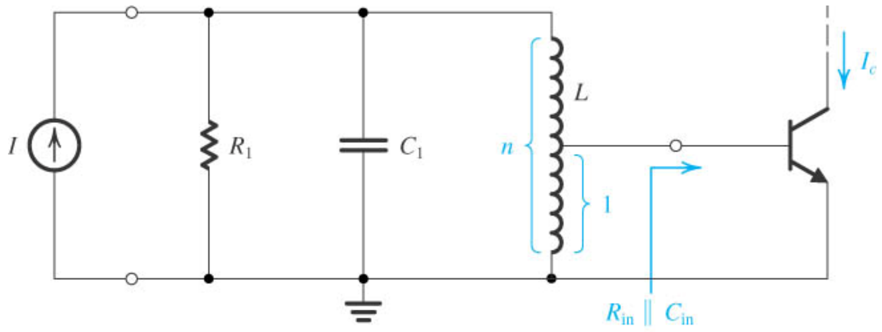
- Impedance transformer
- Examples
  - Allow using a high inductance and a smaller capacitance.



Equivalent circuit:

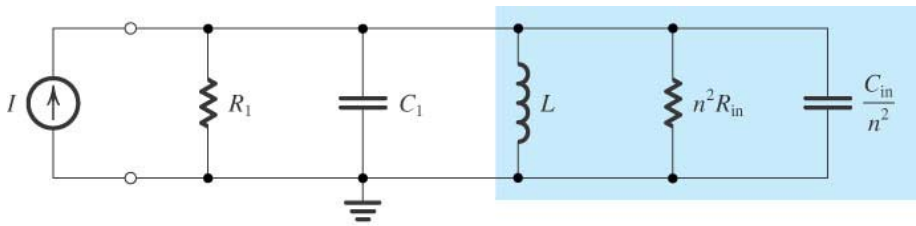


- Increase the effective input impedance of the second stage.



(a)

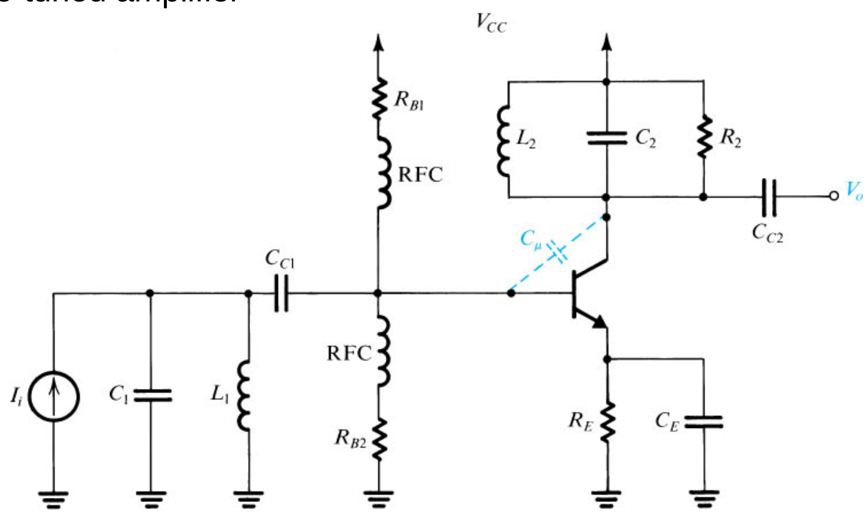
Equivalent circuit:



(b)

Amplifiers with Multiple Tuned Circuits

- Greater selectivity can be obtained.
- Double-tuned amplifier

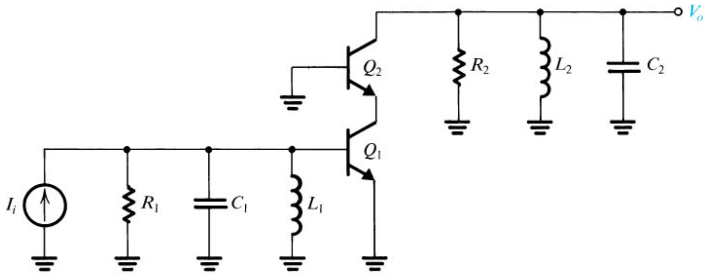


- Miller capacitance will cause
  1. detuning of the input circuit.
  2. skewing of the response of the input circuit.

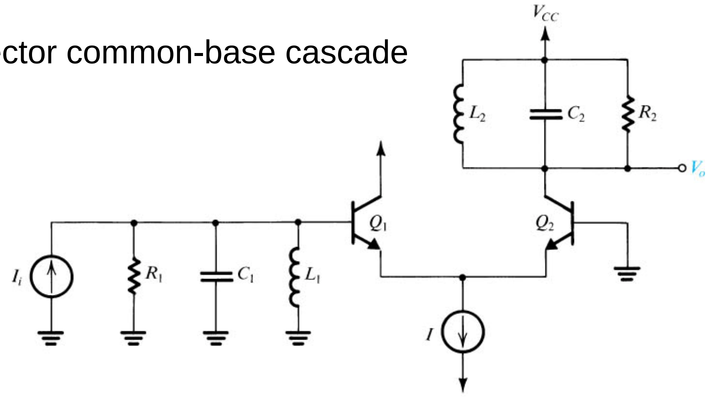
Cascode and CC-CB Cascade

Amplifier configurations do not suffer from the Miller effect.

- Cascode



- Common-collector common-base cascade

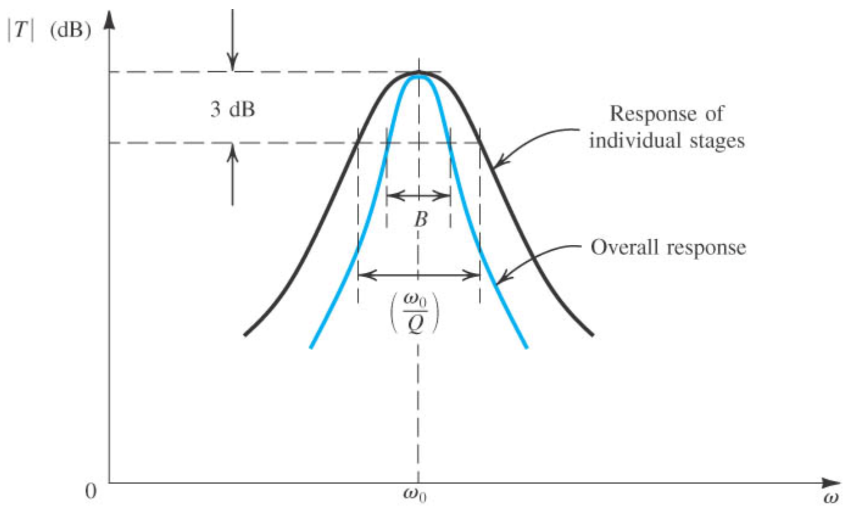


Synchronous Tuning

- N identical tuned circuits (do not interact)
- Bandwidth shrinkage

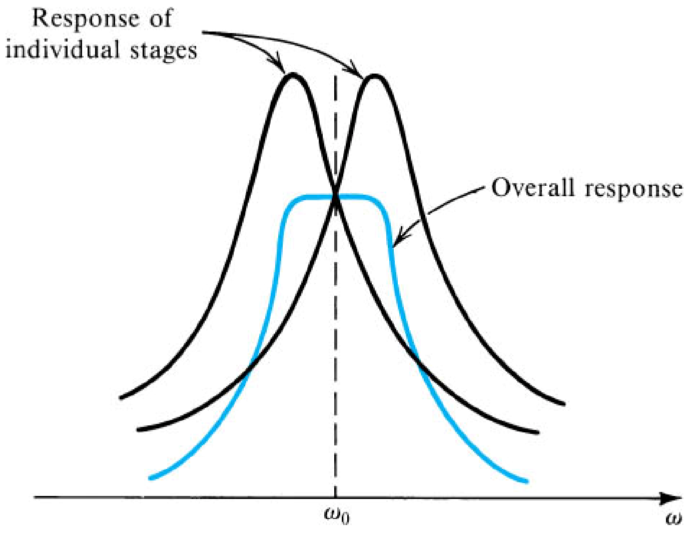
$$B = \frac{\omega_0}{Q}\sqrt{2^{\frac{1}{N}} - 1}$$

where $\sqrt{2^{1/N} - 1}$ is known as the bandwidth-shrinkage factor.



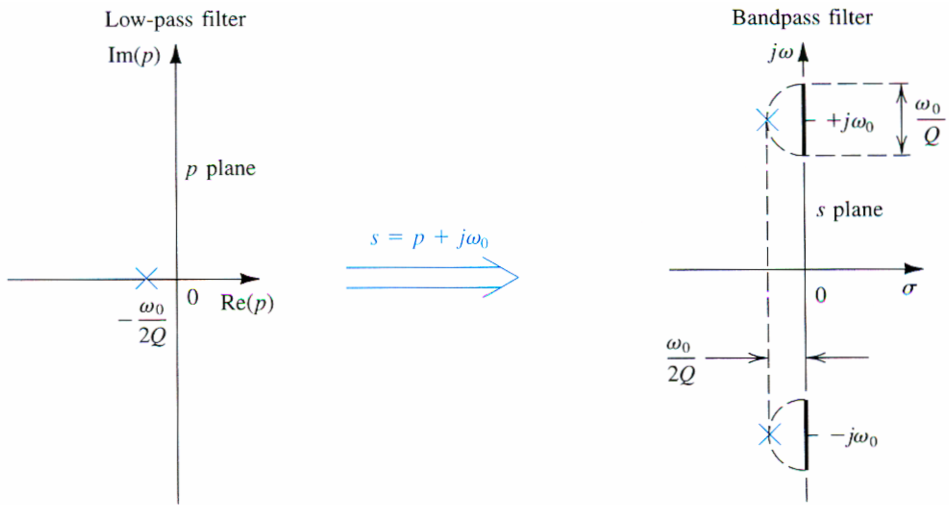
Stagger-Tuning

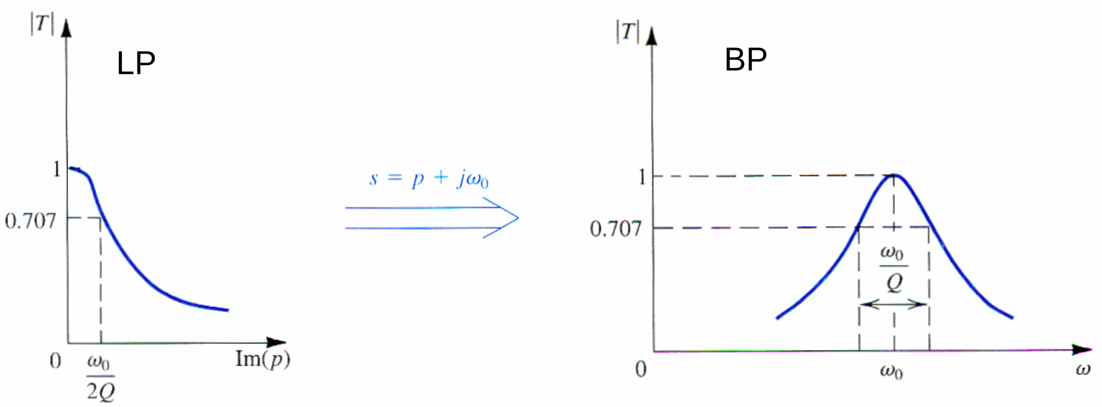
- Maximal flatness around ω_0 (Center frequency)



- The flat response can be obtained by transforming the response of a maximally flat (Butterworth) low-pass filter up the frequency axis to ω_0.

$$T(s)=\frac{a_1 s}{\left(s+\frac{\omega_0}{2Q}-j\omega_0\sqrt{1-\frac{1}{4Q^2}}\right)\left(s+\frac{\omega_0}{2Q}+j\omega_0\sqrt{1-\frac{1}{4Q^2}}\right)}$$





- For a narrow-band filter, $Q >> 1$, and s in the neighborhood of $+j\omega_0$, the transfer function of 2nd-order BP filter can be expressed in terms of its poles as

$$T(s) \approx \frac{a_1/2}{s+\omega_0/2Q - j\omega_0} = \frac{a_1/2}{(s-j\omega_0)+\omega_0/2Q} \quad \text{(Narrow-band approximation)}$$

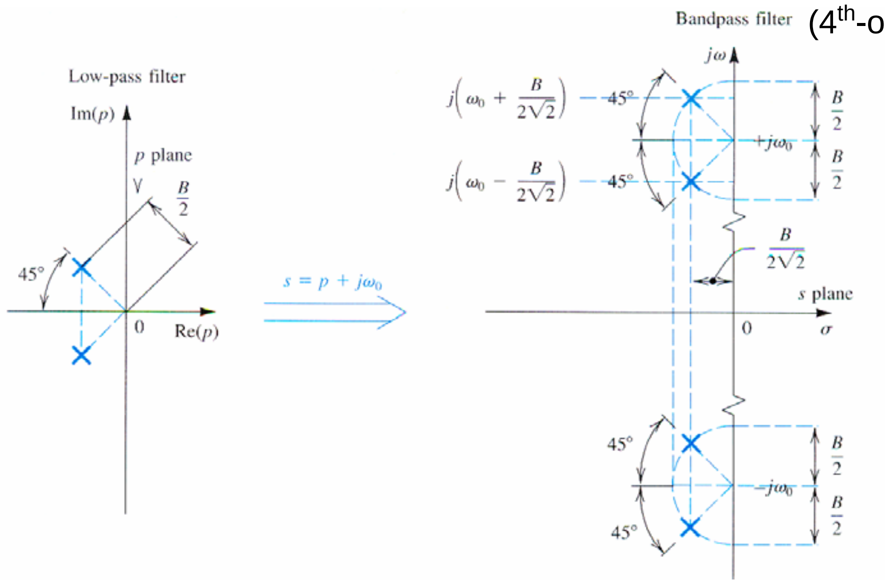
The magnitude response has a peak value of $a_1 Q/\omega_0$ at $\omega = \omega_0$.

- LP to BP transformation for narrow-band filter:

$$T(s) = \left.\frac{K}{p+\omega_0/2Q}\right|_{p=s-j\omega_0}$$

- Transform a maximally flat 2nd-order LP filter ($Q = 1/\sqrt{2}$) to obtain a maximally flat BP filter:

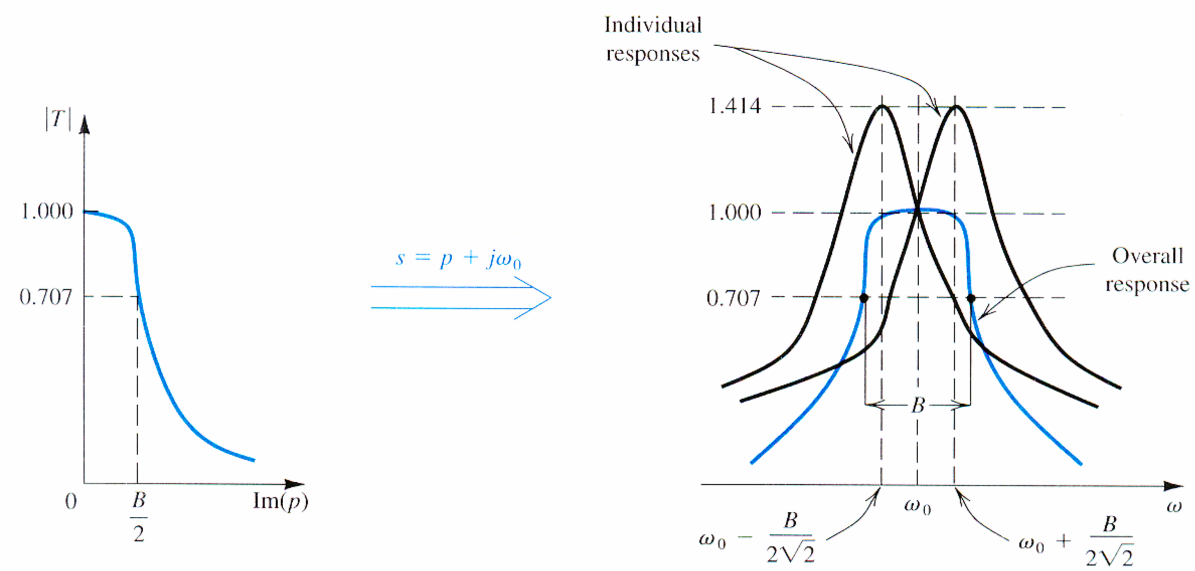
(4th-order)



$$\omega_{01} = \omega_0 + \frac{B}{2\sqrt{2}} \qquad B_1 = \frac{B}{\sqrt{2}} \qquad Q_1 \approx \frac{\sqrt{2}\omega_0}{B}$$

$$\omega_{02} = \omega_0 - \frac{B}{2\sqrt{2}} \qquad B_1 = \frac{B}{\sqrt{2}} \qquad Q_1 \approx \frac{\sqrt{2}\omega_0}{B}$$

Magnitude response:



Homework

- HW2
  Problems 49, 56, 65, 67,74