1. Number Representation

Complete the table below. You must show your work to get full credit for an answer.

Decimal	Binary	Hexadecimal
-243	1111 1111 0000 1101	FF0D
728	0000 0010 1101 1000	02D8
13.4375	1101.0111(unsigned)	D.7
220	11011100 (unsigned)	DC
125.5	0111 1101.1000	7D.8
437	0001 1011 0101	1B5

243 → Binary: 0000 0000 1111 0011

	/ (division)	% (mod function)
243	121	1
121	60	1
60	30	0
30	15	0
15	7	1
7	3	1
3	1	1
1	0	1

-243 → Binary: 1111 1111 0000 1101

0	0	0	0	0	0	0	0	1	1	1	1	0	0	1	1	
1	1	1	1	1	1	1	1	0	0	0	0	1	1	0	0	+
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	

1	1	1	1	1	1	1	1	0	0	0	0	1	1	0	0	
															1	+
1	1	1	1	1	1	1	1	0	0	0	0	1	1	0	1	

-243 → Hexadecimal: FF0D

	Decimal Conversion (2^{n-1})	Hexadecimal Conversion
1111	$2^3 + 2^2 + 2^1 + 2^0 = 15$	F
1111	$2^3 + 2^2 + 2^1 + 2^0 = 15$	F
0000	0	0
1101	$2^3 + 2^2 + 2^0 = 13$	D

728 → Binary: 0000 0010 1101 1000

	/2 (division by 2)	%2 (mod by 2)
728	364	0
364	182	0
182	91	0
91	45	1
45	22	1
22	11	0
11	5	1
5	2	1
2	1	0
1	0	1

728 → Hexadecimal: 02D8

	Decimal Conversion (2^{n-1})	Hexadecimal Conversion
0000	0	0
0010	$2^1 = 2$	2
1101	$2^3 + 2^2 + 2^0 = 13$	D
1000	$2^3 = 8$	8

1101.0111 → Decimal: 13.4375

Binary	1	1	0	1	.	0	1	1	1
Decimal Conversion	2^3	2^2	0	2^0	.	0	2^{-2}	2^{-3}	2^{-4}
Decimal	8	4	0	1	.	0	0.25	0.125	0.0625

$8 + 4 + 1 + 0.25 + 0.125 + 0.0625 = 13.4375$

1101.0111 → Hexadecimal: D.7

	Decimal Conversion (2^{n-1})	Hexadecimal Conversion
1101	$2^3 + 2^2 + 2^0 = 13$	D
0111	$2^2 + 2^1 + 2^0 = 7$	7

11011100 → Decimal: 220

Binary	1	1	0	1	1	1	0	0
Decimal Conversion	2^7	2^6	0	2^4	2^3	2^2	0	0
Decimal	128	64	0	16	8	4	0	0

$128 + 64 + 16 + 8 + 4 = 220$

11011100→ Hexadecimal: DC

	Decimal Conversion (2^{n-1})	Hexadecimal Conversion
1101	$2^3 + 2^2 + 2^0 = 13$	D
1100	$2^3 + 2^2 = 12$	C

7D.8 → Decimal: 125.5

Hexadecimal	7	D	.	8
Decimal Conversion	$7 * 16^1$	$13 * 16^0$	.	$8 * 16^{-1}$
Decimal	112	13	.	0.5

$112 + 13 + 0.5 = 125.5$

7 → Binary = 0111

Decimal value of 7	/2 (division by 2)	%2 (mod of 2)
7	3	1
3	1	1
1	0	1

D → Binary = 1101

Decimal Value of D	/2 (division by 2)	%2 (mod of 2)
13	6	1
6	3	0
3	1	1
1	0	1

8 → Binary = 1000

Decimal Value of 8	/2 (division by 2)	%2 (mod of 2)
8	4	0
4	2	0
2	1	0
1	0	1

7D.8 → Binary: 0111 1101.1000

Hexadecimal	7	D	.	8
Decimal Conversion	0111	1101	.	1000

1B5 → Decimal: 437

Hexadecimal	1	B	5
Decimal Conversion	$1 * 16^2$	$11 * 16^1$	$5 * 16^0$
Decimal	256	176	5

$256 + 176 + 5 = 437$

1 → Binary = 0001

B → Binary = 1011

Decimal Value of B	/2 (division by 2)	%2 (mod of 2)
11	5	1
5	2	1
2	1	0
1	0	1

5 → Binary = 0101

Decimal Value of 5	/2 (division by 2)	%2 (mod of 2)
5	2	1
2	1	0
1	0	1

1B5 → Binary: 0001 1011 0101

Hexadecimal	1	B	5
Decimal Conversion	0001	1011	0101

2. Floating Point Number Representation

i. Represent -76.678595 as an IEEE single precision floating point number (binary and hexadecimal).

76 → Binary (8-bit representation): 0100 1100

	/2 (division by 2)	%2 (mod of 2)
76	38	0
38	19	0
19	9	1
9	4	1
4	2	0
2	1	0
1	0	1

0.678595 → Binary (24-bit representation): 1010 1101 1011 1000 0000

#	Decimal Value	Value multiplied by 2	Ones' Digit
1	.678595	1.35719	1
2	.35719	0.71438	0
3	.71438	1.42876	1
4	.42876	0.85752	0
5	.85752	1.71504	1
6	.71504	1.43008	1
7	.43008	0.86016	0
8	.86016	1.72032	1
9	.72032	1.44064	1
10	.44064	0.88128	0
11	.88128	1.76256	1
12	.76256	1.52512	1
13	.52512	1.05024	1
14	.05024	0.10048	0
15	.10048	0.20096	0
16	.20096	0.40192	0
17	.40192	0.80384	0

76.678595 → Binary (32-bit representation): 0100 1100.1010 1101 1011 1000 0000

76.678595 → Normalized (Rounded off to 23 bits after decimal):
$1.00110010101101101110000 \times 2^6$

Significand: 00110010101101101110000

Sign Bit: 1 (negative)

Exponent Value: 6

Exponent Code: Exponent Value + Bias = 6 +127 = 133

133 → Binary: 10000101

	/2 (division by 2)	%2 (mod of 2)
133	66	1
66	33	0
33	16	1
16	8	0
8	4	0
4	2	0
2	1	0
1	0	1

-76.678595 → IEEE single precision floating point number (Binary):
1 10000101 00110010101101101110000

1 1000 0101 00110010101101101110000 → Hexadecimal: C2995B70

	Decimal Conversion (2^{n-1})	Hexadecimal Conversion
1100	$2^3 + 2^2 = 12$	C
0010	$2^1 = 2$	2
1001	$2^3 + 2^0 = 9$	9
1001	$2^3 + 2^0 = 9$	9
0101	$2^2 + 2^0 = 5$	5
1011	$2^3+2^1 + 2^0 = 11$	B
0111	$2^2+2^1 + 2^0 = 7$	7
0000	0	0

76.678595 → IEEE single precision floating point number (Hexadecimal): 0xC2995B70

ii. Represent 19.459931 as an IEEE single precision floating point number (binary and hexadecimal).

19 → Binary (8-bit representation): 0001 0011

	/2 (division by 2)	%2 (mod of 2)
19	9	1
9	4	1
4	2	0
2	1	0
1	0	1

0.459931 → Binary (24-bit representation): 0111 0101 1011 1110 0000

#	Decimal Value	Value multiplied by 2	Ones' Digit
1	.459931	0.919862	0
2	.919862	1.839724	1
3	.839724	1.679448	1
4	.679448	1.358896	1
5	.358896	0.717792	0
6	.717792	1.435584	1
7	.435584	0.871168	0
8	.871168	1.742336	1
9	.742336	1.484672	1
10	.484672	0.969344	0
11	.969344	1.938688	1
12	.938688	1.877376	1
13	.877376	1.754752	1
14	.754752	1.509504	1
15	.509504	1.019008	1
16	.019008	0.038016	0
17	.038016	0.076032	0
18	.076032	0.152064	0
19	.152064	0.304128	0

19.459931 → Binary (32-bit representation): 0001 0011.0111 0101 1011 1110 0000

19.459931 → Normalized (Rounded off to 23 bits after decimal):
$1.001\ 1011\ 1010\ 1101\ 1111\ 0000 \times 2^4$

Significand: 00110111010110111110000

Sign Bit: 0 (positive)

Exponent Value: 4

Exponent Code: Exponent Value + Bias = 4 +127 = 131

131 → Binary: 10000011

	/2 (division by 2)	%2 (mod of 2)
131	65	1
65	32	1
32	16	0
16	8	0
8	4	0
4	2	0
2	1	0
1	0	1

76.678595 → IEEE single precision floating point number (Binary):
0 10000011 00110111010110111110000

0 10000011 00110010101101101110000→ Hexadecimal: 419BADF0

	Decimal Conversion (2^{n-1})	Hexadecimal Conversion
0100	$2^2 = 4$	4
0001	$2^0 = 1$	1
1001	$2^3 + 2^0 = 9$	9
1011	$2^3 + 2^1 + 2^0 = 11$	B
1010	$2^3 + 2^1 = 10$	A
1101	$2^3+2^2 + 2^0 = 13$	D
1111	$2^3 + 2^2+2^1 + 2^0 = 15$	F
0000	0	0

19.459931→ IEEE single precision floating point number (Hexadecimal): 0x419BADF0

3. Add the signed binary fixed point versions of the above two floating numbers using binary arithmetic and report your answer, showing your working.

To find the binary fixed point version of -76.678595, we can take the binary format of 76.678595 ($(1001100.10101101101110000)_2 \times 2^6)$), invert the bits, and add 0.00000000000000001

$$\begin{array}{l} 0110011.01010010010001111 \\ 0000000.00000000000000001 \\ \hline 0110011.01010010010010000 \end{array}$$

Thus, $(-76.678595)_{10} \approx (0110011.01010010010010000)_2$ And we already know that $(19.45993)_{10} = (10011.0111010110111110000)_2$

We can truncate two bits from the right of that number and add two 0 in fronts so that both numbers have the same number of bits before and after the binary point. Adding them together, we get

$$\begin{array}{l} 0110011.01010010010010000 \\ 0010011.01110101101111100 \\ \hline 1000110.11001000000001100 \end{array}$$

Thus, adding the signed binary fixed points versions of the two floating numbers we found earlier gives $\boxed{1000110.11001000000001100}$

3. Boolean Algebra

(a) $F(A,B,C,D) = (\overline{A} + B \cdot \overline{D}) \cdot (C \cdot B \cdot A + \overline{C} \cdot D)$

i. The truth table

A	B	C	D	$\overline{A}$	$\overline{C}$	$\overline{D}$	$B \cdot \overline{D}$	$\overline{A} + B \cdot \overline{D}$	$C \cdot B \cdot A$	$\overline{C} \cdot D$	$C \cdot B \cdot A + \overline{C} \cdot D$	$F(A,B,C,D)$	$\overline{F(A,B,C,D)}$
0	x	x	0	1	x	1	x	1	0	0	0	0	1
0	x	0	1	1	1	0	0	1	x	1	1	1	0
x	x	1	1	x	0	0	0	x	x	0	0	0	1
1	x	0	x	0	1	x	x	x	0	x	x	0	1
1	0	x	x	0	x	x	0	0	0	x	x	0	1
1	x	x	1	0	x	0	0	0	x	x	x	0	1
1	1	1	0	0	0	1	1	1	1	0	1	1	0

ii. A sum of products expression for F that is minimized.

$$F(A,B,C,D) = (\overline{A} + B \cdot \overline{D}) \cdot (C \cdot B \cdot A + \overline{C} \cdot D)$$

Using the distributive law

$$= (\overline{A} + B \cdot \overline{D})(C \cdot B \cdot A) + (\overline{C} \cdot D)(\overline{A} + B \cdot \overline{D})$$

Using the distributive law again

$$= \overline{A}(C \cdot B \cdot A) + (B \cdot \overline{D})(C \cdot B \cdot A) + \overline{A}(\overline{C} \cdot D) + (B \cdot \overline{D})(\overline{C} \cdot D)$$

Using the associative law

$$= \overline{A} \cdot C \cdot B \cdot A + B \cdot \overline{D} \cdot C \cdot B \cdot A + \overline{A} \cdot \overline{C} \cdot D + B \cdot \overline{D} \cdot \overline{C} \cdot D$$

Using the inverse law for the first and last term, and noticing that the second term contains B twice

$$= \overline{D} \cdot C \cdot B \cdot A + \overline{A} \cdot \overline{C} \cdot D$$

Using commutative law

$$\boxed{= A \cdot B \cdot C \cdot \overline{D} + \overline{A} \cdot \overline{C} \cdot D}$$

iii. A product of sums expression that F is minimized.

Let's first write $\overline{F(A,B,C,D)}$ as a sum-of-products, from the truth table

$$\overline{F(A,B,C,D)} = \overline{A} \cdot \overline{D} + C \cdot D + A \cdot \overline{C} + A \cdot \overline{B} + A \cdot D$$

Let's use the commutative law to reorganize (this will be useful for the next step)

$$= C \cdot D + A \cdot D + A \cdot \overline{C} + A \cdot \overline{B} + \overline{A} \cdot \overline{D}$$

Let's use the inverse law (addition) to multiply the second term by the first letter of the third term added to it's inverse

$$= C \cdot D + A \cdot D(C + \overline{C}) + A \cdot \overline{C} + A \cdot \overline{B} + \overline{A} \cdot \overline{D}$$

Let's use the distributive and associative laws

$$= C \cdot D + C \cdot D(A) + A \cdot \overline{C}(D) + A \cdot \overline{C} + A \cdot \overline{B} + \overline{A} \cdot \overline{D}$$

We can see that the second term is redundant, because if it is true, then the first term was already true. The same thing is true for the third term in regards of the fourth. We can thus remove them

$$= C \cdot D + A \cdot \overline{C} + A \cdot \overline{B} + \overline{A} \cdot \overline{D}$$

Let's negate both sides

$$\overline{\overline{F(A,B,C,D)}} = \overline{C \cdot D + A \cdot \overline{C} + A \cdot \overline{B} + \overline{A} \cdot \overline{D}}$$

Using De Morgan's law

$$= (\overline{C \cdot D}) \cdot (\overline{A \cdot \overline{C}}) \cdot (\overline{A \cdot \overline{B}}) \cdot (\overline{\overline{A} \cdot \overline{D}})$$

Using De Morgan's law again

$$= (\overline{C} + \overline{D}) \cdot (\overline{A} + \overline{\overline{C}}) \cdot (\overline{A} + \overline{\overline{B}}) \cdot (\overline{\overline{A}} + \overline{\overline{D}})$$

Removing the double NOTs

$$\boxed{F(A, B, C, D) = (\overline{C} + \overline{D}) \cdot (\overline{A} + C) \cdot (\overline{A} + B) \cdot (A + D)}$$

(b) $F(W, X, Y, Z) = \overline{(W + \overline{X})(Z\overline{Y} + X)}$

i. The truth table

W	X	Y	Z	$\overline{X}$	$\overline{Y}$	$W + \overline{X}$	$Z \cdot \overline{Y}$	$Z \cdot \overline{Y} + X$	$\overline{F(W, X, Y, Z)}$	$F(W, X, Y, Z)$
0	0	0	0	1	1	1	0	0	0	1
0	0	0	1	1	1	1	1	1	1	0
0	0	1	0	1	0	1	0	0	0	1
0	0	1	1	1	0	1	0	0	0	1
0	1	0	0	0	1	0	0	1	0	1
0	1	0	1	0	1	0	1	1	0	1
0	1	1	0	0	0	0	0	1	0	1
0	1	1	1	0	0	0	0	1	0	1
1	0	0	0	1	1	1	0	0	0	1
1	0	0	1	1	1	1	1	1	1	0
1	0	1	0	1	0	1	0	0	0	1
1	0	1	1	1	0	1	0	0	0	1
1	1	0	0	0	1	1	0	1	1	0
1	1	0	1	0	1	1	1	1	1	0
1	1	1	0	0	0	1	0	1	1	0
1	1	1	1	0	0	1	0	1	1	0

ii. A sum of products expression for F that is minimized.

$$F(W, X, Y, Z) = \overline{(W + \overline{X})(Z\overline{Y} + X)}$$

Using De Morgan's law

$$= (\overline{W + \overline{X}}) + (\overline{Z\overline{Y} + X})$$

Using De Morgan's law again

$$= (\overline{W} \cdot \overline{\overline{X}}) + (\overline{Z\overline{Y}} \cdot \overline{X})$$

Using De Morgan's law once again

$$= (\overline{W} \cdot \overline{\overline{X}}) + ((\overline{Z} + \overline{\overline{Y}}) \cdot \overline{X})$$

Removing double negatives

$$= (\overline{W} \cdot X) + ((\overline{Z} + Y) \cdot \overline{X})$$

Using the distributive law

$$\boxed{= (\overline{W} \cdot X) + (\overline{X} \cdot \overline{Z}) + (\overline{X} \cdot Y)}$$

iii. A product of sums expression that F is minimized.

Let's first write $\overline{F(W,X,Y,Z)}$ as a sum-of-products, from the truth table

$$\begin{aligned}
\overline{F(W,X,Y,Z)} = &\overline{W} \cdot \overline{X} \cdot \overline{Y} \cdot Z + W \cdot \overline{X} \cdot \overline{Y} \cdot Z \\
&+ W \cdot X \cdot \overline{Y} \cdot \overline{Z} + W \cdot X \cdot \overline{Y} \cdot Z \\
&+ W \cdot X \cdot Y \cdot \overline{Z} + W \cdot X \cdot Y \cdot Z
\end{aligned}$$

Using the distributive law

$$\begin{aligned}
= &(\overline{X} \cdot \overline{Y} \cdot Z)(\overline{W} + W) \\
&+ (W \cdot X \cdot \overline{Y})(\overline{Z} + Z) \\
&+ (W \cdot X \cdot Y)(\overline{Z} + Z)
\end{aligned}$$

Using the inverse law

$$= (\overline{X} \cdot \overline{Y} \cdot Z) + (W \cdot X \cdot \overline{Y}) + (W \cdot X \cdot Y)$$

Using the distributive law

$$= (\overline{X} \cdot \overline{Y} \cdot Z) + (W \cdot X \cdot)(\overline{Y} + \cdot Y)$$

Using the inverse law

$$= (\overline{X} \cdot \overline{Y} \cdot Z) + (W \cdot X)$$

Let's negate both sides

$$\overline{\overline{F(W,X,Y,Z)}} = \overline{(\overline{X} \cdot \overline{Y} \cdot Z) + (W \cdot X)}$$

Using De Morgan's law

$$= (\overline{\overline{X} \cdot \overline{Y} \cdot Z}) \cdot (\overline{W \cdot X})$$

Using De Morgan's law again

$$= (\overline{\overline{X}} + \overline{\overline{Y}} + \overline{Z}) \cdot (\overline{W} + \overline{X})$$

Removing double negatives

$$\boxed{F(W, X, Y, Z) = (X + Y + \overline{Z}) \cdot (\overline{W} + \overline{X})}$$

Question 4.1

X	Y	Z	W	$\overline{X}$	$\overline{Y}$	$\overline{X}+\overline{Y}$	$Z+W$	$(\overline{X}+\overline{Y}).(Z+W)$
0	0	0	0	1	1	1	0	0
0	0	0	1	1	1	1	1	1
0	0	1	0	1	1	1	1	1
0	0	1	1	1	1	1	1	1
0	1	0	0	1	0	1	0	0
0	1	0	1	1	0	1	1	1
0	1	1	0	1	0	1	1	1
0	1	1	1	1	0	1	1	1
1	0	0	0	0	1	1	0	0
1	0	0	1	0	1	1	1	1
1	0	1	0	0	1	1	1	1
1	0	1	1	0	1	1	1	1
1	1	0	0	0	0	0	0	0
1	1	0	1	0	0	0	1	0
1	1	1	0	0	0	0	1	0
1	1	1	1	0	0	0	1	0

$(\overline{X}+\overline{Y}).(Z+W).$

$Z(\overline{X}+\overline{Y}) + W(\overline{X}+\overline{Y}).$ Minimum 4 gates!

$Z(\overline{X.Y}) + W(\overline{X.Y}).$ Really BingO!

4.2 Parity Counter

You are asked to design a 4-to-3 parity counter. Such a circuit has 4 input bits, A, B, C, D and 3 output bits F2, F1, F0. The value that the circuit outputs is the number of its input bits that are set to 1. For example, if the input is 1010, then the circuit will output 010 (which is the binary representation of 2 as the unsigned 3-digit binary number F2F1F0). Similarly, if the input is 1111, then the output will be 100. Consider F2 as the highest order bit of the result and F0 as the lowest order bit.

i. Construct the truth table for this circuit

A	B	C	D	$F2$	$F1$	$F0$
0	0	0	0	0	0	0
0	0	0	1	0	0	1
0	0	1	0	0	0	1
0	0	1	1	0	1	0
0	1	0	0	0	0	1
0	1	0	1	0	1	0
0	1	1	0	0	1	0
0	1	1	1	0	1	1
1	0	0	0	0	0	1
1	0	0	1	0	1	0
1	0	1	0	0	1	0
1	0	1	1	0	1	1
1	1	0	0	0	1	0
1	1	0	1	0	1	1
1	1	1	0	0	1	1
1	1	1	1	1	0	0

ii. Write down the Boolean expressions for each of the three outputs in sum of products form. Now, simplify each expression using the laws of Boolean algebra to derive minimized sum-of-products forms.

$$F0 = (\bar{A}\cdot\bar{B}\cdot\bar{C}\cdot D) + (\bar{A}\cdot\bar{B}\cdot C\cdot\bar{D}) + (\bar{A}\cdot B\cdot\bar{C}\cdot\bar{D}) + (\bar{A}\cdot B\cdot C\cdot D) + (A\cdot\bar{B}\cdot\bar{C}\cdot\bar{D}) + (A\cdot\bar{B}\cdot C\cdot D) + (A\cdot B\cdot\bar{C}\cdot D) + (A\cdot B\cdot C\cdot\bar{D})$$

$$\begin{aligned}
F1 &= (A \cdot B \cdot C \cdot \bar{D}) + (A \cdot B \cdot \bar{C} \cdot \bar{D}) + (A \cdot \bar{B} \cdot C \cdot \bar{D}) + (A \cdot \bar{B} \cdot C \cdot D) + (A \cdot B \cdot \bar{C} \cdot D) \\
&\qquad + (A \cdot \bar{B} \cdot \bar{C} \cdot D) + (\bar{A} \cdot B \cdot \bar{C} \cdot D) + (\bar{A} \cdot B \cdot C \cdot D) + (\bar{A} \cdot B \cdot C \cdot \bar{D}) \\
&\qquad + (\bar{A} \cdot \bar{B} \cdot C \cdot D) \\
&= [(A \cdot B \cdot \bar{D}) \cdot (C + \bar{C})] + [(A \cdot \bar{B} \cdot C) \cdot (\bar{D} + D)] + [(A \cdot \bar{C} \cdot D) \cdot (\bar{B} + B)] \\
&\qquad + [(\bar{A} \cdot B \cdot D) \cdot (\bar{C} + C)] + (\bar{A} \cdot B \cdot C \cdot \bar{D}) + (\bar{A} \cdot \bar{B} \cdot C \cdot D) \\
&= [(A \cdot B \cdot \bar{D}) \cdot (1)] + [(A \cdot \bar{B} \cdot C) \cdot (1)] + [(A \cdot \bar{C} \cdot D) \cdot (1)] + [(\bar{A} \cdot B \cdot D) \cdot (1)] \\
&\qquad + (\bar{A} \cdot B \cdot C \cdot \bar{D}) + (\bar{A} \cdot \bar{B} \cdot C \cdot D) \\
&= (A \cdot B \cdot \bar{D}) + (A \cdot \bar{B} \cdot C) + (A \cdot \bar{C} \cdot D) + (\bar{A} \cdot B \cdot D) + (\bar{A} \cdot B \cdot C \cdot \bar{D}) \\
&\qquad + (\bar{A} \cdot \bar{B} \cdot C \cdot D) \\
&= (A \cdot B \cdot \bar{D}) + (A \cdot \bar{C} \cdot D) + [(\bar{A} \cdot B) \cdot (D + (C \cdot \bar{D})] + [(\bar{B} \cdot C) \cdot (A + \bar{A} \cdot D)] \\
&= (A \cdot B \cdot \bar{D}) + (A \cdot \bar{C} \cdot D) + \Big[(\bar{A} \cdot B) \cdot \big((D + C) \cdot (D + \bar{D})\big)\Big] \\
&\qquad + \Big[(\bar{B} \cdot C) \cdot \big((A + \bar{A}) \cdot (A + D)\big)\Big] \\
&= (A \cdot B \cdot \bar{D}) + (A \cdot \bar{C} \cdot D) + \Big[(\bar{A} \cdot B) \cdot \big((D + C) \cdot (1)\big)\Big] + \Big[(\bar{B} \cdot C) \cdot \big((1) \cdot (A + D)\big)\Big] \\
&= (A \cdot B \cdot \bar{D}) + (A \cdot \bar{C} \cdot D) + (\bar{A} \cdot B \cdot D) + (\bar{A} \cdot B \cdot C) + (A \cdot \bar{B} \cdot C) + (\bar{B} \cdot C \cdot D)
\end{aligned}$$

$$F2 = (A \cdot B \cdot C \cdot D)$$

4.3 Full-adders and half-adders

Reasoning

First, since there number is an unsigned 7-bit binary one, there can be between 0 and 7 ones in the number, that is between 000 and 111 in binary. We thus need three 1-bit outputs to show $F_2F_1F_0$. **The minimum number of full-adders that are needed is four** : The first three rightmost bits can be put in the first full-adder, with any of them as carry-in. The next three bits can be put in the second full-adder, once again with any of them as carry-in. Then, we make F_0 by taking the two sums of the previous full-adders as input for the third full-adder, and A_6 as carry-in. **The sum of this is $\boldsymbol{F_0}$**. The carry-out is used as carry-in for the fourth full-adder, and the carry-outs of the first two full-adders are used as input. **The sum of this is $\boldsymbol{F_1}$, and the carry-out of this is $\boldsymbol{F_2}$**