Modern Portfolio Theory

In-depth Concepts and Practical Applications

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Outline

Introduction

- Developed by Harry Markowitz in 1952.
- Optimizes portfolio returns for a given level of risk using diversification.
- Core concepts:
    - **Efficient Frontier: Set of optimal portfolios.
    - **Risk-Return Tradeoff: Balancing expected return and volatility.
    - **Systematic and Unsystematic Risk: Market-wide vs. asset-specific risks.

Understanding Alpha and Beta

Alpha (α):

- **Definition: A measure of an investment's performance relative to a benchmark index.
- **Formula:

$$\alpha = R_p - [R_f + \beta(R_m - R_f)]$$

  Where:
  - R_p: Portfolio return
  - R_f: Risk-free rate
  - R_m: Market return
  - β: Portfolio beta
- **Significance: Positive alpha indicates outperformance, negative alpha suggests underperformance.

Beta (β):

- **Definition: A measure of an asset's sensitivity to market movements.
- **Formula:

$$\beta = \frac{\mathrm{Cov}(R_a, R_m)}{\mathrm{Var}(R_m)}$$

Examples of Alpha and Beta

Example 1 (Alpha):

- Portfolio return $(R_p) = 12$
- Risk-free rate $(R_f) = 3$
- Market return $(R_m) = 10$
- Beta $(\beta) = 1.2$
- Calculate α:

$$\alpha = 12 - [3 + 1.2(10 - 3)] = 12 - 11.4 = 0.6\%$$

- **Interpretation: The portfolio outperformed the benchmark by 0.6

Example 2 (Beta):

- Asset return covariance with market $(\mathrm{Cov}(R_a, R_m)) = 0.015$
- Market variance $(\mathrm{Var}(R_m)) = 0.02$
- Calculate β:

$$\beta = \frac{0.015}{0.02} = 0.75$$

- **Interpretation: The asset is less volatile than the market.

Daily Returns: Expanded Discussion

Practical Use Cases:

- Analyzing short-term asset performance.
- Identifying trends in volatile markets.
- Comparing individual assets to market benchmarks.

Example Problem:

- Price Day 1 $= 150, PriceDay2 =$156
- Calculate daily return:

$$R_t = \frac{156 - 150}{150} \times 100 = 4\%$$

Cumulative Returns: Advanced Applications

Why Cumulative Returns Matter:

- Reflects overall portfolio growth over time.
- Important for long-term investment strategies.

Example Problem:

- Returns: $R_1 = 1\%, R_2 = 5\%, R_3 = -2\%$
- Calculate cumulative return:

$$R_{\text{cumulative}} = (1 + 0.01)(1 + 0.05)(1 - 0.02) - 1 = 4.86\%$$

Monte Carlo Simulation: Expanded Discussion

Practical Uses:

- Identifying efficient portfolios.
- Estimating potential losses (VaR).
- Visualizing risk-return tradeoffs.

Implementation: Use Python to simulate portfolios: "‘python import numpy as np weights = np.random.random(3) weights /= weights.sum() Normalize weights "‘ Visualize results to identify the efficient frontier.

Conclusion

- Modern Portfolio Theory offers a robust framework for risk and return optimization.
- Key metrics like alpha, beta, and returns enhance analysis.
- Monte Carlo simulations provide insights for decision-making in uncertainty.

Thank you!