Econometrics
Assignment II

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Due date: Friday, 4. May 2018

Please submit your answers on Learn@WU to Assignment II by Friday, 4. May.

Part I: Introduction to Bayesian inference

Question 1: Bayesian Normal Linear Regression
Consider the house price data from Gary Koop's textbook "Bayesian Econometrics" companion website:
http://www.wiley.com/legacy/wileychi/koopbayesian/supp/HPRICE.TXT

- Reproduce the results in Table 3.1. in Koop (2003), p.57.
- Discuss how sensitive the results in Table 3.1. are to prior assumptions (prior parameter values, distribution,...)
- The marginal likelihood (ML) $p(Y)$ is a natural measure of model fit/adequacy and explicitly depends on the hyperparameters of the prior. Write down the expression of the marginal likelihood (see Eq. 3.34 in Koop, 2003) and evaluate different values for the prior variance on β, $\underline{V}_\beta = \alpha I$ on a grid of possible values $\alpha \in \{0.0001, 0.1, 0.5, 1, 2, 10\}$. What value of α maximizes the marginal likelihood? (HINT: Check what part of the ML does not depend on $\underline{V}_\beta$ and think about how dropping such a term might influence the ML)
- Write a simple Monte Carlo Integration scheme to simulate from the predictive density $p(y^*|y) = \int \int p(y^*|y, \beta, \sigma^2) p(\beta, \sigma^2|y) d\beta d\sigma^2$

Programs for Question 1: The Matlab scripts for this question are available at: Main MATLAB programs for Koop (2003). If you are not sure on how to approach these problems, try to port the MATLAB code into R.

Part II: Theoretical exercises
Question 1: Normal means model

Suppose that $y \sim \mathcal{N}(\mu, 1)$. Let us specify a normally distributed prior on μ, $\mu \sim \mathcal{N}(\underline{\mu}, \underline{\sigma}^2)$ where $\underline{\mu}$ and $\underline{\sigma}^2$ are prior mean and variance, respectively. Derive the closed form expression for the posterior distribution of $p(\mu|y)$ by applying Bayes theorem.

Question 2: Inverted Gamma distribution

Let $y_j|\sigma^2 \sim \mathcal{N}(0, \sigma^2)$ for $j = 1, \ldots, N$. Impose an inverted Gamma distribution on σ^2 (or equivalently a Gamma distribution on $h = 1/\sigma^2$) and derive the corresponding posterior by applying Bayes theorem.

Question 3: Posterior distributions 2

Let $y = (y_1, \ldots, y_N)'$ be an iid random sample, with

$$p(y_i|\theta) = \begin{cases} \theta^{y_i}(1-\theta)^{y_i} & \text{if } 0 \leq y_i \leq 1 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Derive the posterior for θ assuming a prior of the form

$$p(\theta) = \begin{cases} \frac{\Gamma(\underline{\alpha}+\underline{\beta})}{\Gamma(\underline{\alpha})\Gamma(\underline{\beta})}\theta^{\underline{\alpha}-1}(1-\theta)^{\underline{\beta}-1} & \text{if } 0 < \theta < 1 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

where $\Gamma(\bullet)$ denotes the Gamma function and $\underline{\alpha}$ and $\underline{\beta}$ denote hyperparameters.