STAT 430 - Final Project

Yueqi Chen

10/12/2020

Study 1.1

Factors: X_1 (driving speed): 20-100 km/h and X_2 (engine temprature): level 1-3, from cold to hot.

Motivation: I notice that the engine temperature will continue changing (mostly increasing) while driving, so it's hard to control it at a fixed number and I would like to use levels to represent a range of temperature.

Study 1.2

Y_i, i=1,2,3, is the fuel consumption of each car A, B, and C. We could do the experiment in winter since that would be easier to have a cold engine. We do the experiment on the same day so that the starting engine temperature for each car will nor vary too much. Ask three drivers to drive the three cars with the same type of fuel (gasoline) separately, but they should not accelerate and should only brake at the end of each run. In other words, they cannot brake or accelerate whenever they want so for safety purpose, we will run the experiment on a closed circuit with the same road conditions.

Generally, it takes 5 to 15 minutes for engines to warm up while driving. We will randomly choose a car from A, B, and C. Then choose a random speed from 20, 100, and 60 and a random engine temperature from level 1 - 3. The driver will drive the car with such setting for 5 minutes and record the corresponding consumed fuel and driving distance to calculate fuel consumption.

Study 1.3

Nuisance Variables:

Holding constant:

- type of fuel (gasoline)
- driver's driving style (brake only once at the end and accelerate is not allowed)
- road conditions (same closed circuit for all runs)

Block factor: characteristics of vehicle (three types of car A, B, and C). For each car, we do the same experiments on them. Here, each car is a block so all effeccts are free from confounding. Within a block, runs are randomized.

Study 1.4

I choose a $2^2 + 4$CP design for each car.

Motivation: There are 2 factors with 2 levels so we choose 2^2 factorial design. Since I don't know whether 1_{st} or 2_{nd} order model fits, 4 center points are added for curvature test. After testing curvature, I can decide to whether it is necessary to augment with axial points. (If curvature, augment with 4 axial points and 2 more center points to the central composite design)

Study 1.5

StdOrder	RunOrder	CenterPt	Car	X1	X2
23	1	0	C	60	2
21	2	0	C	60	2
19	3	1	C	20	3
17	4	1	C	20	1
18	5	1	C	100	1
20	6	1	C	100	3
24	7	0	C	60	2
22	8	0	C	60	2
12	9	1	B	100	3
9	10	1	B	20	1
16	11	0	B	60	2
10	12	1	B	100	1
14	13	0	B	60	2
15	14	0	B	60	2
13	15	0	B	60	2
11	16	1	B	20	3
8	17	0	A	60	2
5	18	0	A	60	2
7	19	0	A	60	2
6	20	0	A	60	2
1	21	1	A	20	1
4	22	1	A	100	3
3	23	1	A	20	3
2	24	1	A	100	1

Study 1.6

Use regression model as the statistical model for the results (without curvature):

$$y_i = \beta_{i0} + \beta_{i1}x_{i1} + \beta_{i2}x_{i2} + \beta_{i3}x_{i1}x_{i2} + \epsilon_i$$

Study 2.1

$y_{ijk} = \mu + \tau_i + \beta_{j(i)} + \epsilon_{ijk}$, where i=1,. . . ,10, j=1,2,3, and k = 1,2.

Assumption: $\tau_i \sim N(0, \sigma_\tau^2)$, $\beta_{j(i)} \sim N(0, \sigma_\beta^2)$, and $\epsilon_{ijk} \sim N(0, \sigma^2)$.

Study 2.2

$$\sigma_\tau^2 = \frac{MS_A - MS_{B(A)}}{bn} = \frac{\frac{6\sum_{i=1}^{10}(\bar{y}_{i..}-\bar{y}_{...})^2}{9} - \frac{2\sum_{i=1}^{10}\sum_{j=1}^{3}(\bar{y}_{ij.}-\bar{y}_{i..})^2}{20}}{6} = \frac{\sum_{i=1}^{10}(\bar{y}_{i..} - \bar{y}_{...})^2}{9} - \frac{\sum_{i=1}^{10}\sum_{j=1}^{3}(\bar{y}_{ij.} - \bar{y}_{i..})^2}{60}$$

$$\sigma_\beta^2 = \frac{MS_{B(A)} - MS_E}{n} = \frac{\sum_{i=1}^{10}\sum_{j=1}^{3}(\bar{y}_{ij.} - \bar{y}_{i..})^2}{20} - \frac{\sum_{i=1}^{10}\sum_{j=1}^{3}\sum_{k=1}^{2}(y_{ijk} - \bar{y}_{ij.})^2}{60}$$

$$\sigma^2 = MS_E = \frac{\sum_{i=1}^{10}\sum_{j=1}^{3}\sum_{k=1}^{2}(y_{ijk} - \bar{y}_{ij.})^2}{30}$$

Study 2.3

σ_β^2 is the estimator we offer as a result for this question. If σ_β^2 is big, then the reflectivity of the paint over a test surface is heterogeneous. Conversely, it is homogeneous.

Study 2.4

With only one measurement in each sample area, we can't recognize the measurement error. If we do the replication and the results are different, then this will implies an measurement error.

Study 2.5

- For σ_τ^2, the DF is 10-1 = 9.
- For σ_β^2, the DF is 10(3-1) = 20.
- For σ^2, the DF is 10* 3*(2-1) = 30. As we can see, the DF of σ^2 is already much higher than the DF of the σ_τ^2 and σ_β^2. Minimizing the mean reflectivity in different areas within the same test surface is what we are interested in. Increasing the DF of σ_β^2 or σ_τ^2 can decrease $MS_{B(A)}$ and a smaller $MS_{B(A)}$ will be a more precise estimate of σ_β^2. Consequently, increasing the number of surfaces or the number of areas will be better and it is not useful to do more than 2 measurements per sample area.

Study 3.1

- The objective of the study is to optimize the composition of asphalt (water-to-cement ratio and coarse-aggregate size) so as to maximize its strength.
- Experiment is used to test whether our hypotheses are true or false. It is to study the effects of factors/treatments (X's) onto outcome responses (Y's). Knowing the relationship between X's and Y's can help us optimize products, processes, and treatment protocols.
- The results or conclusions we can get from an experiment are the model between X's and Y's, the corresponding estimation of the effects, hypothesis test results, and the optimation configurations of X's.
- A sequential experimentation strategy is to start with a small experiment and change the settings of experiments and models based on previous results.
- The method of steepest ascent is a strategy for finding a maximum by conducting a swquence of 2- and 3-level experiments. 1) if curvature, then a 2^{nd} order model is needed to locate the optimum. If no curvature, then we are on the slope of the mountain, not in the neighborhood of the optimum. 2) If we are on the slope, then we have to go along with the path of steepest ascent (perpendicular to the contour lines in a contour plot) in order to move towards to the optimum. 3) Along the path, we will see an increase in Y and we will stop the process when there is a decrease in Y and go back one step to the optimum.
- I recommend $X_1 = 2.048157$ and $X_2 = 4.387403$.
- The prediction value for y with settings above is 8.2225.

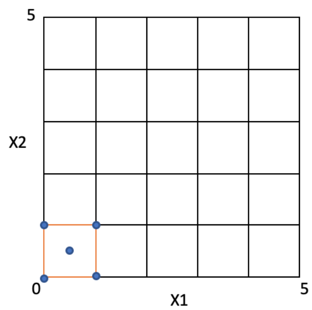
Study 3.2

Experiment #1

Design of the experiment
Factors: X_1 $(0.0-1.0)$, X_2 $(0.0-1.0)$, and X_3 (2.5)
2^2 (2 replicates) + 4cp, 12 runs total
Motivation: we don't know whether we're in the area of the top, so we start with a small experiment.

Table of runs

	X_1	X_2	X_3	Result
1	0.5	0.5	2.5	1.94
2	1	0	2.5	2.26
3	0	1	2.5	1.58
4	1	1	2.5	2.45
5	0	0	2.5	1.32
6	1	0	2.5	2.29
7	0.5	0.5	2.5	1.80
8	1	1	2.5	2.51
9	0	0	2.5	1.39
10	0.5	0.5	2.5	1.94
11	0	1	2.5	1.48
12	0.5	0.5	2.5	1.90

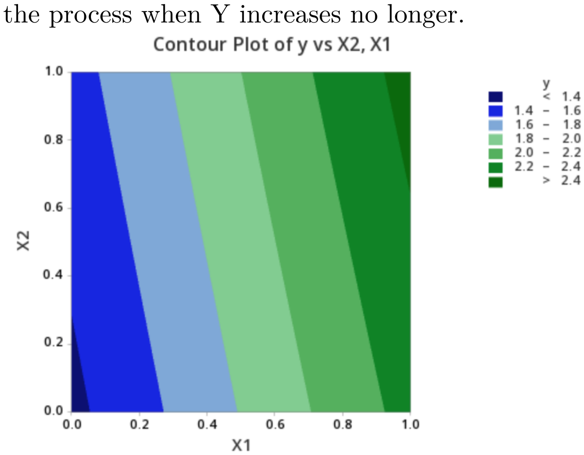


Conclusions from the experiment

- X_1X_2 interaction is not significant with a p-value of 0.721 while X_1 and X_2 are significant with p-values of 0.00 and 0.002 respectively.
- X_1 has the biggest effect.
- $R^2 = 98.76\%$ and no irregularities in the residuals. Good fit.
- No curvature since the p-value of curvature is 0.681. So we are on the slope of the mountain and we need to move the experimental region along the path of steepest ascent.

Next Step

- Based on the contour plot, determine the path of steepest ascent.
- stop the process when Y increases no longer.



Path of steepest ascent #1

Final model (in coded units)

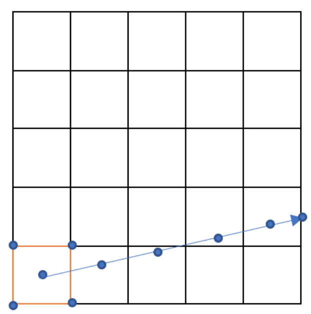
Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		1.9050	0.0149	128.19	0.000	
X1	0.9350	0.4675	0.0182	25.69	0.000	1.00
X2	0.1900	0.0950	0.0182	5.22	0.001	1.00

Points along path

- Start in the center point (0.5, 0.5)
- The biggest effect is X_1. We take a step size of 1.0.
- The step size of X_2 is $1 * \frac{0.095}{0.4675} = 0.2032086$, approximately 0.2.

	X_1	X_2	X_3	Result
1	0.5	0.5	2.5	1.94
2	1.5	0.7	2.5	2.87
3	2.5	0.9	2.5	3.78
4	3.5	1.1	2.5	4.42
5	4.5	1.3	2.5	4.81
6	5	1.5	2.5	4.46



Conclusions

The point (4.5, 1.3) is the best point so far. Either we are over the top now or we crossed a ridge.

Next step

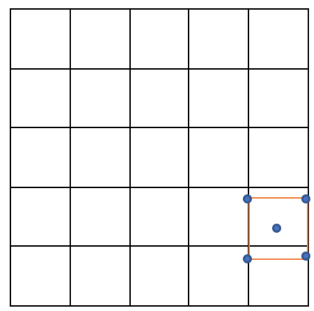
Do a new experiment with (4.5, 1.3) as the center point.

Experiment #2

Design of the experiment
Factors: X_1 $(4.0 - 5.0)$, X_2 $(0.8 - 1.8)$, and X_3 (2.5)
2^2 (2 replicates) + 4cp, 12 runs total
Motivation: we don't know whether we're over the top or crossed a ridge, so we do an experiment with (4.5, 1.3) as the center point.

Table of runs

	X_1	X_2	X_3	Result
1	4.5	1.3	2.5	4.73
2	5	1.8	2.5	4.52
3	5	1.8	2.5	4.53
4	4	1.8	2.5	5.18
5	4.5	1.3	2.5	4.75
6	4.5	1.3	2.5	4.83
7	4.5	1.3	2.5	4.89
8	5	0.8	2.5	4.42
9	5	0.8	2.5	4.36
10	4	0.8	2.5	4.77
11	4	0.8	2.5	4.73
12	4	1.8	2.5	5.07



Conclusions from the experiment

- X_1X_2 interaction is significant with a p-value of 0.025 while X_1 and X_2 are significant with p-values of 0.00 and 0.001 respectively.
- X_2 has the biggest effect.
- $R^2 = 96.27\%$ and no irregularities in the residuals. Good fit.
- Curvature is significant since its p-value is 0.027. We should try a 2^{nd} order model. Augment with axial points.

Next Step - CCD

- add axial points
- add two additional center points

Experiment #3

Design of the experiment
Factors: X_1 $(4.0 - 5.0)$, X_2 $(0.8 - 1.8)$, and X_3 (2.5)
Central Composite Design: 2^2 (2 replicates) + 4cp + 2cp + 4 axial points, 18 runs total
Motivation: we know there is a curvature, so we do a CCD experiment to fit in a 2^{nd} order model.

Table of runs

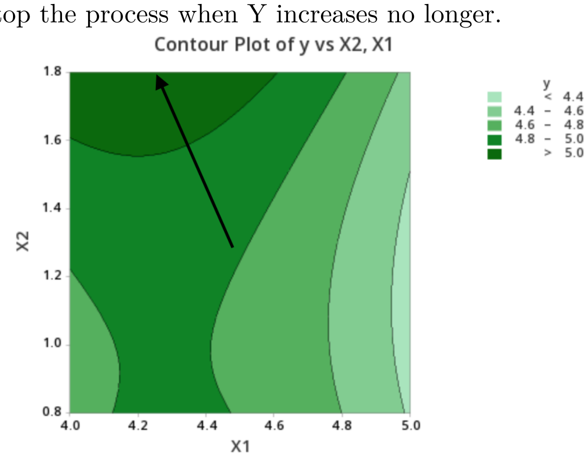
	X_1	X_2	X_3	Result
1	4.5	1.3	2.5	4.73
2	5	1.8	2.5	4.52
3	5	1.8	2.5	4.53
4	4	1.8	2.5	5.18
5	4.5	1.3	2.5	4.75
6	4.5	1.3	2.5	4.83
7	4.5	1.3	2.5	4.89
8	5	0.8	2.5	4.42
9	5	0.8	2.5	4.36
10	4	0.8	2.5	4.77
11	4	0.8	2.5	4.73
12	4	1.8	2.5	5.07
13	4.00	1.30	2.50	4.88
14	5.00	1.30	2.50	4.36
15	4.50	0.80	2.50	4.76
16	4.50	1.80	2.50	5.18
17	4.50	1.30	2.50	4.76
18	4.50	1.30	2.50	4.85

Conclusions from the experiment

- All the terms in this 2^{nd} order model are significant.
- $R^2 = 95.47\%$ and no irregularities in the residuals. Good fit.
- We are near the top. Check the contour plot.

Next Step

- Based on the contour plot, determine the path of steepest ascent.
- stop the process when Y increases no longer.

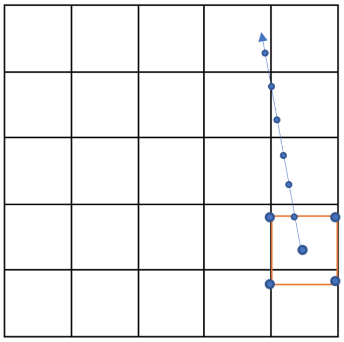


Path of steepest ascent #2

Final model (in coded units)

Coded Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	4.8134	0.0239	201.65	0.000	
X1	-0.2440	0.0198	-12.32	0.000	1.00
X2	0.1440	0.0198	7.27	0.000	1.00
X1*X1	-0.2285	0.0356	-6.42	0.000	1.43
X2*X2	0.1215	0.0356	3.41	0.005	1.43
X1*X2	-0.0600	0.0222	-2.71	0.019	1.00



Points along path

- Start in the center point (4.5, 1.3)
- Based on the contour plot, I decide a path where X_1 takes a step of 0.1 and X_2 takes a step of 0.5.

	X_1	X_2	X_3	Result
1	4.5	1.3	2.5	4.85
2	4.4	1.8	2.5	5.21
3	4.3	2.3	2.5	5.55
4	4.2	2.8	2.5	5.68
5	4.1	3.3	2.5	5.96
6	4	3.8	2.5	6.08
7	3.9	4.3	2.5	6.09
8	3.8	4.8	2.5	5.58

Conclusions

The point (3.9, 4.3) is the best point so far. We are closer to the top.

Next step

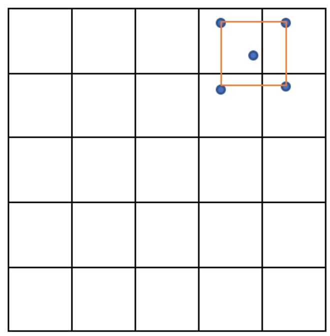
Do a new experiment with (3.9, 4.3) as the center point.

Experiment #4

Design of the experiment
Factors: X_1 $(3.4 - 4.4)$, X_2 $(3.8 - 4.8)$, and X_3 (2.5)
2^2 (2 replicates) + 4cp, 12 runs total
Motivation: we don't know whether we're over the top or crossed a ridge, so we do a factorial experiment with (3.9, 4.3) as the center point.

Table of runs

	X_1	X_2	X_3	Result
1	4.4	4.8	2.5	4.76
2	4.4	4.8	2.5	4.89
3	3.4	4.8	2.5	6.19
4	4.4	3.8	2.5	5.47
5	3.4	3.8	2.5	6.72
6	4.4	3.8	2.5	5.49
7	3.9	4.3	2.5	6.04
8	3.9	4.3	2.5	6.20
9	3.9	4.3	2.5	6.02
10	3.4	4.8	2.5	6.08
11	3.9	4.3	2.5	6.05
12	3.4	3.8	2.5	6.77



Conclusions from the experiment

- X_1X_2 is not significant with a p-value of 0.672. X_1 and X_2 are significant.
- $R^2 = 99.17\%$ and no irregularities in the residuals. Good fit.
- Curvature is very significant since its p-value is 0.000. We should try a 2^{nd} order model. Augment with axial points.

Next Step - CCD

- add axial points
- add two additional center points

Experiment #5

Design of the experiment
Factors: X_1 (3.4 – 4.4), X_2 (3.8 – 4.8), and X_3 (2.5)
Central Composite Design: 2^2 (2 replicates) + 4cp + 2cp + 4 axial points, 18 runs total
Motivation: we know there is a curvature, so we do a CCD experiment to fit in a 2^{nd} order model.

Table of runs

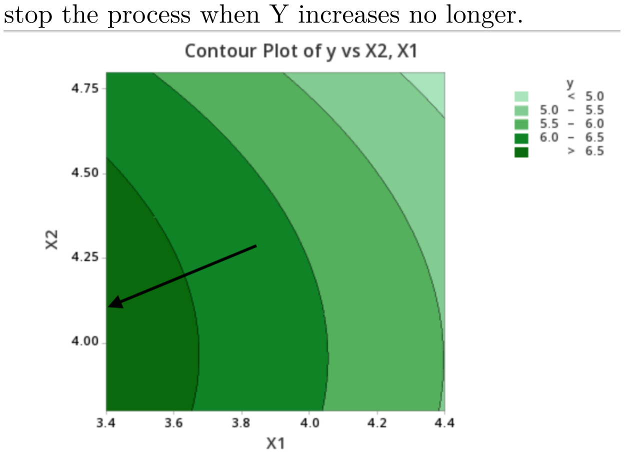
	X_1	X_2	X_3	Result
1	4.4	4.8	2.5	4.76
2	4.4	4.8	2.5	4.89
3	3.4	4.8	2.5	6.19
4	4.4	3.8	2.5	5.47
5	3.4	3.8	2.5	6.72
6	4.4	3.8	2.5	5.49
7	3.9	4.3	2.5	6.04
8	3.9	4.3	2.5	6.20
9	3.9	4.3	2.5	6.02
10	3.4	4.8	2.5	6.08
11	3.9	4.3	2.5	6.05
12	3.4	3.8	2.5	6.77
13	3.4	4.3	2.5	6.87
14	4.4	4.3	2.5	5.29
15	3.9	3.8	2.5	6.27
16	3.9	4.8	2.5	5.51
17	3.90	4.30	2.5	6.12
18	3.90	4.30	2.5	6.10

Conclusions from the experiment

- X_1^2 and X_1X_2 are not significant but X_2^2 is significant, so keep X_1X_2.
- X_1 and X_2 are significant.
- $R^2 = 98.6\%$ and no irregularities in the residuals. Good fit.
- We are near the top. Check the contour plot.

Next Step

- Based on the contour plot, determine the path of steepest ascent.
- stop the process when Y increases no longer.



Path of steepest ascent #3

Final model (in coded units)

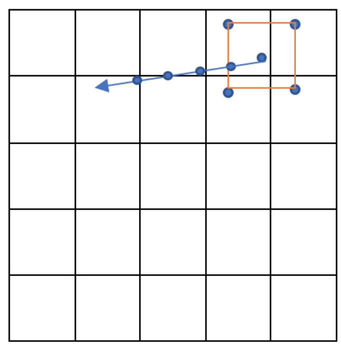
Coded Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	6.0863	0.0299	203.61	0.000	
X1	-0.6730	0.0267	-25.17	0.000	1.00
X2	-0.3290	0.0267	-12.31	0.000	1.00
X2*X2	-0.2712	0.0401	-6.76	0.000	1.00
X1*X2	-0.0113	0.0299	-0.38	0.713	1.00

Points along path

- Start in the center point (3.9, 4.3)
- Based on the contour plot, I decide a path where X_1 takes a step of -0.5 and X_2 takes a step of -0.1.

	X_1	X_2	X_3	Result
1	3.9	4.3	2.5	6.10
2	3.4	4.2	2.5	6.98
3	2.9	4.1	2.5	7.36
4	2.4	4	2.5	7.71
5	1.9	3.9	2.5	7.53



Conclusions

The point (2.4, 4) is the best point so far. We are closer to the top.

Next step

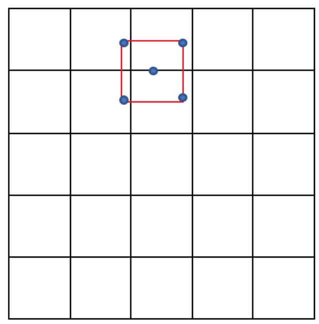
Do a new experiment with (2.4, 4) as the center point.

Experiment #6

Design of the experiment
Factors: X_1 $(1.9 - 2.9)$, X_2 $(3.5 - 4.5)$, and X_3 (2.5)
2^2 (2 replicates) + 4cp, 12 runs total
Motivation: we don't know whether we're over the top or crossed a ridge, so we do a factorial experiment with (2.4, 4) as the center point.

Table of runs

	X_1	X_2	X_3	Result
1	2.4	4	2.5	7.70
2	1.9	3.5	2.5	6.45
3	1.9	3.5	2.5	6.48
4	2.9	4.5	2.5	7.38
5	2.9	3.5	2.5	6.61
6	1.9	4.5	2.5	8.16
7	1.9	4.5	2.5	8.12
8	2.9	3.5	2.5	6.63
9	2.9	4.5	2.5	7.38
10	2.4	4	2.5	7.74
11	2.4	4	2.5	7.77
12	2.4	4	2.5	7.70



Conclusions from the experiment

- All terms are significant.
- $R^2 = 99.89\%$ and no irregularities in the residuals. Good fit.
- Curvature is very significant since its p-value is 0.000. We should try a 2^{nd} order model. Augment with axial points.

Next Step - CCD

- add axial points
- add two additional center points

Experiment #7

Design of the experiment
Factors: X_1 $(1.9 - 2.9)$, X_2 $(3.5 - 4.5)$, and X_3 (2.5)
Central Composite Design: 2^2 (2 replicates) + 4cp + 2cp + 4 axial points, 18 runs total
Motivation: we know there is a curvature, so we do a CCD experiment to fit in a 2^{nd} order model.

Table of runs

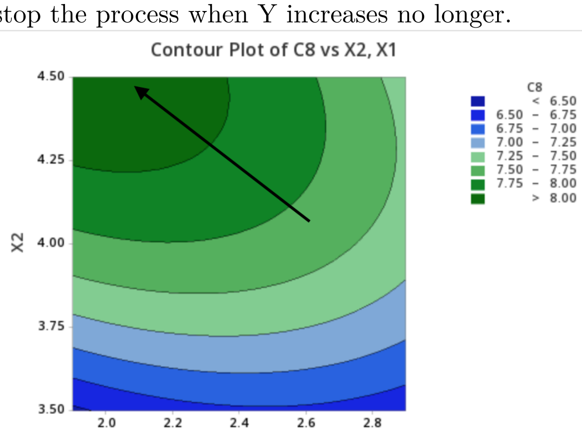
	X_1	X_2	X_3	Result
1	2.4	4	2.5	7.70
2	1.9	3.5	2.5	6.45
3	1.9	3.5	2.5	6.48
4	2.9	4.5	2.5	7.38
5	2.9	3.5	2.5	6.61
6	1.9	4.5	2.5	8.16
7	1.9	4.5	2.5	8.12
8	2.9	3.5	2.5	6.63
9	2.9	4.5	2.5	7.38
10	2.4	4	2.5	7.74
11	2.4	4	2.5	7.77
12	2.4	4	2.5	7.70
13	1.9	4	2.5	7.70
14	2.9	4	2.5	7.28
15	2.4	3.5	2.5	6.59
16	2.4	4.5	2.5	8.04
17	2.40	4.00	2.5	7.67
18	2.40	4.00	2.5	7.72

Conclusions from the experiment

- All terms are significant.
- $R^2 = 99.35\%$ and no irregularities in the residuals. Good fit.
- We are near the top. Check the contour plot.

Next Step

- Based on the contour plot, determine the path of steepest ascent. We notice that it's a ridge.
- stop the process when Y increases no longer.



Path of steepest ascent #4

Final model (in coded units)

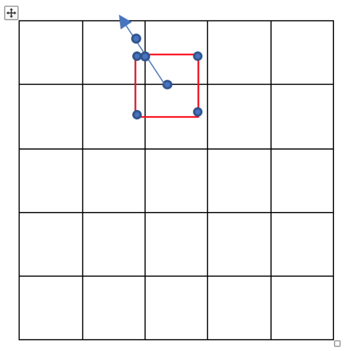
Coded Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	7.7085	0.0214	360.52	0.000	
X1	-0.1630	0.0177	-9.18	0.000	1.00
X2	0.6320	0.0177	35.61	0.000	1.00
X1*X1	-0.1942	0.0319	-6.09	0.000	1.43
X2*X2	-0.3692	0.0319	-11.58	0.000	1.43
X1*X2	-0.2287	0.0198	-11.53	0.000	1.00

Points along path

- Based on the contour plot, we start at (2, 4.5) since it's a ridge.
- I decide a path where X_1 takes a step of -0.1 and X_2 takes a step of 0.1.

	X_1	X_2	X_3	Result
1	2	4.5	2.5	8.20
2	1.9	4.6	2.5	8.13



Conclusions

The point (2, 4.5) is the best point so far. We are closer to the top.

Next step

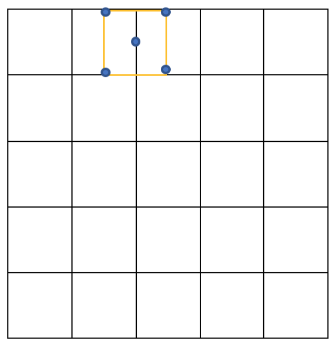
Do a new experiment with (2, 4.5) as the center point.

Experiment #8

Design of the experiment
Factors: X_1 $(1.5 - 2.5)$, X_2 $(4 - 5)$, and X_3 (2.5)
2^2 (2 replicates) + 4cp, 12 runs total
Motivation: we don't know whether we're over the top or crossed a ridge, so we do a factorial experiment with (2, 4.5) as the center point.

Table of runs

	X_1	X_2	X_3	Result
1	2	4.5	2.5	8.22
2	1.5	4	2.5	7.37
3	2	4.5	2.5	8.27
4	1.5	4	2.5	7.34
5	1.5	5	2.5	6.98
6	2.5	4	2.5	7.69
7	2.5	5	2.5	6.78
8	2.5	4	2.5	7.65
9	1.5	5	2.5	7.00
10	2	4.5	2.5	8.12
11	2	4.5	2.5	8.13
12	2.5	5	2.5	6.71



Conclusions from the experiment

- X_1 is not significant but X_1X_2 is significant, so keep X_1
- X_2 significant.
- $R^2 = 99.46\%$ and no irregularities in the residuals. Good fit.
- Curvature is very significant since its p-value is 0.000. We should try a 2^{nd} order model. Augment with axial points.

Next Step - CCD

- add axial points
- add two additional center points

Experiment #9

Design of the experiment
Factors: X_1 $(1.5 - 2.5)$, X_2 $(4 - 5)$, and X_3 (2.5)
Central Composite Design: 2^2 (2 replicates) + 4cp + 2cp + 4 axial points, 18 runs total
Motivation: we know there is a curvature, so we do a CCD experiment to fit in a 2^{nd} order model.

Table of runs

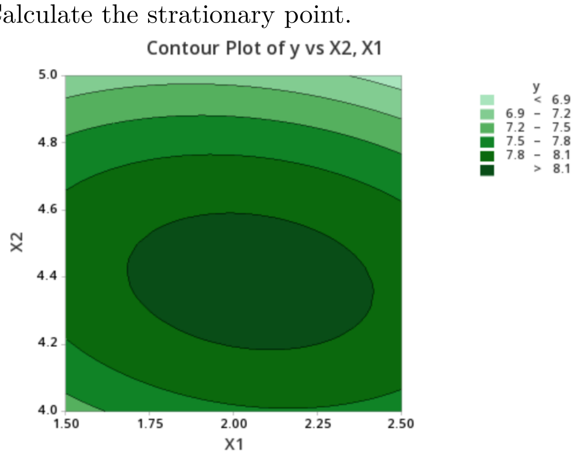
	X_1	X_2	X_3	Result
1	2	4.5	2.5	8.22
2	1.5	4	2.5	7.37
3	2	4.5	2.5	8.27
4	1.5	4	2.5	7.34
5	1.5	5	2.5	6.98
6	2.5	4	2.5	7.69
7	2.5	5	2.5	6.78
8	2.5	4	2.5	7.65
9	1.5	5	2.5	7.00
10	2	4.5	2.5	8.12
11	2	4.5	2.5	8.13
12	2.5	5	2.5	6.71
13	1.5	4.5	2.5	7.94
14	2.5	4.5	2.5	7.94
15	2	4	2.5	7.77
16	2	5	2.5	7.05
17	2.00	4.50	2.5	8.18
18	2.00	4.50	2.5	8.21

Conclusions from the experiment

- X_1 is not significant but X_1X_2 and X_1^2 are significant, so keep X_1.
- X_2 and X_2^2 are significant.
- $R^2 = 99.53\%$ and no irregularities in the residuals. Good fit.
- We are near the top. Check the contour plot.

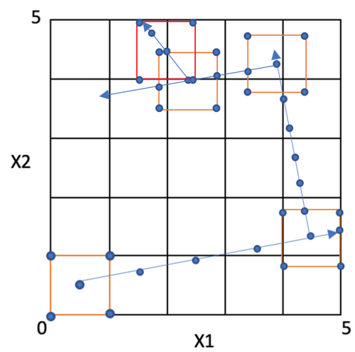
Next Step

- Based on the contour plot, we are at the top.
- Calculate the strationary point.



Study 3.3

Graph of X1 x X2 Space



Location of Stationary point

Regression Equation in Uncoded Units

y = -59.54 + 6.287 X1 + 27.954 X2 - 0.935 X1*X1 - 3.055 X2*X2 - 0.5600 X1*X2

From the equation, we observe that

$$b = \begin{pmatrix} 6.287 \\ 27.954 \end{pmatrix}, \quad B = \begin{pmatrix} -0.935 & -0.28 \\ -0.28 & -3.055 \end{pmatrix}$$

```r
## Calculation
b = as.matrix(c(6.287, 27.954))
B = cbind(c(-0.935, -0.28), c(-0.28, -3.055))
xs = (-1/2) * solve(B) %*% b
xs
```

```
##          [,1]
## [1,] 2.048157
## [2,] 4.387403
```

Then

$$x_s = -\frac{1}{2}B^{-1}b = \begin{pmatrix} 2.048 \\ 4.387 \end{pmatrix}$$

Thefeore, the optimal setting is when $X_1 = 2$ and $X_2 = 4.4$. The location of the stationary point is (2, 4.4).

Mean and Standard Deviation of y in the stationary point

Response	Fit	SE Fit	95% CI	95% PI
y	8.2225	0.0167	(8.1861, 8.2589)	(8.1193, 8.3256)

The mean of y in the stationary point is 8.2225 and the standard deviation is 0.0167.

Rescale

Similar to the idea of coded and uncoded units.

$$\frac{x_1 - 0.34}{0.06} = \frac{2.048157 - 2.5}{2.5}$$
$$x_1 = 0.3292$$

$$\frac{x_2 - 11}{1.5} = \frac{4.387403 - 2.5}{2.5}$$
$$x_2 = 12.1324$$

Therefore, the stationary point in the original scales is (0.3292, 12.1324).