Pycroft, Jonathan; Vergano, Lucia; Hope, Chris; Paci, Daniele; Ciscar, Juan Carlos

Working Paper

A tale of tails: Uncertainty and the social cost of carbon dioxide

Economics Discussion Papers, No. 2011-36

Provided in Cooperation with:
Kiel Institute for the World Economy (IfW)

Discussion Paper
No. 2011-36 | September 5, 2011 | http://www.economics-ejournal.org/economics/discussionpapers/2011-36

A Tale of Tails: Uncertainty and the Social Cost of Carbon Dioxide

Jonathan Pycroft
IPTS, European Commission, Seville
Lucia Vergano
IPTS, European Commission, Seville
Chris Hope
Cambridge Judge Business School, University of Cambridge
Daniele Paci
IPTS, European Commission, Seville
Juan Carlos Ciscar
IPTS, European Commission, Seville

Abstract Recent thinking about the economics of climate change has concerned the uncertainty about the upper bound of both climate sensitivity to greenhouse gases and the damages that might occur at high temperatures. This argument suggests that the appropriate probability distributions for these factors may be fat-tailed. The matter of tail shape has important implications for the calculation of the social cost of carbon dioxide (SCCO2). In this paper a probabilistic integrated assessment model is adapted to allow for the possibility of a thin, intermediate or fat tail for both (i) the climate sensitivity parameter and (ii) the damage function exponent. Results show that depending on the tail shape of the climate sensitivity parameter the mean SCCO2 rises by 29 to 85 percent. Changes in the mean SCCO2 due to the adjustments to the damage function alone range from a reduction of 7 percent to a rise of 12 percent. The combination of both leads to rises of 33 to 115 percent. Greater rises occur for the upper percentiles of the SCCO2 estimates. Given the uncertainties in both the science and the economics of climate change different tail shapes deserve consideration due to their important implications for the range of possible values for the SCCO2.

Paper submitted to the special issue
The Social Cost of Carbon

JEL Q54

Keywords Climate change; integrated assessment models; social cost of carbon dioxide; uncertainty

Correspondence Jonathan Pycroft, Economics of Climate Change, Energy and Transport Unit, Institute for Prospective Technological Studies, Joint Research Centre, European Commission, Edificio EXPO, C/ Inca Garcilaso, 3, 41092 Seville, Spain; e-mail: Jonathan.PYCROFT@ec.europa.eu.

Introduction

In the study of the economics of climate change, the issue of how to deal with catastrophic events has recently received a great deal of attention. The possibility that climate change could cause catastrophic outcomes is of deep concern, even if such an outcome is unlikely. From a scientific perspective, it has long been clear that radically changing the composition of the atmosphere, effectively instantaneously in geological terms, could have large, irreversible effects on ecosystems and highly undesirable consequences for humankind. This leads to calls for urgent action to limit the concentrations of greenhouse gases (GHGs) (IPCC, 2007; European Commission, 2007) or even to reduce concentrations below their current levels (Hansen et al., 2008).

There is some controversy within the economics literature regarding how to deal with climate impacts and what the policy implications are (e.g. Weitzman, 2009; Nordhaus, 2009). On the one hand, many of the economic assessments of the damage from carbon emissions report fairly modest figures (for an overview, see Tol, 2008; for a recent estimate, see Interagency Working Group on Social Cost of Carbon, U.S. Government, 2010). When these estimates are placed into a cost-benefit analysis framework, which balances *inter alia* medium-term prosperity against longer-term damages, the resulting recommendations can be for GHG concentrations to rise above the limits recommended by the IPCC.[1] On the other hand, a more recent literature (especially Weitzman, 2007 and 2009) has suggested that the possibility of catastrophic events could be a key factor (perhaps *the* key factor) in climate economics, even if such events are unlikely.

It is not the intention of this article to make a comprehensive review of this discussion. Instead, the investigation focuses on the treatment of catastrophic events within the economic estimates of the damage from GHGs. Though some previous economic estimates have allowed for some possibility of catastrophic damage, recent research (Weitzman, 2010; Dietz et al., forthcoming) proposes further directions in which the relevant models can be adapted to better account for this factor.

Climate damages are typically estimated with integrated assessment models (IAMs), which take into account contributions to climate policy from various disciplines, from climatology to economics. These model the most significant interactions and feedback mechanisms of the human-climate system. They also deal with intergenerational fairness, income regional distribution and, some of them, at least to a certain extent, risk and uncertainty management (Dietz et al., 2007).

A typical application of IAMs is the computation of the social cost of carbon dioxide (SCCO2), i.e., the cost to society caused by one additional tonne of carbon dioxide released into the atmosphere. The SCCO2 is a prominent indicator within both the literature and the policy debate. In principle, it summarizes climate policy benefits in a single dimension variable taking into account all possible biophysical and economic impacts in all world regions and in all future time periods. The

[1] This criticism is made in Weitzman (2010), with reference to results from Nordhaus' DICE model.

concept is particularly useful in project appraisal putting a value to the benefits of avoided GHGs emissions.[2] The value of the SCCO2 can be useful in judging policies, which in economic terms are only justified when their marginal benefit is at least equal to their marginal cost. It can also be used as a guide for the level of a Pigouvian tax on emissions.

Inevitably, IAMs rely on a series of simplifying assumptions,[3] using highly aggregated variables and data (Ackerman et al., 2009a; Patt et al., 2010), and the limitations of the methodology have been noted (e.g. Warren et al., 2006, Dietz et al., 2007). Nevertheless, IAMs provide a useful conceptual framework for exploring the implications of alternative specifications. Furthermore, many IAMs are designed to be flexible (Nordhaus and Boyer, 2000), allowing users the opportunity to enter alternative parameters. As noted in Dietz et al. (2007), IAMs can be thought of as a "canvas" on which debates about the parameters can be "painted". We take advantage of this feature in this research.

This article moves from the discussion in Weitzman (2010), questioning the extent to which uncertain extreme values of climate sensitivity and the damage functions have been accounted for in IAMs. In particular it explores a method of introducing thin, intermediate and fat tails for these key parameters in a specific IAM, the PAGE09 Model. With respect to the climate sensitivity, the alternative tails proposed in that paper are inputted, and the damage functions are extended in the spirit of the analysis. The focus is specifically on the implications for these changes on the SCCO2.

The article is organised as follows: Section 2 is devoted to methodological issues and explains the model used and the changes introduced on both the sensitivity parameter and the damage function exponents; Section 3 presents the main results and Section 4 concludes.

1 Methodology

1.1 The PAGE09 Model

The latest version of the model, PAGE09, keeps unchanged the general structure of the version used for the Stern Review (Stern, 2007), but introduces further developments reflecting the IPCC Fourth Assessment Report (2007). Exogenous assumptions for economic and population growth and GHGs emissions reflect the IPCC SRES A1B scenario (Hope, 2011).

PAGE09 uses a simple economic module (Hope et al., 1993; Plambeck et al., 1997; Hope, 2006; Hope, 2008; Hope, 2011) and expands it to consider climate issues and the linkages between the economic and the climate systems through some stylized equations within the climate module. Uncertainty is taken into

[2] For a comprehensive discussion of the SCCO2, including its main weaknesses, see for instance Ackerman and Stanton (2010).

[3] DICE 1990 model is based for instance on twenty equations (Nordhaus and Boyer, 2000).

account through Latin Hypercube sampling[4]. Functional forms are assumed to be known with certainty, while each of the uncertain model parameters (approximately 80) is represented by a probability distribution. A full run of the model involves repeating the calculations of the following output variables: global warming over time, damages, adaptive costs and abatement costs.

Four impact categories, specified as the percentage loss of GDP and subtracted from consumption, are defined within the economic module: sea level impact, economic and non-economic impacts based on regional temperature rise and discontinuity impact. As most IAMs, damage is defined as a non-linear function (Bosello and Roson, 2007). The total effect of climate change is equal to the sum of impacts, abatement costs and adaptive costs.[5]

1.2 Uncertainty in the climate sensitivity parameter (SENS)

1.2.1 Background and Literature

Weitzman (2010) provides a methodology to stress the robustness of modelling highly uncertain extreme consequences induced by catastrophic climate change events. Due to their unknown and potentially huge consequences for humankind, even low probability events associated with highly-negative impacts need to be taken into account in the economics of climate change. Little is known about the upper end of this distribution (meaning the possibility of extreme temperature rises), but the peer-reviewed studies[6] mentioned by Weitzman (2009) suggest that the probability of the most extreme few percent could be higher than current IAMs allow for. Though it is impossible to know precisely the "true" probability distribution, the general notion that there are small – but decidedly nonzero – probabilities of extreme events is certainly one that can be incorporated. This is a belief that can be "painted" onto the "canvas" of an IAM.

The intention is to take account of the uncertainty surrounding both the physical processes governing temperatures and the economic evaluation of the welfare losses associated with catastrophic events. The first type of uncertainty might be captured by the so called equilibrium climate sensitivity (SENS) parameter. As stated in the IPCC – AR4 Synthesis Report (2007), this provides "a measure of the climate system response to sustained radiative forcing" and "is defined as the global average surface warming induced by a doubling of carbon dioxide atmospheric concentration after a new equilibrium of the climate system has been reached. It is likely to be in the range 2 to 4.5°C with a best estimate of 3°C and is very unlikely to be less than

[4] Latin Hypercube sampling is preferred to “random” Monte Carlo sampling since it provides a better coverage of the underlying PDFs.

[5] Since in standard welfare models with constant and strictly positive relative risk aversion marginal utility tends to infinity as consumption tends to zero, if climate damages can reach 100 percent of consumption, then they need to be in some way bounded (Dietz et al., forthcoming). Following a suggestion in Weitzman (2009), total damages are capped if they exceed the statistical value of civilisation.

[6] These are the 22 estimates of climate sensitivity included in IPCC-AR4.

1.5°C. Values substantially higher than 4.5°C cannot be excluded, but agreement of models with observations is not as good for those values".

With respect to this research, it is more accurate to interpret climate sensitivity as a summary for the consequences of climate change, many of which are highly uncertain.[7] Focusing on climate sensitivity is, therefore, a reductionist approach. With respect to the science, it can be justified because, as well as the importance of climate sensitivity in itself, it is also correlated with many aspects of climate change effects (Knutti and Hegerl, 2008). This also justifies the prominent role it plays in IAMs, such as PAGE09.

Due to their uncertain nature, temperature changes induced by GHGs atmospheric concentration can only be described in terms of probabilities. In identifying the climate sensitivity probability distribution function (PDF), Weitzman refers to the language of tail probabilities. While the existing literature on cost-benefit analysis and IAMs of climate change mainly focuses on super thin tailed point mass PDFs, he takes into account tails of varying degrees of fatness: thin tailed probabilities, declining exponentially or faster; fat-tailed probabilities, declining polynomially or slower; intermediate-tailed probabilities, declining slower than exponentially but faster than polynomially. As will be shown below, for the upper 50 percentiles his proposed PDFs are implemented: 1) the thin-tailed normal distribution; 2) the fat-tailed Pareto distribution; and 3) the intermediate-tailed lognormal distribution. All three are calibrated so as to have a median of 3°C, which is the best estimate from the IPCC-AR4 (2007). The probability of the value being between 2°C and 4.5°C is reported to be above 66 percent and below 90 percent, which leads Weitzman (2010) to propose an 85th percentile of 4.5°C.[8]

1.2.2 Model adjustments

In PAGE09, the SENS parameter is determined by two input variables: transient climate response (TCR) and the feedback response time (FRT). The former refers to the temperature change (°C) at the time of CO2 concentration doubling. The latter indicates how many years GHGs persist in the atmosphere. The relationship among the three variables is indicated in Equation (1):[9]

$$SENS = \frac{TCR}{1-\left[\left(\frac{FRT}{70}\right)*\left(1-e^{\left(\frac{-70}{FRT}\right)}\right)\right]} \tag{1}$$

[7] The same argument was made in Weitzman (2010) with respect to the use of climate sensitivity in that paper.

[8] The IPCC likely probability definition implies a probability between 5 and 17 percent of climate sensitivity being greater than 4.5°C. Weitzman justifies edging towards the high end of that range as the "earth system sensitivity" probably matters more than the "fast equilibrium sensitivity" over the relevant time frame. Zickfield, Morgan, Frame and Keith (2010) estimate the same probability to be equal to 23 percent, while in Pindyck (**?**) it is equal to 10 percent.

[9] The calculation of SENS in this way is an innovation of PAGE09 based on IPCC 4AR and research by Andrews and Allen (2008).

In other words, the probabilistic distributions of TCR and FRT affect the variable capturing the global temperature increase due to a doubling of CO2 concentration in the atmosphere. In PAGE09, these variables are assumed to follow a triangular probability distribution. In the modified version of PAGE09, the original distribution of the considered variables is kept, but a different definition of SENS has been introduced, in order to take into account Weitzman's suggestions. In doing so, the SENS distribution has been modified in such a way that up to its median it is distributed according to a rescaled version of the original PDF, whose median was equal to 2.87°C,[10] while for its upper-half tail assumes the distributions discussed by Weitzman (2010).

(a) Lower 50 percentiles

For the lower 50 percentiles, the standard calibration is kept and therefore the standard PAGE09 values are retained: TCR sampled between 1 and 2.8°C – triangle (1, 1.3, 2.8) – and FRT between 10 and 65 years – triangle (10, 30, 65). The values below 3°C are drawn proportionally for the lower 50 percentiles. In the standard version of the model, 55.62 percent[11] of the results are less than 3°C. Therefore, to obtain 50 percent of the final draws, this distribution is sampled whenever the value is (i) below 3°C and (ii) a uniform distribution [0, 1] is below 0.8990.[12]

(b) Upper 50 percentiles

For the upper 50 percentiles of SENS, the thin, intermediate and fat-tailed distributions proposed in Weitzman (2010) are inputted, with the corresponding parameter values. As he suggests, the median is fixed at 3°C, which is consistent with the best estimate of the IPCC-4AR (2007), and the 85th percentile at 4.5°C. This comes from the "likely" range given by the IPCC of 2 to 4.5°C, where "likely" is defined as a probability greater than 66 percent but less than 90 percent. Defining "likely" as 70 percent, gives the 85th percentile of 4.5°C. Given these two parameter values, the three distributions proposed by Weitzman, the thin-tailed normal distribution, the fat-tailed Pareto distribution and the intermediate-tailed lognormal distribution, are taken into account.[13] Fitting these distributions to the specified 50th and 85th percentiles gives the following values:

$$\text{Normal: } f_N(SENS) = \frac{1}{1.447\sqrt{2\pi}} e^{\left(-\frac{(SENS-3)^2}{2\times(1.447)^2}\right)} \tag{2}$$

[10] The median value is based on 10,000 runs of PAGE09.
[11] Based on one million runs.
[12] 55.62 percent * 89.90 percent = 50.00 percent. Note that this procedure slightly raises the median value of SENS relative to the standard model.
[13] In his empirical analysis of risk in the economics of climate change, Dietz (forthcoming) takes into account the previous version of the model (PAGE2002) and considers a log-logistic distribution for the climate sensitivity parameter and a lognormal distribution for the damage function. These two are the distributions better fitting in terms of the lowest root-mean-square error.

$$\text{Lognormal: } f_L(SENS) = \frac{1}{0.3912\sqrt{2\pi}SENS} e^{\left(-\frac{(lnSENS-1.099)^2}{2\times(0.3912)^2}\right)} \tag{3}$$

$$\text{Pareto: } f_P(SENS) = 38.76 \times SENS^{-3.969} \tag{4}$$

The three distributions are compared graphically in Figure1 below.

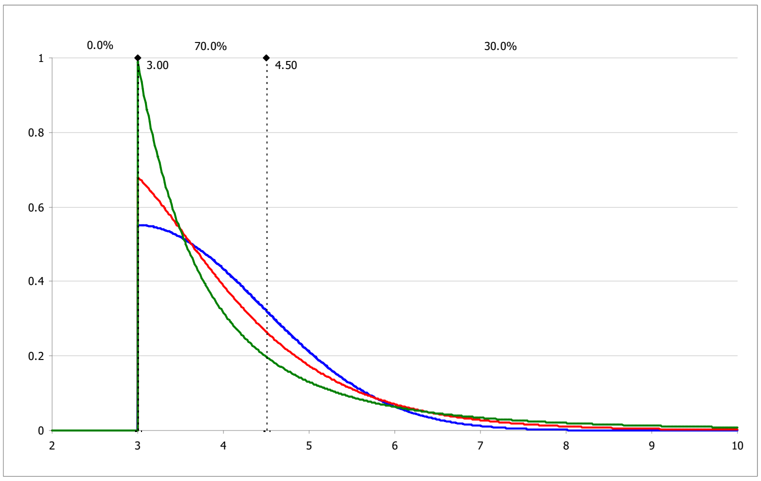

Figure 1: The upper 50 percentiles of the climate sensitivity parameter: normal, lognormal and Pareto

The graphs show the differences in tail thickness, with the Pareto (green line) being the fattest tail, followed by the lognormal (red line) and the normal (blue line). Note that both the lognormal and Pareto draw more often from the lower range of values (close to 3) than the normal does.

1.3 Uncertainty in the damage functions

1.3.1 Background and Literature

There is considerable uncertainty about the correct shape of damage functions. The argument is similar to that made for the climate sensitivity parameter above: whilst reasonable estimates can be made for the lower end of the distribution, the high end of the distribution is uncertain, and possibly unknowable.

The damage functions relate to the economic consequences caused by the physical response of the climate system. When attempting to quantify climate change damages, one is trying to estimate the net cost of damage from sources such as population movements, damage to property, agricultural productivity, access to fresh water and, generally, access to what can be termed bio-system services. This approach is appropriate for relatively small temperature change. However, it is unclear whether the damage from a large temperature change is simply an extension of that from a small temperature change. Various tipping points can be envisaged (Lenton et al., 2008; Kriegler et al., 2009), which would lead to severe sudden damages. Furthermore, the consequent political or community responses could be even more serious.

Rapid climate change will stress many economic, social and political systems. Of course, it is impossible to predict the result of such events, especially the extreme negative tail, which is why the possibility of very high damages ought to be included in the analysis. The opposite viewpoint – insisting that dramatic consequences will not occur – seems more difficult to justify.

1.3.2 Model adjustments

At the core of the damage function in PAGE09 is the Equation (5).[14]

$$d = \alpha \left(\frac{T_{ACT}}{T_{CAL}} \right)^{\beta} \tag{5}$$

where d is the damage, α is the damage at the calibration temperature, T_{CAL} is the calibration temperature rise, and T_{ACT} is the actual temperature rise, β is the damage exponent.

The calibration temperature is on average 3°C.[15] Therefore, if the actual temperature rise is 3 °C, on average, the damage equals α. The damage exponent, β, becomes more important as temperatures rise above T_{CAL}. In the standard model, β is entered as triangle (1.5, 2, 3). Therefore, on average, the exponent is 2.167 (slightly above a quadratic), meaning that at twice the calibration temperature (on average, T_{ACT} equals 6°C), the damage will be 4.5 times α. With the maximum value for β, which is 3, the damage would be 8 times α.

This shows that the standard PAGE09 Model does allow for the possibility of reasonably high damages. However, the arguments above suggest that these bounds may not adequately take into account the possibility of extreme damages. In the same spirit as for the changes in SENS, three distributions are proposed for the damage exponent, β.

(a) Lower 50 percentiles

The median value for β is chosen to be 2 (i.e. quadratic), which is the most common value for β in the literature.[16] The lower 50 percentiles are entered as the standard model distribution for values below 2. This is simply triangle (1.5, 2, 2).

(b) Upper 50 percentiles

For the upper percentiles, we follow a suggestion of Dietz et al. (forthcoming), who propose incorporating a 10 percent probability that the β exceeds 3. Otherwise, the

[14] The full estimates of damages in PAGE09 take into account regional differentials, discounting, weighting for income inequality, saturation of damage effects and the capacity for adaptation. All such considerations are unchanged from the standard model.

[15] It is entered as a triangle distribution (2.5, 3, 3.5).

[16] Note that the median in the standard PAGE09 model is slightly higher at 2.134.

distributions are fitted to have a median of 2. As for the SENS parameter, three distributions are fitted to these criteria: a thin-tailed normal, an intermediate-tailed lognormal, and a fat-tailed Pareto. Fitting these distributions as specified gives the following PDFs:

$$\text{Normal: } f_N(\beta) = \frac{1}{0.7803\sqrt{2\pi}} e^{\left(-\frac{(\beta-2)^2}{2\times(0.7803)^2}\right)} \tag{6}$$

$$\text{Lognormal: } f_L(\beta) = \frac{1}{0.3164\sqrt{2\pi}\beta} e^{\left(-\frac{(lnSENS-0.6931)^2}{2\times(0.3164)^2}\right)} \tag{7}$$

$$\text{Pareto: } f_P(\beta) = 31.09 \times \beta^{-4.969} \tag{8}$$

The tails of the distributions are shown in Figure 2 below, which compares the normal distribution (blue line), the lognormal (red line) and the Pareto (green line).

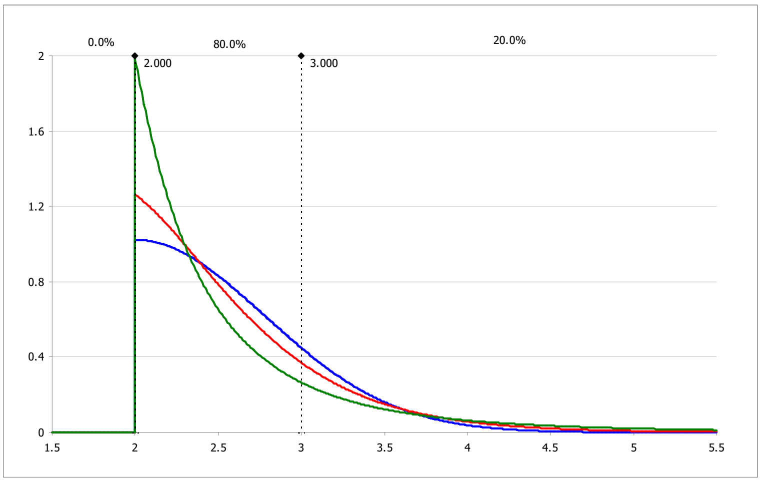

Figure 2: The upper 50 percentiles of the damage exponent: normal, lognormal and Pareto

These distributions were entered for both the economic and non-economic damage functions. The third type of damage in PAGE09 comes from sea-level rise. The exponent for sea-level damages in the standard model is triangle (0.5, 0.7, 1). Analogous reasoning to that for economic and non-economic damage exponent is used to adjust the sea-level damage exponent. The resulting distributions all have a median of 0.7 and a 90th percentile of 1. The lower 50 percentiles are triangle distributions (0.5, 0.7, 0.7) and the upper 50 percentiles are normal, lognormal or Pareto.

The fourth damage category in PAGE09 is discontinuity damages, which is intended to account for uncertain damages not elsewhere accounted for in the model. As the justification for discontinuity damages partially overlaps with the justification for extending the tails on the damage exponents, we have switched them off here to be sure to avoid double-counting.[17]

2 Results

The results show significant changes in the estimated value of the SCCO2 when different tails are inputted for the relevant PDFs. Following the steps of our methodology, the results are presented as follows: (i) the effects of changing the climate sensitivity parameter alone, (ii) the effects of changing the damage functions alone, and (iii) both effects together. For reference, the full set of all results is provided in a single table in the Appendix.

2.1 *Climate sensitivity parameter (SENS)*

Table 1 shows the results of changing the probability distribution of the climate sensitivity parameter (SENS) only. For comparison, the first column refers to the standard PAGE09 Model, with the default assumptions. The standard model provides a mean value of 102 \$/tCO2, which is already higher than many of the estimates in the literature.[18]

Table 1: Alternative SENS - SCCO2 in US\$/tCO2

	Standard (triangle)	Thin tail (normal)	Interm. tail (logn.)	Fat tail (Pareto)
Mean	**102**	**131**	**146**	**188**
5th perc.	11	12	11	12
50th perc.	49	57	57	54
95th perc.	231	374	409	564
99th perc.	447	841	1,095	2,797

Source: Authors' calculations, each based on 10,000 runs of the modified PAGE09.

When the tail for climate sensitivity is normally distributed the SCCO2 mean value rises to 131 \$/tCO2 (29 percent above the standard model). In the case of lognormal distribution for SENS the mean value is 146 \$/tCO2 (44 percent above), while when a Pareto distribution is used the SCCO2 is estimated to be 188 \$/tCO2 on average (85 percent above).

It is worth emphasising the asymmetry of the effects on the SCCO2 range. As expected, the values of the 5th and 50th percentiles do not change significantly, while there is a large increase for the 95th and 99th percentiles. Using a normal distribution for climate sensitivity implies that SCCO2 would be larger than 374 \$/tCO2 with a

[17] The same is done in Dietz et al. (forthcoming).
[18] Many of the reasons for this (especially the differences between PAGE2002 and PAGE09) are explained in Ackerman et al. (2009b) and Hope (2011).

5 percent probability, and there would be a probability of 1 percent that the value of SCCO2 is above 841 \$/tCO2 (which is 88 percent higher than the 99th percentile of the standard PAGE09 Model). In the case of the Pareto distribution the value of the 95th percentile is 564 \$/tCO2 and that of the 99th percentile[19] is 2,797 \$/tCO2, which corresponds to an increase of 144 percent and 525 percent respectively, with respect to the estimates of the standard PAGE09 Model.

2.2 *Damage exponents*

Table 2 reports the results of the runs when the distribution of the damage exponents is modified, without changing the distribution of the sensitivity parameter. In these cases, as discussed in the previous section, the probability of discontinuity damages is switched off, to avoid double counting. In order to distinguish which part of the changes is due to removing the discontinuity damages and which part is due to adding the tails, the standard PAGE09 Model is run with the discontinuity damages switched off (second column of Table 2). This alone gives a significantly lower mean value for the SCCO2, equal to 76 \$/tCO2.

Table 2: Alternative damage exponents - SCCO2 in US\$/tCO2

	standard (triangle)	standard (triangle) Disc. OFF	thin tail (normal)	interm. tail (logn.)	fat tail (Pareto)
Mean	**102**	**76**	**99**	**94**	**114**
5th perc.	11	11	11	11	11
50th perc.	49	48	50	50	50
95th perc.	231	226	300	300	358
99th perc.	447	418	658	762	1,421

Source: Authors' calculations, each based on 10,000 runs of the modified PAGE09.

Modifying the PDFs of the damage exponents raises the SCCO2 (in a similar way as for changing climate sensitivity) relative to the standard model without discontinuity damages. When comparing to the full standard model (with discontinuity damages), adding thin, intermediate or fat tails either lowers the mean value of the SCCO2 by 3 or 7 percent or raises it by 12 percent respectively (99, 94 and 114 \$/tCO2 compared to 102).

[19] The 99thpercentile is rarely reported for the estimates of the SCCO2 in the literature. We include it here because in the context of the discussion about climate uncertainty, we believe it is important to acknowledge the extreme values, even if they are highly unlikely. Roughly speaking extended tails for the PDF of the inputs to the model leads to extended tails in the outputs. This is a relevant result, which we wish to show. Nevertheless, it should also be acknowledged that, as one would expect, the values for the 99thpercentiles relatively imprecise, varying fairly considerably if the same scenario is rerun.

2.3 *SENS and damage exponents together*

Table 3 shows the results of the model when both changes (on SENS and the damage exponents) are done in combination.

Table 3: Alternative SENS and damage exponents - SCCO2 in US$/tCO2

	standard (triangle)	thin tail (normal)	interm. tail (logn.)	fat tail (Pareto)
Mean	**102**	**135**	**147**	**218**
5th perc.	11	12	12	11
50th perc.	49	58	57	55
95th perc.	231	489	551	839
99th perc.	447	1,276	1,660	3,082

Source: Authors' calculations, each based on 10,000 runs of the modified PAGE09.

The adjusted model, with both SENS and the damage exponents normally distributed, estimates the average SCCO2 to be 135 $/tCO2 on average, 147 $/tCO2 with the lognormal distribution and 216 $/tCO2 with the Pareto distribution. The new SCCO2 is 33 to 115 percent higher than the standard PAGE09 Model.

As in the previous cases, the lower half of the distribution is essentially unchanged (adjusting the tail has little impact on the bulk of the results). The upper percentiles, however, are greatly extended. The 95th percentile shows rises from between 110 to 260 percent, while the 99th percentile shows even greater rises. The reason for this behaviour of the model has partly to do with changes in the way extreme damages are modelled. By adding tails, in place of the original discontinuity damages, the very few, very large "tipping point" damages of the standard model are removed. In the SCCO2 PAGE09, the marginal change is only responsible for a change in discontinuity damage in less than one percent of the runs (when it does occur, the impact is very large). Therefore, these very large damages do not influence even the 99th percentile in the standard models results.

2.4 *Comparison with existing estimates*

Many estimates of the SCCO2[20] have been proposed in the literature.[21] Tol (2005) gathered over 100 estimates from 28 published studies and combined them to form a probability density function with a median of 4 $/tCO2, a mean of 25 $/tCO2,

[20] Note that many of the results quoted here were originally reported as estimates of the social cost of carbon (not of carbon dioxide). These have been converted into SCCO2 units. The multiplier for doing so is the relative molecular weight of CO2 to carbon, which is 3.67 (44 g per mole/12 g per mole;Interagency Working Group on Social Cost of Carbon, U.S. Government (2010)). For example, this means that 100 $/tCO2 is equivalent to 367 $/tC.

[21] Some early estimates include Frankhauser (1994) that reports marginal impacts of between 2 and 12 $/tCO2 with a mean value of 5 $/tCO2 (figures in US$1990). The Second Assessment Report from the IPCC (1996) estimates range from 1 to 34 $/tCO2 (US$1990). Tol (1999) estimates the marginal impact to be between 2 and 6 $/tCO2 (US$1990).

and a 95thpercentile of 95 \$/tCO2. In an updated version of this meta-analysis, Tol (2008) considered 211 estimates of the SCC, including the Stern Review (Stern, 2007), and found higher estimates than in the previous studies. Adjusting alternative kernel density estimators to data points, the author found that when the Gaussian distribution and the sample coefficient of variation is used (which is the case closest to the 2005 study), the distribution of the estimates has a median of 4 \$/tCO2, a mean of 28 \$/tCO2 and a 95th(99th) percentile of 162 \$/tCO2 (552 \$/tCO2).

A recent report of the US Government Interagency Working Group on Social Cost of Carbon (2010) presents SCCO2 estimates resulting from three IAMs, the DICE, PAGE 2002 and FUND models. The SCCO2 estimates from the average of the three IAMs are 35, 21 and 5 \$/tCO2, at discount rates of 2.5 percent, 3 percent and 5 percent, respectively.

Our results appear to be significantly higher than the average SCCO2 estimates provided in the literature so far. Nevertheless, comparing our results with past estimates is not straightforward, as differences in the model structure and parameter values,[22] emissions and socioeconomic scenarios and discount factor assumptions might bias the effects of introducing uncertainties in key parameters. While we can directly compare the results of the PAGE09 model before and after fat-tail adjustments, further research would be needed to isolate the impact of these adjustments with respect to the different models and model assumptions used in previous studies.

3 Conclusions

There are large uncertainties surrounding the catastrophic impacts climate change might cause. These uncertainties call into question the appropriate weight in the tails of the probability distributions in climate models. Following recent literature and taking advantage of the flexible nature of IAMs, we have adjusted the tails of two key areas of uncertainty in the PAGE09 Model: the climate sensitivity parameter and the damage exponents. For each, we considered a normal (thin), a lognormal (intermediate) and a Pareto (fat) tail. Though there are some doubts about the probabilities for the bulk of these distributions, we focused on the extreme values which are the most uncertain.

Which of the three tail shapes is the most credible can be debated. Under uncertainty, the Bayesian approach leads one towards the presumption of a fat tail (in the absence of reliable information to the contrary),[23] which would suggest a bias towards the Pareto tails in this research. However, perhaps the more important approach would be to seek to estimate which distribution gives the most plausible weight for extreme negative events. The importance of the functional forms used for the tails in this paper is in the weight given to extreme negative events, as opposed to

[22] For instance, *ceteris paribus* the new features of the standard PAGE09 model alone result in at least a threefold increase of the SCCO2 estimates with respect to the earlier PAGE 2002 model.
[23] The position is explained in Weitzman (2009).

the mathematical shape. This type of thinking[24] is more relevant whenever damages are bounded (as they are in IAMs).[25]

The potential for a change in the tail probabilities to cause up to an approximate doubling of the mean value of the SCCO2 and sevenfold increase in the 99$^{\text{th}}$percentile is an important result from our analysis. As constructed here, the effect of adjusting the climate sensitivity parameter exceeds that of adjusting the damage function. Not only do both impact on the SCCO2 mean values, but more especially on its higher percentiles. It is worth noting the high values for the SCCO2 emerging from a minority of runs. For example, our results suggest that it is highly unlikely that the damage done by emitting a single tonne of CO2 is in excess of a thousand dollars, but the possibility is not vanishingly small. Indeed, if the Pareto tails are accepted for both climate sensitivity and the damage exponents, the probability that the SCCO2 exceeds US$1,000 is 4 percent.

The limitations of IAMs should be taken into account when interpreting the results. Specifically in relation to the dismal theorem (Weitzman, 2009), the methodology developed in this paper goes some way towards incorporating uncertainty into some key elements of the model, but does not attempt to test the limit of this particular critique.[26] Furthermore, the impact of the discount rate has not been investigated, and PAGE09 standard values have been used. It is clear from the literature, and from experimentation with the model, that raising (lowering) the pure time preference time and/or the elasticity of the marginal utility of consumption would lower (raise) the SCCO2.

The relationship of the SCCO2 to Pigouvian taxation makes it an important figure for policy makers. The weight placed on extreme outcomes for policy purposes depends the level of risk aversion. In fact, as there is uncertainty about the PDF of many parameters, the concept of ambiguity aversion (beyond risk aversion) is also applicable in this context. A higher risk/ambiguity aversion gives more consideration to the negative extremes, which leads to the notion of climate policy being justified, in part, as insurance against catastrophe.

[24] This argument is expanded upon in Pindyck (forthcoming).
[25] It would, of course, have been possible to introduce a thin-tailed distribution that would have caused higher values for the SCCO2, simply by adjusting the mean and standard deviation (though the justification for doing so would have been somewhat arbitrary).
[26] Weitzman (forthcoming) emphasizes that the key "fat tail" is that of the PDF of the log of overall disutility of climate change, resulting from a chain of uncertain, interacting components.

References

Ackerman, F., De Canio, S. J., Howarth, R. B., and Sheeran, K. (2009a). Limitations of the integrated assessment models of climate change. *Climatic Change*, 95: 297–315. URL http://www.springerlink.com/content/c85v5581x7n74571/fulltext.pdf.

Ackerman, F., and Stanton, E. (2010). The social cost of carbon. *real-world economics review*, 53: 129–143. URL http://www.paecon.net/PAEReview/issue53/whole53.pdf.

Ackerman, F., Stanton, E., Hope, C., and Alberth, S. (2009b). Did the Stern Review underestimate US and global climate damages? *Energy Policy*, 37(7): 2717–2721. URL http://www.sciencedirect.com/science/article/pii/S0301421509001529.

Bosello, F., and Roson, R. (2007). Estimating a Climate Change Damage Function through General Equilibrium Modeling. *Working Paper Department of Economics Ca' Foscari University of Venice*, (08/WP/2007). URL http://www.dse.unive.it/fileadmin/templates/dse/wp/WP_2007/WP_DSE_bosello_roson_08_07.pdf.

Dietz, S. (forthcoming). The Treatment of Risk and Uncertainty in the US Social Cost of Carbon for Regulatory Impact Analysis. *Economics: The Open-Access, Open-Assessment E-Journal*. URL http://www.economics-ejournal.org/economics/discussionpapers/2011-30.

Dietz, S., Hope, C., and Patmore, N. (2007). Some economics of "dangerous" climate change: Reflections on the Stern Review. *Global Environmental Change*, 17(3-4): 311–325. URL http://www.sciencedirect.com/science/article/pii/S095937800700043X.

Dietz, S., Hope, C., and Patmore, N. (forthcoming). High impact, low probability? An empirical analysis of risk in the economic of climate change. *Climatic Change*. URL http://www.springerlink.com/content/3451552923k4144w/fulltext.pdf.

European Commission (2007). Communication from the Commission to the Council, the European Parliament, the European Economic and Social Committee and the Committee of the Regions. Limiting Global Climate Change to 2 degrees Celsius: The way ahead for 2020 and beyond. URL http://eur-lex.europa.eu/LexUriServ/LexUriServ.do?uri=COM:2007:0002:FIN:EN:PDF.

Frankhauser, S. (1994). The social costs of greenhouse gas emissions: an expected value approach. *Energy Journal*, 15(2).

Hansen, J., Sato, M., Kharecha, P., Beerling, D., Berner, R., Masson-Delmotte, V., Pagani, M., Raymo, M., Royer, D., and Zachos, J. (2008). Target atmospheric CO2: Where should humanity aim? *Atmospheric and Oceanic Physics*, 2: 217–231. URL http://benthamscience.com/open/toascj/articles/V002/217TOASCJ.pdf.

Hope, C. (2006). The marginal impact of CO2 from PAGE2002: An integrated assessment model incorporating the IPCC's five reasons for concern. *Integrated Assessment*, 6(1): 19–56. URL http://journals.sfu.ca/int_assess/index.php/iaj/article/view/227/190.

Hope, C. (2008). Discount rates, equity weights and the social cost of carbon. *Energy Economics*, 30(3): 1011–1019. URL http://www.sciencedirect.com/science/article/pii/S0140988306001459.

Hope, C. (2011). The PAGE09 integrated assessment model: A technical description. *Cambridge Judge Business School Working Paper*, 4/11. URL http://www.jbs.cam.ac.uk/research/working_papers/2011/wp1104.pdf.

Hope, C., Anderson, J., and Wenman, P. (1993). Policy analysis of the greenhouse effect: an application of the PAGE model. *Energy Policy*, 21(3): 327–338.

Interagency Working Group on Social Cost of Carbon, U.S. Government (2010). Technical Support Document: Social Cost of Carbon for regulatory Impact Analysis. Under Executive Order 12866. URL http://www.epa.gov/oms/climate/regulations/scc-tsd.pdf.

IPCC (1996). *Climate Change 1995: Impacts, Adaptation and Mitigation of Climate Change: Scientific-Technical Analysis. Contribution of Working Group II to the Second Assessment Report of the Intergovernmental Panel on Climate Change*. Cambridge University Press, Cambridge, UK. URL http://www.ipcc.ch/ipccreports/sar/wg_II/ipcc_sar_wg_II_full_report.pdf.

IPCC (2007). Climate Change 2007: Synthesis Report. Contribution of Working Groups I, II and III to the Fourth Assessment Report of the Intergovernmental Panel on Climate Change. URL http://www.ipcc.ch/publications_and_data/ar4/syr/en/contents.html.

Knutti, R., and Hegerl, G. C. (2008). The equilibrium sensitivity of the Earth's temperature to radiation chages. *Nature Geoscience*, 1: 735–743. URL http://www.iac.ethz.ch/people/knuttir/papers/knutti08natgeo.pdf.

Kriegler, E., Hall, J., Held, H., Dawson, R., and Schellnhube, H. (2009). Imprecise Probability Assessment of Tipping Points in the Climate System. *Proceedings of the National Academy of Science of the USA*, 106(13): 5041–5046. URL http://www.pnas.org/content/early/2009/03/13/0809117106.full.pdf.

Lenton, T., Held, H., Kriegler, E., Hall, J., Lucht, W., Rahmstorf, S., and Schellnhuber, H. J. (2008). Tipping elements in the Earth's climate system. *Proceedings of the National Academy of Sciences of the USA*, 105(6): 1786–1793. URL http://www.pnas.org/content/105/6/1786.full.pdf.

Nordhaus, W. (2009). An analysis of the Dismal Theorem. *Cowles Foundation Discussion Paper*, 1686. URL http://cowles.econ.yale.edu/P/cd/d16b/d1686.pdf.

Nordhaus, W., and Boyer, J. (2000). *Warming the World: Economic Models of Global Warming*. The MIT Press, Cambridge, Massachusetts, USA. URL http://www.econ.yale.edu/~nordhaus/homepage/web%20table%20of%20contents%20102599.htm.

Patt, A., van Vuuren, D., Berkhout, F., Aaheim, A., Hof, A., Isaac, M., and Mechler, R. (2010). Adaptation in integrated assessment modelling: where do we stand? *Climatic Change*, 99(3-4): 383–402. URL http://www.springerlink.com/content/q0026gu420415704/fulltext.pdf.

Pindyck, R. (forthcoming). Fat tails, thin tails and climate change policy. *Economics: The Open-Access, Open-Assessment E-Journal*. URL http://web.mit.edu/rpindyck/www/Papers/FatTailsREEP0910.pdf.

Plambeck, E., Hope, C., and Anderson, J. (1997). The PAGE95 model: Integrating the science and economics of global warming. *Energy Economics*, 19(1): 77–101.

Stern, N. (2007). *Stern Review on the Economics of Climate Change*. HM Treasury, UK. URL http://webarchive.nationalarchives.gov.uk/+/http://www.hm-treasury.gov.uk/stern_review_report.htm.

Tol, R. (1999). The marginal costs of greenhouse gas emissions. *The Energy Journal*, 20(1): 61–81.

Tol, R. (2005). The marginal damage costs of carbon dioxide emissions: an assessment of uncertainties. *Energy Policy*, 33(16): 2064–2074. URL http://www.fnu.zmaw.de/fileadmin/fnu-files/publication/tol/enpolmargcost.pdf.

Tol, R. (2008). The Social Cost of Carbon: Trends, Outliers and Catastrophes. *Economics: The Open-Access, Open-Assessment E-Journal*, 2(25): 1–22. URL http://dx.doi.org/10.5018/economics-ejournal.ja.2008-25.

Warren, R., Hope, C., Mastrandrea, M., Tol, R., Adger, N., and Lorenzoni, I. (2006). Spotlighting the Impacts Functions in Integrated Assessment. Research Report Prepared for the Stern Review on the Economics of Climate Change. *Tyndall Centre for Climate Change Research Working Paper*, (91). URL http://www.dfld.de/Presse/PMitt/2006/061030c4.pdf.

Weitzman, M. (2007). A review of the stern review on the economics of climate change. *Journal of Economic Literature*, 45(3): 703–724. URL http://ejournals.ebsco.com/direct.asp?ArticleID=475EB1FEC45E5A284E36.

Weitzman, M. (2009). On modelling and interpreting the economics of catastrophic climate change. *The Review of Economics and Statistics*, 91(1): 1–19. URL http://www.mitpressjournals.org/doi/pdfplus/10.1162/rest.91.1.1.

Weitzman, M. (2010). GHG Targets as Insurance Against catastrophic Climate Change. URL http://www.economics.harvard.edu/faculty/weitzman/files/1A1A.InsuranceCatastrophicRisks.pdf.

Weitzman, M. (forthcoming). Fat-Tailed Uncertainty in the Economics of Catastrophic Climate Change. *Review of Environmental and Economic Policy*. URL http://www.economics.harvard.edu/faculty/weitzman/files/REEP2011%2Bfat-tail.pdf.

Zickfield, K., Morgan, M., Frame, D., and Keith, D. (2010). Expert judgments about transient climate response to alternative trajectories of radiative forcing. *Proc. of the National Aademy of Sciences of the United States of America*, pages 1–6. URL http://www.pnas.org/content/early/2010/06/24/0908906107.full.pdf+html.

A Full Results Table

Table 4 details all the results for the SCCO2 each based on 10,000 runs. Most of these results appear in Tables 1, 2 and 3, which are also accompanied by more detailed explanations. Here the results are placed all together to allow direct comparisons and also to report three "intermediate" results for SENS with a normal, lognormal and Pareto tail with the discontinuity damages switched off.

Table 4: Full Results Table – SCCO2 in US$/tCO2 each based on 10,000 runs of PAGE09

			DAMAGE EXPONENTS				
			disc. ON	discontinuity damages OFF			
			standard	standard	normal	lognorm.	Pareto
SENS	standard	**Mean**	**102**	**76**	**99**	**94**	**114**
		5^{th}perc.	11	11	11	11	11
		50^{th}perc.	49	48	50	50	50
		95^{th}perc.	231	226	300	300	358
		99^{th}perc.	447	418	658	762	1,421
	normal	**Mean**	**131**	**107**	**135**		
		5^{th}perc.	12	12	12		
		50^{th}perc.	57	57	58		
		95^{th}perc.	374	374	489		
		99^{th}perc.	841	744	1,276		
	lognormal	**Mean**	**146**	**120**		**147**	
		5^{th}perc.	11	11		12	
		50^{th}perc.	57	56		57	
		95^{th}perc.	409	412		551	
		99^{th}perc.	1,095	1,045		1,660	
	Pareto	**Mean**	**188**	**162**			**218**
		5^{th}perc.	12	12			11
		50^{th}perc.	54	53			55
		95^{th}perc.	564	549			839
		99^{th}perc.	2,797	1,996			3,082

Source: Authors' calculations, each based on 10,000 runs of the modified PAGE09.