A DERIVATION OF EQ. 11

Detailed derivation is as follows:

$$
\begin{aligned}
IP =& CP - SP \\
=& \left[\frac{1}{k}\sum_{i=1}^{k} DiffEntropy(H_i)\right] - DiffEntropy(H_\mu) \\
=& \frac{1}{k}\sum_{i=1}^{k}\left[\frac{rank(\Sigma_i)}{2} + \frac{rank(\Sigma_i)}{2}\ln(2\pi) + \frac{1}{2}\ln|\Sigma_i|\right] \\
& - \left[\frac{rank(\Sigma_\mu)}{2} + \frac{rank(\Sigma_\mu)}{2}\ln(2\pi) + \frac{1}{2}\ln|\Sigma_\mu|\right] \\
=& \frac{1}{k}\left[\sum_{i=1}^{k}\frac{rank(\Sigma_i)}{2} + \sum_{i=1}^{k}\frac{rank(\Sigma_i)}{2}\ln(2\pi) + \sum_{i=1}^{k}\frac{1}{2}\ln|\Sigma_i|\right] \\
& - \left[\frac{rank(\Sigma_\mu)}{2} + \frac{rank(\Sigma_\mu)}{2}\ln(2\pi) + \frac{1}{2}\ln|\Sigma_\mu|\right] \\
=& \frac{1}{2k}\left\{\sum_{i=1}^{k}\left[1+\ln(2\pi)\right] rank(\Sigma_i) + \sum_{i=1}^{k}\ln|\Sigma_i|\right\} \\
& - \frac{1}{2}\left\{\left[1+\ln(2\pi)\right] rank(\Sigma_\mu) + \ln|\Sigma_\mu|\right\} \\
=& \frac{1}{2}\left(\frac{1}{k}\sum_{i=1}^{k}\ln|\Sigma_i| - \ln|\Sigma_\mu|\right) + \frac{1+\ln(2\pi)}{2}\left[\frac{1}{k}\sum_{i=1}^{k} rank(\Sigma_i)\right. \\
& \left. - rank(\Sigma_\mu)\right]
\end{aligned}
$$

B CLUSTERING EVALUATION RESULTS UNDER A SINGLE CLUSTERING ALGORITHM

As before, we first present the statistical results of each clustering algorithm and report them in Table 11. Obviously, compared with other 13 indices, our IP index has significant advance in achieving the best results under each clustering algorithm. Specifically, for all 60 cases, our IP index can produce 33 best results, while the second top index SD only has 11 best results in K-Means. Our IP index can produce 38 best results, while the second top index S only has 12 best results in GMM. Our index can produce 28 best results, while the second top index SD only has 14 best results in AHC. Although CH can produce 42 best results while our index only has 20 best results in DBSCAN, our index is better than CH in the other three clustering algorithms. Finally, IP index outperforms other indices greatly with respect to different domains, i.e., text and image. Then the specific results of each clustering algorithm can been in B.1, B.2, B.3 and B.4, respectively.

B.1 The Clustering Results on K-Means

In this section, we only use K-Means to evaluate the clustering results. Since the random initialization nature of K-Means, the experimental results are averaged over five random runs for each validation index. The evaluation results based on external indices of ACC and ARI are shown in Table 12, 13, 14 and 15, the evaluation results in terms of NMI are shown in Table 16, 17, 18 and 19, where the best results are highlighted in bold. Moreover, the optimal k results each index select can be seen in Fig. 3, 4, 5, and 6. Obviously, for almost all cases, our IP index outperforms other indices and is close to the real k value represented in red dash line.

B.2 The Custering Results on GMM

In this section, we only use GMM to evaluate the clustering results. Since the random initialization nature of GMM, the experimental results are averaged over five random runs for each validation index. The evaluation results based on external indices of ACC and ARI are shown in 20, 21, 22 and 23, the evaluation results in terms of NMI are shown in Table 24, 25, 26 and 27 where the best results are highlighted in bold and the optimal k value each index selecet is provided in Fig. 7, 8, 9 and 10. Obviously, for almost all cases, our IP index outperforms other indices and is close to the real k value represented in red dash line.

B.3 The Custering Results on AHC

In this section, we only use AHC to evaluate the clustering results. The evaluation results based on external indices of ACC, ARI and NMI are shown in Table 28, 29, 30 and 31, where the best results are highlighted in bold. Moreover, the optimal k each index select, i.e. opt_k, and the true cluster number in each dataset, i.e. $dataset\text{-}k$, are provided in table. Obviously, for almost all cases, our IP index outperforms other indices and is close to the real k value.

B.4 The Custering Results on DBSCAN

In this section, we only use DBSCAN to evaluate the clustering results. The evaluation results based on external indices of ACC, ARI and NMI are shown in Table 32, 33, 34 and 35 , where the best results are highlighted in bold and Top-3 best results are underlined. Moreover, the optimal k each index select, i.e. opt_k, and the true cluster number in each dataset, i.e. $dataset\text{-}k$, are provided in table. Our index is either on par or slightly better than competing indices.

Table 11: Number of best clustering results based on counting over K-Means, GMM, AHC and DBSCAN

	SD	Dunn	I	XB	S	CH	DB	S_Dbw	CVNN	DCVI	DBCV	AIC	BIC	**IP**
K-Means														
Text datasets	4	2	0	0	1	0	0	1	0	2	1	0	0	**18**
Image datasets	7	1	0	6	2	4	5	4	3	0	6	0	0	**15**
All datasets	11	3	0	6	3	4	5	5	3	2	7	0	0	**33**
GMM														
Text datasets	1	0	0	0	3	0	0	0	0	0	0	7	0	**16**
Image datasets	2	0	0	2	9	0	2	1	0	0	2	0	0	**12**
All datasets	3	0	0	2	12	0	2	1	0	0	2	7	0	**38**
AHC														
Text datasets	2	8	0	0	2	0	2	3	0	5	5	0	0	**13**
Image datasets	12	0	0	3	4	3	3	4	3	1	6	0	0	**15**
All datasets	14	8	0	3	6	3	5	7	3	6	11	0	0	**28**
DBSCAN														
Text datasets	0	0	0	0	0	**18**	0	0	1	0	4	0	0	7
Image datasets	0	0	0	0	3	**24**	0	0	0	0	1	0	0	13
All datasets	0	0	0	0	3	**42**	0	0	1	0	5	0	0	20

Table 12: BERT based K-Means clustering results on five text datasets.

	SearchSnippets		**Biomedical**		**StackOverflow**		**WebofScience**		**Yahoo!Answers**	
	ACC	ARI	ACC	ARI	ACC	ARI	ACC	ARI	ACC	ARI
SD	27.98±10.23	17.91±6.21	29.31±0.01	14.64±0.01	8.42±0.01	0.90±0.00	40.66±0.01	24.36±0.02	19.97±0.03	2.59±0.02
Dunn	27.39±12.76	17.35±7.89	12.79±0.76	7.4±0.70	7.65±3.05	1.47±2.91	8.48±0.44	6.24±0.27	**25.98±8.58**	**10.18±3.11**
I	33.63±0.03	12.22±0.06	9.66±0.00	2.52±0.01	6.29±0.00	0.17±0.00	29.39±0.01	17.99±0.01	12.90±0.03	0.70±0.01
XB	17.22±3.37	11.19±2.12	29.31±0.01	14.64±0.01	8.42±0.01	0.90±0.00	29.39±0.01	17.99±0.01	12.90±0.03	0.70±0.01
S	10.05±0.59	6.66±0.24	9.66±0.00	2.52±0.01	8.42±0.01	0.90±0.00	29.39±0.01	17.99±0.01	12.90±0.03	0.70±0.01
CH	33.63±0.03	12.22±0.06	9.66±0.00	2.52±0.01	6.29±0.00	0.17±0.00	29.39±0.01	17.99±0.01	12.90±0.03	0.70±0.01
DB	10.22±0.39	6.67±0.18	29.31±0.01	14.64±0.01	10.96±0.01	2.41±0.01	40.66±0.01	24.36±0.02	16.52±0.02	1.89±0.01
S_Dbw	9.85±0.51	6.60±0.15	11.38±0.43	6.58±0.23	12.92±0.66	**7.05±0.23**	8.20±0.39	6.09±0.12	12.59±1.53	5.19±0.56
CVNN	33.63±0.03	12.22±0.06	9.66±0.00	2.52±0.01	6.29±0.00	0.17±0.00	29.39±0.01	17.99±0.01	12.90±0.03	0.70±0.01
DCVI	10.18±0.93	6.70±0.41	11.59±0.29	6.74±0.30	13.14±0.65	6.98±0.35	8.60±0.21	6.22±0.19	11.91±0.94	5.03±0.27
DBCV	9.73±0.59	6.45±0.13	11.87±0.81	6.90±0.46	12.58±0.27	6.87±0.08	8.49±0.20	6.26±0.26	11.93±0.76	4.99±0.25
AIC	33.63±0.03	12.22±0.06	9.66±0.00	2.52±0.01	6.29±0.00	0.17±0.00	29.39±0.01	17.99±0.01	12.90±0.03	0.70±0.01
BIC	33.63±0.03	12.22±0.06	9.66±0.00	2.52±0.01	6.29±0.00	0.17±0.00	29.39±0.01	17.99±0.01	12.90±0.03	0.70±0.01
IP	**54.18±5.71**	**33.58±4.04**	**31.48±0.55**	**15.63±0.12**	**15.98±0.14**	4.09±0.04	**47.93±2.40**	**31.90±1.75**	19.97±0.03	2.59±0.02

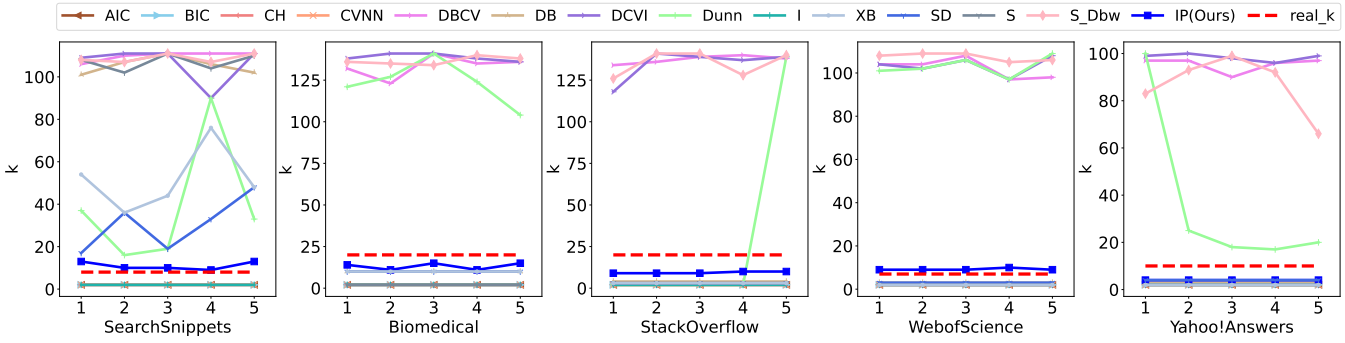

Figure 3: The optimal k value found by each index on BERT representations for text datasets.

Table 13: SimCSE based K-Means clustering results on five text datasets.

	SearchSnippets		Biomedical		StackOverflow		WebofScience		Yahoo!Answers	
	ACC	ARI	ACC	ARI	ACC	ARI	ACC	ARI	ACC	ARI
SD	10.77±0.80	7.35±0.69	21.06±8.20	9.47±6.35	61.42±2.06	46.07±0.99	**49.01±1.77**	**34.49±1.44**	17.29±1.72	8.74±0.83
Dunn	24.24±13.22	15.72±8.27	23.10±1.61	11.34±1.41	38.60±14.71	28.25±6.46	22.47±17.29	16.78±11.43	30.93±11.3	12.3±3.89
I	30.44±0.01	6.29±0.01	9.75±0.00	3.21±0.00	9.21±0.02	3.10±0.02	31.69±0.00	21.95±0.00	17.70±0.04	4.22±0.02
XB	11.25±1.54	7.49±0.94	17.39±0.01	6.63±0.01	60.40±3.08	45.69±1.47	43.59±0.00	28.19±0.01	11.22±1.66	5.49±0.97
S	10.52±0.62	6.82±0.44	9.75±0.00	3.21±0.00	63.36±1.61	**46.99±0.71**	31.69±0.00	21.95±0.00	17.70±0.04	4.22±0.02
CH	30.44±0.01	6.29±0.01	9.75±0.00	3.21±0.00	9.21±0.02	3.10±0.02	31.69±0.00	21.95±0.00	17.70±0.04	4.22±0.02
DB	10.82±0.67	6.98±0.19	14.76±1.06	8.93±0.54	63.25±3.11	46.71±1.33	43.59±0.00	28.19±0.01	10.26±0.25	4.89±0.10
S_Dbw	10.32±0.42	6.75±0.09	13.67±0.28	8.14±0.20	19.57±0.55	17.19±0.29	8.64±0.62	6.98±0.24	10.27±0.5	5.07±0.26
CVNN	30.44±0.01	6.29±0.01	17.39±0.01	6.63±0.01	9.21±0.02	3.10±0.02	31.69±0.00	21.95±0.00	17.70±0.04	4.22±0.02
DCVI	10.26±0.31	6.65±0.05	14.47±0.28	8.79±0.21	21.49±1.03	18.53±1.13	9.04±0.19	7.13±0.17	10.38±0.5	5.07±0.21
DBCV	10.04±0.22	6.67±0.20	13.99±0.74	8.44±0.52	21.71±1.54	18.57±1.72	9.32±0.48	7.17±0.23	10.41±0.46	5.06±0.24
AIC	30.44±0.01	6.29±0.01	9.75±0.00	3.21±0.00	9.21±0.02	3.10±0.02	31.69±0.00	21.95±0.00	17.70±0.04	4.22±0.02
BIC	30.44±0.01	6.29±0.01	9.75±0.00	3.21±0.00	9.21±0.02	3.10±0.02	31.69±0.00	21.95±0.00	17.70±0.04	4.22±0.02
IP	**50.33±0.82**	**30.74±0.99**	**37.96±0.19**	**20.06±0.10**	**68.74±1.38**	46.44±1.36	48.98±3.73	33.74±2.07	**37.03±0.15**	**14.75±0.16**

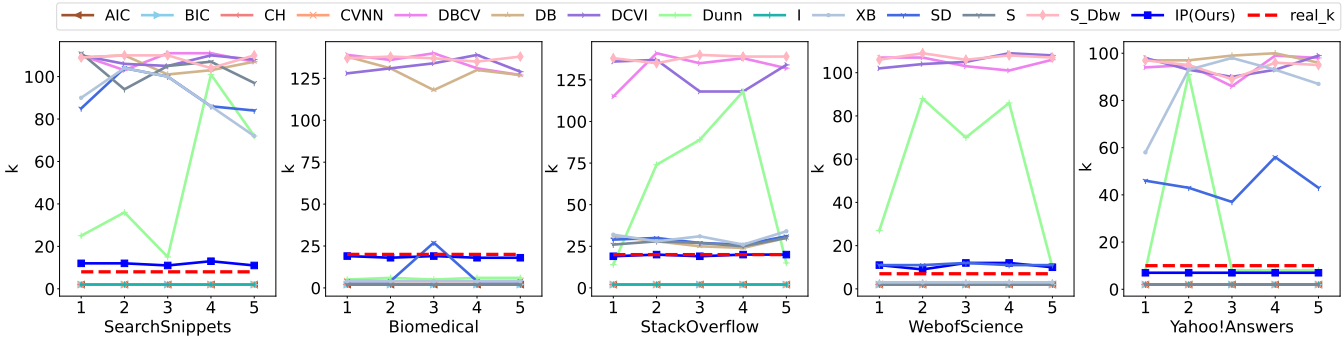

Figure 4: The optimal k value found by each index on SimCSE representations for text datasets.

Table 14: ViT based K-Means clustering results on five image datasets.

	CIFAR-10		MNIST		FashionMNIST		ImageNet-10		CINIC-10	
	ACC	ARI	ACC	ARI	ACC	ARI	ACC	ARI	ACC	ARI
SD	**76.07±0.96**	58.84±1.63	31.17±0.23	11.71±0.29	33.40±0.01	21.72±0.02	80.06±12.91	71.44±20.37	17.66±1.71	13.29±1.35
Dunn	23.10±4.86	10.71±3.00	29.19±0.89	10.64±0.31	35.25±2.63	20.33±0.51	57.15±38.37	51.34±42.69	29.28±3.54	20.03±2.69
I	19.55±0.00	8.52±0.00	15.87±0.00	1.90±0.00	19.81±0.00	11.31±0.00	23.68±5.32	9.90±4.10	19.02±0.00	7.60±0.00
XB	23.10±4.86	10.71±3.00	28.78±0.03	10.5±0.03	19.81±0.00	11.31±0.00	92.21±5.75	89.12±6.89	19.02±0.00	7.60±0.00
S	68.07±2.29	57.42±0.96	15.87±0.00	1.90±0.00	30.14±0.00	18.93±0.00	89.13±2.84	88.55±1.85	53.86±1.07	**41.14±0.69**
CH	19.55±0.00	8.52±0.00	15.87±0.00	1.90±0.00	19.81±0.00	11.31±0.00	92.21±5.75	89.12±6.89	19.02±0.00	7.60±0.00
DB	23.10±4.86	10.71±3.00	30.82±0.07	11.44±0.40	19.81±0.00	11.31±0.00	68.95±40.62	60.95±48.98	19.02±0.00	7.60±0.00
S_Dbw	18.60±4.88	16.85±4.55	7.50±0.21	4.41±0.07	8.92±0.24	6.16±0.07	66.31±16.12	50.61±24.24	15.33±1.65	11.99±1.1
CVNN	23.10±4.86	10.71±3.00	15.87±0.00	1.90±0.00	19.81±0.00	11.31±0.00	52.64±8.71	31.27±9.35	19.02±0.00	7.60±0.00
DCVI	19.70±4.89	15.08±0.63	7.61±0.33	4.38±0.13	8.89±0.11	6.21±0.19	84.74±30.79	82.49±31.77	15.05±0.76	11.73±0.43
DBCV	17.36±1.05	15.35±0.69	7.73±0.26	4.39±0.12	9.10±0.31	6.21±0.13	**92.36±8.83**	**91.64±7.17**	14.62±0.76	11.38±0.39
AIC	19.55±0.00	8.52±0.00	15.87±0.00	1.90±0.00	19.81±0.00	11.31±0.00	19.80±0.01	8.40±2.17	19.02±0.00	7.60±0.00
BIC	19.55±0.00	8.52±0.00	15.87±0.00	1.90±0.00	19.81±0.00	11.31±0.00	19.80±0.01	8.40±2.17	19.02±0.00	7.60±0.00
IP	75.87±0.03	**58.89±0.09**	**31.37±0.36**	**11.73±0.33**	**39.75±1.08**	**21.87±0.23**	72.14±13.20	58.87±20.65	**63.39±0.96**	40.94±2.68

Table 15: Swin based K-Means clustering results on five image datasets.

	CIFAR-10		**MNIST**		**FashionMNIST**		**ImageNet-10**		**CINIC-10**	
	ACC	ARI	ACC	ARI	ACC	ARI	ACC	ARI	ACC	ARI
SD	82.98±5.17	79.22±3.93	**27.69±0.03**	9.13±0.02	30.85±0.00	20.65±0.00	**99.84±0.00**	**99.64±0.00**	63.12±3.42	51.65±2.45
Dunn	81.19±6.46	78.84±6.64	22.27±0.60	**10.56±0.10**	19.99±0.00	13.57±0.00	83.78±15.16	82.85±16.46	35.64±13.51	29.44±11.03
I	20.00±0.00	17.35±0.00	19.47±0.01	4.94±0.00	19.99±0.00	13.57±0.00	19.81±0.06	4.56±0.07	19.89±0.00	14.32±0.01
XB	86.76±0.02	82.11±0.06	21.12±3.69	5.78±1.89	34.27±0.01	**20.76±0.03**	**99.84±0.00**	**99.64±0.00**	56.69±15.68	44.02±12.84
S	91.44±1.36	88.85±1.13	19.47±0.01	4.94±0.00	24.65±0.00	15.22±0.00	88.86±2.29	90.75±1.70	65.29±2.47	**52.94±2.19**
CH	20.00±0.00	17.35±0.00	19.47±0.01	4.94±0.00	19.99±0.00	13.57±0.00	**99.84±0.00**	**99.64±0.00**	19.89±0.00	14.32±0.01
DB	86.76±0.02	82.11±0.06	**27.69±0.03**	9.13±0.02	30.85±0.00	20.65±0.00	**99.84±0.00**	**99.64±0.00**	**68.87±0.11**	52.62±0.17
S_Dbw	20.07±0.51	19.90±0.41	8.32±0.37	4.38±0.32	9.05±0.44	6.20±0.13	**99.84±0.00**	**99.64±0.00**	20.72±3.52	16.93±3.28
CVNN	86.76±0.02	82.11±0.06	19.47±0.01	4.94±0.00	19.99±0.00	13.57±0.00	**99.84±0.00**	**99.64±0.00**	27.01±3.98	19.72±3.02
DCVI	20.66±0.86	20.28±1.29	8.19±0.34	4.25±0.12	9.70±0.68	6.62±0.28	89.81±14.18	89.18±15.15	18.52±2.05	14.81±1.31
DBCV	21.14±1.04	20.88±1.02	8.37±0.34	4.35±0.19	9.34±0.29	6.44±0.08	**99.84±0.00**	**99.64±0.00**	20.43±1.39	16.30±1.25
AIC	20.00±0.00	17.35±0.00	19.47±0.01	4.94±0.00	19.99±0.00	13.57±0.00	19.81±0.06	4.56±0.07	19.89±0.00	14.32±0.01
BIC	20.00±0.00	17.35±0.00	19.47±0.01	4.94±0.00	19.99±0.00	13.57±0.00	19.81±0.06	4.56±0.07	19.89±0.00	14.32±0.01
IP	**95.38±0.02**	**90.12±0.05**	27.30±0.01	8.60±0.01	**35.18±0.30**	20.10±0.67	**99.84±0.00**	**99.64±0.00**	68.05±1.77	51.80±1.74

Table 16: BERT based K-Means clustering results in terms of NMI on five text datasets.

	SearchSnippets	**Biomedical**	**StackOverflow**	**WebofScience**	**Yahoo!Answers**
SD	37.51±0.89	26.6±0.02	2.51±0.01	36.26±0.07	6.9±0.02
Dunn	37.43±0.69	28.84±0.16	5.93±12.07	35.78±0.17	19.03±0.82
I	16.68±0.1	8.21±0.02	0.53±0.0	28.63±0.01	1.85±0.01
XB	36.82±0.24	26.6±0.02	2.51±0.01	28.63±0.01	1.85±0.01
S	37.5±0.35	8.21±0.02	2.51±0.01	28.63±0.01	1.85±0.01
CH	16.68±0.1	8.21±0.02	0.53±0.0	28.63±0.01	1.85±0.01
DB	37.05±0.05	26.6±0.02	6.99±0.01	36.26±0.07	4.79±0.02
S_Dbw	37.47±0.47	**28.95±0.1**	28.42±0.46	35.96±0.25	19.87±0.26
CVNN	16.68±0.1	8.21±0.02	0.53±0.0	28.63±0.01	1.85±0.01
DCVI	37.41±0.56	29.01±0.17	28.34±0.59	35.67±0.28	**20.08±0.37**
DBCV	37.30±0.37	28.95±0.14	**28.44±0.37**	35.72±0.20	19.98±0.20
AIC	16.68±0.10	8.21±0.02	0.53±0.00	28.63±0.01	1.85±0.01
BIC	16.68±0.10	8.21±0.02	0.53±0.00	28.63±0.01	1.85±0.01
IP	**42.31±3.01**	27.09±0.23	10.56±0.12	**40.83±0.21**	6.90±0.02

Table 17: SimCSE based K-Means clustering results in terms of NMI on five text datasets.

	SearchSnippets	**Biomedical**	**StackOverflow**	**WebofScience**	**Yahoo!Answers**
SD	37.06±0.53	20.81±6.61	58.68±0.86	**44.6±0.26**	**23.82±0.3**
Dunn	37.78±0.9	23.05±1.48	52.1±1.7	41.89±2.24	22.03±0.79
I	10.4±0.01	10.21±0.01	9.67±0.06	35.76±0.0	7.92±0.04
XB	36.84±0.36	17.85±0.03	58.69±0.9	43.53±0.02	23.13±0.31
S	36.69±0.44	10.21±0.01	59.28±0.35	35.76±0.0	7.92±0.04
CH	10.4±0.01	10.21±0.01	9.67±0.06	35.76±0.0	7.92±0.04
DB	37.13±0.5	32.08±0.18	59.12±0.9	43.53±0.02	23.3±0.14
S_Dbw	37.06±0.19	32.02±0.28	51.4±0.16	39.39±0.12	23.33±0.24
CVNN	10.4±0.01	17.85±0.03	9.67±0.06	35.76±0.0	7.92±0.04
DCVI	37.02±0.29	**32.1±0.21**	51.55±0.17	39.55±0.18	23.27±0.39
DBCV	37.12±0.67	31.92±0.25	51.25±0.50	39.55±0.19	23.12±0.24
AIC	10.40±0.01	10.21±0.01	9.67±0.06	35.76±0.00	7.92±0.04
BIC	10.40±0.01	10.21±0.01	9.67±0.06	35.76±0.00	7.92±0.04
IP	**39.79±0.57**	31.55±0.08	**59.22±0.79**	44.50±1.09	20.93±0.11

Table 18: ViT based K-Means clustering results in terms of NMI on five image datasets.

	CIFAR-10	MNIST	FashionMNIST	ImageNet-10	CINIC-10
SD	**71.26±0.6**	20.38±0.42	33.31±0.02	87.49±7.95	45.4±0.57
Dunn	34.92±6.81	18.6±0.58	33.15±1.3	67.19±30.54	42.84±3.18
I	29.94±0.0	3.53±0.0	25.14±0.0	35.96±9.22	26.43±0.01
XB	34.92±6.81	18.34±0.04	25.14±0.0	**93.75±2.19**	26.43±0.01
S	67.94±1.29	3.53±0.0	29.67±0.0	90.82±0.92	55.92±0.55
CH	29.94±0.0	3.53±0.0	25.14±0.0	**93.75±2.19**	26.43±0.01
DB	34.92±6.81	19.86±0.37	25.14±0.0	70.62±35.5	26.43±0.01
S_Dbw	53.7±2.06	23.77±0.33	33.31±0.13	78.58±10.88	44.83±0.45
CVNN	34.92±6.81	3.53±0.0	25.14±0.0	68.42±8.85	26.43±0.01
DCVI	50.72±4.67	24.09±0.28	33.29±0.11	87.89±18.48	44.79±0.26
DBCV	52.80±0.56	**24.18±0.24**	33.20±0.19	93.22±4.06	44.76±0.12
AIC	29.94±0.00	3.53±0.00	25.14±0.00	31.51±4.42	26.43±0.01
BIC	29.94±0.00	3.53±0.00	25.14±0.00	31.51±4.42	26.43±0.01
IP	70.24±0.07	20.74±0.42	**34.88±0.71**	82.93±8.36	**56.89±0.73**

Table 19: SWin based K-Means clustering results in terms of NMI on five image datasets.

	CIFAR-10	MNIST	FashionMNIST	ImageNet-10	CINIC-10
SD	87.75±1.38	14.19±0.03	34.89±0.0	**99.47±0.0**	**62.9±1.44**
Dunn	86.77±2.03	17.9±0.11	33.58±0.0	93.33±6.56	53.86±3.46
I	40.25±0.0	9.45±0.01	33.58±0.0	23.7±0.31	30.65±0.03
XB	88.77±0.05	10.41±2.14	**35.76±0.08**	**99.47±0.0**	58.06±8.9
S	89.9±0.59	9.45±0.01	29.48±0.01	94.61±0.47	63.88±0.51
CH	40.25±0.0	9.45±0.01	33.58±0.0	**99.47±0.0**	30.65±0.03
DB	88.77±0.05	14.19±0.03	34.89±0.0	**99.47±0.0**	62.41±0.08
S_Dbw	63.45±0.26	**20.07±0.18**	33.42±0.29	**99.47±0.0**	50.18±1.46
CVNN	88.77±0.05	9.45±0.01	33.58±0.0	**99.47±0.0**	39.91±5.19
DCVI	63.44±0.55	19.76±0.11	33.63±0.2	95.77±5.36	49.21±0.4
DBCV	63.64±0.52	19.85±0.26	33.63±0.24	**99.47±0.00**	49.78±0.60
AIC	40.25±0.00	9.45±0.01	33.58±0.00	23.70±0.31	30.65±0.03
BIC	40.25±0.00	9.45±0.01	33.58±0.00	23.70±0.31	30.65±0.03
IP	**90.14±0.02**	13.69±0.02	34.10±0.33	**99.47±0.00**	61.97±0.97

Table 20: BERT based GMM clustering results on five text datasets.

	SearchSnippets		Biomedical		StackOverflow		WebofScience		Yahoo!Answers	
	ACC	ARI	ACC	ARI	ACC	ARI	ACC	ARI	ACC	ARI
SD	27.19±7.62	17.25±4.15	29.45±0.05	14.73±0.15	13.98±1.96	3.48±0.66	40.88±0.01	24.69±0.01	19.76±1.95	3.06±1.00
Dunn	30.72±15.00	15.60±6.96	14.31±8.35	5.56±5.12	7.18±1.94	0.59±0.98	12.54±5.19	8.99±3.76	26.13±3.34	9.76±3.00
I	34.65±0.11	14.06±0.27	9.82±0.00	2.94±0.00	6.32±0.00	0.15±0.00	29.42±0.01	18.43±0.06	12.23±0.26	0.24±0.05
XB	24.46±10.60	15.42±6.13	26.04±3.78	12.38±2.60	8.01±0.00	0.65±0.00	29.42±0.01	18.43±0.06	12.23±0.26	0.24±0.05
S	9.87±0.52	6.58±0.22	9.82±0.00	2.94±0.00	8.01±0.00	0.65±0.00	29.42±0.01	18.43±0.06	12.23±0.26	0.24±0.05
CH	34.65±0.11	14.06±0.27	9.82±0.00	2.94±0.00	6.32±0.00	0.15±0.00	29.42±0.01	18.43±0.06	12.23±0.26	0.24±0.05
DB	9.84±0.76	6.62±0.34	27.38±3.58	13.55±2.17	10.73±0.01	2.34±0.01	40.88±0.01	24.69±0.01	14.78±2.58	1.25±0.97
S_Dbw	9.44±0.51	6.35±0.29	11.66±0.38	6.78±0.26	12.54±0.46	6.71±0.26	8.23±0.20	5.93±0.10	13.52±3.18	5.66±1.31
CVNN	34.65±0.11	14.06±0.27	14.88±3.35	6.65±2.36	6.32±0.00	0.15±0.00	29.42±0.01	18.43±0.06	20.80±2.99	3.82±1.98
DCVI	9.63±0.52	6.44±0.24	11.78±0.42	6.85±0.27	12.73±0.40	6.85±0.28	8.73±0.80	6.30±0.33	12.04±0.35	4.98±0.14
DBCV	9.73±0.58	6.57±0.35	11.98±0.65	6.99±0.52	12.59±0.39	6.73±0.23	8.68±0.23	6.25±0.21	12.51±0.60	5.22±0.19
AIC	27.84±1.16	18.61±0.62	24.50±1.69	13.88±1.33	**22.34±0.48**	**9.99±0.30**	40.88±0.01	24.69±0.01	**29.52±1.72**	**11.86±0.62**
BIC	34.65±0.11	14.06±0.27	9.82±0.00	2.94±0.00	6.32±0.00	0.15±0.00	29.42±0.01	18.43±0.06	12.23±0.26	0.24±0.05
IP	**50.68±2.20**	**30.71±2.89**	**31.96±0.11**	**15.86±0.19**	18.07±2.63	5.05±1.00	**45.54±2.47**	**29.92±1.96**	20.54±2.65	3.70±1.80

Table 21: SimCSE based GMM clustering results on five text datasets.

	SearchSnippets		Biomedical		StackOverflow		WebofScience		Yahoo!Answers	
	ACC	ARI	ACC	ARI	ACC	ARI	ACC	ARI	ACC	ARI
SD	15.83±4.84	10.15±3.17	32.91±4.88	18.41±3.51	61.59±6.52	43.41±5.43	55.10±9.34	36.36±4.02	16.35±2.34	8.28±1.13
Dunn	33.02±17.80	14.36±11.62	18.74±8.19	8.88±5.44	26.20±21.59	15.46±16.05	32.04±0.01	22.47±0.00	21.70±7.96	6.84±4.06
I	31.38±0.07	9.01±0.08	9.79±0.01	2.99±0.05	7.96±0.39	1.15±0.30	32.04±0.01	22.47±0.00	18.14±0.02	5.02±0.03
XB	16.25±8.42	10.77±5.87	30.73±4.27	16.05±2.12	63.69±3.62	45.64±0.57	39.03±6.38	26.00±3.23	16.02±2.16	8.14±1.12
S	10.49±1.13	6.76±0.43	9.79±0.01	2.99±0.05	62.76±3.02	**46.48±1.35**	32.04±0.01	22.47±0.00	18.14±0.02	5.02±0.03
CH	31.38±0.07	9.01±0.08	9.79±0.01	2.99±0.05	12.76±0.07	6.87±0.16	32.04±0.01	22.47±0.00	18.14±0.02	5.02±0.03
DB	10.03±0.58	6.53±0.18	14.11±0.94	8.77±0.60	62.63±3.41	45.37±1.95	47.91±6.26	32.42±5.57	10.03±0.40	4.92±0.18
S_Dbw	10.06±0.60	6.50±0.14	13.53±0.61	8.39±0.31	19.82±0.58	17.14±0.33	8.70±0.33	6.85±0.06	9.98±0.43	4.85±0.19
CVNN	31.38±0.07	9.01±0.08	13.63±2.10	5.86±0.69	12.76±0.07	6.87±0.16	32.04±0.01	22.47±0.00	18.14±0.02	5.02±0.03
DCVI	10.82±0.51	6.94±0.21	13.51±0.90	8.39±0.47	19.81±0.95	17.15±0.51	9.06±0.59	7.16±0.33	9.89±0.47	4.88±0.16
DBCV	10.39±0.41	6.61±0.18	13.94±0.79	8.64±0.45	19.65±0.67	17.01±0.29	9.50±0.93	7.42±0.55	11.92±1.91	5.91±0.98
AIC	24.42±0.68	16.24±0.67	26.95±0.12	16.24±0.18	41.91±1.33	34.70±1.16	**55.65±3.13**	**38.40±0.60**	22.73±1.23	11.32±0.63
BIC	31.38±0.07	9.01±0.08	9.79±0.01	2.99±0.05	7.96±0.39	1.15±0.30	32.04±0.01	22.47±0.00	18.14±0.02	5.02±0.03
IP	**48.29±3.91**	**28.32±2.62**	**37.99±0.19**	**20.13±0.27**	**67.49±2.51**	46.21±1.64	46.41±3.61	31.83±1.66	**35.25±2.82**	**13.63±2.11**

Table 22: ViT based GMM clustering results on five image datasets.

	CIFAR-10		MNIST		FashionMNIST		ImageNet-10		CINIC-10	
	ACC	ARI	ACC	ARI	ACC	ARI	ACC	ARI	ACC	ARI
SD	69.11±7.08	53.05±7.04	29.68±1.74	10.71±1.32	32.48±1.42	20.68±1.42	82.27±11.64	74.07±17.78	29.36±12.92	20.61±7.21
Dunn	47.27±22.57	37.26±20.70	27.63±0.78	9.12±0.86	36.40±2.84	20.57±0.59	29.66±14.00	20.69±18.36	31.04±4.50	22.81±4.38
I	21.56±4.01	13.31±3.64	16.02±0.00	1.99±0.00	19.87±0.00	11.66±0.00	23.54±5.34	7.83±5.13	19.50±0.26	11.93±2.41
XB	47.29±18.92	34.67±18.36	28.50±0.94	10.13±0.95	19.87±0.00	11.66±0.00	88.33±10.28	83.08±14.68	33.74±8.85	21.73±8.83
S	67.83±3.34	55.12±2.36	16.02±0.00	1.99±0.00	30.11±0.05	18.94±0.02	**93.07±4.07**	**91.18±4.83**	56.74±1.70	**41.36±0.87**
CH	19.78±0.12	12.16±2.03	16.02±0.00	1.99±0.00	19.87±0.00	11.66±0.00	91.79±8.84	87.97±12.49	19.50±0.26	11.93±2.41
DB	56.61±21.38	44.40±21.24	30.00±1.93	11.01±1.52	19.87±0.00	11.66±0.00	55.09±40.42	43.88±48.49	20.44±8.11	12.69±3.87
S_Dbw	17.48±2.12	16.01±2.14	7.37±0.25	4.23±0.15	8.80±0.28	6.19±0.08	68.18±25.51	61.39±24.20	16.34±2.99	12.91±2.31
CVNN	31.12±16.21	19.98±12.77	16.02±0.00	1.99±0.00	19.87±0.00	11.66±0.00	54.54±17.79	37.92±24.98	21.15±3.77	13.71±5.17
DCVI	20.83±5.34	19.66±5.79	7.38±0.29	4.23±0.15	8.87±0.32	6.29±0.18	51.29±25.36	45.91±28.08	15.33±1.13	12.02±0.88
DBCV	16.33±0.37	14.75±0.39	7.49±0.28	4.34±0.12	8.97±0.38	6.35±0.15	81.05±19.39	79.76±19.90	15.65±0.94	12.07±0.80
AIC	28.21±1.97	26.67±2.46	13.55±0.94	7.57±0.50	16.94±0.72	11.68±0.51	33.01±3.63	36.74±4.01	27.31±1.26	21.82±0.62
BIC	19.78±0.12	12.16±2.03	16.02±0.00	1.99±0.00	19.87±0.00	11.66±0.00	19.74±0.18	6.44±2.80	19.50±0.26	11.93±2.41
IP	**73.60±3.53**	**56.39±3.39**	**30.78±1.05**	**12.05±0.25**	**38.90±1.80**	**21.32±0.61**	80.96±9.90	73.42±16.84	**59.47±4.88**	38.70±5.14

Table 23: Swin based GMM clustering results on five image datasets.

	CIFAR-10		MNIST		FashionMNIST		ImageNet-10		CINIC-10	
	ACC	ARI	ACC	ARI	ACC	ARI	ACC	ARI	ACC	ARI
SD	74.07±21.13	68.91±25.07	**28.46±0.55**	**10.12±0.60**	30.82±0.04	20.60±0.09	71.76±26.80	62.11±34.91	64.82±6.10	50.99±4.16
Dunn	39.90±20.25	36.86±22.86	20.17±1.05	7.16±2.97	14.74±5.20	10.06±3.56	43.87±25.01	39.57±29.21	19.89±0.00	14.65±0.01
I	23.84±5.40	14.03±6.86	19.49±0.00	4.99±0.00	17.47±2.30	6.73±6.26	29.87±17.30	18.27±27.22	19.89±0.00	14.65±0.01
XB	72.37±16.35	68.17±19.63	19.49±0.00	4.99±0.00	34.26±0.02	**20.77±0.01**	83.72±11.50	79.47±16.85	55.96±16.19	43.67±13.05
S	**83.33±8.91**	**81.36±7.57**	19.49±0.00	4.99±0.00	24.65±0.02	15.29±0.02	**89.65±9.85**	**91.17±8.50**	50.51±15.02	43.66±9.85
CH	23.87±8.67	19.32±8.78	19.49±0.00	4.99±0.00	22.79±2.56	14.62±0.94	88.53±13.27	84.43±19.69	19.89±0.00	14.65±0.01
DB	82.76±8.17	78.84±5.87	28.32±0.59	9.92±0.79	30.82±0.04	20.60±0.09	69.77±29.93	64.12±36.75	68.13±1.34	**53.41±0.74**
S_Dbw	22.27±2.93	21.81±4.03	8.48±0.77	4.45±0.37	9.10±0.15	6.25±0.05	60.25±34.43	54.26±43.70	20.37±4.74	16.66±4.43
CVNN	58.54±18.89	51.50±22.25	19.49±0.00	4.99±0.00	22.79±2.56	14.62±0.94	61.81±35.55	51.92±43.60	19.89±0.00	14.65±0.01
DCVI	21.19±1.21	20.73±1.67	8.33±0.53	4.46±0.30	9.35±0.53	6.47±0.37	59.82±24.42	57.67±31.36	17.27±0.42	13.75±0.43
DBCV	22.19±2.53	21.36±2.61	8.37±0.61	4.43±0.34	9.49±0.37	6.50±0.23	88.31±10.26	87.30±11.75	17.52±1.38	13.92±1.18
AIC	43.81±2.50	47.01±3.22	16.28±0.72	8.50±0.46	21.25±1.51	13.79±0.80	50.88±6.23	55.04±6.82	35.38±1.05	29.84±0.77
BIC	19.94±0.13	13.46±6.12	19.49±0.00	4.99±0.00	17.47±2.30	6.73±6.26	19.93±0.10	10.43±7.96	19.89±0.00	14.65±0.01
IP	80.96±8.35	76.12±7.75	28.42±0.74	9.84±0.73	**35.63±0.63**	20.43±0.60	71.76±26.80	62.11±34.91	**68.57±2.84**	52.89±1.34

Table 24: BERT based GMM clustering results in terms of NMI on five text datasets.

	SearchSnippets	Biomedical	StackOverflow	WebofScience	Yahoo!Answers
SD	37.58±0.78	26.71±0.09	9.22±1.41	36.58±0.02	7.46±1.89
Dunn	29.92±9.06	13.73±7.16	1.66±2.67	36.36±0.80	17.02±4.09
I	19.73±0.38	9.93±0.03	0.46±0.00	29.34±0.09	1.01±0.08
XB	37.21±0.62	23.74±3.16	1.95±0.00	29.34±0.09	1.01±0.08
S	37.28±0.53	9.93±0.03	1.95±0.00	29.34±0.09	1.01±0.08
CH	19.73±0.38	9.93±0.03	0.46±0.00	29.34±0.09	1.01±0.08
DB	37.18±0.56	25.25±2.76	6.83±0.02	36.58±0.02	3.31±2.19
S_Dbw	37.14±0.59	**29.06±0.16**	**28.23±0.34**	35.57±0.20	**20.10±0.24**
CVNN	19.73±0.38	16.23±3.96	0.46±0.00	29.34±0.09	8.02±2.56
DCVI	37.19±0.54	29.04±0.16	28.19±0.50	35.71±0.29	20.00±0.19
DBCV	37.25±0.49	28.99±0.22	28.21±0.43	35.65±0.27	19.98±0.16
AIC	37.90±1.35	28.20±0.42	22.61±0.90	36.58±0.02	19.31±0.93
BIC	19.73±0.38	9.93±0.03	0.46±0.00	29.34±0.09	1.01±0.08
IP	**39.72±3.34**	27.29±0.22	10.24±0.75	**39.99±0.86**	7.90±2.39

Table 25: SimCSE based GMM clustering results in terms of NMI on five text datasets.

	SearchSnippets	Biomedical	StackOverflow	WebofScience	Yahoo!Answers
SD	36.56±0.74	30.05±3.48	56.74±3.70	45.44±1.32	23.67±0.08
Dunn	24.91±11.65	18.64±8.52	25.18±22.26	36.68±0.03	11.93±5.44
I	14.31±0.07	9.59±0.23	3.42±0.87	36.68±0.03	9.49±0.05
XB	37.34±1.95	28.08±1.78	58.44±0.42	40.85±3.82	23.62±0.27
S	36.53±0.43	9.59±0.23	58.79±0.62	36.68±0.03	9.49±0.05
CH	14.31±0.07	9.59±0.23	17.38±0.35	36.68±0.03	9.49±0.05
DB	36.64±0.49	31.99±0.16	58.22±1.31	44.47±1.17	23.28±0.17
S_Dbw	36.52±0.41	31.98±0.15	51.26±0.25	39.13±0.16	23.17±0.28
CVNN	14.31±0.07	15.47±1.66	17.38±0.35	36.68±0.03	9.49±0.05
DCVI	36.58±0.27	31.95±0.18	51.31±0.17	39.33±0.26	23.23±0.17
DBCV	36.49±0.45	32.01±0.17	51.22±0.20	39.43±0.33	23.34±0.30
AIC	37.81±0.82	**32.03±0.22**	55.67±0.49	**45.72±1.02**	**23.95±0.36**
BIC	14.31±0.07	9.59±0.23	3.42±0.87	36.68±0.03	9.49±0.05
IP	**37.53±2.35**	31.62±0.16	**59.04±1.18**	44.09±0.67	19.74±2.35

Table 26: ViT based GMM clustering results in terms of NMI on five image datasets.

	CIFAR-10	MNIST	FashionMNIST	ImageNet-10	CINIC-10
SD	68.10±2.73	19.27±2.05	31.80±2.09	88.97±5.79	48.07±2.57
Dunn	54.67±16.57	16.84±1.01	33.67±1.32	45.84±22.36	44.33±3.45
I	34.39±5.86	3.69±0.00	26.08±0.02	31.42±11.87	29.52±1.71
XB	54.94±14.94	17.74±1.14	26.08±0.02	91.94±4.53	43.49±9.78
S	68.52±1.01	3.69±0.00	29.65±0.06	**93.39±2.09**	55.65±0.73
CH	31.93±1.09	3.69±0.00	26.08±0.02	93.29±3.84	29.52±1.71
DB	60.54±17.25	19.38±2.10	26.08±0.02	58.61±36.52	41.16±8.23
S_Dbw	53.23±1.10	**23.85±0.17**	33.36±0.22	83.84±9.27	45.17±1.16
CVNN	42.37±15.17	3.69±0.00	26.08±0.02	68.65±15.96	31.99±6.20
DCVI	54.70±2.32	23.77±0.11	33.44±0.15	69.08±22.06	44.67±0.44
DBCV	52.47±0.29	23.82±0.24	33.41±0.31	87.92±8.31	44.53±0.24
AIC	57.69±0.95	22.90±0.32	33.89±0.59	72.87±1.03	48.74±0.35
BIC	31.93±1.09	3.69±0.00	26.08±0.02	27.37±6.07	29.52±1.71
IP	**69.58±1.44**	20.59±0.70	**34.09±0.54**	88.17±4.96	**54.43±2.33**

Table 27: SWin based GMM clustering results in terms of NMI on five image datasets.

	CIFAR-10	MNIST	FashionMNIST	ImageNet-10	CINIC-10
SD	81.29±11.95	15.46±0.89	34.84±0.11	81.69±22.68	61.36±2.92
Dunn	60.62±15.95	13.06±4.80	33.67±0.37	63.54±26.27	31.41±0.02
I	37.72±11.88	9.56±0.00	15.88±16.19	40.07±26.62	31.41±0.02
XB	81.82±7.77	9.56±0.00	**35.70±0.16**	92.86±5.52	57.20±8.49
S	**86.39±2.44**	9.56±0.00	29.63±0.02	95.33±3.89	57.42±6.34
CH	43.57±11.96	9.56±0.00	31.23±2.17	94.35±6.23	31.41±0.02
DB	86.36±2.29	15.16±1.04	34.84±0.11	78.83±30.79	**63.05±1.19**
S_Dbw	64.05±1.55	19.84±0.34	33.34±0.22	69.50±33.65	50.15±1.84
CVNN	74.76±10.36	9.56±0.00	31.23±2.17	69.56±33.83	31.41±0.02
DCVI	63.69±0.82	19.86±0.27	33.47±0.19	75.52±28.87	48.98±0.52
DBCV	63.83±1.10	**19.87±0.28**	33.54±0.22	**95.25±3.81**	48.98±0.70
AIC	73.79±1.01	18.35±0.23	34.68±0.31	79.93±2.47	54.39±0.22
BIC	35.08±9.18	9.56±0.00	15.88±16.19	32.40±11.47	31.41±0.02
IP	85.57±2.73	15.05±0.82	34.10±0.34	81.69±22.68	62.36±0.71

Table 28: BERT based AHC clustering results on five text datasets.

	SearchSnippets - 8				Biomedical - 20				StackOverflow - 20				WebofScience - 7				Yahoo!Answers - 10			
	ACC	ARI	NMI	opt_k	ACC	ARI	NMI	opt_k	ACC	ARI	NMI	opt_k	ACC	ARI	NMI	opt_k	ACC	ARI	NMI	opt_k
SD	13.08	8.07	38.17	98	25.61	11.87	22.53	7	10.86	1.56	7.69	4	42.58	25.51	**40.41**	3	18.5	2.77	6.42	3
Dunn	19.21	12.5	39.08	57	12.26	6.93	**27.95**	141	13.8	**7.16**	**28.78**	141	20.48	13.75	36.2	50	**29.73**	**9.68**	16.78	15
I	37.27	12.84	24.83	3	9.49	2.77	8.89	2	6.57	0.18	0.64	2	31.52	21.39	34.22	2	18.3	2.75	6.38	4
XB	13.08	8.07	38.17	98	28.52	13.24	25.08	10	10.86	1.56	7.69	4	31.52	21.39	34.22	2	18.5	2.77	6.42	3
S	30.79	8.77	18.2	2	9.49	2.77	8.89	2	8.57	0.7	3.12	3	31.52	21.39	34.22	2	15.76	2.06	3.84	2
CH	30.79	8.77	18.2	2	9.49	2.77	8.89	2	6.57	0.18	0.64	2	31.52	21.39	34.22	2	15.76	2.06	3.84	2
DB	10.93	7.06	37.96	110	28.52	13.24	25.08	10	10.86	1.56	7.69	4	42.58	25.51	**40.41**	3	19.25	2.56	8.9	6
S_Dbw	10.75	7.02	37.92	111	12.9	7.22	27.84	126	13.8	**7.16**	**28.78**	141	9.69	6.5	35.2	109	20.66	8.12	19.12	45
CVNN	49.09	25.14	37.89	5	12.88	5.13	13.94	3	6.57	0.18	0.64	2	31.52	21.39	34.22	2	15.76	2.06	3.84	2
DCVI	10.75	7.02	37.92	111	12.26	6.93	**27.95**	141	13.8	**7.16**	**28.78**	141	9.69	6.5	35.2	109	14.18	5.64	**19.63**	100
DBCV	10.75	7.02	37.92	111	12.26	6.93	**27.95**	141	13.8	**7.16**	**28.78**	141	9.69	6.5	35.2	109	14.18	5.64	**19.63**	100
AIC	30.79	8.77	18.2	2	9.49	2.77	8.89	2	6.57	0.18	0.64	2	31.52	21.39	34.22	2	15.76	2.06	3.84	2
BIC	30.79	8.77	18.2	2	9.49	2.77	8.89	2	6.57	0.18	0.64	2	31.52	21.39	34.22	2	15.76	2.06	3.84	2
IP	**64.6**	**33.29**	**43.96**	7	**28.8**	**13.93**	25.38	15	**14.49**	3.95	10.54	7	**53.72**	**31.44**	39.73	7	18.5	2.77	6.42	3

Table 29: SimCSE based AHC clustering results on five text datasets.

	SearchSnippets - 8				Biomedical - 20				StackOverflow - 20				WebofScience - 7				Yahoo!Answers - 10			
	ACC	ARI	NMI	opt_k	ACC	ARI	NMI	opt_k	ACC	ARI	NMI	opt_k	ACC	ARI	NMI	opt_k	ACC	ARI	NMI	opt_k
SD	12.1	6.77	37	111	16.26	4.07	16.28	4	35.94	26.34	50.39	76	50.25	33.08	44.24	10	**25.96**	4.37	17.99	4
Dunn	**48.7**	**26.19**	**40.03**	12	17.69	10.48	29.97	112	8.97	2.31	15.85	2	28.92	19.04	30.28	2	16.36	1.74	8.25	2
I	43.23	14.67	28.97	4	12.68	3.59	12.91	3	12.86	4.38	25.03	3	28.92	19.04	30.28	2	21.54	3.21	13.79	3
XB	12.1	6.77	37	111	16.26	4.07	16.28	4	8.97	2.31	15.85	2	40.13	24.49	38.35	3	16.36	1.74	8.25	2
S	12.1	6.77	37	111	9.7	2.97	9.91	2	54.28	**36.8**	**53.09**	30	28.92	19.04	30.28	2	16.36	1.74	8.25	2
CH	32.21	8.82	15.97	2	9.7	2.97	9.91	2	8.97	2.31	15.85	2	28.92	19.04	30.28	2	16.36	1.74	8.25	2
DB	12.1	6.77	37	111	16.26	4.07	16.28	4	**57.99**	23.79	54.46	19	50.25	33.08	44.24	10	9.55	4.41	21.78	100
S_Dbw	12.1	6.77	37	111	14.44	8.69	**30.09**	141	24.81	19.33	48.93	141	10.79	7.28	38.48	109	10.3	**4.65**	**21.84**	95
CVNN	32.21	8.82	15.97	2	9.7	2.97	9.91	2	8.97	2.31	15.85	2	28.92	19.04	30.28	2	21.54	3.21	13.79	3
DCVI	12.1	6.81	37.01	110	14.44	8.69	**30.09**	141	24.81	19.33	48.93	141	10.79	7.28	38.48	109	9.55	4.41	21.78	100
DBCV	12.1	6.81	37.01	110	14.44	8.69	**30.09**	141	24.81	19.33	48.93	141	10.79	7.28	38.48	109	9.55	4.41	21.78	100
AIC	32.21	8.82	15.97	2	9.7	2.97	9.91	2	8.97	2.31	15.85	2	28.92	19.04	30.28	2	16.36	1.74	8.25	2
BIC	32.21	8.82	15.97	2	9.7	2.97	9.91	2	8.97	2.31	15.85	2	28.92	19.04	30.28	2	16.36	1.74	8.25	2
IP	38.82	13.99	25.74	3	**28.86**	**13.73**	25.62	11	36.31	15.93	43.76	10	**52.45**	**34.44**	**44.56**	9	21.54	3.21	13.79	3

Table 30: ViT based AHC clustering results on five image datasets.

	CIFAR-10 - 10				MNIST - 10				FashionMNIST - 10				ImageNet-10 - 10				CINIC-10 - 10			
	ACC	ARI	NMI	opt_k	ACC	ARI	NMI	opt_k	ACC	ARI	NMI	opt_k	ACC	ARI	NMI	opt_k	ACC	ARI	NMI	opt_k
SD	26.44	10.03	36.34	3	**31.07**	**11.74**	19.67	9	30.38	16.69	**35.1**	4	87.52	82.8	91.19	9	**61.4**	35.04	**52.82**	10
Dunn	50.37	38.95	56.87	6	29.57	10.74	17.82	6	30.38	16.69	**35.1**	4	19.85	9.56	33.86	2	18.43	6.32	23.59	2
I	18.47	6.02	24.6	2	23.52	5.43	9.22	4	19.79	9.91	28.48	2	29.65	16.54	49.2	3	18.43	6.32	23.59	2
XB	26.44	10.03	36.34	3	21.82	3.78	8.22	3	19.79	9.91	28.48	2	87.52	82.8	91.19	9	18.43	6.32	23.59	2
S	60.92	49.9	63.19	16	16.88	2.69	5.33	2	19.79	9.91	28.48	2	**91.85**	**92.36**	**92.99**	12	57.36	**35.96**	52.69	9
CH	18.47	6.02	24.6	2	16.88	2.69	5.33	2	19.79	9.91	28.48	2	87.52	82.8	91.19	9	18.43	6.32	23.59	2
DB	26.44	10.03	36.34	3	30.01	11.45	19.61	8	19.79	9.91	28.48	2	87.52	82.8	91.19	9	18.43	6.32	23.59	2
S_Dbw	22.16	18.31	52.76	81	8.81	4.76	**24.04**	99	9.96	6.26	32.55	99	87.52	82.8	91.19	9	17.01	12.04	43.28	100
CVNN	26.44	10.03	36.34	3	16.88	2.69	5.33	2	22.97	11.36	27.83	3	59.23	49.23	78.3	6	18.43	6.32	23.59	2
DCVI	50.37	38.95	56.87	6	8.81	4.71	24.02	100	9.96	6.16	32.52	100	39.58	25.55	61.52	4	17.06	12.18	43.31	99
DBCV	20.17	16.16	51.81	100	8.81	4.71	24.02	100	9.96	6.16	32.52	100	**91.85**	**92.36**	**92.99**	12	17.01	12.04	43.28	100
AIC	18.47	6.02	24.6	2	16.88	2.69	5.33	2	19.79	9.91	28.48	2	19.85	9.56	33.86	2	18.43	6.32	23.59	2
BIC	18.47	6.02	24.6	2	16.88	2.69	5.33	2	19.79	9.91	28.48	2	19.85	9.56	33.86	2	18.43	6.32	23.59	2
IP	**70.92**	**51.77**	**64.26**	10	29.57	10.74	17.82	6	**38.57**	**22.17**	34.24	8	87.52	82.8	91.19	9	46.74	28.84	48.71	7

Table 31: Swin based AHC clustering results on five image datasets.

	CIFAR-10 - 10				MNIST - 10				FashionMNIST - 10				ImageNet-10 - 10				CINIC-10 - 10			
	ACC	ARI	NMI	opt_k	ACC	ARI	NMI	opt_k	ACC	ARI	NMI	opt_k	ACC	ARI	NMI	opt_k	ACC	ARI	NMI	opt_k
SD	85.92	80.08	86.43	9	19.36	4.09	7.85	3	31.8	**21.96**	**37.75**	4	**99.66**	**99.25**	**98.94**	10	**64.77**	47.09	**58.26**	12
Dunn	19.94	18.12	42.4	2	23.54	6.38	9.96	4	19.97	13.79	34.16	2	39.93	18.49	57.59	4	19.35	14.62	33.03	2
I	19.94	18.12	42.4	2	16.93	2.01	4.46	2	19.97	13.79	34.16	2	29.93	10.73	43.02	3	19.35	14.62	33.03	2
XB	85.92	80.08	86.43	9	16.93	2.01	4.46	2	34.43	20.54	36.51	5	**99.66**	**99.25**	**98.94**	10	27.79	18.64	43.35	3
S	89	83.9	85.83	11	16.93	2.01	4.46	2	19.97	13.79	34.16	2	91.03	91.92	94.97	14	43.62	38.8	52.97	5
CH	19.94	18.12	42.4	2	16.93	2.01	4.46	2	19.97	13.79	34.16	2	**99.66**	**99.25**	**98.94**	10	19.35	14.62	33.03	2
DB	85.92	80.08	86.43	9	16.93	2.01	4.46	2	34.43	20.54	36.51	5	**99.66**	**99.25**	**98.94**	10	27.79	18.64	43.35	3
S_Dbw	46.3	46.79	72.51	34	8.66	4.35	**19.86**	99	9.81	6.36	33.21	100	**99.66**	**99.25**	**98.94**	10	20.7	15.57	48.79	86
CVNN	76.7	72.92	83.99	8	16.93	2.01	4.46	2	19.97	13.79	34.16	2	**99.66**	**99.25**	**98.94**	10	27.79	18.64	43.35	3
DCVI	21.2	19.46	62.25	100	8.66	4.34	**19.86**	100	9.81	6.36	33.21	100	39.93	18.49	57.59	4	18.74	14.08	48.21	100
DBCV	19.94	18.12	42.4	2	8.66	4.34	**19.86**	100	9.81	6.36	33.21	100	**99.66**	**99.25**	**98.94**	10	18.74	14.08	48.21	100
AIC	19.94	18.12	42.4	2	16.93	2.01	4.46	2	19.97	13.79	34.16	2	19.97	9.85	35.28	2	19.35	14.62	33.03	2
BIC	19.94	18.12	42.4	2	16.93	2.01	4.46	2	19.97	13.79	34.16	2	19.97	9.85	35.28	2	19.35	14.62	33.03	2
IP	**93.62**	**86.56**	**87.01**	10	**24.03**	**7.17**	12.3	8	**35.03**	21.09	35.54	6	**99.66**	**99.25**	**98.94**	10	61.49	**47.21**	57.78	8

Table 32: BERT based DBSCAN clustering results on five text datasets.

	SearchSnippets - 8				Biomedical - 20				StackOverflow - 20				WebofScience - 7				Yahoo!Answers - 10			
	ACC	ARI	NMI	opt_k	ACC	ARI	NMI	opt_k	ACC	ARI	NMI	opt_k	ACC	ARI	NMI	opt_k	ACC	ARI	NMI	opt_k
SD	21.56	0	0.01	2	5	0	0.01	2	5.01	0	0.01	2	17.6	0	0.02	2	10.01	0	0.02	2
Dunn	21.56	0	0.01	2	5	0	0.01	2	5.01	0	0.01	2	17.61	0	0.02	2	10.04	0	0.05	2
I	21.56	0	0.01	2	5	0	0.01	2	5.01	0	0.01	2	17.6	0	0.02	2	10.01	0	0.02	2
XB	21.56	0	0.01	2	5	0	0.01	2	5.01	0	0.01	2	17.6	0	0.02	2	10.01	0	0.02	2
S	21.56	0	0.01	2	5	0	0.01	2	5.01	0	0.01	2	17.6	0	0.02	2	10.01	0	0.02	2
CH	**24.63**	0.29	**5.39**	2	9.22	**1.45**	5.27	2	**6.45**	**0.28**	**0.91**	2	**25.86**	**3.49**	**14.52**	4	11.86	0.18	0.94	2
DB	21.56	0	0.01	2	5	0	0.01	2	5.01	0	0.01	2	17.6	0	0.02	2	10.01	0	0.02	2
S_Dbw	21.64	0.01	0.18	3	5	0	0.01	2	5.01	0	0.01	2	17.6	0	0.02	2	10.05	0.01	0.07	3
CVNN	21.67	0.01	1.01	14	5.34	0	0.66	2	5.62	0.02	0.43	2	17.85	0.02	0.46	2	10.04	0	0.05	2
DCVI	21.56	0	0.01	2	5	0	0.01	2	5.01	0	0.01	2	17.6	0	0.02	2	10.01	0	0.02	2
DBCV	23.41	**0.5**	1.13	3	8.28	0.74	3.87	6	5.95	0.06	0.58	3	19.43	0.14	0.8	3	10.01	0	0.02	2
AIC	21.56	0	0.03	2	5.01	0	0.03	2	5.01	0	0.01	2	17.6	0	0.02	2	10.02	0	0.04	2
BIC	21.56	0	0.03	2	5.01	0	0.03	2	5.01	0	0.01	2	17.6	0	0.02	2	10.02	0	0.04	2
IP	21.74	-0.38	4.34	2	**9.23**	1.36	**7.2**	2	5.38	0.01	0.39	2	18.38	0.05	0.34	2	**13.65**	**0.95**	**2.86**	2

Table 33: SimCSE based DBSCAN clustering results on five text datasets.

	SearchSnippets - 8				Biomedical - 20				StackOverflow - 20				WebofScience - 7				Yahoo!Answers - 10			
	ACC	ARI	NMI	opt_k	ACC	ARI	NMI	opt_k	ACC	ARI	NMI	opt_k	ACC	ARI	NMI	opt_k	ACC	ARI	NMI	opt_k
SD	21.65	0.05	1.03	11	8.86	0.25	7.34	12	6.64	0.02	3.18	10	17.62	0	0.02	2	10.01	0	0.02	2
Dunn	21.56	0	0.01	2	5	0	0.02	2	5.02	0	0.03	2	17.62	0	0.02	2	10.01	0	0.02	2
I	21.56	0	0.01	2	5	0	0.02	2	5.04	0	0.08	2	17.62	0	0.02	2	10.01	0	0.02	2
XB	21.56	0	0.01	2	5	0	0.02	2	5.04	0	0.08	2	17.62	0	0.02	2	10.01	0	0.02	2
S	21.56	0	0.01	2	5.03	0	0.08	2	5.02	0	0.06	2	17.62	0	0.02	2	10.01	0	0.02	2
CH	**26.29**	**3.09**	5.79	2	9.18	**2.27**	7.02	2	7.54	0.37	1.77	2	**26.46**	**6.05**	**15.68**	3	**14.78**	**0.95**	**6.26**	2
DB	21.56	0	0.01	2	5.1	0	0.43	9	5.04	0	0.08	2	17.62	0	0.02	2	10.01	0	0.02	2
S_Dbw	21.6	0	0.09	2	6.1	0.03	2.85	22	5.45	0	1.12	14	17.62	0	0.02	2	10.04	0.01	0.07	3
CVNN	21.65	-0.01	0.52	4	9.15	0.34	7.9	5	8.86	0.21	**7.41**	4	18.13	0.04	1.01	2	10.03	0	0.04	2
DCVI	21.56	0	0.01	2	5	0	0.02	2	6.33	0.01	3.89	42	17.62	0	0.02	2	10.01	0	0.02	2
DBCV	21.04	0.27	**13.25**	114	8.91	1.93	6.03	3	**9.29**	**1.65**	5.86	3	18.9	0.27	0.83	3	10.09	0	0.2	4
AIC	21.56	0	0.01	2	5	0	0.02	2	5.02	0	0.03	2	17.62	0	0.02	2	10.01	0	0.02	2
BIC	21.56	0	0.01	2	5	0	0.02	2	5.02	0	0.03	2	17.62	0	0.02	2	10.01	0	0.02	2
IP	21.6	-0.29	1.88	2	**11.31**	0.99	**12.65**	3	5.4	0.01	0.31	2	25.9	5.66	15.16	3	14.72	0.92	6.19	2

Table 34: ViT based DBSCAN clustering results on five image datasets.

	CIFAR-10 - 10				MNIST - 10				FashionMNIST - 10				ImageNet-10 - 10				CINIC-10 - 10			
	ACC	ARI	NMI	opt_k	ACC	ARI	NMI	opt_k	ACC	ARI	NMI	opt_k	ACC	ARI	NMI	opt_k	ACC	ARI	NMI	opt_k
SD	10.49	0	1.39	7	11.34	0	0.02	2	10.01	0	0.02	2	34.34	5.11	36.03	9	10.01	0	0.02	2
Dunn	10.01	0	0.02	2	11.34	0	0.02	2	10.01	0	0.02	2	10.03	0	0.06	2	10.01	0	0.02	2
I	10.01	0	0.02	2	11.34	0	0.02	2	10.01	0	0.02	2	10.04	0	0.08	2	10.01	0	0.02	2
XB	10.01	0	0.02	2	11.34	0	0.02	2	10.01	0	0.02	2	10.04	0	0.08	2	10.01	0	0.02	2
S	10.01	0	0.02	2	11.34	0	0.02	2	10.02	0	0.04	2	**75.64**	**62.69**	**79.51**	10	10.01	0	0.02	2
CH	32.68	**16.62**	40.09	5	**15.55**	**0.82**	**3.99**	2	**19.68**	**3.19**	**12.92**	3	75.63	59.05	78.06	9	**38.18**	**13.33**	**42.86**	7
DB	10.01	0	0.02	2	11.34	0	0.02	2	10.01	0	0.02	2	10.04	0	0.08	2	10.01	0	0.02	2
S_Dbw	11.52	0.02	4.32	16	11.34	0	0.02	2	10.01	0	0.02	2	20.41	1.38	18.56	14	10.03	0	0.08	2
CVNN	11.09	0.02	2.75	7	12.95	0.12	2.46	2	12.82	0.34	4.56	3	15.5	0.43	10.18	4	10.86	0.03	1.71	2
DCVI	10.01	0	0.02	2	11.34	0	0.02	2	10.01	0	0.02	2	10.03	0	0.06	2	10.01	0	0.02	2
DBCV	11.46	0.13	1.51	3	14.66	1.24	3.17	3	15.69	1.79	4.44	3	67.96	45.14	71.31	10	28.71	4.25	30.43	7
AIC	10.01	0	0.03	2	11.35	0	0.03	2	10.01	0	0.03	2	10.03	0	0.06	2	10.02	0	0.04	2
BIC	10.01	0	0.03	2	11.35	0	0.03	2	10.01	0	0.03	2	10.03	0	0.06	2	10.02	0	0.04	2
IP	37.38	11.91	**44.58**	8	14.52	0.43	3.28	2	12.48	0.39	4.11	2	53.08	17.88	56.16	8	25.01	3.63	26.38	5

Table 35: Swin based DBSCAN clustering results on five image datasets.

	CIFAR-10 - 10				MNIST - 10				FashionMNIST - 10				ImageNet-10 - 10				CINIC-10 - 10			
	ACC	ARI	NMI	opt_k	ACC	ARI	NMI	opt_k	ACC	ARI	NMI	opt_k	ACC	ARI	NMI	opt_k	ACC	ARI	NMI	opt_k
SD	10.01	0	0.02	2	15.31	0.78	8.24	3	10.01	0	0.02	2	56.1	21.67	59.71	10	10.01	0	0.02	2
Dunn	10.01	0	0.02	2	11.37	0	0.04	2	10.01	0	0.02	2	10.02	0	0.03	2	10.01	0	0.02	2
I	10.01	0	0.02	2	11.37	0	0.04	2	10.01	0	0.02	2	10.02	0	0.03	2	10.01	0	0.02	2
XB	10.01	0	0.02	2	11.37	0	0.04	2	10.01	0	0.02	2	10.02	0	0.03	2	10.01	0	0.02	2
S	10.01	0	0.02	2	11.37	0	0.04	2	10.04	0	0.08	2	10.02	0	0.03	2	10.04	0	0.08	2
CH	**33.74**	**18.58**	**42.17**	5	**18.98**	**3.28**	5.89	2	**19.23**	**3.09**	**16.7**	3	**73.78**	**43.41**	**73.46**	10	14.56	1.27	2.48	2
DB	10.01	0	0.02	2	11.15	-0.01	0.38	2	10.01	0	0.02	2	10.02	0	0.03	2	10.01	0	0.02	2
S_Dbw	13.62	0.13	7.21	11	13.77	0.31	6.63	4	10.52	0	1.16	7	13.92	0.2	8.88	13	10.03	0	0.08	2
CVNN	12.9	0.11	5.5	5	13.63	0.29	6.37	3	11.31	0.03	2.25	4	12.82	0.09	5.55	6	14.87	0.27	9.23	8
DCVI	10.01	0	0.02	2	11.38	0	0.07	2	10.01	0	0.02	2	10.12	0	0.27	2	10.01	0	0.02	2
DBCV	23.44	1.51	23.05	15	16.18	1.23	**9.44**	3	13.68	0.36	7.42	3	57.78	22.57	60.58	10	22.38	1.95	24.05	12
AIC	10.01	0	0.03	2	11.37	0	0.04	2	10.02	0	0.04	2	10.02	0	0.03	2	10.04	0	0.08	2
BIC	10.01	0	0.03	2	11.37	0	0.04	2	10.02	0	0.04	2	10.02	0	0.03	2	10.04	0	0.08	2
IP	**33.74**	**18.58**	**42.17**	5	16.74	1.48	8.57	2	**19.23**	**3.09**	**16.7**	3	**73.78**	**43.41**	**73.46**	10	**26.19**	**4.29**	**28.26**	13

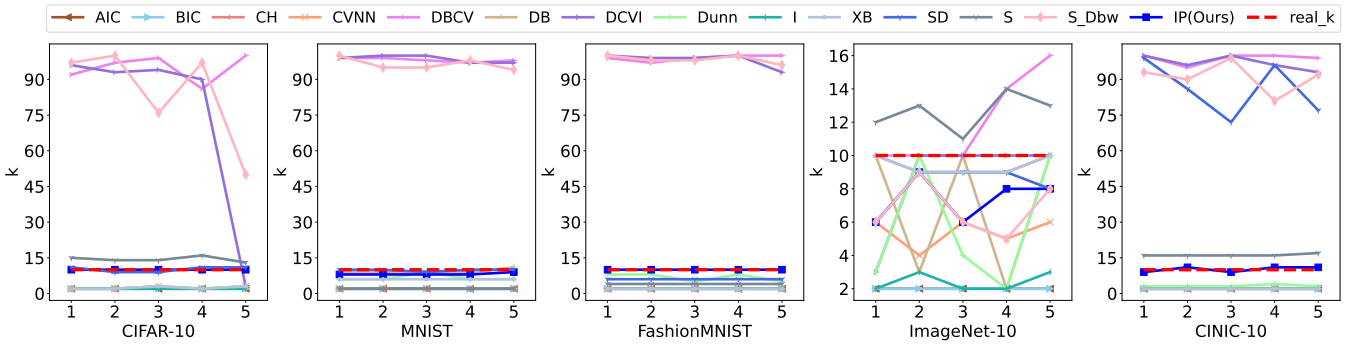

Figure 5: The optimal k value found by each index on ViT representations for image datasets.

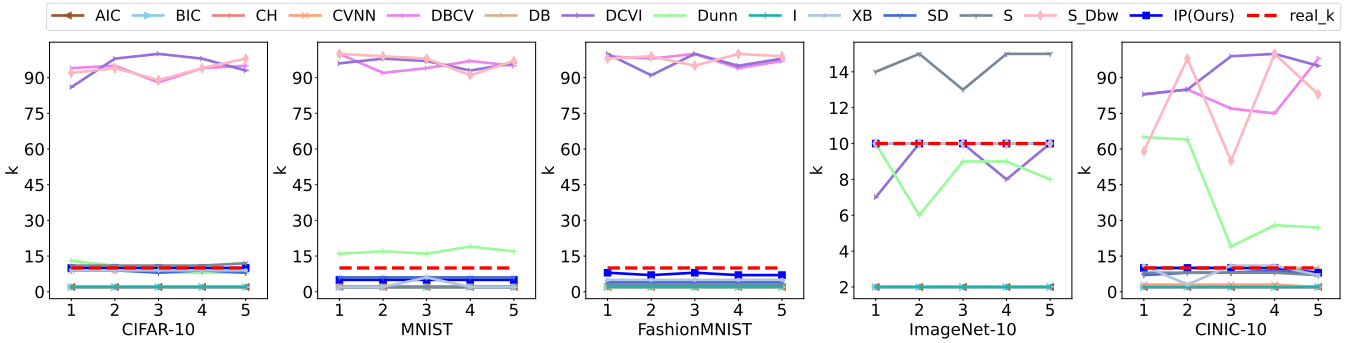

Figure 6: The optimal k value found by each index on Swin representations for image datasets.

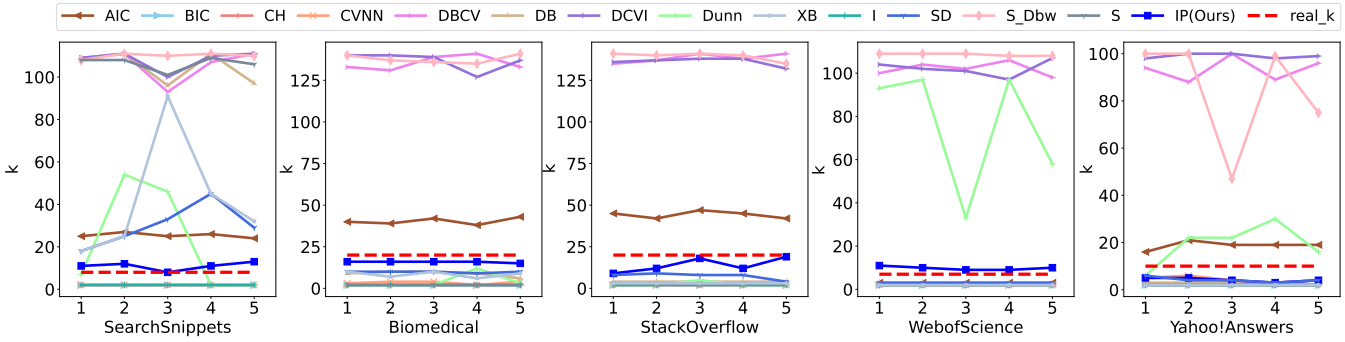

Figure 7: The optimal k value found by each index on BERT representations for text datasets.

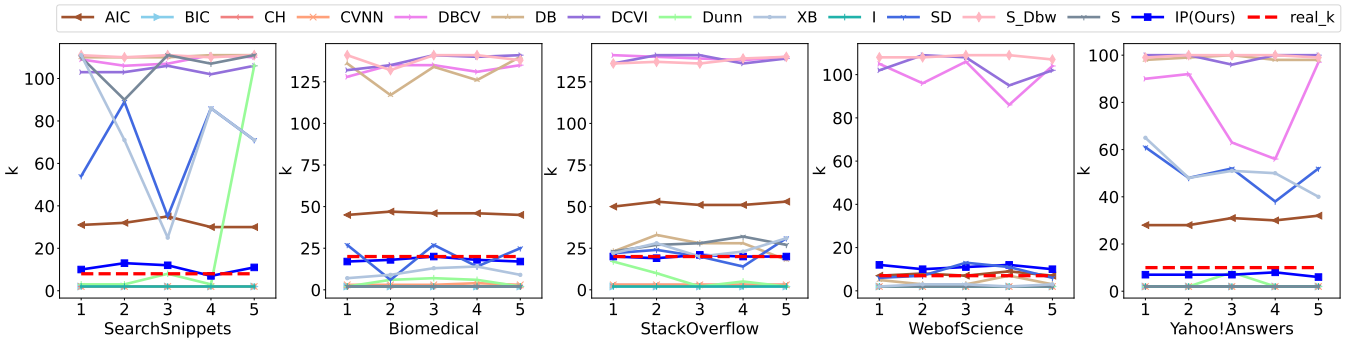

Figure 8: The optimal k value found by each index on SimCSE representations for text datasets.

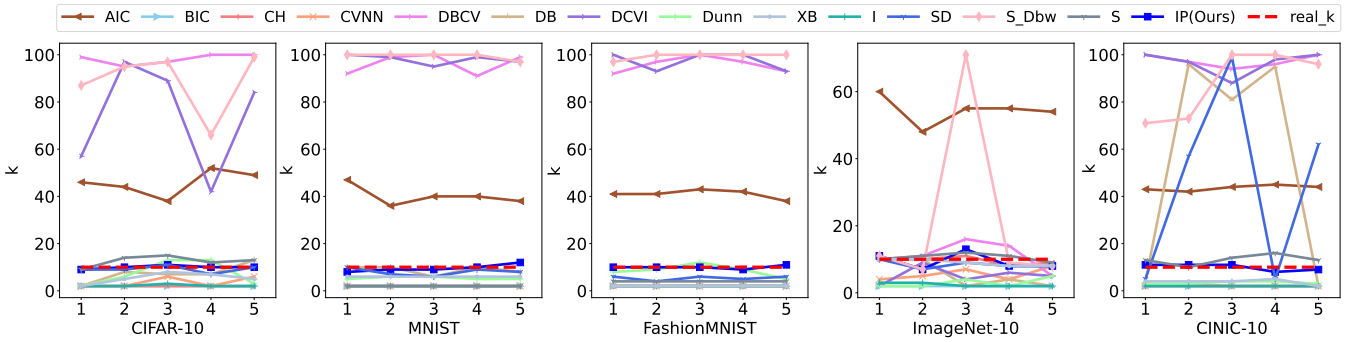

Figure 9: The optimal k value found by each index on ViT representations for image datasets.

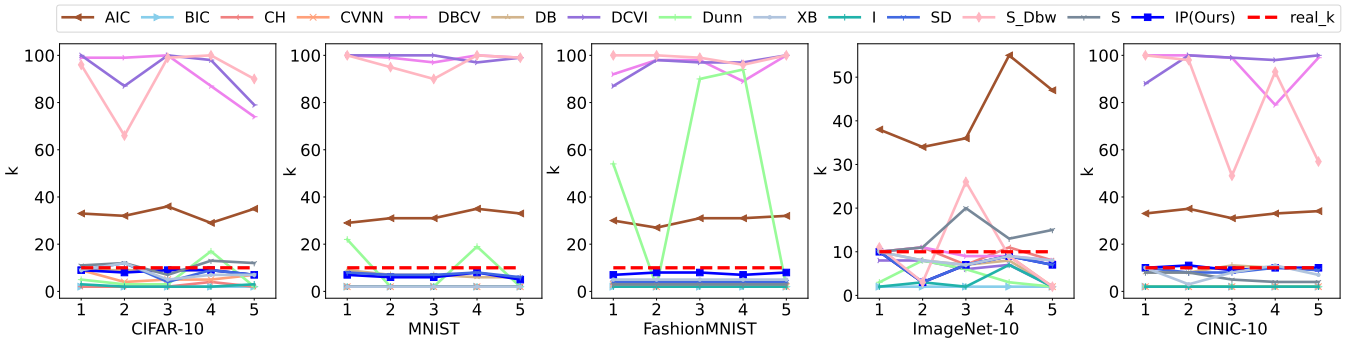

Figure 10: The optimal k value found by each index on Swin representations for image datasets.