Exercise Solution

Classifying Mathematical Optimization Problems

Use the table below to match the four mathematical optimization problems **a** through **d** with decision variables x, y, and z to the four problem classes: LP (linear programming), ILP (integer linear programming), MILP (mixed-integer linear programming), and NLP (nonlinear programming).

Linear Programming (LP)	all continuous variables all linear functions
Integer Linear Programming (ILP)	all integer variables all linear functions
Mixed-Integer Linear Programming (MILP)	some integer variables all linear functions
Nonlinear Programming (NLP)	all continuous variables some nonlinear functions

a. minimize $4x + 5y + 2z$
subject to $7x + 8y + 3z \geq 20$
$x \geq 0, y \geq 0, z \geq 0$
x, y, z integers

ILP: All variables are required to be integers. This ILP has two optimal solutions: $x = 3$, $y = 0$, $z = 0$, and $x = 2$, $y = 0$, $z = 2$. (Both solutions are feasible and have objective value 12.) The first constraint can be used to show that the objective value is at least 20(4/7) = 11 3/7.

b. maximize $3x + 5y - 4z$
subject to $6x^2 + 2y^2 + z^2 \leq 17$
$x + y + z = 3$
$-1 \leq x \leq 1$

NLP: Although the objective function is linear, the inequality constraint contains nonlinear (quadratic) terms. This NLP has a unique optimal solution: $x = 0.67689$, $y = 2.6588$, $z = -0.33567$. (Apart from the bound on x, the feasible region is a cross section of an ellipsoid and hence convex.)

c. maximize $12x + 19y - 4z$
subject to $x + 3y + z = 225$
$x + y - z \leq 117$
$x \geq 0, y \geq 0, z \geq 0$

LP: Linear constraints can be equations or inequalities. This LP has the unique optimal solution: $x = 171$, $y = 0$, $z = 54$.

d. maximize $4x + 5y$

subject to
$$
\begin{aligned}
& x + y + 2z \leq 10 \\
& 2x + 3y - 3z \leq 5 \\
& x \geq 0,\ y \geq 0 \\
& x,\ y \text{ integers}
\end{aligned}
$$

MILP: Only some of the variables (x and y, but not z) are required to be integer, but otherwise the constraints are linear. Every point with $x = 3$, $y = 2$, and $2\frac{1}{3} \leq z \leq 2\frac{1}{2}$ is an optimal solution to the MILP with objective value 22. You can think of z as a parameter that enables you to exchange one resource for another.