

1. Bellman-Ford (verificar um ciclo negativo)

(início: vertice 0)

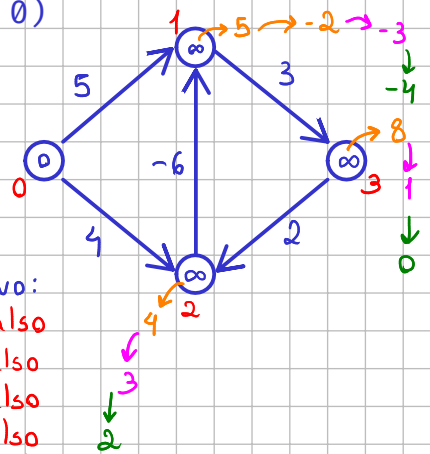
0-1:(5)

0-2:(4)

1-3:(3)

2-1:(-6)

3-2:(2)



Iterações	v[0]	v[1]	v[2]	v[3]
	0	∞	∞	∞
1	0	5	4	8
2	0	-2	4	1
3	0	-3	2	0

Teste Ciclo negativo:

① $-4 > 0 + 5$ Falso

$-4 > 2 - 6$ Falso

② $2 > 0 + 4$ Falso

$2 > 0 + 2$ Falso

③ $0 > -4 + 3$ Verdade

Logo, há ciclo negativo

2. Algoritmos de Bellman-Ford e Floyd-Warshall

(início: vertice 1)

1-5:(1)

1-2:(1)

2-3:(1)

2-4:(2)

3-4:(4)

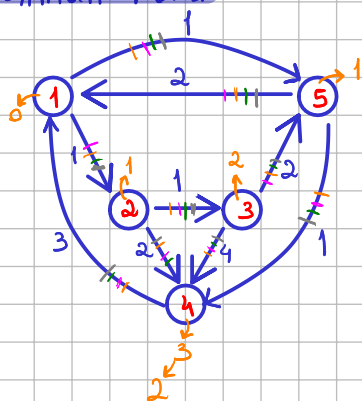
3-5:(2)

4-1:(3)

5-1:(2)

5-4:(1)

Bellman-Ford



Iterações	v[1]	v[2]	v[3]	v[4]	v[5]
	0	∞	∞	∞	∞
1	0	1	2	3	1
2	0	1	2	2	1
3	0	1	2	2	1
4	0	1	2	2	1

Teste ciclo neg.

① $0 > 2 + 3$ X

$0 > 1 + 2$ X

② $1 > 0 + 1$ X

③ $2 > 1 + 1$ X

④ $2 > 1 + 2$ X

$2 > 2 + 4$ X

$2 > 1 + 1$ X

Floyd-Warshall

$M^0 = \begin{bmatrix} 0 & 1 & \infty & \infty & 1 \\ \infty & 0 & 1 & 2 & \infty \\ \infty & \infty & 0 & 4 & 2 \\ 3 & \infty & \infty & 0 & \infty \\ 2 & \infty & \infty & 1 & 0 \end{bmatrix}$	$\pi^0 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 & 4 \\ 5 & 5 & 5 & 5 & 5 \end{bmatrix}$	$\rightarrow M^1 = \begin{bmatrix} 0 & 1 & \infty & \infty & 1 \\ \infty & 0 & 1 & 2 & \infty \\ \infty & \infty & 0 & 4 & 2 \\ 3 & 4 & \infty & 0 & 4 \\ 2 & 3 & \infty & 1 & 0 \end{bmatrix}$	$\pi^1 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 & 3 \\ 4 & 1 & 4 & 4 & 1 \\ 5 & 1 & 5 & 5 & 5 \end{bmatrix}$	$\rightarrow M^2 = \begin{bmatrix} 0 & 1 & 2 & 3 & 1 \\ \infty & 0 & 1 & 2 & \infty \\ \infty & \infty & 0 & 4 & 2 \\ 3 & 4 & 5 & 0 & 4 \\ 2 & 3 & 4 & 1 & 0 \end{bmatrix}$	$\pi^2 = \begin{bmatrix} 1 & 1 & 2 & 2 & 1 \\ 2 & 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 & 3 \\ 4 & 1 & 2 & 4 & 1 \\ 5 & 1 & 2 & 5 & 5 \end{bmatrix}$
$\rightarrow M^3 = \begin{bmatrix} 0 & 1 & 2 & 3 & 1 \\ \infty & 0 & 1 & 2 & 3 \\ \infty & \infty & 0 & 4 & 2 \\ 3 & 4 & 5 & 0 & 4 \\ 2 & 3 & 4 & 1 & 0 \end{bmatrix}$	$\pi^3 = \begin{bmatrix} 1 & 1 & 2 & 2 & 1 \\ 2 & 2 & 2 & 2 & 3 \\ 3 & 3 & 3 & 3 & 3 \\ 4 & 1 & 2 & 4 & 1 \\ 5 & 1 & 2 & 5 & 5 \end{bmatrix}$	$\rightarrow M^4 = \begin{bmatrix} 0 & 1 & 2 & 3 & 1 \\ 5 & 0 & 1 & 2 & 3 \\ 7 & 8 & 0 & 4 & 2 \\ 3 & 4 & 5 & 0 & 4 \\ 2 & 3 & 4 & 1 & 0 \end{bmatrix}$	$\pi^4 = \begin{bmatrix} 1 & 1 & 2 & 2 & 1 \\ 4 & 2 & 2 & 2 & 3 \\ 4 & 4 & 3 & 3 & 3 \\ 1 & 1 & 2 & 4 & 1 \\ 5 & 1 & 2 & 5 & 5 \end{bmatrix}$	$\rightarrow M^5 = \begin{bmatrix} 0 & 1 & 2 & 2 & 1 \\ 5 & 0 & 1 & 2 & 3 \\ 4 & 5 & 0 & 3 & 2 \\ 3 & 4 & 5 & 0 & 4 \\ 2 & 3 & 4 & 1 & 0 \end{bmatrix}$	$\pi^5 = \begin{bmatrix} 1 & 1 & 2 & 5 & 1 \\ 4 & 2 & 2 & 2 & 3 \\ 5 & 1 & 3 & 5 & 3 \\ 4 & 1 & 2 & 4 & 1 \\ 5 & 1 & 2 & 5 & 5 \end{bmatrix}$

matriz de distância:

$[[0, 1, 2, 2, 1]$

$[5, 0, 1, 2, 3]$

$[4, 5, 0, 3, 2]$

$[3, 4, 5, 0, 4]$

$[2, 3, 4, 1, 0]$

matriz de antecessores:

$[[None, 1, 2, 5, 1]$

$[4, None, 2, 2, 3]$

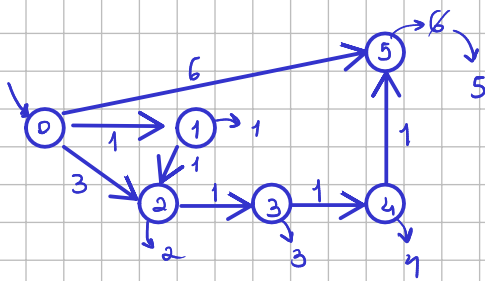
$[5, 1, None, 5, 3]$

$[4, 1, 2, None, 1]$

$[5, 1, 2, 5, None]$

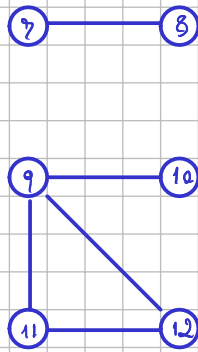
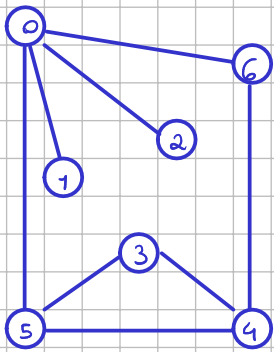
4. Algoritmo de Dijkstra (inicio: vertice 0)

0-1:(1)
1-2:(1)
2-3:(1)
3-4:(1)
4-5:(1)
0-2:(3)
0-5:(6)



S	0	1	2	3	4	5
\emptyset	0	∞	∞	∞	∞	∞
{0}		1	3			6
{0,1}			2			
{0,1,2}				3		
{0,1,2,3}					4	
{0,1,2,3,4}						5
{0,1,2,3,4,5}	0	1	2	3	4	5

Componentes conectadas



DFS(0)

marcar(0)

id[0] = 0

DFS(1)

marcar(1)

id[1] = 0

DFS(2)

marcar(2)

id[2] = 0

DFS(5)

marcar(5)

id[5] = 0

DFS(3)

marcar(3)

id[3] = 0

DFS(4)

marcar(4)

id[4] = 0

DFS(6)

marcar(6)

id[6] = 0

cont++

DFS(7)

marcar(7)

id[7] = 1

DFS(8)

marcar(8)

id[8] = 1

cont++

DFS(9)

marcar(9)

id[9] = 2

DFS(10)

marcar(10)

id[10] = 2

DFS(11)

marcar(11)

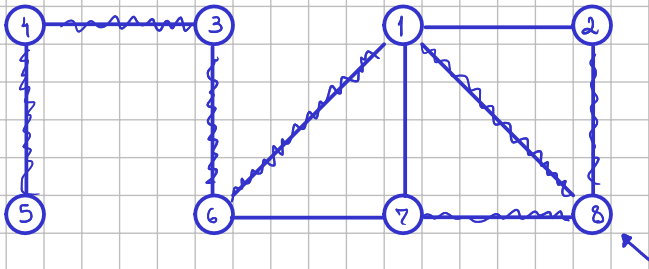
id[11] = 2

DFS(12)

marcar(12)

id[12] = 2

BFS



Q
5 3 6 7 2 1 8

8

$cor[8] = \text{cinza}$

$\pi[8] = \text{Null}$

$d[8] = 0$

$cor[1] = \text{cinza}$

$\pi[1] = 8$

$d[1] = d[8] + 1 = 1$

$cor[2] = \text{cinza}$

$\pi[2] = 8$

$d[2] = d[8] + 1 = 1$

$cor[7] = \text{cinza}$

$\pi[7] = 8$

$d[7] = d[8] + 1 = 1$

$cor[8] = \text{preto}$

1 $cor[6] = \text{cinza}$

$\pi[6] = 1$

$d[6] = d[1] + 1 = 2$

$cor[1] = \text{preto}$

2 $cor[2] = \text{preto}$

7 $cor[7] = \text{preto}$

6 $cor[3] = \text{cinza}$

$\pi[3] = 6$

$d[3] = d[6] + 1 = 3$

$cor[6] = \text{preto}$

3 $cor[4] = \text{cinza}$

$\pi[4] = 3$

$d[4] = d[3] + 1 = 4$

$cor[3] = \text{preto}$

4 $cor[5] = \text{cinza}$

$\pi[5] = 4$

$d[5] = d[4] + 1 = 5$

$cor[4] = \text{preto}$

5 $cor[5] = \text{preto}$

Fecho Transitivo - Warshall

$$W_0 = G = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$W_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$W_2 = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$W_3 = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$W_4 = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$W_0 = G = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$W_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

$$W_2 = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

$$W_3 = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

$$W_4 = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

$$W_5 = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

Fechamento transitivo com multiplicação de matrizes

$$G = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \times \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = G^2 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = G^3 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} =$$

$$= G^4 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = G^* = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$