Package Delivery:
A Mathematical Analysis of Path Selection

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Parcel Delivery Problem

The problem consists of a network of nodes with a number of packages, distinct transport costs between nodes and fixed costs for establishing hubs (distribution centres). What is the cheapest way to place hubs and connect all the nodes, such that all packages get delivered?

Main Methods

- ILP
- CNC heuristic: connect a node to the cheapest hub
- Exhaustive heuristic algorithm: brute-force all possible hub selections
- Greedy heuristic algorithm: iteratively opt for the optimal hub selection at each step

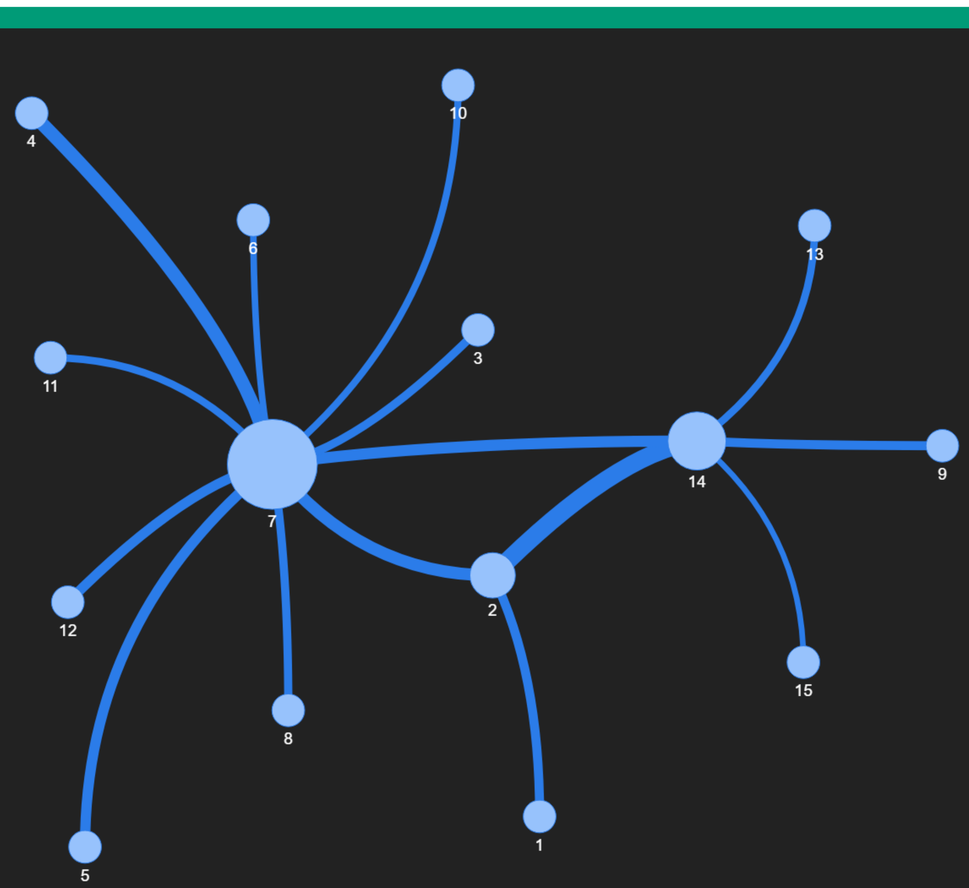

Figure 1: Results of applying the ILP on the large data set.

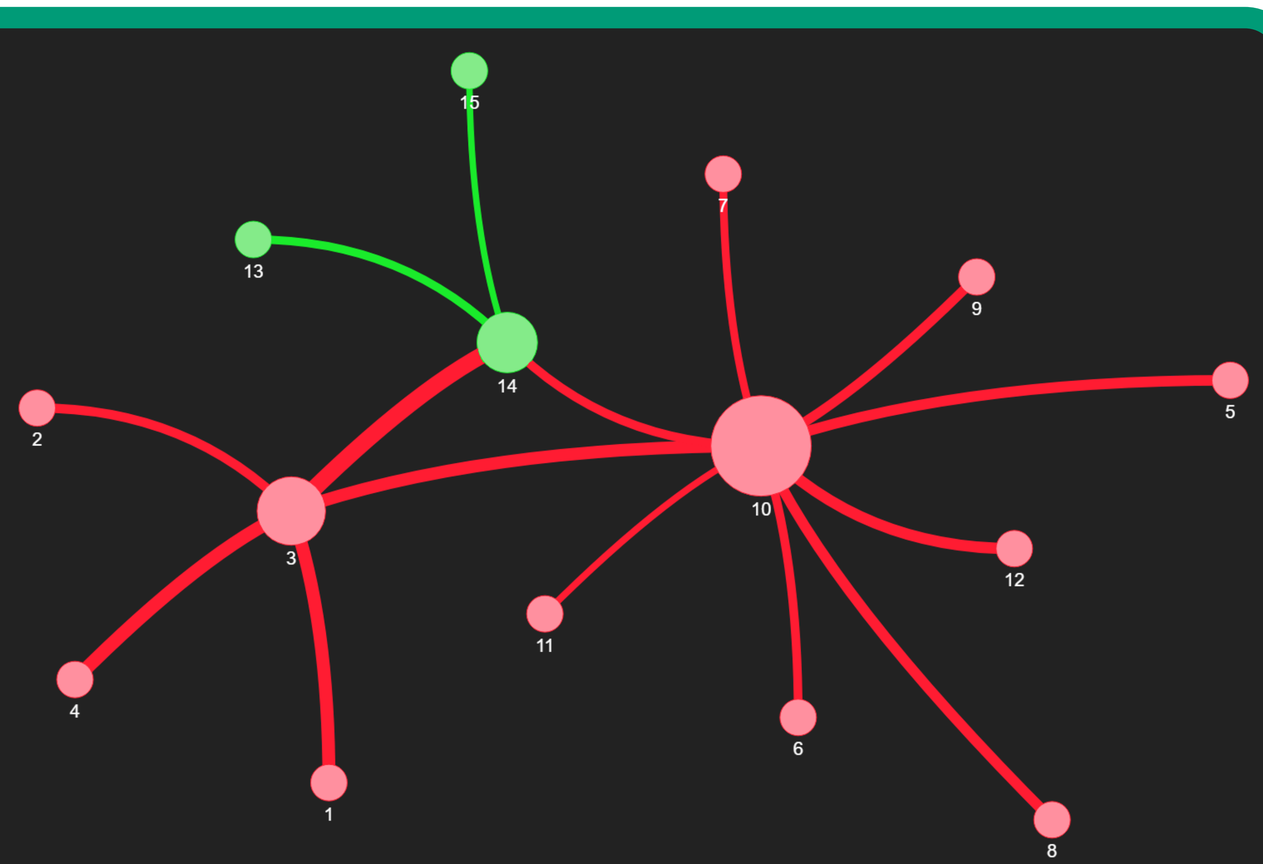

Figure 2: Results of applying the heuristic algorithms on the large data set. Differences with Figure 1 are highlighted in red and similarities in green.

Modelling Transportation Vehicles

Trucks (HTMs) transport packages between two assigned hubs with a discount ξ. Smaller vehicles like cargo bikes (NTMs) transport all packages between a hub and its directly linked non-hub nodes with the same discount of ξ.

Method	Minimum cost	Execution time
ILP Model (PuLP)	136.832	54 years*
ILP Model (Gurobi)	136.832	00:44:33
ILP Model (CPLEX)	136.832	17:37:35
Exhaustive Heuristic Algorithm	139.184	00:39:03
Greedy Heuristic Algorithm	139.184	00:00:04

Table 1: Results of applying the various algorithms on the large data set.

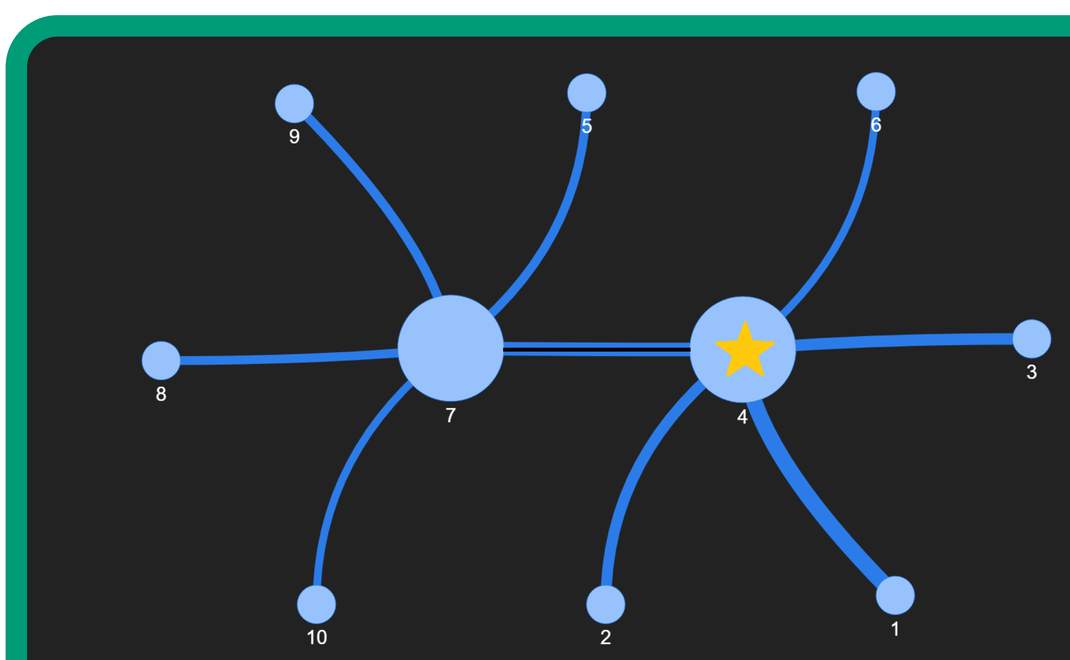

Figure 3: Results of applying the ILP on the small data set with a discount ξ of 15%. The star represents an NTM and the double line an HTM.

Conclusion

On average, the solution from the greedy heuristic algorithm is 3.18% more expensive for data sets of size 10 and 5.13% for data sets of size 15 compared to the ILP. With a discount ξ of 15% for the HTMs and NTMs, the total cost for the small data set is 221.358, 5.6% cheaper.

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