Exercise sheet 1: Econometrics SS 2018

Please complete this assignment in groups up to three people.

General instructions: Use R (or any other statistics/computing environment, e.g. Matlab, Python) to carry out the exercises below. For the answer sheet, please do not simply include the corresponding R code but make sure that the corresponding outputs and solutions are i) well discussed and ii.) properly organized. Most importantly: Make sure to include all group members on top of the exercise sheet (including both names and student ID numbers)

Exercise 1) Monte Carlo-Simulations for linear models

Aim of this exercise is to check if the ols- command provided by R is working correctly. To achieve that we will use Monte Carlo simulations:

i.) The true model is given by: $Y = X \cdot \beta + \varepsilon = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \\ 1 & 6 \\ 1 & 7 \\ 1 & 8 \\ 1 & 9 \\ 1 & 10 \\ 1 & 11 \\ 1 & 12 \\ 1 & 13 \\ 1 & 14 \\ 1 & 15 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1/4 \end{pmatrix} + \varepsilon, \varepsilon\ N(0, I_{15} \cdot \sigma^2)$

First we will generate a vector labeled with „e“, which is drawn from $N(0, I_{15})$-distribution, where we use the command: „rnorm(15,0,1)“ to simulate 15 draws from a standard normal distribution. Given that vector we can calculate the vector “Y”. This vector Y is then inserted into the function „lm(Y~X)“.

Store the resulting betas of the lm function into a row vector (2 by 1).

Repeat the whole process (generating the “e” and “Y”-vector, inserting it into the lm function,…) 1000 times by using a loop and save in each loop the beta estimates. Therefore your resulting beta-estimate-vector (which we label „b_hat“) should have a size of 1000 by 2.

ii.) Calculate the average and the standard deviation of the vector „b_hat“. Furthermore, run summary(lm(Y~X)) for a given replication of the experiment. Which numbers (roughly) correspond in the “prt” output to „mean(b_hat)./std(b_hat)“?

iii.) Interpret one OLS-Output.

Exercise 2)

In this exercise we take a deeper look into the properties of densities. First, we will look at the normal distribution.

i.) We will now simulate a normally distributed vector with an expected value E[x]=-2 and variance Var[x]=9. The resulting vector should have a length of 100 and we will label the vector with „E_n“. The command „rnorm(1,0,1)“ creates a scalar which is standard-normally distributed (hence the scalar has an expected value of 0 and a variance of 1.). The following equations might be useful:

- Let $Y = a \cdot x + b \wedge x \sim N(0,1) \rightarrow E[Y] = E[a \cdot x + b] = a \cdot E[x] + E[b] = b$
- Let $Y = a \cdot x + b \wedge x \sim N(0,1) \rightarrow Var[Y] = Var[a \cdot x + b] = a^2 \cdot Var[x] = a^2$
- It follows that if $x \sim N(0,1)$ and $Y = a \cdot x + b, thatY \sim N(b, a^2)$

ii.) We will now look if the average estimator is converging if we increase the sample size:

a. First, create a new vector „M_E_n“, where the i-th entry is the average of the vectors „E_n“ first i-th entries.
b. In the next step, create a vector labeled with „N“. This vectors i-th entry has the value i.
c. We now want visualize the convergence of the mean estimator. For that we use the command „ts.plot(cbind(N,M_E_n))“. Can you spot the convergence of mean estimator given increasing vector length.
d. Try the same for a vector with length 10000.

Now we will look at the convergence of the average given a Cauchy- distribution:

iii.) We can create a Cauchy- distribution by dividing two standard- normal distributed random variables. Hence we can create a Cauchy-distributed random variable by using the following command: „rnorm(1,0,1)/rnorm(1,0,1)“. Create a vector of length 10000 and try to verify graphically, if the average estimator is converging. What can you see?

iv.) The Cauchy- distribution has the following density:

$$f(x)_X = \frac{1}{\pi} \frac{1}{1 + x^2} where - \infty < x < \infty$$

Try to illustrate this density graphically.