Assignment 1.

1. 243

= 121 × 2 + 1

60	1
30	0
15	0
7	1
3	1
1	1
0	1

0000 0000 1111 0011

invert ⇓

1111 1111 0000 1100

+ 1

$(1111\ 1111\ 0000\ 1101)_2$

Hexadecimal ⇓

FF0D

728

= 364 × 2 + 0

182	0
91	0
45	1
22	1
11	0
5	1
2	1
1	0
0	1

$(1011011000)_2$

Hexadecimal ⇓

2D8

1101.0111

$= 2^3 + 2^2 + 2^0 + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4}$

$= 13.4375$

$1101.0111 = (11010111)_2 \times 2^{-4}$

$= (D7)_{16} \times 16^{-1}$

$= (D.7)_{16}$

$(11011100)_2$

$= 2^7 + 2^6 + 2^4 + 2^3 + 2^2$

$= 128 + 64 + 16 + 8 + 4$

$= 220$

$(11011100)_2$

$= (DC)_{16}$

$7D.8$

$= (0111\,1101.1000)_2$

$= 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^0 + 2^{-1}$

$= 64 + 32 + 16 + 8 + 4 + 1 + 0.5$

$= 125.5$

$1B5$

$= (0001\,1011\,0101)_2$

1135

$= 16^2 + 16 \times 11 + 5$

$= 437$

2. 1. 76

$= 38 \times 2 + 0$

19	0
9	1
4	1
2	0
1	0
0	1

$(1001100)_2 . (678595)_{10}$

$= (10011001)_2 . (35719)_{10} \times 2^{-1}$

$= (100110010)_2 . (71438)_{10} \times 2^{-2}$

$= (1001100101)_2 . (42876)_{10} \times 2^{-3}$

$= (10011001010)_2 . (85752)_{10} \times 2^{-4}$

$\vdots$

$= (1.00110010101101101110001)_2 \times 2^{6}$

Exp = 6.

$6+127=133=(10000101)_2$

That gives:

1 10000101 00110010101 10110111 0001

sign Exp mantissa

$\Rightarrow$ 0x42995B71

2. $19.459931 = (1.0011011101011011110000)_2 \times 2^4$

0 10000011 0011011101011011110000

sign Exp mantissa

$\Rightarrow$ 0x419BADF0

3. $(1.0011011101011011110000)_2 \times 2^4$

$(1.00110010101 10110111 0001)_2 \times 2^6$

$=$ 100.1100101011011011 10001 $\times 2^6$

$$
\begin{array}{r}
100.110010101101101110001\,00 \\
-\quad 1.0011011101011011110000 \\
\hline
11.1001001101111111101010\,0 \\
\end{array}
$$

$\Rightarrow -76.678595 + 19.459931 = (1.1100100110111111101010 \times 2^5)$

$5+127=132$

$\Rightarrow$ 1 10000100 1100100110111111101010

$\Rightarrow$ 0xC264DFEA

3. a)

ii) $F = \bar{A}\cdot C\cdot B\cdot A + \bar{A}\cdot\bar{C}\cdot D$

$+ B\cdot\bar{D}\cdot C\cdot B\cdot A + B\cdot\bar{D}\cdot\bar{C}\cdot D$

$= \bar{A}\cdot\bar{C}\cdot D + A\cdot C\cdot\bar{D}\cdot B$

i)

A	B	C	D	F
0	X	0	1	1
1	1	1	0	1
⋮	⋮	⋮	⋮	0 ⋮ 0

iii) $\bar{F} = \overline{\bar{A}\cdot\bar{C}\cdot D + A\cdot C\cdot\bar{D}\cdot B}$

$= \overline{\bar{A}\cdot\bar{C}\cdot D} \cdot \overline{A\cdot C\cdot\bar{D}\cdot B}$

$= (A+C+\bar{D})\cdot(\bar{A}+\bar{C}+D+\bar{B})$

$F = \bar{\bar{F}} = \overline{(A+C+\bar{D})\cdot(\bar{A}+\bar{C}+D+\bar{B})}$

b). $F = \overline{W+\overline{X}} + \overline{Z\overline{Y}+X}$

$= \overline{W} \cdot X + \overline{Z\cdot\overline{Y}} \cdot \overline{X}$

$= \overline{W} \cdot X + (\overline{Z} + Y) \cdot \overline{X}$

ii) $= \overline{W} \cdot X + \overline{Z} \cdot \overline{X} + Y \cdot \overline{X}$

i)

W	X	Y	Z	F
0	1	X	X	1
X	0	X	0	1
X	0	1	X	1
⋮	⋮	⋮	⋮	0 ⋮ 0

iii) $\overline{F} = W \cdot X + W \cdot \overline{X} + \overline{W} \cdot \overline{X}$

$+ X \cdot Z + \overline{X} \cdot Z + X \cdot \overline{Z}$

$+ X \cdot Y + \overline{X} \cdot \overline{Y} + X \cdot \overline{Y}$

$F = \overline{\overline{F}} = (\overline{W}+\overline{X}) \cdot (\overline{W}+X) \cdot (W+X) \cdot$

$(\overline{X}+\overline{Z}) \cdot (X+\overline{Z}) \cdot (\overline{X}+Z) \cdot$

$(\overline{X}+\overline{Y}) \cdot (X+Y) \cdot (\overline{X}+Y)$

4.1

$$F = (\bar{x}+\bar{y})\cdot z + (\bar{x}+\bar{y})\cdot w$$

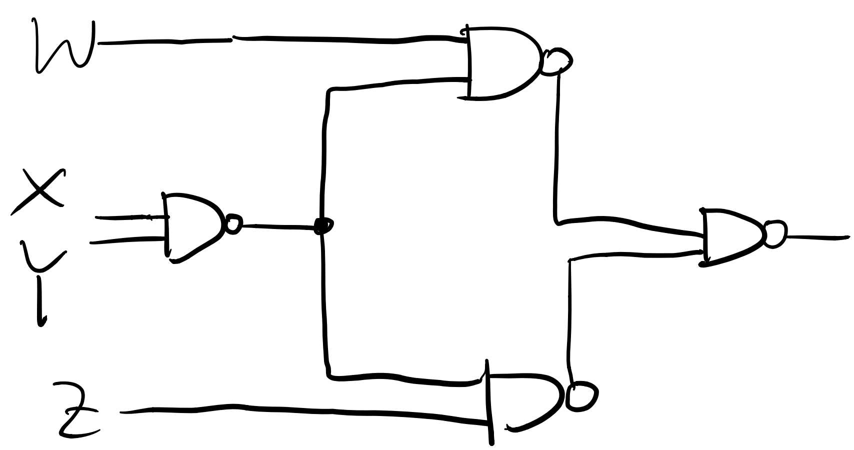
$$= \overline{\overline{(\bar{x}+\bar{y})\cdot z + (\bar{x}+\bar{y})\cdot w}}$$

$$= \overline{\overline{((\bar{x}+\bar{y})\cdot z)} \cdot \overline{((\bar{x}+\bar{y})\cdot w}}$$

$$= \overline{\left(\overline{(\bar{x}+\bar{y})} + \bar{w}\right)} + \overline{\left(\overline{(\bar{x}+\bar{y})} + \bar{z}\right)}$$

NAND required to express $\bar{?} + \bar{?}$: 1

$\Rightarrow$ in total, 4 NANDs are required.



4.2 i)

A	B	C	D	F_2	F_1	F_0
1	1	1	1	1	0	0
1	1	1	0	0	1	1
1	1	0	1	0	1	1
1	1	0	0	0	1	0
1	0	1	1	0	1	1
1	0	1	0	0	1	0
1	0	0	1	0	1	0
1	0	0	0	0	0	1
0	1	1	1	0	1	1
0	1	1	0	0	1	0
0	1	0	1	0	1	0
0	1	0	0	0	0	1
0	0	1	1	0	1	0
0	0	1	0	0	0	1
0	0	0	1	0	0	1
0	0	0	0	0	0	0

ii) $F_2 = A \cdot B \cdot C \cdot D$

$$F_1 = ABC\overline{D} + AB\overline{C}D + A\overline{B}CD + \overline{A}BCD + \overline{A}\overline{B}CD + \overline{A}B\overline{C}D + \overline{A}BC\overline{D} + A\overline{B}\overline{C}D + A\overline{B}C\overline{D} + AB\overline{C}\overline{D}$$

$$= \overline{B}CD + \overline{A}BD + BC\overline{D} + A\overline{C}D + A\overline{B}C\overline{D} + AB\overline{C}\overline{D}$$

$$F_0 = ABC\overline{D} + AB\overline{C}D + A\overline{B}CD + \overline{A}BCD + A\overline{B}\overline{C}\overline{D} + \overline{A}B\overline{C}\overline{D} + \overline{A}\overline{B}C\overline{D} + \overline{A}\overline{B}\overline{C}D$$

4.3 Full adder sums up to 3 digits while half adder only sums up to 2. So I chose full adder to minimize the total adder number.

Connect A_6, A_5, A_4 to Full Adder (FA) 1.
A_3, A_2, A_1 to FA2.

for FA1 and FA2, if S, there're 2 1s
if C, there's 1 1.
if S and C, there're 3 1s

Connect C_{FA1}, C_{FA2} and A_0 to FA3, making FA3 function just like FA1 or FA2

Connect S_{FA1}, S_{FA2} and S_{FA3} to FA4,
so C_{FA4} means 2 1s.
S_{FA4} means 4 1s

Finally, connect S_{FA4} to Four.
C_{FA4} to Two
C_{FA3} to One