The see-saw mechanism: Neutrino mixing, leptogenesis and lepton flavour violation

WERNER RODEJOHANN

Max-Planck-Institut für Kernphysik, Postfach 103980, D-69029 Heidelberg, Germany
E-mail: werner.rodejohann@mpi-hd.mpg.de

Abstract. The see-saw mechanism to generate small neutrino masses is reviewed. After summarizing our current knowledge about the low energy neutrino mass matrix, we consider reconstructing the see-saw mechanism. Indirect tests of see-saw are leptogenesis and lepton flavour violation in supersymmetric scenarios, which together with neutrino mass and mixing define the framework of see-saw phenomenology. Several examples are given, both phenomenological and GUT-related.

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1. Introduction: The neutrino mass matrix

Non-trivial lepton mixing in the form of neutrino oscillations proves that neutrinos are massive and that the Standard Model (SM) of elementary particles is incomplete. At low energy, all phenomenologies can be explained by the neutrino mass matrix

$$m_\nu = U \, m_\nu^{\mathrm{diag}} \, U^T, \tag{1}$$

where $m_\nu^{\mathrm{diag}} = \mathrm{diag}(m_1, m_2, m_3)$ contains the individual neutrino masses. In the basis in which the charged lepton mass matrix is real and diagonal, U is the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) mixing matrix. We will work in this very basis throughout the text. The PMNS matrix can explicitly be parametrized as

$$U = \begin{pmatrix} c_{12} \, c_{13} & s_{12} \, c_{13} & s_{13} \, \mathrm{e}^{-i\delta} \\ -s_{12} \, c_{23} - c_{12} \, s_{23} \, s_{13} \, \mathrm{e}^{i\delta} & c_{12} \, c_{23} - s_{12} \, s_{23} \, s_{13} \, \mathrm{e}^{i\delta} & s_{23} \, c_{13} \\ s_{12} \, s_{23} - c_{12} \, c_{23} \, s_{13} \, \mathrm{e}^{i\delta} & -c_{12} \, s_{23} - s_{12} \, c_{23} \, s_{13} \, \mathrm{e}^{i\delta} & c_{23} \, c_{13} \end{pmatrix} P \, , \tag{2}$$

where P contains the Majorana phases. All in all, nine physical parameters are present in m_ν. Neutrino physics deals with explaining and determining them. To very good precision the angles θ_{12}, θ_{23} and θ_{13} correspond to the mixing angles in solar (and long-baseline reactor), atmospheric (and long-baseline accelerator) and

short-baseline reactor neutrino experiments, respectively. The analyses of neutrino experiments revealed the following best-fit values and 3σ ranges of the oscillation parameters [1]:

$$\Delta m_\odot^2 \equiv m_2^2 - m_1^2 = \left(7.67^{+0.67}_{-0.61}\right) \cdot 10^{-5} \text{ eV}^2, \quad \sin^2\theta_{12} = 0.32^{+0.08}_{-0.06},$$

$$\Delta m_A^2 \equiv \left|m_3^2 - m_1^2\right| = \begin{cases} \left(2.46^{+0.47}_{-0.42}\right) \cdot 10^{-3} \text{ eV}^2 & \text{for } m_3^2 > m_1^2 \\ \left(2.37^{+0.43}_{-0.46}\right) \cdot 10^{-3} \text{ eV}^2 & \text{for } m_3^2 < m_1^2 \end{cases}, \quad (3)$$

$$\sin^2\theta_{23} = 0.45^{+0.20}_{-0.13}, \quad |U_{e3}|^2 = 0^{+0.050}_{-0.000}.$$

The overall scale of neutrino masses is not known, except for the upper limit of order 1 eV coming from direct mass search experiments and cosmology. The hierarchy of the light neutrinos, at least between the two heaviest ones, is moderate.

The current data for the mixing angles can accurately be described by tri-bimaximal mixing [2], i.e., $\sin^2\theta_{12} = \frac{1}{3}$, $\sin^2\theta_{23} = \frac{1}{2}$ and $\sin^2\theta_{13} = 0$. Tri-bimaximal mixing is a special case of μ–τ symmetry, which implies $\theta_{23} = -\pi/4$ and $\theta_{13} = 0$. The mass matrices for μ–τ symmetry and for tri-bimaximal mixing are

$$(m_\nu)^{\mu-\tau} = \begin{pmatrix} A & B & B \\ \cdot & D & E \\ \cdot & \cdot & D \end{pmatrix},$$

$$(m_\nu)^{\text{TBM}} = \begin{pmatrix} \tilde{A} & \tilde{B} & \tilde{B} \\ \cdot & \frac{1}{2}(\tilde{A} + \tilde{B} + \tilde{D}) & \frac{1}{2}(\tilde{A} + \tilde{B} - \tilde{D}) \\ \cdot & \cdot & \frac{1}{2}(\tilde{A} + \tilde{B} + \tilde{D}) \end{pmatrix}, \quad (4)$$

where the $\overset{(\sim)}{A}, \overset{(\sim)}{B}, \overset{(\sim)}{D}, E$ are functions of the neutrino masses, Majorana phases, and in case of μ–τ symmetry, θ_{12}.

Obviously, there are many models and ansätz for the neutrino mass matrix, simply due to the fact that many of the low energy parameters are currently unknown. Future precision data will sort out many possibilities [3] and shed more light on the flavour structure in the lepton sector.

2. The see-saw mechanism and its reconstruction: The see-saw degeneracy

The most important question in this framework is about the origin of the neutrino mass matrix. One possibility to accommodate m_ν is to introduce SM singlets which can couple to the left-handed ν_L and the (up-type) Higgs doublet. Usually, these singlets are right-handed neutrinos N_{Ri}, and the corresponding Lagrangian is

$$\mathcal{L} = \frac{1}{2}\overline{N_{Ri}^c}(M_R)_{ij}N_{Rj} + \overline{L_\alpha}(Y_D)_{i\alpha}N_{Ri}\Phi$$

$$= \frac{1}{2}\overline{N_R^c}M_R N_R + \overline{\nu_L}m_D N_R. \quad (5)$$

Here m_{D} is the Dirac mass matrix expected to be related to the known SM masses, and M_{R} is a (symmetric) Majorana mass matrix. Integrating out the heavy $N_{\mathrm{R}i}$ (M_{R} is not constrained by the electroweak scale because $N_{\mathrm{R}i}$ are SM singlets) gives the see-saw formula [4]

$$m_\nu = -m_{\mathrm{D}} M_{\mathrm{R}}^{-1} m_{\mathrm{D}}^T. \tag{6}$$

It is also known as the 'conventional', or Type I, see-saw formula. Taking the neutrino mass scale as $\sqrt{\Delta m_{\mathrm{A}}^2}$ and the scale of m_{D} as $v = 174$ GeV gives $M_{\mathrm{R}} \simeq 10^{15}$ GeV. We will assume in what follows that the see-saw particles are very heavy.

The main ingredient of the see-saw mechanism is the vertex $\overline{L_\alpha}\,(Y_{\mathrm{D}})_{i\alpha} N_{\mathrm{R}i} \Phi$. Testing this vertex is obviously crucial for testing and reconstructing see-saw. In this respect, note that the number of physical parameters in m_{D} and M_{R} is 18, six of which are phases. Comparing this with the number of parameters in m_ν we see that half of the see-saw parameters get lost when the heavy degrees of freedom are integrated out. To put it in another way, we hardly know m_ν and we know neither m_{D} nor M_{R}. Reconstructing the see-saw mechanism is therefore a formidable task [5–7], even more so when one notes that the see-saw scale of $M_{\mathrm{R}} \simeq 10^{15}$ GeV is 11 orders of magnitude above the LHC centre-of-mass energy. Leaving aside for now observables which indirectly depend on the see-saw parameters (see below), we have two possibilities to facilitate the reconstruction: (i) making assumptions about m_{D} and/or M_{R} and (ii) parametrize our ignorance.

(i) *Making assumptions*

The most simple semi-realistic example is to assume that m_{D} is the up-quark mass matrix. This can happen in $SO(10)$ models with a 10 Higgs representation. We can in this case use the see-saw formula to find $M_{\mathrm{R}} = -m_{\mathrm{up}} m_\nu^{-1} m_{\mathrm{up}}$ and diagonalize M_{R} to obtain the heavy masses. Assuming that m_{D} is diagonal, and inserting tri-bimaximal mixing and no CP phases gives [8,9]

$$M_1 \simeq 3\,\frac{2m_u^2}{m_2}, \quad M_2 \simeq \frac{2m_c^2}{m_3}, \quad M_3 \simeq \frac{1}{3}\,\frac{m_t^2}{2m_1}. \tag{7}$$

The naive see-saw expectation $m_3 \propto m_t^2$, $m_2 \propto m_c^2$ and $m_1 \propto m_u^2$ is completely changed due to the large neutrino mixing. Note that $M_1 \propto m_u^2$, $M_2 \propto m_c^2$ and $M_3 \propto m_t^2$, i.e., the hierarchy of the heavy neutrinos is the hierarchy of the up-quarks squared. This is necessary, in particular, to 'correct' the strong up-quark hierarchy into the very mild light neutrino hierarchy.

The simple picture presented changes already in the presence of CP phases [9]. Even more modification occurs in realistic $SO(10)$ models. In table 1, taken from ref. [10], predictions for the smallest neutrino mass of different $SO(10)$ models, which differ in their Higgs content and in their flavour structure, are given (see also table 2, which is taken from ref. [11]). The value of M_1 in the simple example leading to eq. (7) was about 10^5 GeV, obviously very different from the values in the table, which also differ a lot for the various models. The reason for this large spread in seemingly similar models is connected to the next issue.

(ii) *Parametrizing our ignorance: The see-saw degeneracy*

The impossibility to make unambiguous statements about the see-saw parameters becomes very obvious when we parametrize our ignorance. This can be done with the so-called Casas–Ibarra parametrization [12]:

Table 1. Higgs content, predicted mass M_1 of the lightest right-handed neutrino and baryon asymmetry η_B in various $SO(10)$ models. The prediction for $|U_{e3}|$ is also given (taken from [10] and slightly modified).

	BPW	GMN	JLM	DMM	AB
Higgs	10, 16, $\overline{16}$, 45	10, 210, $\overline{126}$	10, 16, $\overline{16}$, 45	10, 210, $\overline{126}$, 120	10, 16, $\overline{16}$, 45
M_1 (GeV)	10^{10}	10^{13}	$3.77 \cdot 10^{10}$	10^{13}	$5.4 \cdot 10^{8}$
η_B	$12 \cdot 10^{-10} \sin 2\phi$	$5 \cdot 10^{-10}$	$6.2 \cdot 10^{-10}$	$10^{-9} \sin 2\phi$	$2.6 \cdot 10^{-10}$
$|U_{e3}|$	≤ 0.16	0.18	$0.12 \div 0.15$	$0.06 \div 0.11$	0.05

$$m_{\mathrm{D}} = i\, U\, \sqrt{m_\nu^{\mathrm{diag}}}\, R\, \sqrt{M_{\mathrm{R}}}. \tag{8}$$

Here R is a complex and orthogonal matrix which contains the unknown see-saw parameters. Usually the parametrization in eq. (8) is considered in the basis in which M_{R} is real and diagonal. In the already pretty ideal situation in which we knew m_ν and M_{R}, there would still be an infinite number of allowed Dirac mass matrices. We will refer to this unpleasant feature as 'see-saw degeneracy'. We can parametrize the parametrization of our ignorance by writing R as

$$R = R_{12}R_{13}R_{23}, \tag{9}$$

where R_{ij} is a rotation around the ij-axis with complex angle $\omega_{ij} = \rho_{ij} + i\sigma_{ij}$, ρ_{ij} and σ_{ij} being real. Actually, this parametrization does not include 'reflections' [12], i.e., it should be multiplied by $\tilde{R} \equiv \mathrm{diag}(\pm 1, \pm 1, \pm 1)$ from the left, where $\tilde{R}$ contains an odd number of minus signs. However, in many cases the implied additional forms of R do not lead to different textures in m_{D} and the parametrization in eq. (9) is general enough.

3. See-saw at work: Lepton flavour violation and leptogenesis

We conclude from the above that reconstructing see-saw requires more than low energy neutrino physics. One observable which can in principle be used is the baryon asymmetry of the universe. Lepton flavour violation (LFV) in supersymmetric scenarios can also depend on the see-saw parameters. Here we will focus on the rare decays $\ell_i \to \ell_j \gamma$, with $\ell_{3,2,1} = \tau, \mu, e$.

3.1 *Lepton flavour violation*

LFV in supersymmetric see-saw scenarios allows decays like $\ell_i \to \ell_j \gamma$, triggered by off-diagonal entries in the slepton mass matrix $\tilde{m}_{\mathrm{L}}^2$. The branching ratios for radiative decays of the charged leptons $\ell_i = e, \mu, \tau$ are [13]

$$\mathrm{BR}(\ell_i \to \ell_j \gamma) = \mathrm{BR}(\ell_i \to \ell_j\, \nu\bar{\nu})\, \frac{\alpha^3}{G_{\mathrm{F}}^2\, m_{\mathrm{S}}^8}\, |(\tilde{m}_{\mathrm{L}}^2)_{ij}|^2\, \tan^2\beta\,, \tag{10}$$

where m_S is a typical mass scale of SUSY particles. Current limits on the branching ratios for $\ell_i \to \ell_j \gamma$ are $BR(\mu \to e\gamma) \leq 1.2 \cdot 10^{-11}$, $BR(\tau \to e\gamma) \leq 1.1 \cdot 10^{-7}$ and $BR(\tau \to \mu\gamma) \leq 6.8 \cdot 10^{-8}$. One expects to improve these bounds by two to three orders of magnitude for $BR(\mu \to e\gamma)$ and by one to two orders of magnitude for the other branching ratios.

To satisfy the requirement that the LFV branching ratios $BR(\ell_i \to \ell_j \gamma)$ be below their experimental upper bounds, one typically assumes that $\tilde{m}_L^2$ and all other slepton mass and trilinear coupling matrices are diagonal at the scale M_X. Such a situation occurs for instance in the CMSSM. Off-diagonal terms get induced at low energy scales radiatively, which explains their smallness. In this case a very good approximation for the typical SUSY mass appearing in eq. (10) is [14] $m_S^8 = 0.5 m_0^2 m_{1/2}^2 (m_0^2 + 0.6 m_{1/2}^2)^2$, where m_0 is the universal scalar mass and $m_{1/2}$ is the universal gaugino mass at M_X. The well-known result for the slepton mass matrix entries is [13]

$$(\tilde{m}_L^2)_{ij} = -\frac{(3m_0^2 + A_0^2)}{8\pi^2 v_u^2}(m_D L m_D^\dagger)_{ij}, \tag{11}$$

where

$$L_{ij} = \delta_{ij} \ln \frac{M_X}{M_i}.$$

Here $v_u = v \sin\beta$ and A_0 is the universal trilinear coupling.

Inserting the Casas–Ibarra parametrization from eq. (8) in $m_D m_D^\dagger$ reveals that, in general, in addition to the high energy parameters, LFV depends on all the parameters in the light neutrino mass matrix, including the Majorana phases, all three light neutrino masses and the mass ordering.

We stress here that $(\tilde{m}_L^2)_{ij}$ factorizes in a term containing SUSY parameters and a term containing parameters of the Yukawa coupling matrix m_D. Therefore, the ratios of the branching ratios are independent of the SUSY parameters and contain information only on the flavour structure. For instance,

$$\frac{BR(\mu \to e\gamma)}{BR(\tau \to e\gamma)} \simeq \frac{1}{BR(\tau \to e\nu\bar{\nu})} \left| \frac{(m_D L m_D^\dagger)_{12}}{(m_D L m_D^\dagger)_{13}} \right|^2. \tag{12}$$

We will mostly consider these ratios of branching ratios from now on. Note that LFV (and later on leptogenesis) should be evaluated on the basis in which the heavy neutrino and the charged leptons are real and diagonal. If they are not diagonal, then m_D should be replaced by $U_\ell^\dagger m_D V_R^*$, where $m_\ell m_\ell^\dagger = U_\ell (m_\ell^{\text{diag}})^2 U_\ell^\dagger$ and $V_R^\dagger M_R V_R^*$.

One simple example is the following: suppose both m_D and M_R obey a 2–3 exchange symmetry:

$$m_D = \begin{pmatrix} a & b & b \\ d & e & f \\ d & f & e \end{pmatrix} \quad \text{and} \quad M_R = \begin{pmatrix} X & Y & Y \\ \cdot & Z & W \\ \cdot & \cdot & Z \end{pmatrix}. \tag{13}$$

Obviously m_ν will be μ–τ symmetric, i.e., look like eq. (4), in this case. Ignoring logarithmic corrections, one finds that $(m_{\rm D}m_{\rm D}^\dagger)_{21} = (m_{\rm D}m_{\rm D}^\dagger)_{31}$ and consequently $\mathrm{BR}(\mu \to e\gamma)/\mathrm{BR}(\tau \to e\gamma) \simeq 1/\mathrm{BR}(\tau \to e\,\nu\bar\nu) \simeq 5.7$. Up to the normalization factor the branching ratios are equal, which is so-to-speak a consequence of the fact that μ–τ symmetry makes here no difference between muon and tau flavour.

Recall the current limit of $1.2 \cdot 10^{-11}$ on $\mathrm{BR}(\mu \to e\gamma)$, and an expected improvement of two orders of magnitude on the limit of $\mathrm{BR}(\tau \to e\gamma) \leq 1.1 \cdot 10^{-7}$. Therefore, in this example it follows that $\tau \to e\gamma$ will not be observed in a foreseeable future. The decay $\tau \to \mu\gamma$ is not constrained.

Leaving this model-independent approach aside now, let us perform a GUT inspired estimate of the ratio of the branching ratios: suppose $m_{\rm D}$ coincides with the mass matrix of up-type quarks $m_{\rm up}$. In addition, we will follow [9] and assume that the mismatch between the left-handed rotations diagonalizing the Dirac-type neutrino mass matrix $m_{\rm D}$ and the mass matrix of charged leptons m_ℓ is the same as the mismatch of the left-handed rotations diagonalizing the up-type and down-type quark matrices, i.e., is given by $V_{\rm CKM}$. This includes the special case in which $m_{\rm D} = m_{\rm up}$ is diagonal and m_ℓ is diagonalized by the CKM matrix. This in turn occurs in a scenario leading to quark–lepton complementarity [15,16], sometimes called QLC 1. In either realization of this possibility, heavy neutrino masses very similar to the ones in eq. (7) will result. The overall result is that $m_{\rm D}m_{\rm D}^\dagger \simeq V_{\rm CKM}^\dagger \,\mathrm{diag}(m_u^2, m_c^2, m_t^2)\,V_{\rm CKM}$. Adopting the Wolfenstein parametrization of the CKM matrix and taking into account that the up-type quark masses satisfy $m_u : m_c : m_t \simeq \lambda^8 : \lambda^4 : 1$, we find

$$\mathrm{BR}(\mu \to e\gamma) \propto A^4 \left(\eta^2 + (1-\rho)^2\right)\lambda^{10}, \tag{14}$$

$$\mathrm{BR}(\tau \to e\gamma) \propto \mathrm{BR}(\tau \to e\nu\bar\nu) A^2 \left(\eta^2 + (1-\rho)^2\right)\lambda^6, \tag{15}$$

$$\mathrm{BR}(\tau \to \mu\gamma) \propto \mathrm{BR}(\tau \to \mu\nu\bar\nu) A^2 \lambda^4. \tag{16}$$

The relative size of the branching ratios can very well be described by

$$\mathrm{BR}(\mu \to e\gamma) : \mathrm{BR}(\tau \to e\gamma) : \mathrm{BR}(\tau \to \mu\gamma) \simeq \lambda^5 : \lambda^2 : 1. \tag{17}$$

Here we have taken into account the normalization factors $\mathrm{BR}(\tau \to e\,\nu\bar\nu) \simeq \mathrm{BR}(\tau \to \mu\,\nu\bar\nu) \sim \lambda$. The relation in eq. (17) implies that if $\mathrm{BR}(\mu \to e\gamma)$ lies close to its current upper limit, then both $\tau \to e\gamma$ and $\tau \to \mu\gamma$ decays are observable. To give a feeling of the numerical values, we can use the parameters $m_0 = 100$ GeV, $m_{1/2} = 600$ GeV and $A_0 = 0$, for which $\mathrm{BR}(\mu \to e\gamma) \simeq 5 \cdot 10^{-19} \tan^2\beta$.

Again, we can consider the situation in realistic SUSY $SO(10)$ models. Recently, a comparison of the predictions for LFV was performed in ref. [11]. Table 2 summarizes the findings, where we have for convenience rewritten the numerical values from [11] in terms of powers of λ. Note that only in one model $\mu \to e\gamma$ is not the rarest decay, and that the ratio of $\tau \to e\gamma$ and $\tau \to \mu\gamma$ is usually not too far away from our naive estimate in eq. (17). In general the branching ratio for $\tau \to \mu\gamma$ is the largest. The prediction for $\mu \to e\gamma$ in the models CM (roughly $8 \cdot 10^{-19} \tan^2\beta$ for $m_0 = 100$ GeV, $m_{1/2} = 600$ GeV and $A_0 = 0$) and CY (roughly $2 \cdot 10^{-19} \tan^2\beta$) is very close to our naive estimate. The other models predict a sizably larger branching ratio, $\mathrm{BR}(\mu \to e\gamma)$ for DR is more than two orders of magnitude larger, whereas model AB (GK) predict a branching ratio larger by five (six) orders of magnitude.

Table 2. Higgs content, predicted mass M_1 of the lightest right-handed neutrino, $\mathrm{BR}(\mu \to e\gamma)$ divided by $\tan^2 \beta$ for $m_0 = 100$ GeV, $m_{1/2} = 600$ GeV, $A_0 = 0$, and the ratio of $\mathrm{BR}(\mu \to e\gamma) : \mathrm{BR}(\tau \to e\gamma) : \mathrm{BR}(\tau \to \mu\gamma)$ in various SUSY $SO(10)$ models. The prediction for $|U_{e3}|$ is also given (taken from [11] and slightly modified).

	AB	CM	CY	DR	GK	Naive
Higgs	10, 16, $\overline{16}$, 45	10, $\overline{126}$	10, $\overline{126}$	10, 45	10, 120, $\overline{126}$	10
M_1 (GeV)	$4.5 \cdot 10^8$	$1.1 \cdot 10^7$	$2.4 \cdot 10^{12}$	$1.1 \cdot 10^{10}$	$6.7 \cdot 10^{12}$	$2.0 \cdot 10^5$
$|U_{e3}|$	0.05	0.11	0.05	0.05	0.02	–
$\dfrac{\mathrm{BR}(\mu \to e\gamma)}{\tan^2 \beta}$	$5 \cdot 10^{-14}$	$8 \cdot 10^{-19}$	$2 \cdot 10^{-19}$	$1 \cdot 10^{-16}$	$2 \cdot 10^{-13}$	$5 \cdot 10^{-19}$
Ratio	$\lambda^2 : \lambda^3 : 1$	$\lambda^7 : \lambda^3 : 1$	$\lambda^4 : \lambda^3 : 1$	$\lambda^5 : \lambda^3 : 1$	$\lambda : \lambda : 1$	$\lambda^5 : \lambda^2 : 1$

3.2 *Leptogenesis*

See-saw is connected to heavy particles, and heavy masses correspond in cosmology to early times. The see-saw vertex of leptons, Higgs and heavy neutrinos shows up here in the form of a decay of the heavy neutrinos [17]. The decay asymmetry is then (for a recent review, see [18])

$$
\begin{aligned}
\varepsilon_i^\alpha &= \frac{\Gamma(N_i \to \Phi \bar{l}_\alpha) - \Gamma(N_i \to \Phi^\dagger l_\alpha)}{\Gamma(N_i \to \Phi \bar{l}) + \Gamma(N_i \to \Phi^\dagger l)} \\
&= \frac{1}{8\pi v_u^2} \frac{1}{(m_{\mathrm{D}}^\dagger m_{\mathrm{D}})_{ii}} \sum_{j \neq i} \mathrm{Im}[(m_{\mathrm{D}}^\dagger)_{i\alpha}(m_{\mathrm{D}})_{\alpha j}(m_{\mathrm{D}}^\dagger m_{\mathrm{D}})_{ij}] f(M_j^2/M_i^2), \quad (18)
\end{aligned}
$$

where $f(x) = \sqrt{x}(\frac{2}{1-x} - \ln(\frac{1+x}{x}))$. We have indicated here that flavour effects [19–24] might play a role, i.e., ε_i^α describes the decay of the heavy neutrino of mass M_i into leptons of flavour $\alpha = e, \mu, \tau$. In the case when the lowest-mass heavy neutrino is much lighter than the other two, i.e., $M_1 \ll M_{2,3}$, the lepton asymmetry is dominated by the decay of this lightest neutrino and $f(M_j^2/M_1^2) \simeq -3M_1/M_j$. We have omitted additional terms in ε_i^α which vanish when summed over flavours and which are suppressed by an additional power of M_1/M_j when neutrinos are hierarchical. The sum over flavours reads

$$
\begin{aligned}
\varepsilon_i &= \sum_\alpha \frac{\Gamma(N_i \to \Phi \bar{l}_\alpha) - \Gamma(N_i \to \Phi^\dagger l_\alpha)}{\Gamma(N_i \to \Phi \bar{l}) + \Gamma(N_i \to \Phi^\dagger l)} \equiv \frac{\Gamma(N_i \to \Phi \bar{l}) - \Gamma(N_i \to \Phi^\dagger l)}{\Gamma(N_i \to \Phi \bar{l}) + \Gamma(N_i \to \Phi^\dagger l)} \\
&= \frac{1}{8\pi v_u^2} \frac{1}{(m_{\mathrm{D}}^\dagger m_{\mathrm{D}})_{ii}} \sum_{j \neq i} \mathrm{Im}[(m_{\mathrm{D}}^\dagger m_{\mathrm{D}})_{ij}^2] f(M_j^2/M_i^2). \quad (19)
\end{aligned}
$$

The expressions we gave for the decay asymmetries are valid in the case of the MSSM. Their flavour structure is however identical to the case of just the Standard Model. Also important in leptogenesis are the effective mass parameters responsible for the wash-out. We will not discuss this issue here and refer to [19,20,26] for details. The final baryon asymmetry is

$$Y_{\mathrm{B}} \simeq \begin{cases} -0.01\varepsilon_1\eta(\tilde{m}_1) & \text{one-flavour,} \\ -\frac{12}{37g^*}\left((\varepsilon_1^e + \varepsilon_1^\mu)\left(\frac{417}{589}(\tilde{m}_1^e + \tilde{m}_1^\mu)\right) + \varepsilon_1^\tau\left(\frac{390}{589}(\tilde{m}_1^\tau)\right)\right) & \text{two-flavour,} \\ -\frac{12}{37g^*}\left(\varepsilon_1^e\left(\frac{151}{179}\tilde{m}_1^e\right) + \varepsilon_1^\mu\left(\frac{344}{537}\tilde{m}_1^\mu\right) + \varepsilon_1^\tau\left(\frac{344}{537}(\tilde{m}_1^\tau)\right)\right) & \text{three-flavour.} \end{cases}$$

(20)

Here $g^* = 228.75$ and we gave the expressions valid in the case of one-, two- and three-flavoured leptogenesis. The three-flavour case occurs for $M_1(1+\tan^2\beta) \leq 10^9$ GeV, the one-flavour case for $M_1(1+\tan^2\beta) \geq 10^{12}$ GeV, and the two-flavour case applies in between. The quantity Y_{B} is defined as the number density of baryons divided by the entropy density: $Y_{\mathrm{B}} = n_{\mathrm{B}}/s$, which is related to $\eta_{\mathrm{B}} = n_{\mathrm{B}}/n_\gamma$ via $\eta_{\mathrm{B}} = 7.04 Y_{\mathrm{B}}$. The measured value $Y_{\mathrm{B}} = (0.87 \pm 0.03) \cdot 10^{-10}$.

One interesting possible feature of leptogenesis is the connection of low energy CP violation to the CP violation necessary for leptogenesis. Without flavour effects, ε_1 in eq. (19) is relevant. After inserting the Casas–Ibarra parametrization in ε_1 it becomes clear that U, and therefore the low energy CP phases, do not show up in the decay asymmetry [7,25]. Very frequently, however, specific models have a connection between high and low energy CP violation, originating from relations between mass matrix entries, zero textures, etc. There are countless examples for this.

In general, reproducing the observed value of Y_{B}, and its sign, is rarely a problem in models, including $SO(10)$ scenarios (see table 1). The naive GUT-inspired framework leading to the heavy neutrino masses in eq. (7) and the ratio of branching ratios from eq. (17) can also lead to leptogenesis [9,16]. However, recall that M_1 is typically well below 10^6 GeV in eq. (7). Therefore, it lies below the minimal mass value required for successful thermal leptogenesis (see below). Hence, tuning via CP phases is necessary in order to make M_1 and M_2 quasi-degenerate and to generate the baryon asymmetry via 'resonant leptogenesis'.

The general situation in what regards the connection of low and high energy CP violation slightly changes in case of flavoured leptogenesis [19–23]. This can be understood by inserting the Casas–Ibarra parametrization in the expression for the decay asymmetries ε_1^α in eq. (18). Note that they contain individual terms $(m_{\mathrm{D}})_{\alpha j}$ and $(m_{\mathrm{D}}^\dagger)_{1\alpha}$. Consequently, terms in which U explicitly shows up are present in ε_1^α. Hence, if the low energy phases are non-trivial, they contribute to Y_{B}. Their effect can however be partly cancelled by the high energy CP phases in the complex orthogonal matrix R. In addition, flavoured leptogenesis works perfectly well when the low energy phases vanish ($\alpha = \beta = \delta = 0$) [24]. Connecting low and high energy CP violation is therefore similar, but not identical, to the case of unflavoured leptogenesis: a certain amount of input/assumptions is necessary.

The other interesting question in the framework of leptogenesis regards the required values of light and heavy neutrino masses. Most of the results depend on the wash-out and the Boltzmann equations, and we refer to [19,20,26] for details. An important point is that there is an upper limit on $|\varepsilon_1|$ which decreases with the light neutrino mass scale [27], a property not shared by $|\varepsilon_1^\alpha|$. Hence, there is an upper limit on neutrino masses for unflavoured leptogenesis, but not for flavoured leptogenesis. The upper limit on M_1 is basically not affected by the presence of flavour effects.

3.3 *Combining LFV and leptogenesis*

One can try to combine now everything and try to understand the interplay of neutrino mass and mixing, LFV and leptogenesis [5–7,28]. The following example [28] shows that indeed interesting information on the flavour structure at high energy can be obtained and that the see-saw degeneracy can partly be broken: let us assume the SUSY parameters $m_0 = m_{1/2} = 250$ GeV and $A_0 = -100$ GeV. They correspond to

$$\mathrm{BR}(\mu \to e\gamma) \simeq 9.1 \cdot 10^{-9} |(m_\mathrm{D} L m_\mathrm{D}^\dagger)_{12}|^2 \frac{1}{v_u^4} \tan^2 \beta. \tag{21}$$

Using the Casas–Ibarra parametrization implies that we can express $(m_\mathrm{D} L m_\mathrm{D}^\dagger)_{12}$ in terms of the heavy neutrino masses, the light neutrino parameters and the complex angles contained in R. The term proportional to M_3 will be the leading one. It can be found by setting $M_1 = M_2 = m_1 = 0$ and, for simplicity, inserting tri-bimaximal mixing:

$$(m_\mathrm{D} L m_\mathrm{D}^\dagger)_{12} \simeq -\frac{1}{6} L_3 M_3 \sqrt{m_2} \cos \omega_{13} \cos \omega_{13}^*$$
$$\times \left(\sqrt{6} e^{i(\alpha-\beta)} \sqrt{m_3} \cos \omega_{23} + 2\sqrt{m_2} \sin \omega_{23} \right) \sin \omega_{23}^*. \tag{22}$$

We have parametrized R here as $R = R_{23} R_{13} R_{12}$. For a natural value of $M_3 = 10^{15}$ GeV it turns out that the branching ratio of $\mu \to e\gamma$ is too large by at least three orders of magnitude. We can get rid of the potentially dangerous terms proportional to M_3 by setting $\omega_{13} = \pi/2$. If we would set $\omega_{23} = 0$ then terms of order $|U_{e3}| m_3 L_3 M_3 \cos \omega_{13} \cos \omega_{13}^*$ can lead to dangerously large $\mathrm{BR}(\mu \to e\gamma)$. For the value of $\omega_{13} = \pi/2$ the matrix R simplifies to

$$R = \begin{pmatrix} 0 & 0 & 1 \\ -\sin \omega & \cos \omega & 0 \\ -\cos \omega & -\sin \omega & 0 \end{pmatrix} \quad \text{with} \quad \omega = \omega_{12} + \omega_{23} . \tag{23}$$

There is only one free complex parameter, which can be written as $\omega = \rho + i\sigma$ with real ρ and σ. One can go on to study in this framework the constraints on ω from leptogenesis and also the implications for LFV (see figure 1).

4. Summary

The neutrino mass matrix and its origin are an exciting field of research, with overlap to many fields of (astro)particle physics, including SUSY phenomenology and cosmology. The see-saw mechanism (or any one of its many variants) and its challenging reconstruction represent the crucial link between these fields. Future data will help us draw a clearer picture of the flavour structure in the lepton sector, and if we are lucky we could test and reconstruct the see-saw. The hope is that in the not too far future only a limited number of theories/scenarios survive which are able to explain all observations.
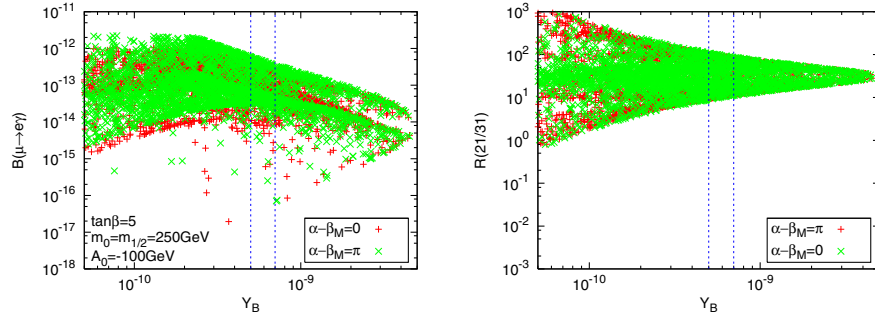
Figure 1. Phenomenology of the scenario defined by eq. (23). Shown are the correlations between Y_B and the rate of $\mu \to e\gamma$ and between Y_B and $\mathrm{BR}(\mu \to e\gamma)/\mathrm{BR}(\tau \to e\gamma)$.

Acknowledgements

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